Non-Gaussianity from LQC

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Work in collaboration with:

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Sreenath Vijayakumar: Postdos in LSU 2015-2017. At present, postdoc at IUCAA, India.

They deserve all merits. I own responsibilities.

Main Reference: "Non-Gaussianity in LQC", 1712.08148, PRD.

First Part: General Overview

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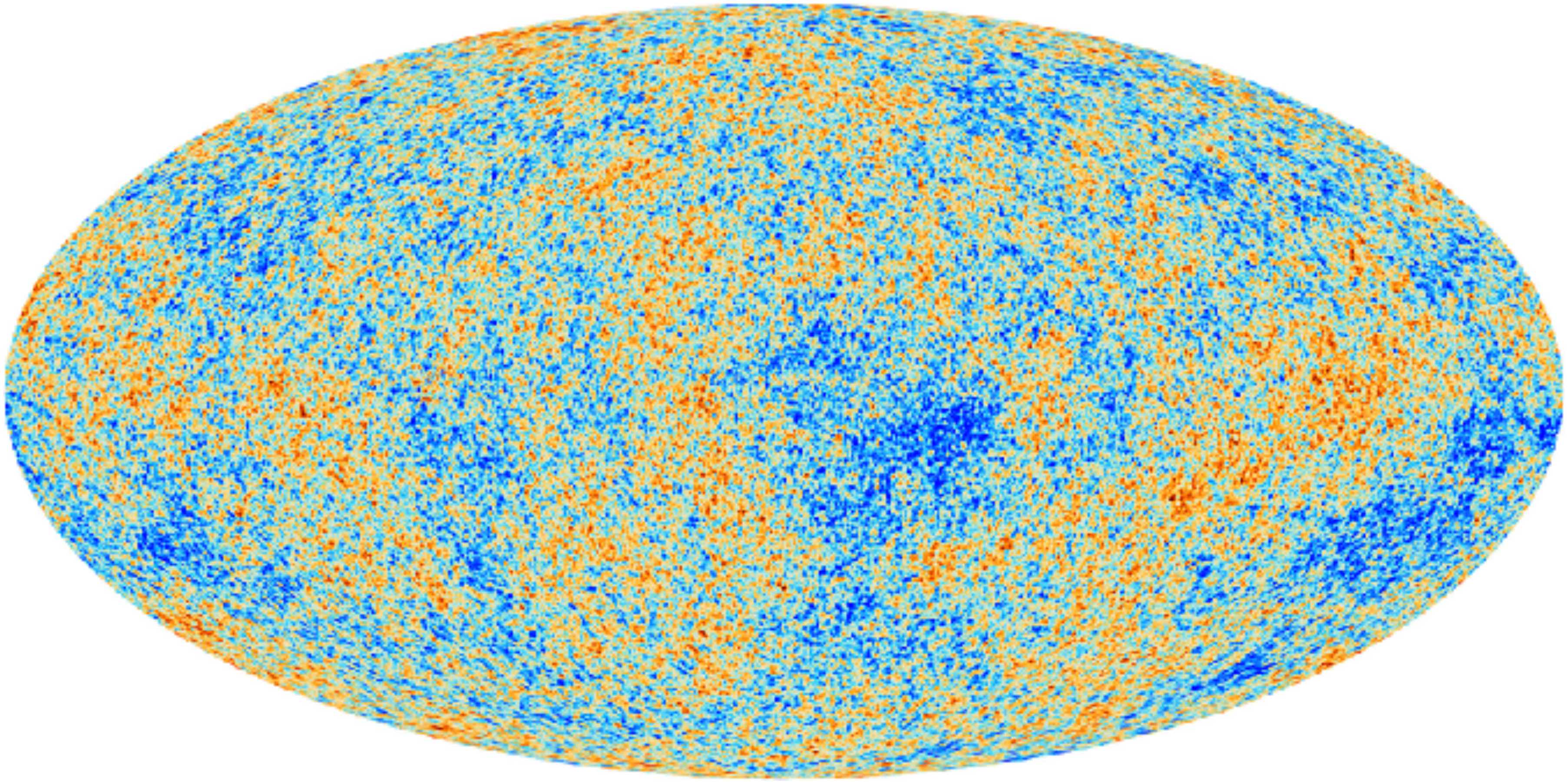
1. Non-Gaussianity: What and Why?

First Part: General Overview

1. Non-Gaussianity: What and Why?

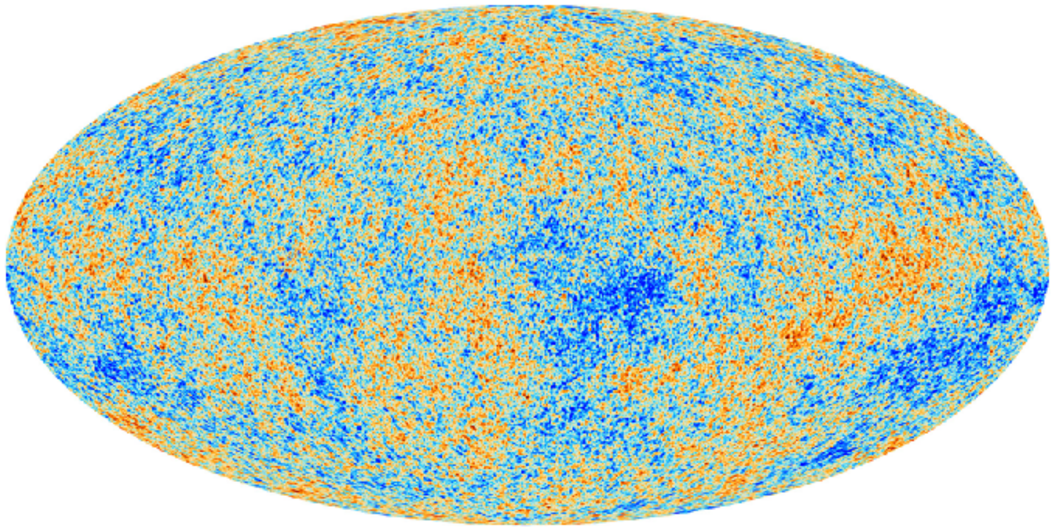
What?

CMB: our source of info about the early universe



(The first selfie ever)

CMB: info encoded in the statistics of temperature anisotropies



1. Average temperature

$$\bar{T} = 2.726$$

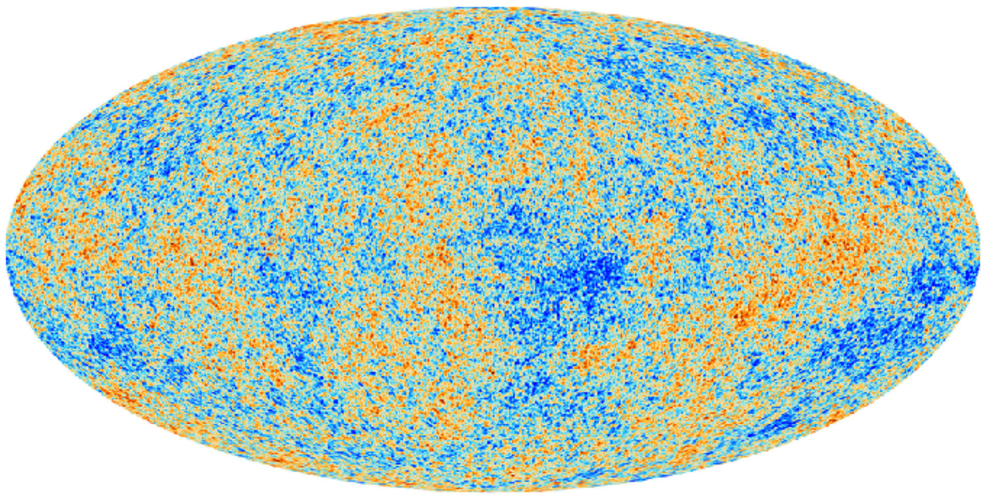
2. Temperature anisotropies

$$\Delta T(\hat{n}) \equiv T(\hat{n}) - \bar{T}$$

3. Root mean square

$$\frac{\sqrt{\langle \Delta T^2 \rangle}}{\bar{T}} \sim 10^{-5}$$

CMB: info encoded in the statistics of temperature anisotropies



4. Two point function

- **In real space:**

Legendre pols.

$$\langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle_c = \sum_\ell \frac{4\pi}{2\ell+1} C_\ell \, P_\ell(\cos\theta)$$

(assuming isotropic distribution)

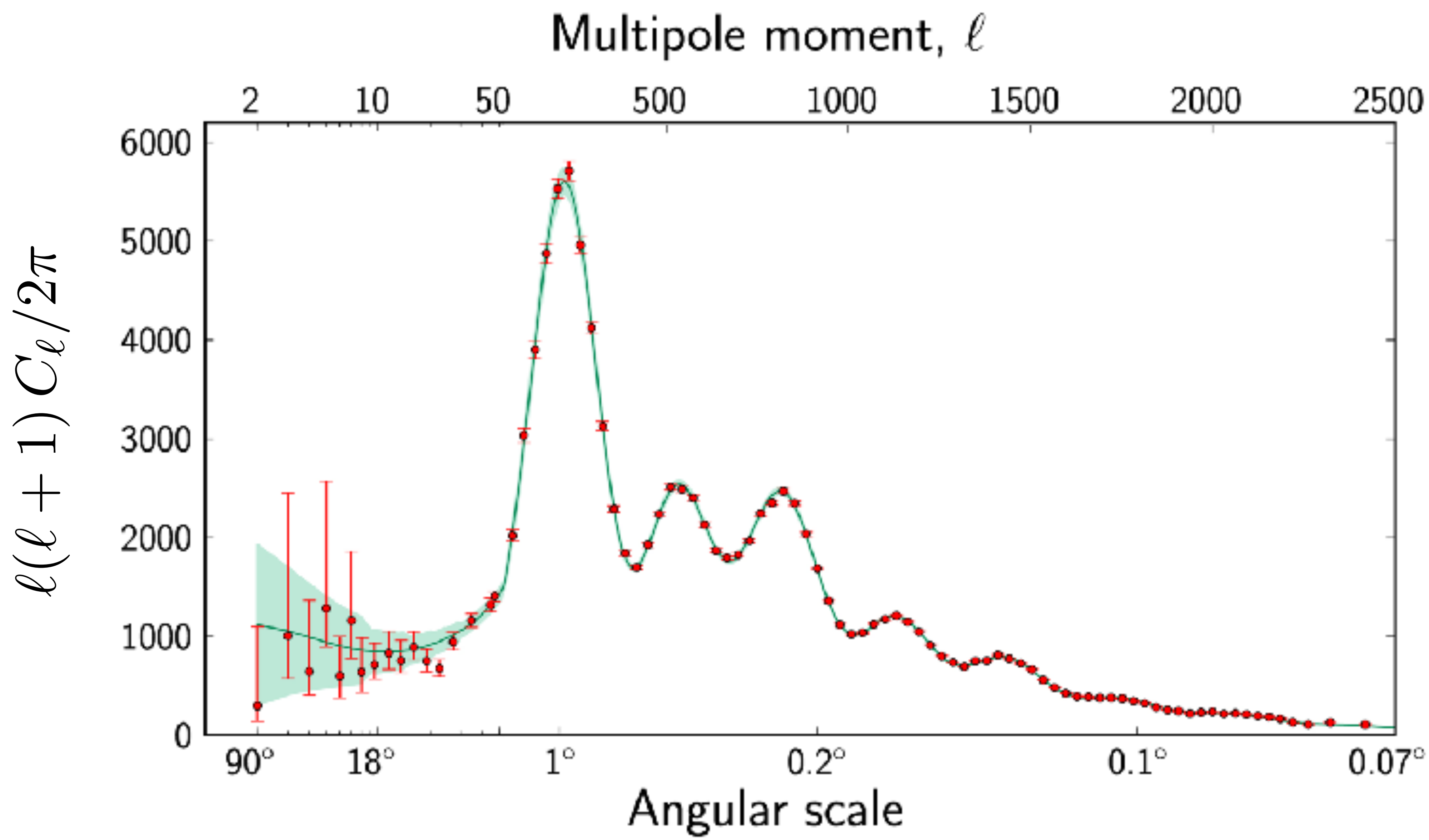
classical statistical average

$\boxed{C_\ell \text{ angular Power Spectrum}}$

- **Equivalently, in angular Fourier space:** $\langle a_{\ell m} a^\star_{\ell' m'} \rangle_c = \delta_{\ell\ell'} \delta_{mm'} C_\ell$

where $a_{\ell m}$ **is defined from:** $\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$

Observations (Planck 2015)



Rich structure!

A remarkably beautiful idea about the origin of the statistics of temp. anisotropies:

Vacuum quantum fluctuations in the early universe!

We need to compute the quantum statistics (i.e. correlation functions) of a field

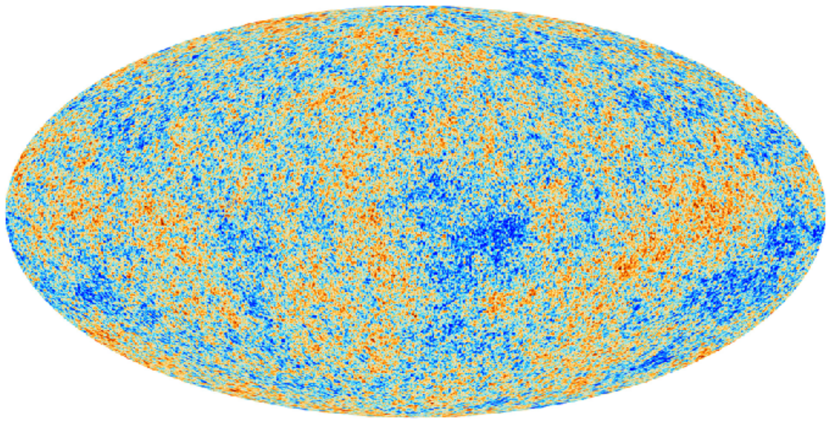
- In angular Fourier space:

$$\langle 0|\hat{\delta\phi}_{\vec{k}}\hat{\delta\phi}_{\vec{k}'}|0\rangle \equiv (2\pi)^3\delta^{(3)}(\vec{k}+\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_{\delta\phi}(k)$$

(for a hom+isot state)

$\mathcal{P}_{\delta\phi}(k)$ Primordial Power Spectrum

Propagating until the instant of last scattering, and projecting on a sphere, produces C_ℓ



5. Three-point function

- **In real space:**

$$\langle \Delta T(\hat{n}_1) \Delta T(\hat{n}_2) \Delta T(\hat{n}_2) \rangle_c$$

- **In angular Fourier space:**

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle_c = \mathcal{G}^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \, b_{\ell_1 \ell_2 \ell_3}$$

(assuming isotropic distribution)

where $\mathcal{G}^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \equiv \int d\hat{n} \, Y_{\ell_1 m_1}(\hat{n}) \, Y_{\ell_2 m_2}(\hat{n}) \, Y_{\ell_3 m_3}(\hat{n})$

$b_{\ell_1 \ell_2 \ell_3}$ **Bispectrum**

This is what cosmologists call non-Gaussianity

(because in a Gaussian probability distribution the three-point function vanishes)

Primordial origin: three-point quantum correlation function

- **In angular Fourier space:**

$$\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{k}_3}|0\rangle \equiv (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,\mathcal{B}_{\delta\phi}(k_1,k_2,k_3)$$ **(for a hom+isot state)**

$\mathcal{B}_{\delta\phi}(k_1,k_2,k_3)$ **Primordial Bispectrum**

Propagating until the instant of last scattering, and projecting on a sphere, produces $b_{\ell_1\ell_2\ell_3}$

Therefore, studying primordial non-Gaussianity simply means computing the three-point correlation function of scalar perturbations

First Part: General Overview

1. Non-Gaussianity: What and Why?

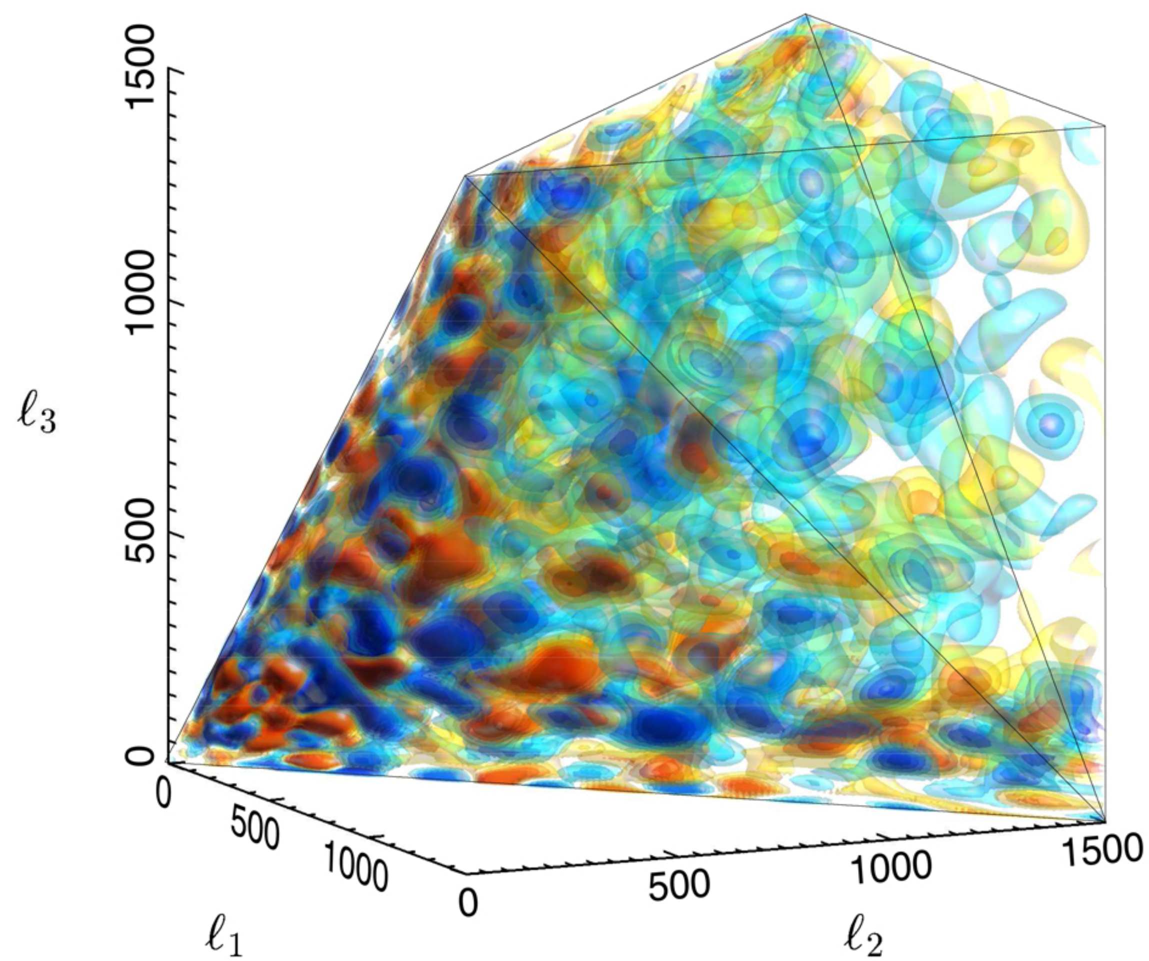
Why?

Because we have data!!!

Data on non-Gaussianity provides valuable information about the mechanism generating the primordial perturbations

Observations (Planck 2015)

$b_{\ell_1 \ell_2 \ell_3}$ **Bispectrum**



Again, rich structure!

Although most of this structure is not primordial

Forthcoming observations of the Large Scale Structure (galaxy distribution) is expected to provide richer observational data on non-Gaussianity

First Part: General Overview

2. Non-Gaussianity and LQC

Last few years: lot of interest in LQC-phenomenology

Reason: contact between quantum gravity and observations

Framework: LQC + Inflation

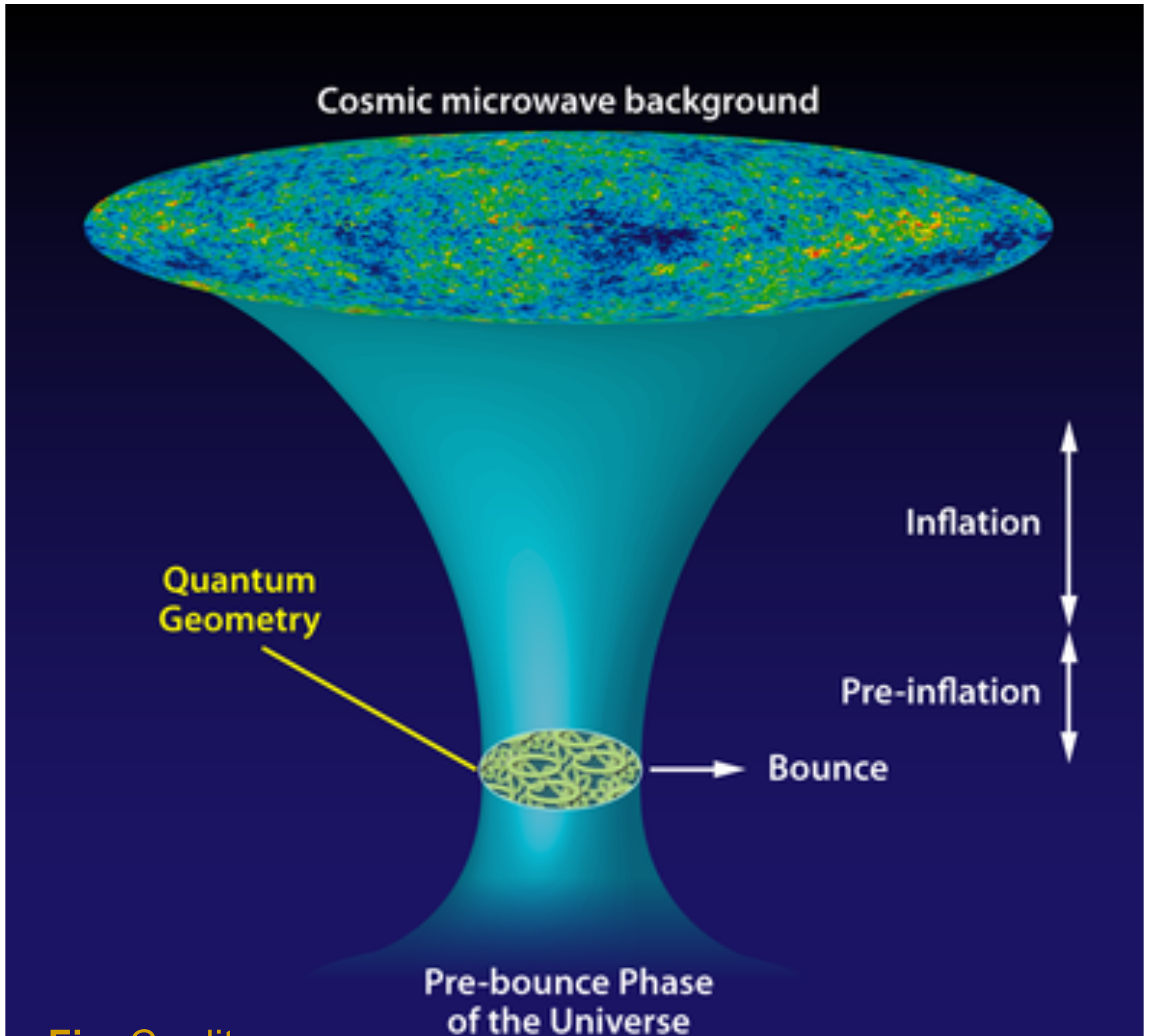
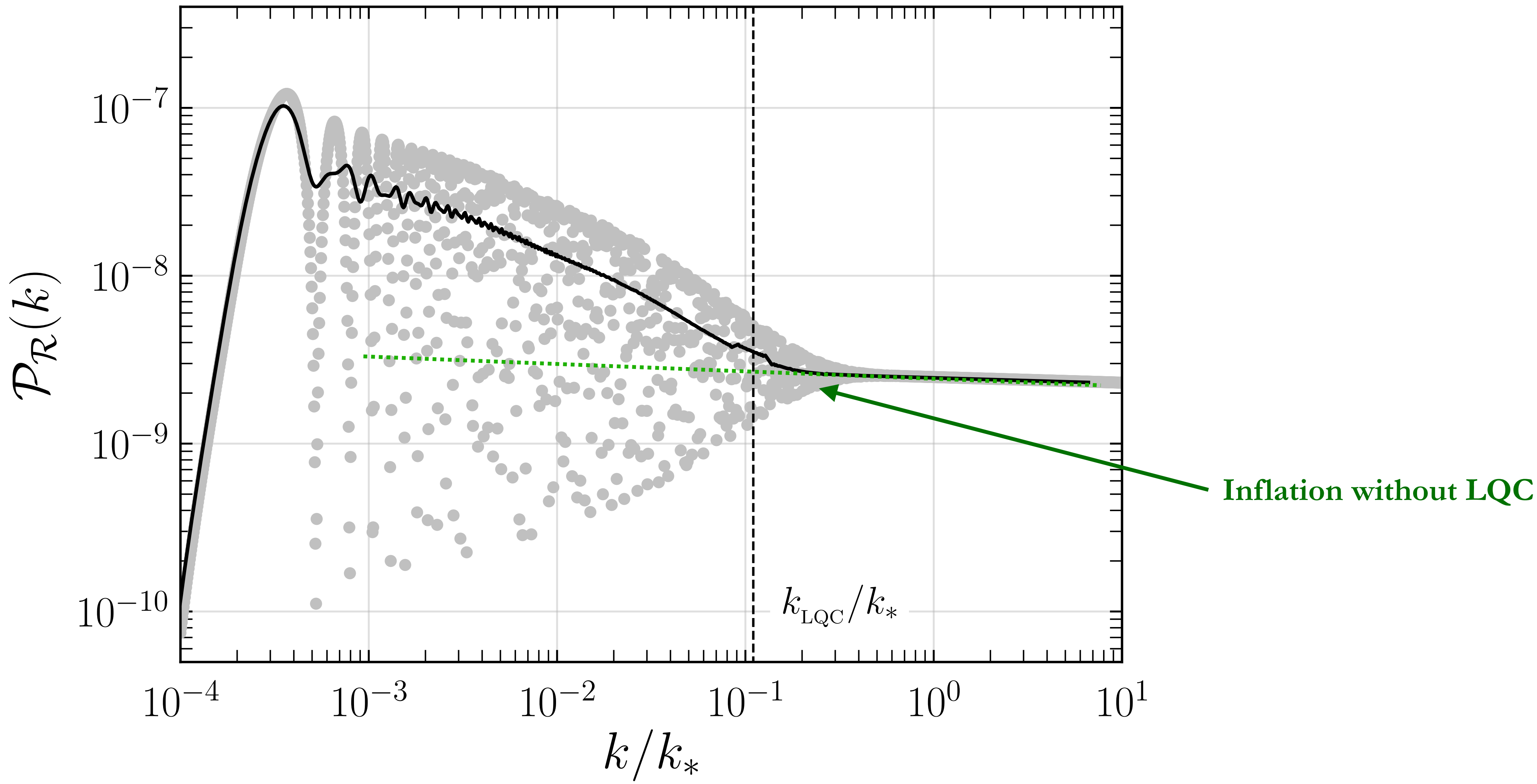


Fig. Credits:
P. Singh, Physics 5, 142 (2012)

Focus so far: Power spectrum



Gray points: Power spectrum numerically computed for individual values of k

Black points: Average of gray points

Similar results obtained by different groups:

I.A., Ashtekar, Barrau, Bojowald, Bonga, Bolliet, Brizuela, Calcagni, Castelló-Gomar, Grain, Gupt, Martin-Benito, Martin de Blas, Mena-Marugan, Mielczarek, Linsefors, Nelson, Olmedo, Vijayakumar, Wilson-Ewing,…

For other choices of initial conditions (for perturbations) the infrared part of the spectrum is suppressed rather than enhanced (Martin de Blas-Olmedo, Ashtekar-Gupt)

Comparing with data:

(1) Restriction to the parameter space to the region that makes results compatible with observation

(2) In this region of the parameter space, look for LQC-specific signatures, both for scalar and tensor modes.

In this talk, we argue that this (the analysis of the power spectrum) is only half of the story

It remains to be answered:

How big is the non-Gaussianity generated by the bounce?

(a) If it is large enough, the perturbative expansion used to compute the power spectrum would break down.

This is a real possibility, because the bounce takes place at the Planck scale

(b) Even if perturbation theory turns out to be OK, there are strong observational upper bounds we must satisfy to claim compatibility with data.

An analysis of non-Gaussianity is therefore needed (a) to claim self-consistency and (b) to make statements about predictions.

What we need to do (now schematically, more details later):

(1) Second order classical cosmological perturbation theory:

$$\mathcal{H} = \mathcal{H}^{(2)} + \mathcal{H}^{(3)}$$

$\mathcal{H}^{(2)}$: piece quadratic in perturbation. Dictates the free evolution (linear equations of motion)

$\mathcal{H}^{(3)}$: cubic piece. Interaction Hamiltonian

(2) Quantize, and use interaction picture

Operators evolve with the free Hamiltonian $\mathcal{H}^{(2)}$: $\hat{\delta\phi}^I$

States evolve with the interaction Hamiltonian $\mathcal{H}^{(3)}$: $U(\eta,\eta_0) = T\exp\left(-i/\hbar\int_{\eta_0}^{\eta} d\eta'\,\hat{\mathcal{H}}^{(3)\mathrm{I}}(\eta')\right),$

(3) Power Spectrum

$$\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}|0\rangle = \langle 0|U^\dagger\hat{\delta\phi}^I_{\vec{k}_1}\hat{\delta\phi}^I_{\vec{k}_2}U^\dagger|0\rangle = \langle 0|\hat{\delta\phi}^I_{\vec{k}_1}\hat{\delta\phi}^I_{\vec{k}_2}|0\rangle - i/\hbar\int_{\eta_0}^{\eta} d\eta'\,\langle 0|\left[\hat{\delta\phi}^I_{\vec{k}_1}\hat{\delta\phi}^I_{\vec{k}_2},\hat{\mathcal{H}}^{(3)\mathrm{I}}_{\mathrm{int}}(\eta')\right]|0\rangle + \mathcal{O}(\mathcal{H}^{(3)})^2 .$$

Next-to-leading order correction

It must be small for the perturbative expansion to make sense

(4) Non-Gaussianity

$$\langle 0|\hat{\delta\phi}_{\vec{k}_1}(\eta)\hat{\delta\phi}_{\vec{k}_2}(\eta)\hat{\delta\phi}_{\vec{k}_3}(\eta)|0\rangle = \underbrace{\cancel{\langle 0|\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta)|0\rangle}}_{0}$$

$$-i/\hbar\int d\eta'\langle 0|\left[\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta),\hat{\mathcal{H}}^{(3){\rm I}}(\eta')\right]|0\rangle + \mathcal{O}(\mathcal{H}^{(3)})^2$$

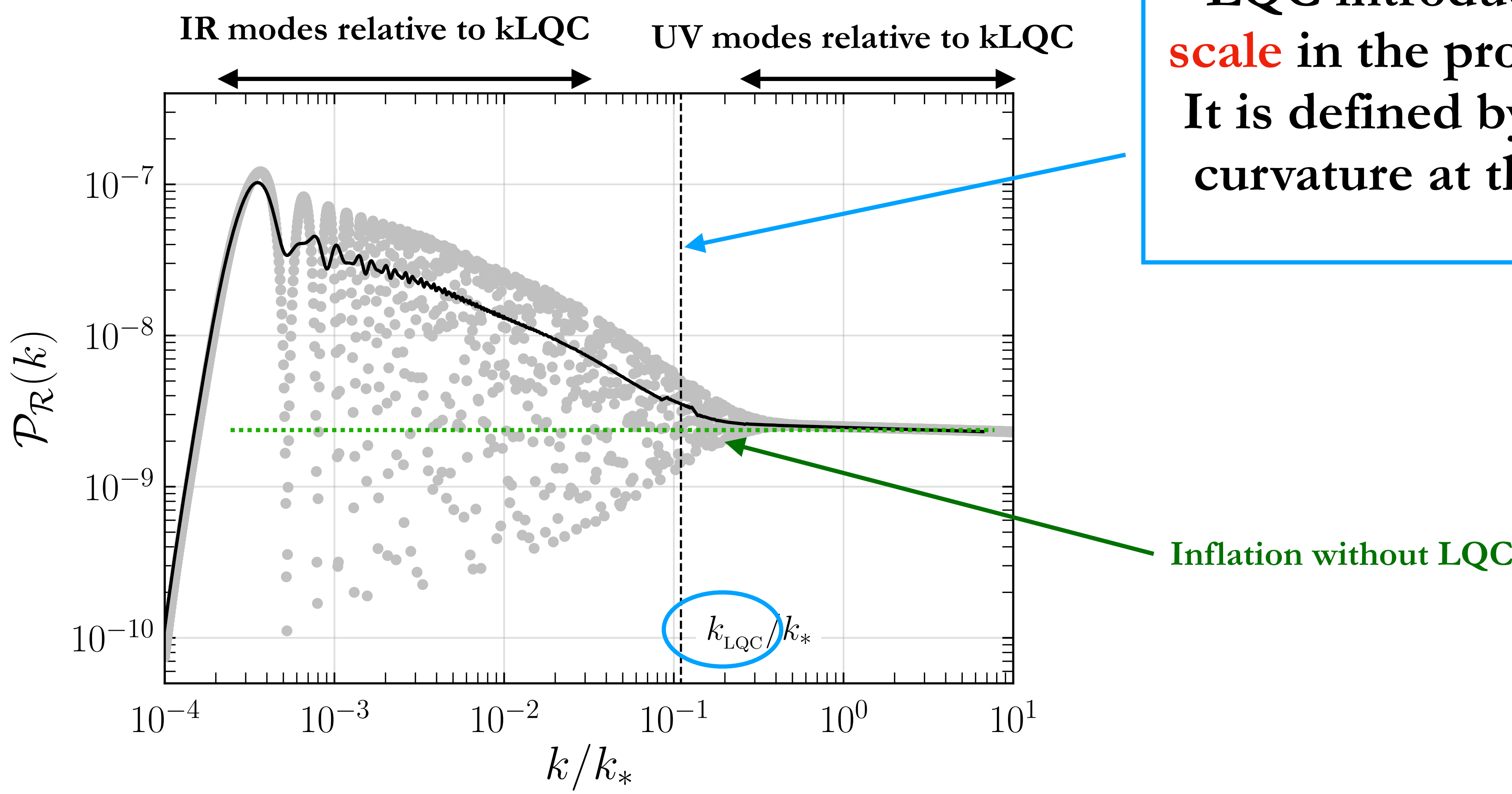
This is what we need to compute

First Part: General Overview

3. Results

- **We have developed a numerical code to compute non-Gaussianity in generic FRLW spacetime**
- **Embedded in the numerical infrastructure CLASS**
- **Codes for non-Gaussianity already existed, but their range of applicability is not generic enough for our problem**
- **We have made it publicly available:** https://github.com/borisbolliet/class_lqc_public
- **This code will be useful beyond LQC**

(1) Power Spectrum



LQC introduces a new scale in the problem kLQC. It is defined by the Ricci curvature at the bounce

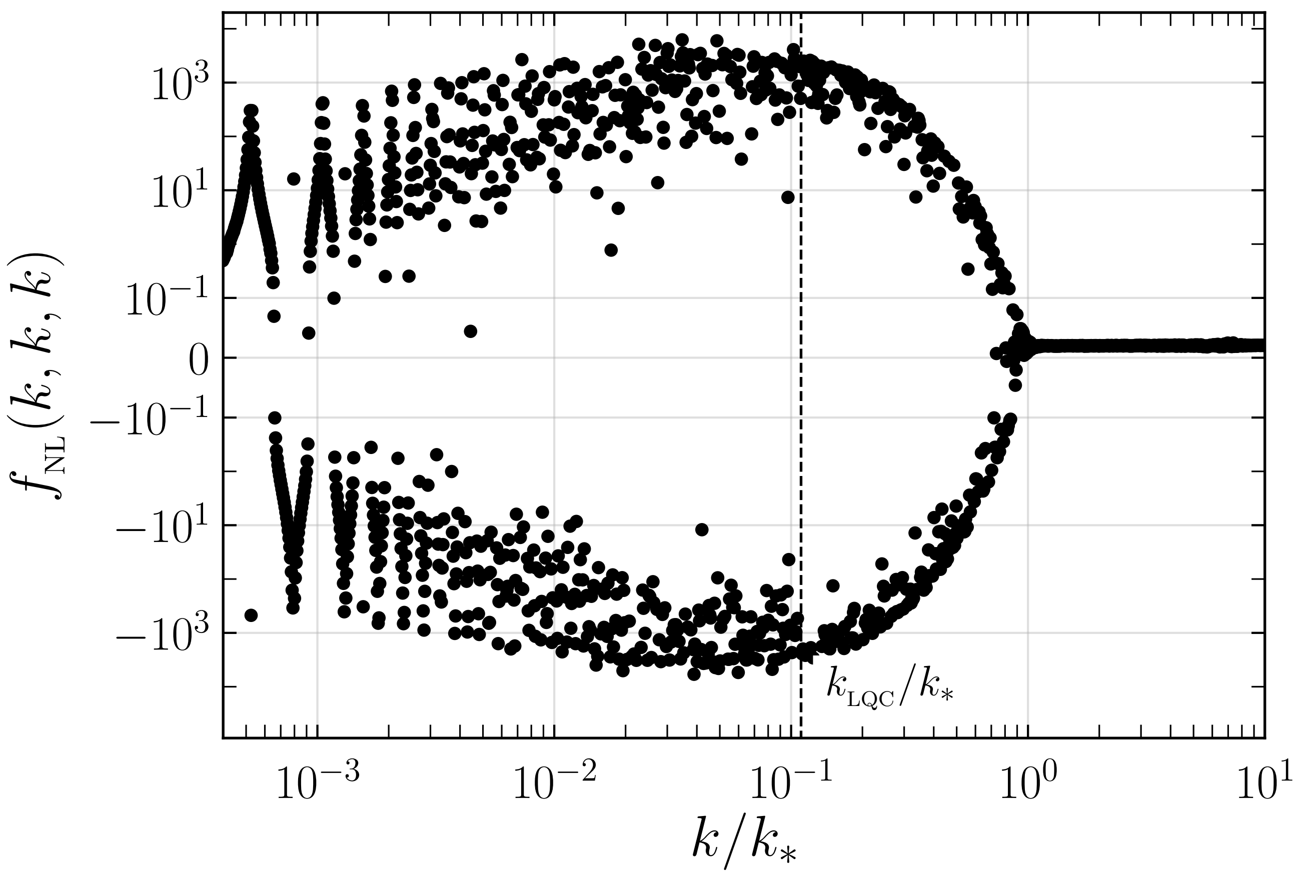
Agreement with standard inflation for UV modes (these modes don't "feel" the bounce")

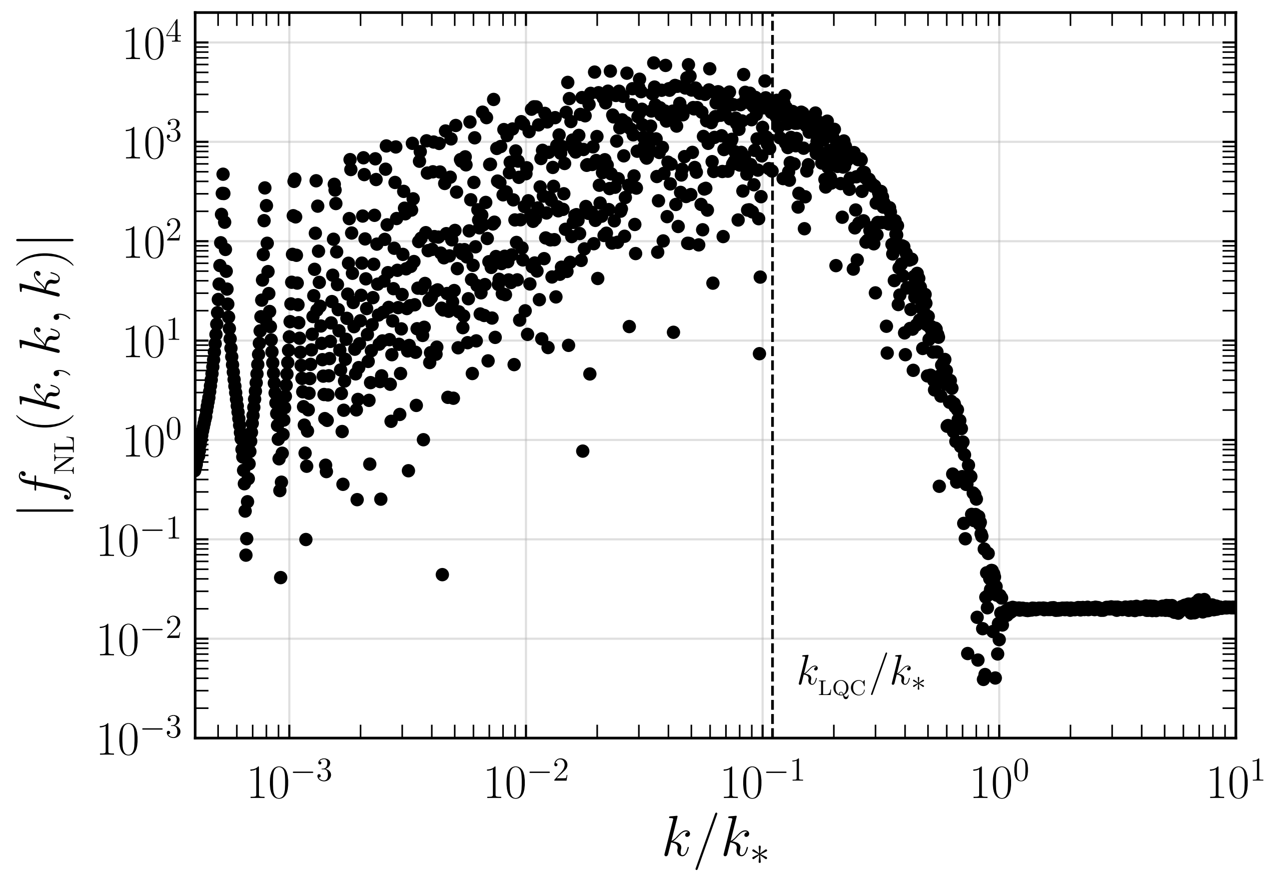
The bounce affects the IR part of the spectrum

(2) Non-Gaussianity

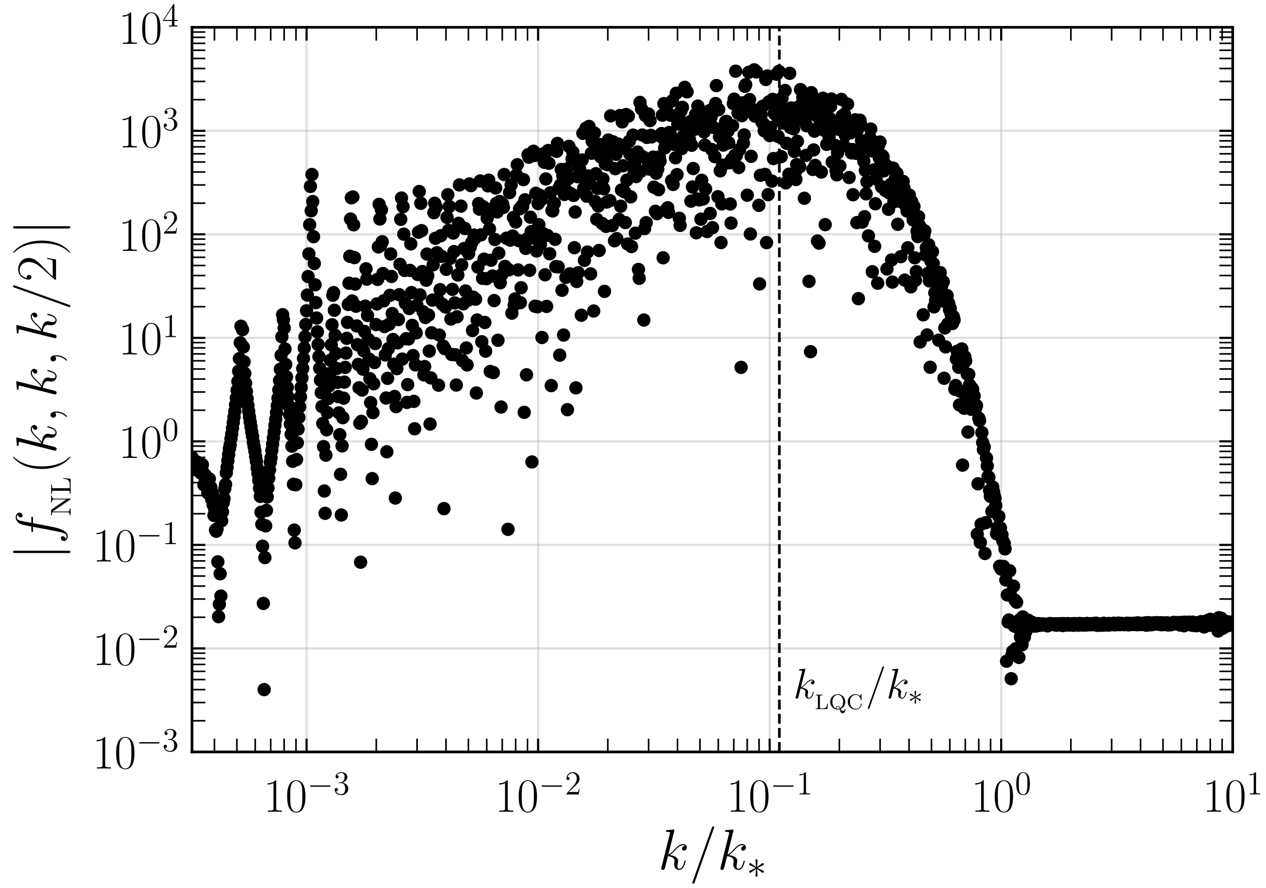
$f_{NL}(k_1, k_2, k_3)$: **Bispectrum in "units" of the power spectrum. See later for exact definition**

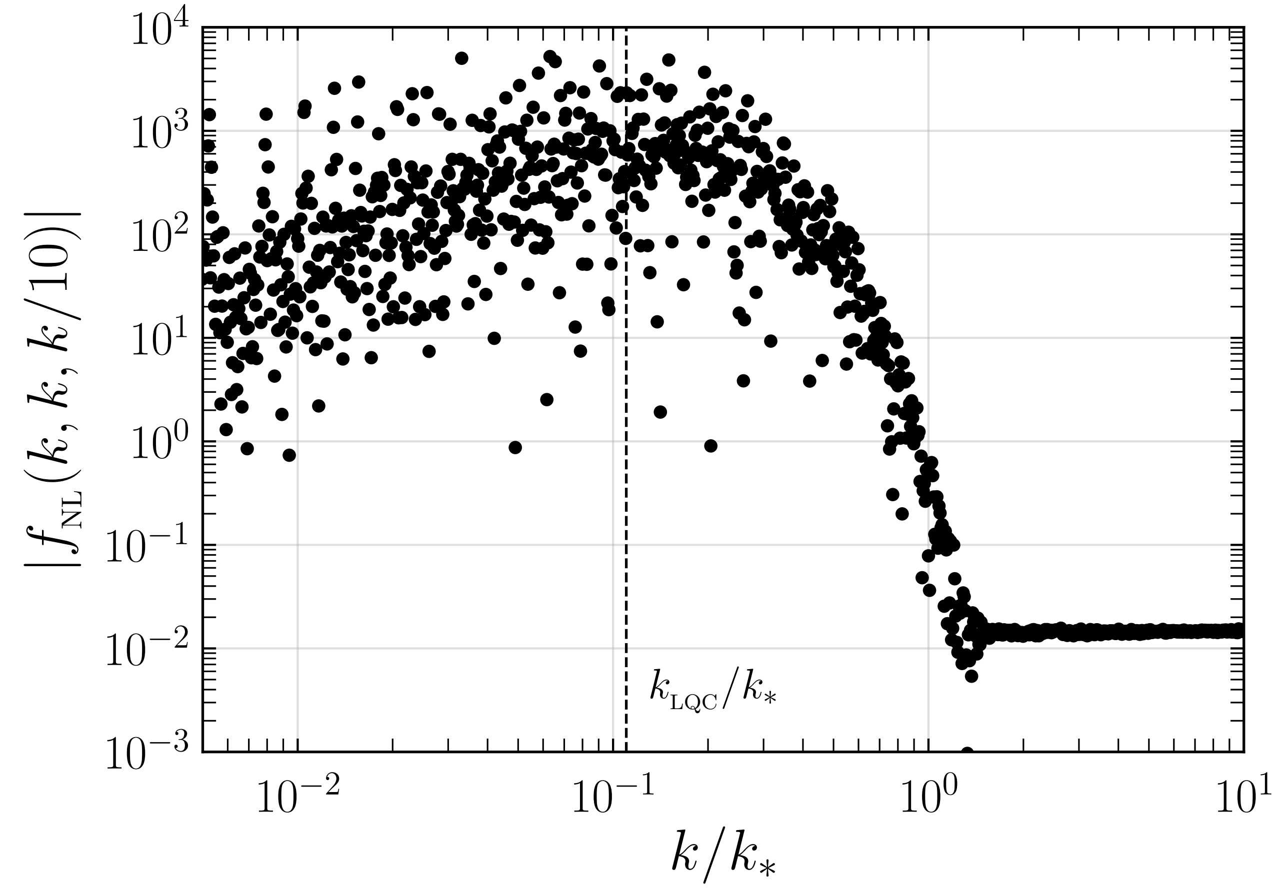
First, we show equilateral configurations, i.e. k1=k2=k3

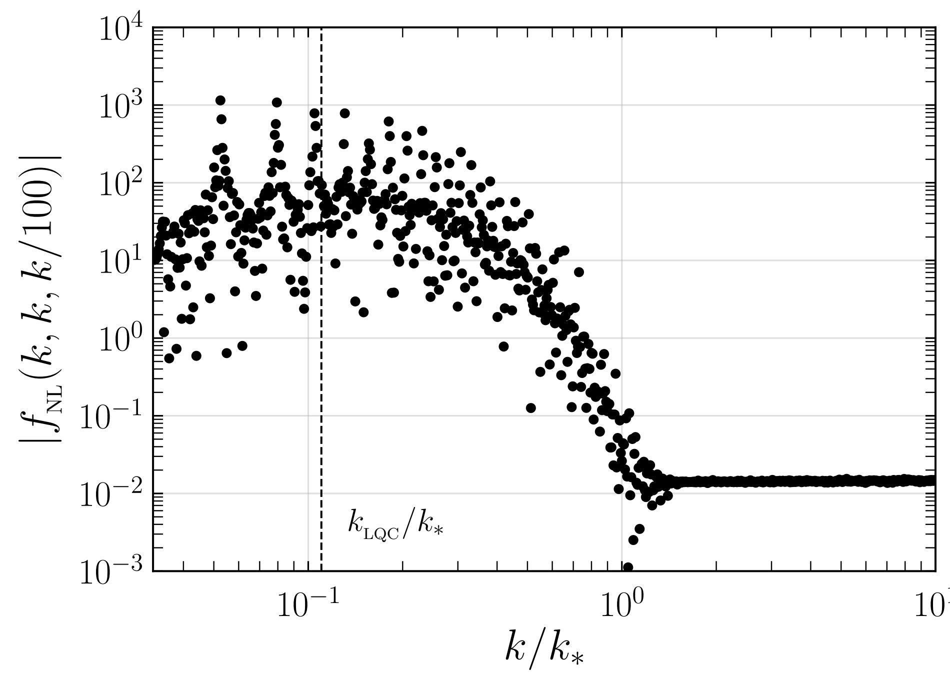


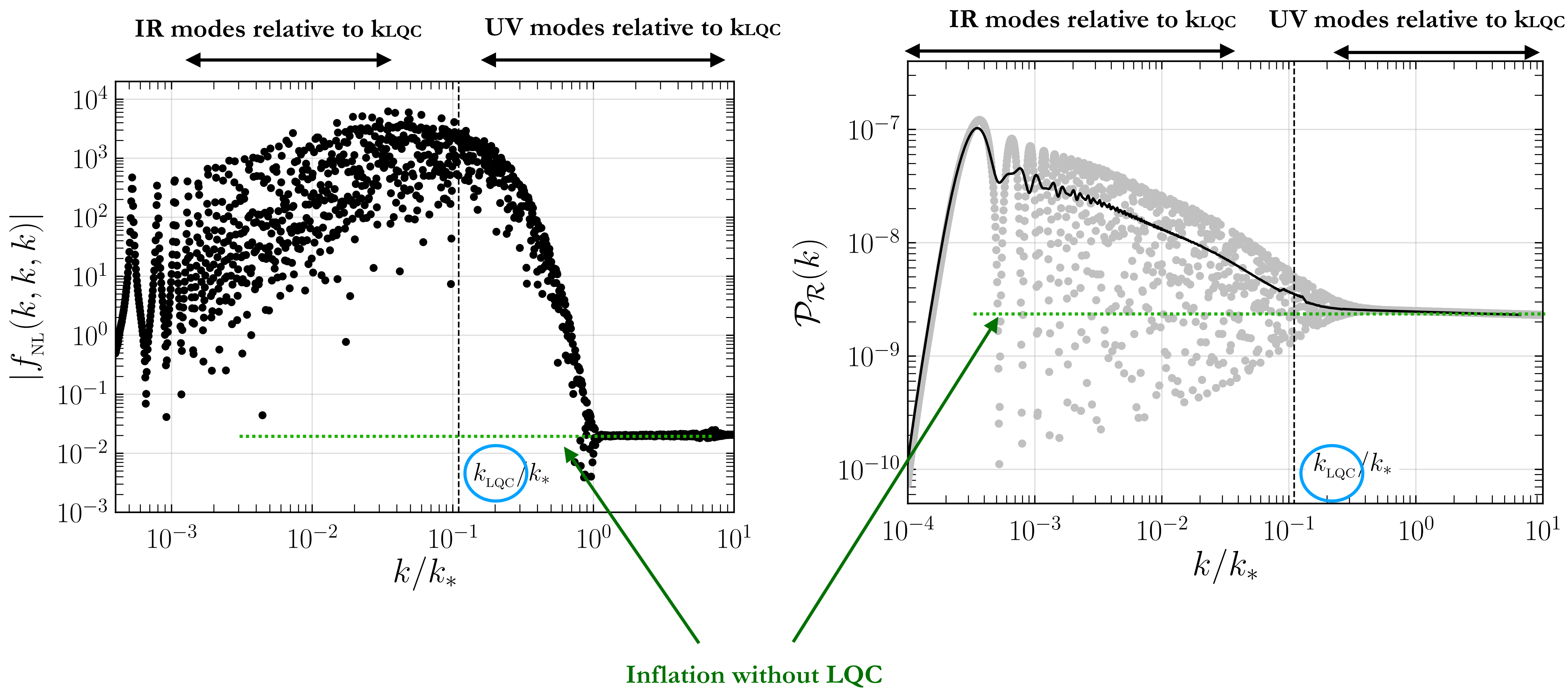


Similar results for other configurations









Qualitative understanding: similar to the power spectrum

The bounce amplifies non-Gaussianity significantly, for modes that are of the same order or more infrared than the curvature radius at the bounce

Non-Gaussianty in LQC are strongly scale dependent, in contrast to a majority of models in the market

Summary of the results:

(1) The results of standard inflation exactly recovered for UV modes (nice check)

(2) Non-Gaussianity is very oscillatory

(3) The amplitude largely enhanced by the bounce for IR modes

(4) The origin of this enhancement understood analytically

(5) Non-Gaussianity "more sensitive" to the bounce than the power spectrum (see previous slide)

(6) We have check that, despite the large enhancement, perturbation theory is under control

(7) Comparison with observations modifies the range of parameters that makes the predictions of LQC compatible with data

Second Part:

Additional Details

Additional Details

1. Second order classical perturbation theory

Well-known in the Lagrangian framework (Maldacena's paper on non-Gaussianity in inflation 2003)

For LQC we need Hamiltonian approach

We couldn't find this in the literature, so we did it ourselves

We took advantage of the very useful Mathematica package xAct, written by Brizuela, Martin, and Mena-Marugán

Perturbative strategy

Expansion at second order. Two strategies:

$$\Phi(\vec{x}) = \phi + \delta\phi^{(1)}(\vec{x}) + \frac{1}{2!}\delta\phi^{(2)}(\vec{x}) + \dots$$
$$P_\Phi(\vec{x}) = p_\phi + \delta p_\phi^{(1)}(\vec{x}) + \frac{1}{2!}\delta p_\phi^{(2)}(\vec{x}) + \dots$$
$$q_{ij}(\vec{x}) = \mathring{q}_{ij} + \delta q_{ij}^{(1)}(\vec{x}) + \frac{1}{2!}\delta q_{ij}^{(2)}(\vec{x}) + \dots$$
$$\pi^{ij}(\vec{x}) = \mathring{\pi}^{ij} + \delta\pi^{ij\,(1)}(\vec{x}) + \frac{1}{2!}\delta\pi^{ij\,(2)}(\vec{x}) + \dots$$

Linear e.o.m. with sources

Versus

$$\Phi(\vec{x}) = \phi + \delta\phi(\vec{x}),$$
$$P_\Phi(\vec{x}) = p_\phi + \delta p_\phi(\vec{x}),$$
$$q_{ij}(\vec{x}) = \mathring{q}_{ij} + \delta q_{ij}(\vec{x}),$$
$$\pi^{ij}(\vec{x}) = \mathring{\pi}^{ij} + \delta\pi^{ij}(\vec{x})$$

Non-linear e.o.m.

where: $\mathring{q}_{ij} = a^2\,\delta_{ij}\,;$ $\quad \mathring{\pi}^{ij} = \frac{\pi_a}{6\,a}\,\delta^{ij}$

We choose the second strategy, because it is easier to implement in the quantum theory via standard perturbative techniques in the interaction picture.

Gauge choice for scalar perturbations

- First question, Gauge invariant versus Gauge fixed

- Remember: no matter what your choice is, the results must be reported in terms of the comoving curvature perturbations $\mathcal{R}$ (it remains constant at super-Hubble scales)

- But $\mathcal{R}$ is ill-defined a bit before inflation starts, when $\dot{\phi}=0$

- We use $\delta\phi$ to parameterized scalar perturbations and fix the gauge so $\delta q_{ij}=0$

- We compute correlation functions for $\delta\phi$ and translate them to $\mathcal{R}$ at the end of inflation, using the relation:

$$\mathcal{R}(\vec{x},\eta)=-\frac{a}{z}\,\delta\phi(\vec{x},\eta)+\left[-\frac{3}{2}+3\frac{V_\phi\,a^2}{\kappa\,p_\phi\,\pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]\left(\frac{a}{z}\,\delta\phi(\vec{x},\eta)\right)^2+\cdots$$

subdominant terms at the end of inflation

where $z:=-\frac{6}{\kappa}\frac{p_\phi}{\pi_a}$

Second order Hamiltonian (free evolution)

$$\mathcal{H}^{(2)} = N\frac{1}{2}\int \mathrm{d}^3x \left[\frac{1}{a^3}\,\delta p_\phi^2 + a^3\,(\vec{\partial}\delta\phi)^2 + a^3\,\mathfrak{A}\,\delta\phi^2\right]$$

Potential:

$$\mathfrak{A} = -9\frac{p_\phi^4}{a^8\pi_a^2} + \frac{3}{2}\kappa\frac{p_\phi^2}{a^6} - \frac{6\,p_\phi}{a\,\pi_a}V_\phi + V_{\phi\phi} + 6\frac{p_\phi\dot{p}_\phi}{a^4\,\pi_a} - 3\frac{p_\phi^2\,\dot{\pi}_a}{a^4\,\pi_a^2} - 3\,\frac{\dot{a}\,p_\phi^2}{a^5\,\pi_a}$$

Produces equations of motion: $\left(\Box - \mathfrak{A}(t)\right)\delta\phi(\vec{x},t) = 0$

Third order Hamiltonian (self-interactions)

$$\begin{aligned}\mathcal{H}^{(3)} &= N\int d^3x\Bigg[\left(\frac{9\,\kappa\,p_\phi^3}{4\,a^4\,\pi_a}-\frac{27\,p_\phi^5}{2\,a^6\pi_a^3}-\frac{3\,a^2\,p_\phi\,V_{\phi\phi}}{2\,\pi_a}+\frac{a^3\,V_{\phi\phi\phi}}{6}\right)\delta\phi^3\\ &-\frac{3\,p_\phi}{2\,a^4\,\pi_a}\,\delta p_\phi^2\,\delta\phi-\frac{9\,p_\phi^3}{a^5\pi_a^2}\,\delta p_\phi\delta\phi^2-\frac{3\,a^2\,p_\phi}{2\,\pi_a}\delta\phi\,(\vec{\partial}\delta\phi)^2+\frac{3\,p_\phi^2}{N\,a\,\pi_a}\,\delta\phi^2\partial^2\chi\\ &+\frac{1}{N}\,\delta p_\phi\,\partial_i\delta\phi\,\partial^i\chi+\frac{3\,a^2\,p_\phi}{N^2\,2\,\kappa\,\pi_a}\,\delta\phi\,\partial^2\chi\,\partial^2\chi-\frac{3\,a^2\,p_\phi}{N^2\,2\,\kappa\,\pi_a}\,\delta\phi\,\partial_i\partial_j\chi\,\partial^i\partial^j\chi\Bigg].\end{aligned}$$

with: $\tilde{\chi} = N\,\dfrac{3\,\kappa}{k^2a^3}\left[\left(\dfrac{p_\phi}{2}-\dfrac{a^5\,V_\phi}{\kappa\,\pi_a}\right)\delta\tilde{\phi}-\dfrac{p_\phi}{\kappa\,a\,\pi_a}\delta\tilde{p}_\phi\right]$

Note: **most of terms are independent of** $V(\phi)$ **. These are self-interactions mediated by gravity**

Check:

After a Legendre transformation, agreement with Maldacena's third order Lagrangian

Additional Details

2. Extension of the dressed metric approach

Dressed metric approach (Ashtekar-Kaminski-Lewandowski, I.A., Dapor, Nelson, Puchta, Tavakoli)

The formal derivation is the same as at it is when working at leading order in perturbations. I don't repeat it here, but only emphasize the differences

Under the test field approx. the e.o.m. for perturbation turn out to be:

$$i\hbar\,\partial_\phi \delta\Psi = \langle\Psi_0|\hat{\mathcal{H}}_{\rm pert}|\Psi_0\rangle\,\delta\Psi$$

State of perturbations **FLRW background**

Main difference: the Hamiltonian for perturbations now acquires a new term (in red):

$$\hat{\mathcal{H}}_{\rm pert} = \mathcal{V}_0\left((\hat{H}_0)^{-1/2}\,(\hat{\mathcal{H}}^{(2)}[N_\tau] + \hat{\mathcal{H}}^{(3)}[N_\tau])\,(\hat{H}_0)^{-1/2}\right)$$

First, free evolution:

Same as in previous papers: $(\tilde{\Box} - \tilde{\mathfrak{A}})\, \hat{\delta\phi}(\vec{x}, \tilde{\eta}) = 0$

$\tilde{\Box}$ d'Alembertian of the dressed metric $\quad \tilde{g}_{ab} dx^a dx^b = \tilde{a}^2(\tilde{\eta})\,(-d\tilde{\eta}^2 + d\vec{x}^2)$

- with scale factor: $\tilde{a}^4 = \dfrac{\langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle}{\langle \hat{H}_0^{-1} \rangle}$
- and relation conformal time-internal time $d\tilde{\eta} = \mathcal{V}_0\, \langle \hat{H}_0^{-1} \rangle^{1/2}\, \langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle^{1/2}\, d\phi$
- and dressed potential $\quad \tilde{\mathfrak{A}} = \dfrac{\langle \hat{H}_0^{-\frac{1}{2}}\, \hat{a}^2\, \hat{\mathfrak{A}}\, \hat{a}^2\, \hat{H}_0^{-\frac{1}{2}} \rangle}{\langle \hat{H}_0^{-\frac{1}{2}}\, \hat{a}^4\, \hat{H}_0^{-\frac{1}{2}} \rangle}$

Three moments of Ψ_0 affect the dynamics of perturbations

If a sharply peaked state Ψ_0 is used for the background, the dressed metric reduces effective metric

I.A., Ashtekar, Gupt 2017, studied phenomenology with states with large dispersions

Interactions:

Evolution operator: $$U(\eta,\eta_0) = T\exp\left(-i/\hbar\int_{\eta_0}^{\eta} d\eta'\,\hat{\mathcal{H}}^{\rm I}_{\rm int}(\eta')\right)$$

Where: $$\hat{\mathcal{H}}_{\rm int} = \langle\Psi_0|\mathcal{V}_0\left((\hat{H}_0)^{-1/2}\,\hat{\mathcal{H}}^{(3)}[N_\tau]\,(\hat{H}_0)^{-1/2}\right)|\Psi_0\rangle$$

A bunch of new moments of Ψ_0 enter in the dynamics

One can compute the new moments using the same techniques as in I.A., Ashtekar, Gupt 2017

We will focus here on sharply peaked states

- **Definition of the bispectrum:**

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle =: (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, B_{\mathcal{R}}(k_1,k_2,k_3)$$

- **Definition of** $f_{NL}(k_1,k_2,k_3)$

$$B_{\mathcal{R}}(k_1,k_2,k_3) \equiv -\frac{6}{5}\, f_{\rm NL}(k_1,k_2,k_3)\, \times (\Delta_{k_1}\Delta_{k_2}+\Delta_{k_1}\Delta_{k_3}+\Delta_{k_2}\Delta_{k_3})$$

- **We have to compute**

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle = \left(-\frac{a}{z}\right)^3 \langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{k}_3}|0\rangle$$
$$+\left(-\frac{3}{2}+3\frac{V_\phi\, a^5}{\kappa\, p_\phi\, \pi_a}+\frac{\kappa}{4}\frac{z^2}{a^2}\right)\left(-\frac{a}{z}\right)^4\Big[\int \frac{d^3p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_3-\vec{p}}|0\rangle + (\vec{k}_1\leftrightarrow\vec{k}_3)+(\vec{k}_2\leftrightarrow\vec{k}_3)$$
$$+\cdots\Big].$$

Additional Details

3. Results

To compute fNL we need to specify:

A choice of potential $V(\phi)$

Initial conditions for the background field $\phi(t_B)$

Energy density at the bounce ρ_B

Initial quantum state for perturbations

First choice

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

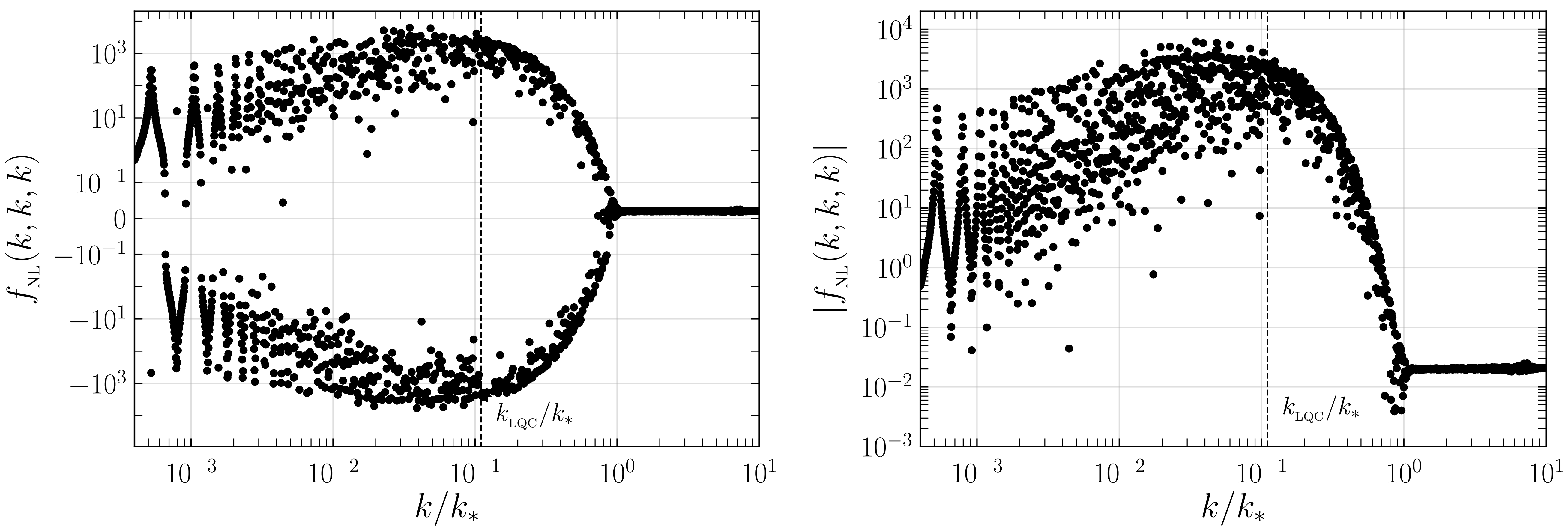
$$\phi(t_B) = 7.62\, M_{P\ell}$$

$$\rho_B = 1\, M_{P\ell}^4$$

Minkowski vacuum in the past, well before the bounce

Later, we will explore other choices

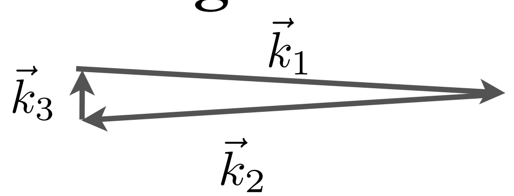
Equilateral configurations, i.e. k1=k2=k3

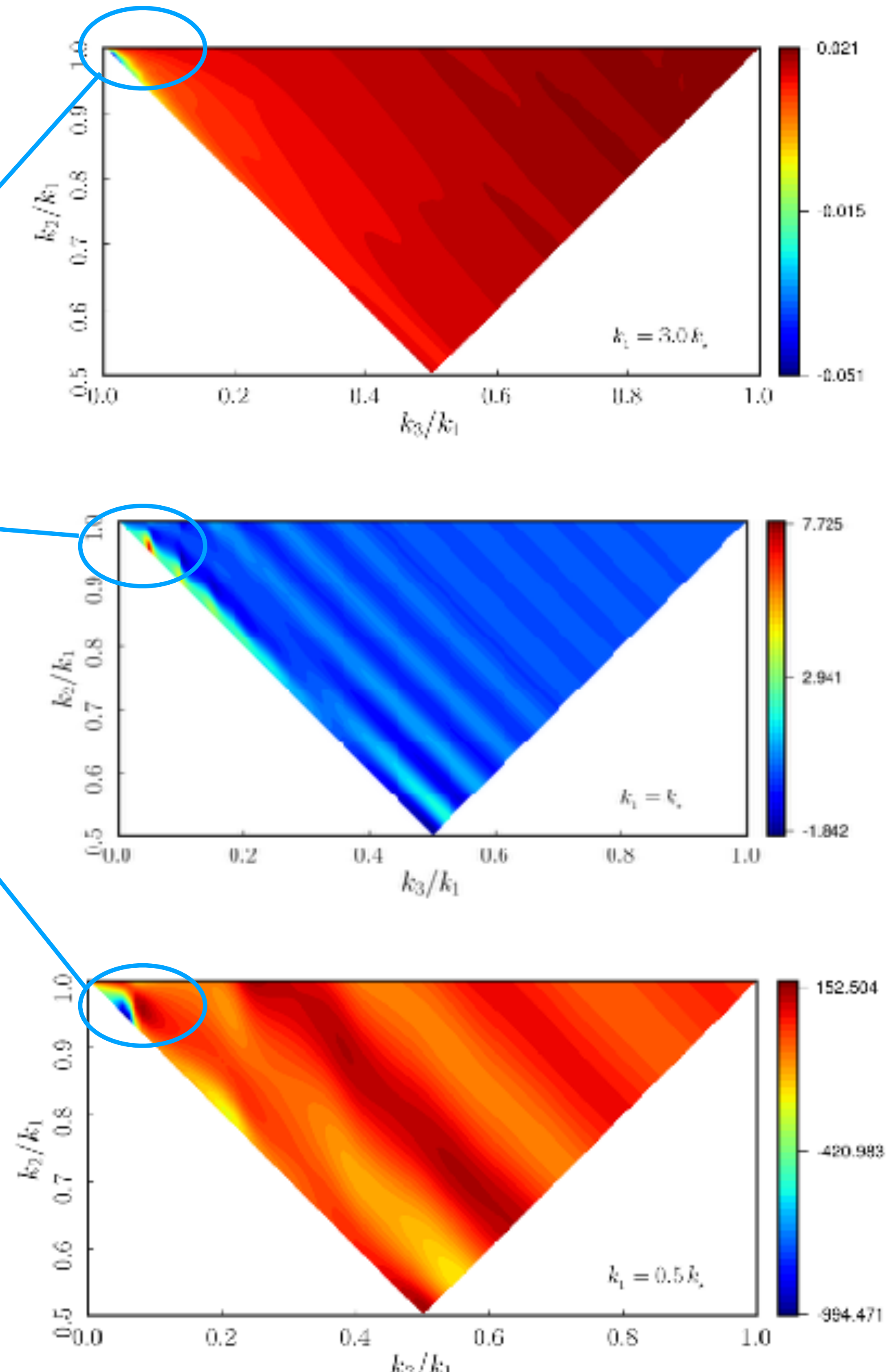


Similar for other configurations (see slide number 27 , and next slide)

Two dimensional plots: fNL vs k2 and k3, for fixed k1

The amplitude of fNL is quite uniform, although larger in squeezed configurations





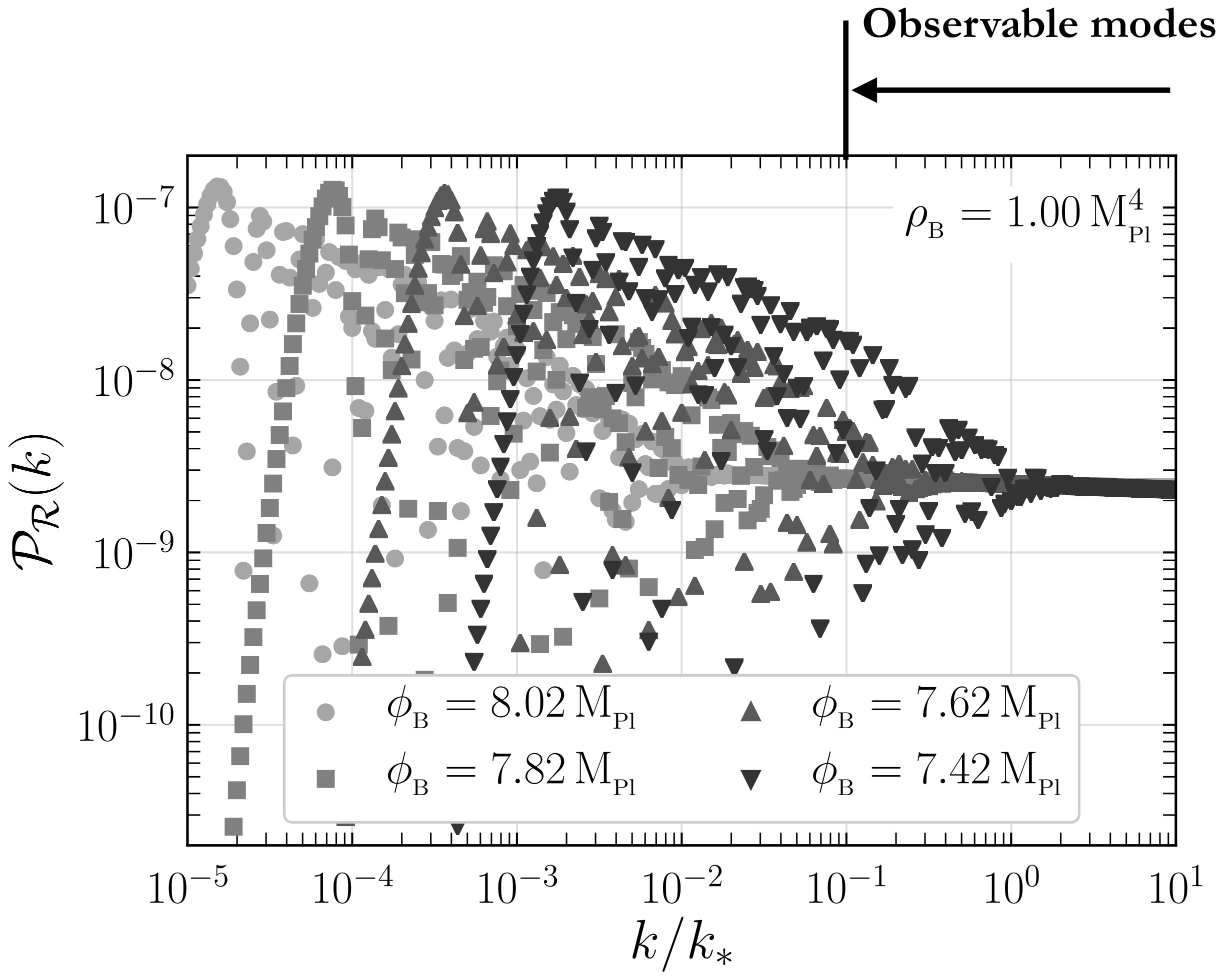
$k_1 = 3\,k_*$

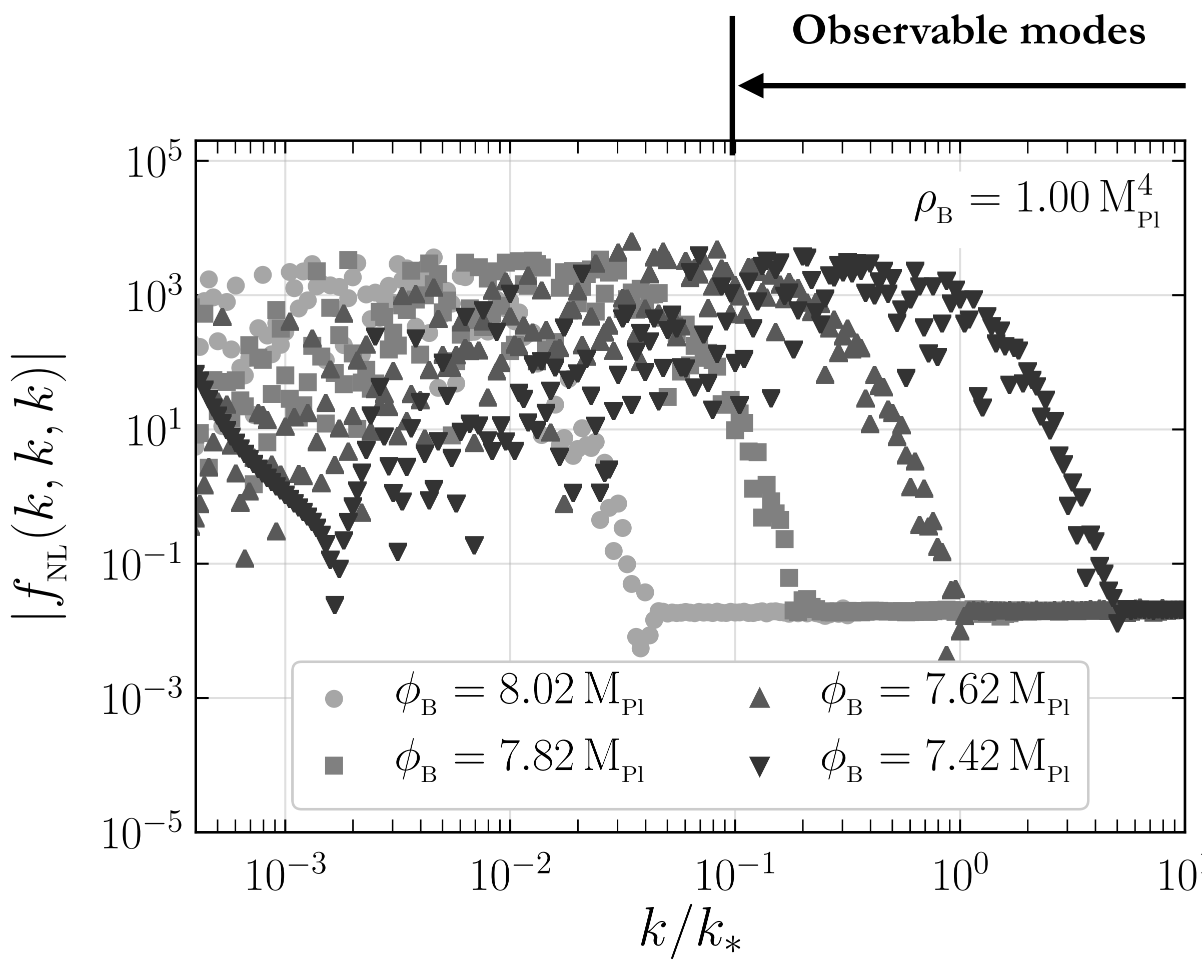
$k_1 = k_*$

$k_1 = 0.5\,k_*$

Different values of $\phi(t_B)$

Physically, $\phi(t_B)$ dictates the amount of expansion from the bounce to the end of inflation



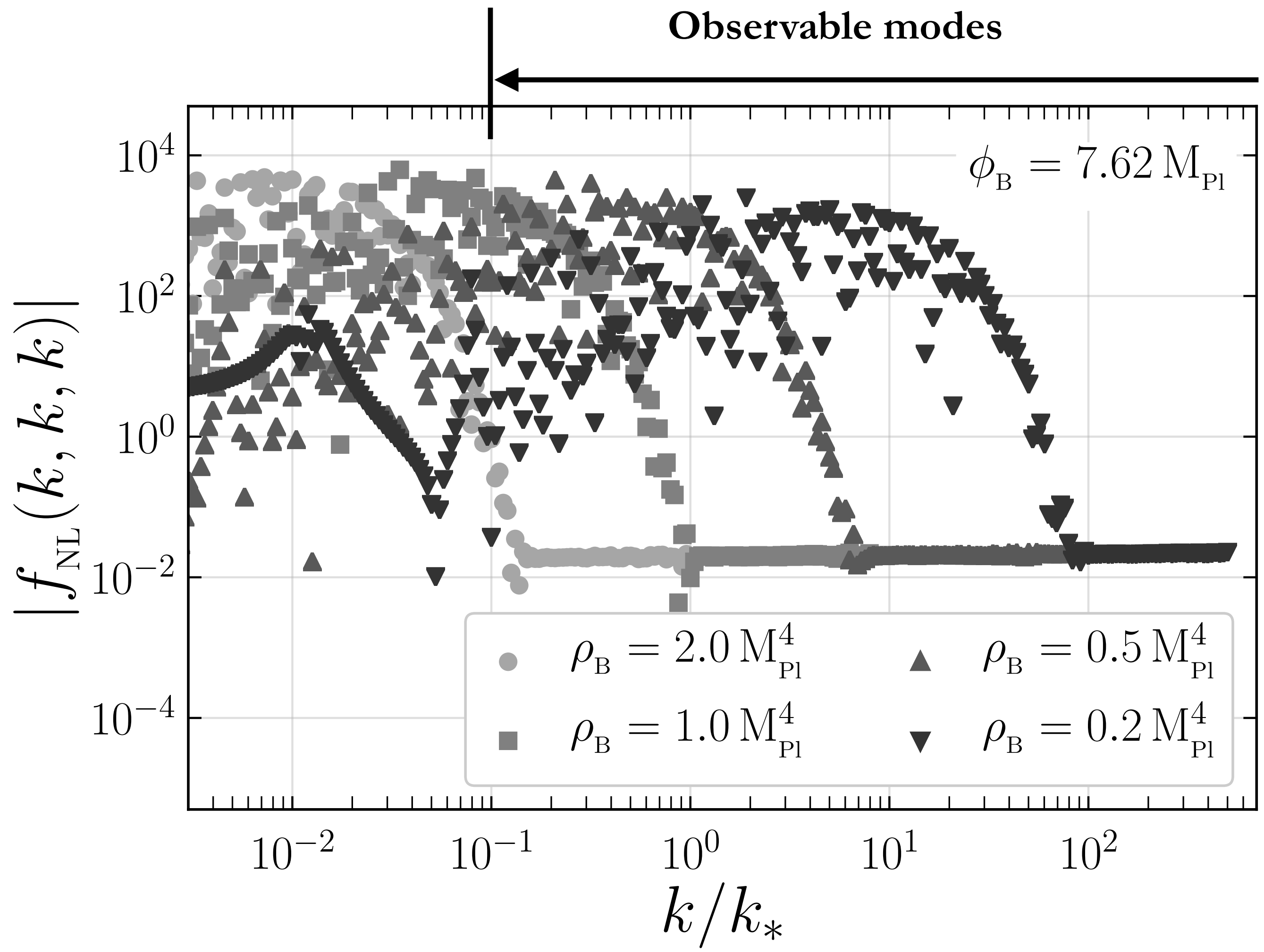


Changing $\phi(t_B)$ produces a shift w.r.t. the observable window, but does not change the shape

Different values of the energy density at the bounce

Changing ρ_B changes the amount of expansion from bounce to end of inflation

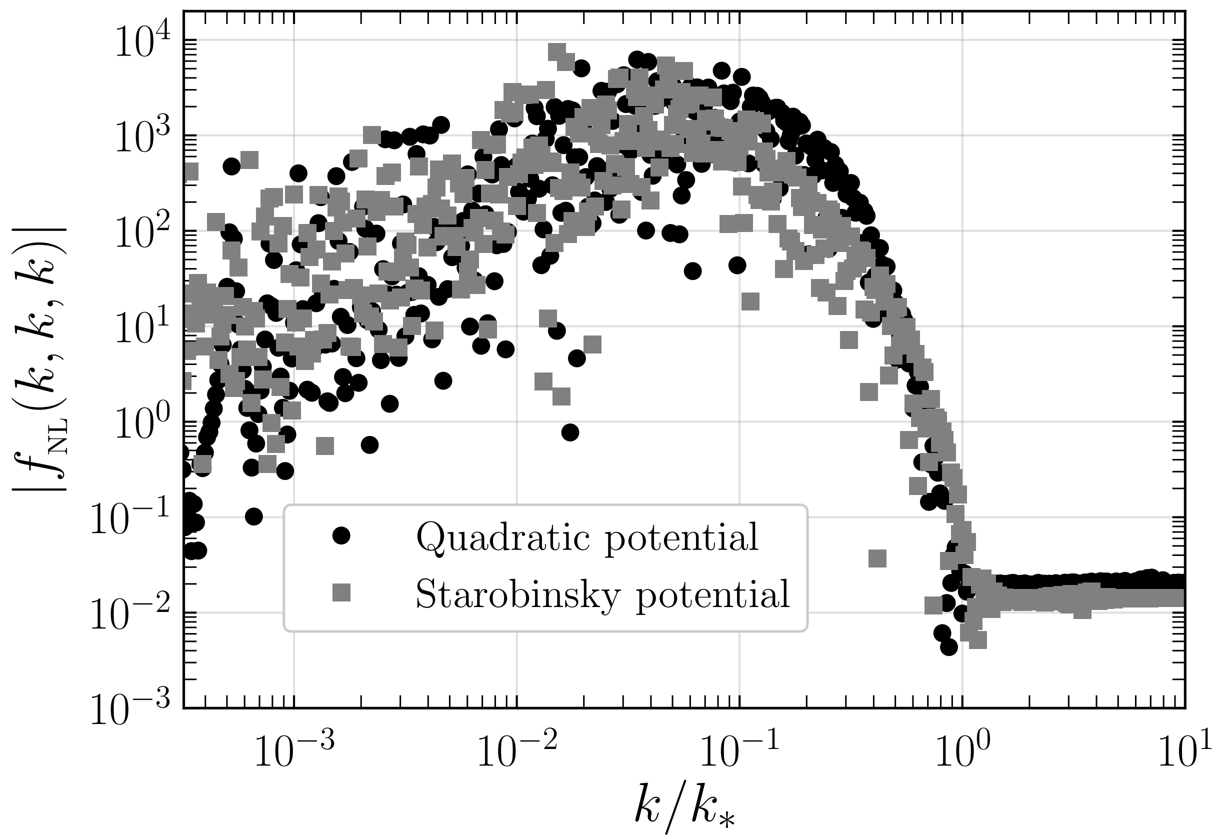
But it also changes the energy scale of the bounce, and this is related to the effective strength of interactions among perturbations

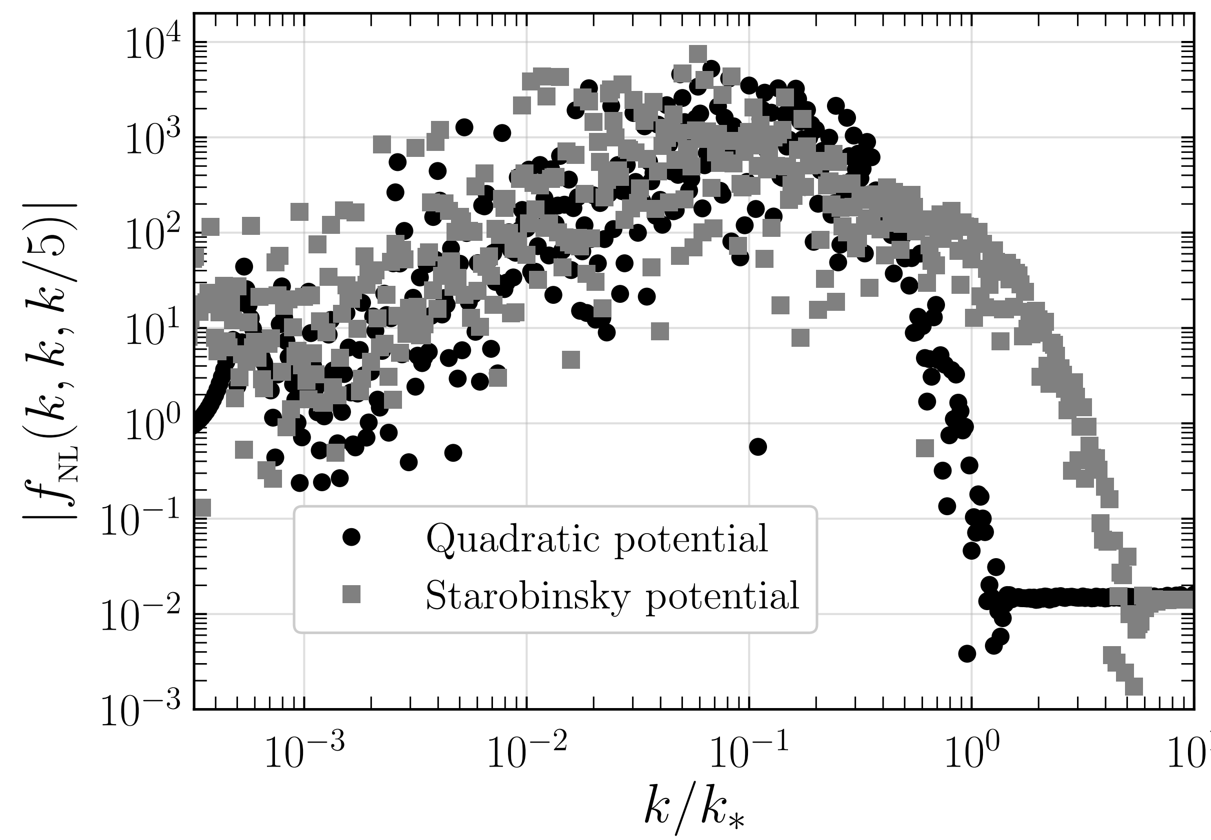


Changing ρ_B produces both a shift relative to the observable window, and a different maximum value of non-Gaussianity

Different potential $V(\phi)$

Starobinsky potential $$V(\phi) = \frac{3\,M^2}{4\,\kappa}\left(1 - e^{-\sqrt{\frac{2\,\kappa}{3}}\phi}\right)^2 \qquad M = 2.51\times 10^{-6} M_{P\ell}$$

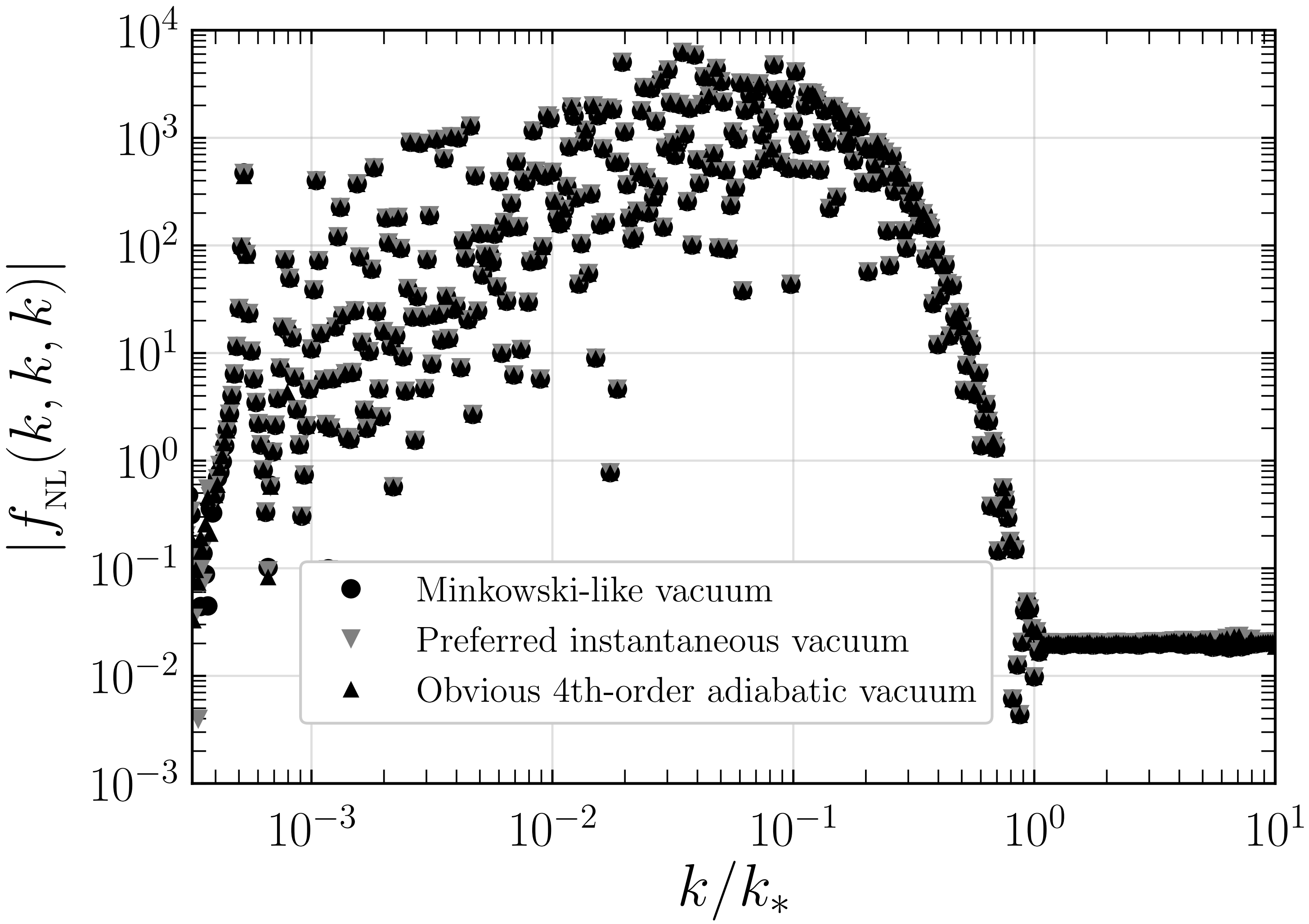




Plots obtained for: $\phi_{\rm B} = -4.88\,M_{P\ell}$, $\rho_{\rm B} = 1\,M_{P\ell}^4$

Conclusion: non-Gausssianity from the bounce the same. Differences come from differences in the background spacetime during inflation, and also well before the bounce

Different choices of initial vacuum in the past, before the bounce



All plots for initial time:

$$\eta_0 = 2.842 \times 10^3\, T_{P\ell}.$$

or, in cosmic time $t_0 = -10^5 T_{P\ell}$

Conclusions: same results

Results are also the same if we choose the initial time to be different, as long as modes start evolution well inside the curvature radius

On the contrary, if we start evolution very close or at the bounce, when some modes are outside the curvature radius, results are very sensitive to the specific initial time chosen.

Additional Details

4. Analytical argument

Our calculations are numerical, and that makes difficult to understand the origin of the enhancement of non-Gaussianity

Goal here: get further understanding using analytical approximations

The argument is very simple, and approximation may look crude, but the result turns out to be an excellent approximation for the numerics

I will be brief here (more details in 1712.08148).

Contribution of non-Gaussianity generated by the bounce is given by integrals of this type:

$$I(k_1,k_2,k_3) = \int_{-\Delta\eta}^{\Delta\eta} d\eta\, \underbrace{f_1(\eta)}_{\text{Background functions}}\, \underbrace{\varphi_{k_1}(\eta)\varphi_{k_2}(\eta)\varphi_{k_3}(\eta)}_{\text{Perturbations}}$$

Extend the integration rate by introducing a window function:

$$I(k_1,k_2,k_3) = \int_{-\infty}^{\infty} d\eta\, W(\eta)\, f_1(\eta)\, \varphi_{k_1}(\eta)\varphi_{k_2}(\eta)\varphi_{k_3}(\eta)$$

where $W(\eta) = \begin{cases} 1 \text{ for } \eta > \Delta\eta \\ 0 \text{ for } |\eta| > \Delta\eta \end{cases}$ **and smooth**

The mode functions are oscillatory (for large enough k):

$$\varphi_{k_1}(\eta)\varphi_{k_2}(\eta)\varphi_{k_3}(\eta) \propto e^{-i\,(k_1+k_2+k_3)\,\eta}$$

So the integrals involved in the calculations are of the type:

$$I(k_1, k_2, k_3) = \int_{-\infty}^{\infty} d\eta \, g(\eta) \, e^{-i\,(k_1+k_2+k_3)\,\eta}$$

Cauchy intergal theorem+ asymptotic analysis:

The behavior of the integral for large enough values of $k_t = k_1 + k_2 + k_3$ **is dominated by the pole of the integrand with the largest imaginary part**

Such pole is the pole of the inverse of the scale factor $a(\eta)^{-1}$

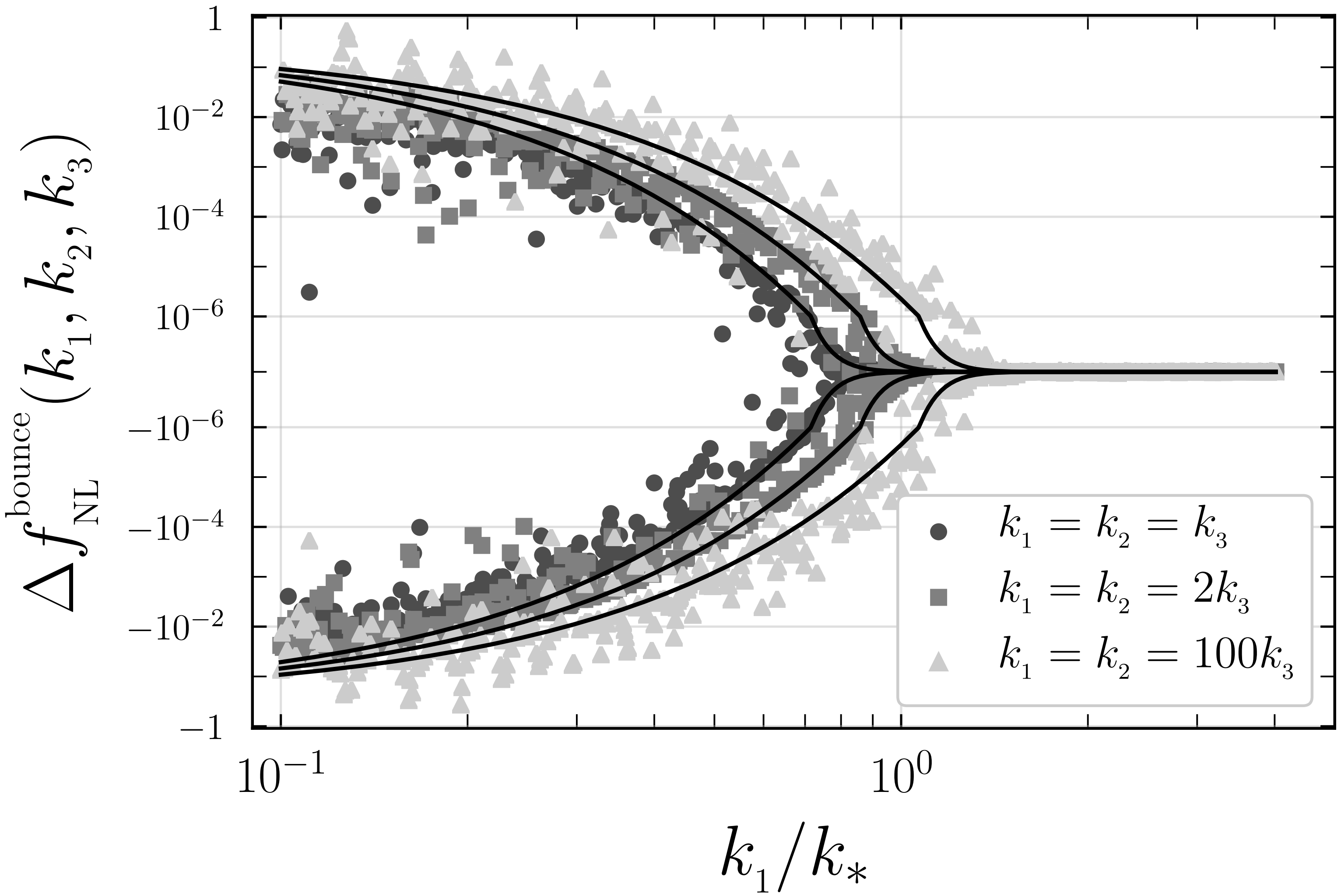
Using an analytical approx. valid near the bounce $a(t) = a_B \left(1 + \frac{1}{2} R_B \, t^2\right)^{1/6}$

The Pole at the bounce is (in conformal time) $\eta_p = i\,\sqrt{\pi/3}\,\frac{\Gamma[5/6]}{2\Gamma[4/3]}\,\frac{1}{a_B\,\sqrt{R_B/6}} = i\,\frac{\alpha}{k_{\rm LQC}}$

where: $\alpha \simeq 0.64677$ $k_{\rm LQC} = \sqrt{R_B/6}$

Therefore, the integral is estimated to behave as: $e^{-\alpha(k_1+k_2+k_3)/k_{\rm LQC}}$, when $(k_1 + k_2 + k_3) \gtrsim k_{\rm LQC}$

Comparison with numerics for different configurations:



Points: numerics

Solid line: curve $e^{-\alpha(k_1+k_2+k_3)/k_{\rm LQC}}$ **with** $\alpha \simeq 0.64677$

Additional Details

5. Stability of perturbation theory

(The results of this sections were already mentioned in the first part of the talk. Here a few more details)

We have seen that the three-point function is several orders of magnitud larger than in standard inflation.

Second order effects may be too large $\longrightarrow$ breakdown of perturbation theory

We can check this explicitly

Next to leading order contribution to the two-point function

Next-to-leading order

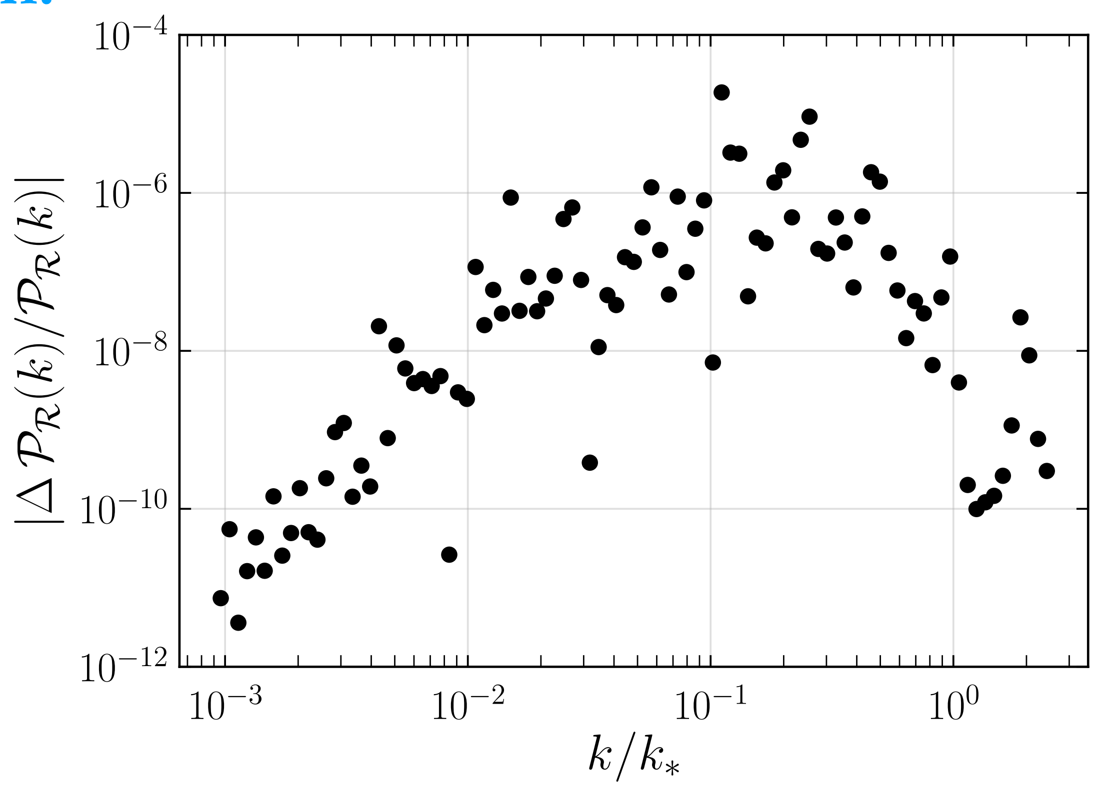
$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}|0\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)\,\frac{2\pi^2}{k_1^3}\,[\mathcal{P}_{\mathcal{R}}(k_1) + \Delta\mathcal{P}_{\mathcal{R}}(k_1)]$$

Where

$$\Delta\mathcal{P}_{\mathcal{R}}(k_1) = \hbar\,\frac{k_1^3}{\pi^2}\Bigg[\left(-\frac{a}{z}\right)^3\left[-\frac{3}{2}+3\frac{V_\phi\, a^5}{\kappa\, p_\phi\, \pi_a}+\frac{\kappa}{4}\frac{z^2}{a^2}\right]\int\frac{d^3p}{(2\pi)^3}\,B_{\delta\phi}(\vec{k}_1,\vec{p},-\vec{k}_1-\vec{p})$$
$$+\left(-\frac{a}{z}\right)^4\left[-\frac{3}{2}+3\frac{V_\phi\, a^5}{\kappa\, p_\phi\, \pi_a}+\frac{\kappa}{4}\frac{z^2}{a^2}\right]^2\int\frac{d^3p}{(2\pi)^3}\,|\varphi_p|^2\,|\varphi_{|\vec{k}_1-\vec{p}|}|^2\Bigg]+\mathcal{O}(\mathcal{H}_{\rm int}^2)$$

The three-point fnc enters here

Numerical evaluation:



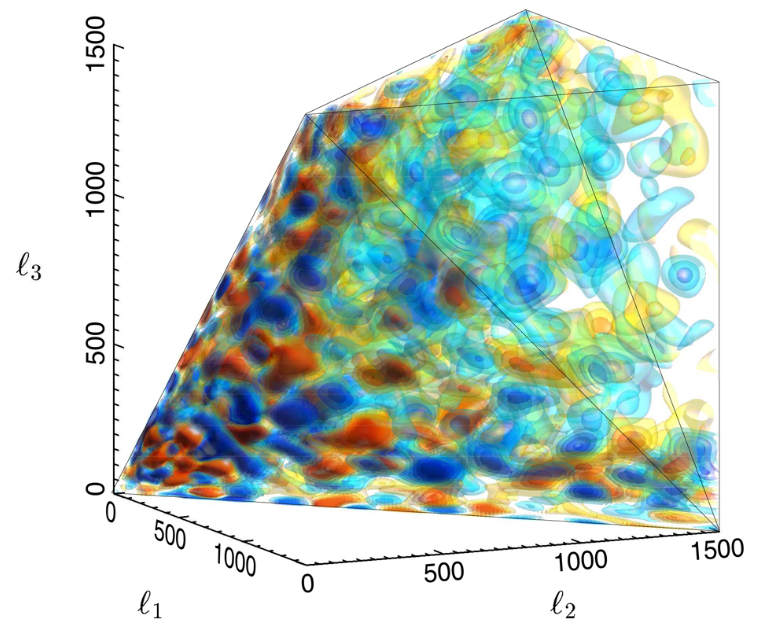
Perturbation theory is under control

Additional Details

6. Contrast with observations

The Planck collaboration reported results for non-Gaussianity in 2015

P. A. R. Ade *et al.* (Planck), "Planck 2015 results. XVII. Constraints on primordial non-Gaussianity," Astron. Astrophys. **594**, A17 (2016), arXiv:1502.01592 [astro-ph.CO].

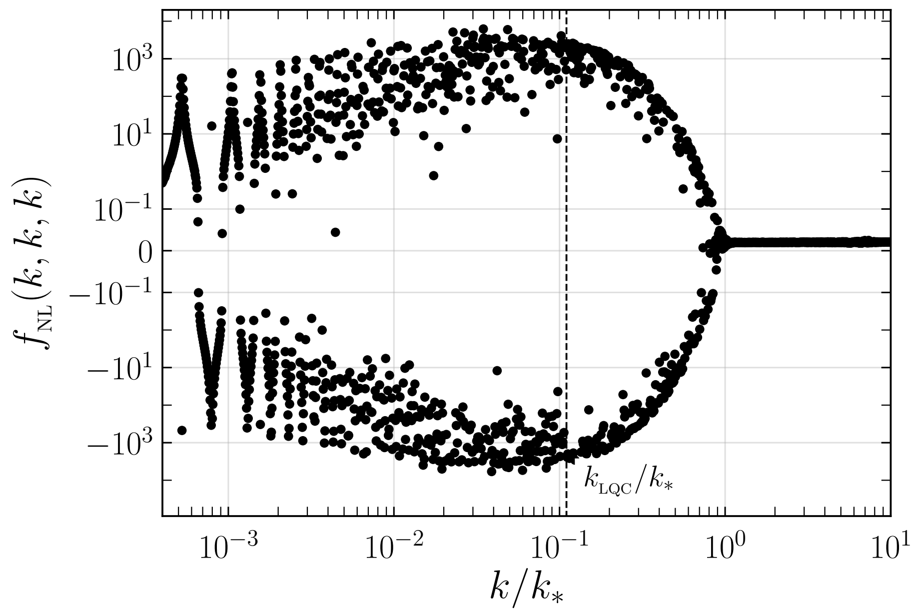


Their results constrained models in the market, and did not confirm any.

Planck sensitivity is good at large multiples:

$$|f_{NL}| \lesssim 10 \text{ for } \ell \gtrsim 1000$$

and it gets worse as for low multipoles, growing as $1/\sqrt{\ell}$



LQC-non-Gaussianity has precisely the shape needed to respect observational constraints on large multipoles (large k), and still to produce some observable effect at low multipoles

Here, we obtain an estimate of constraints on our free parameters coming from data

I will consider here the most restrictive scenario, forgetting about the oscillations in f_{NL}

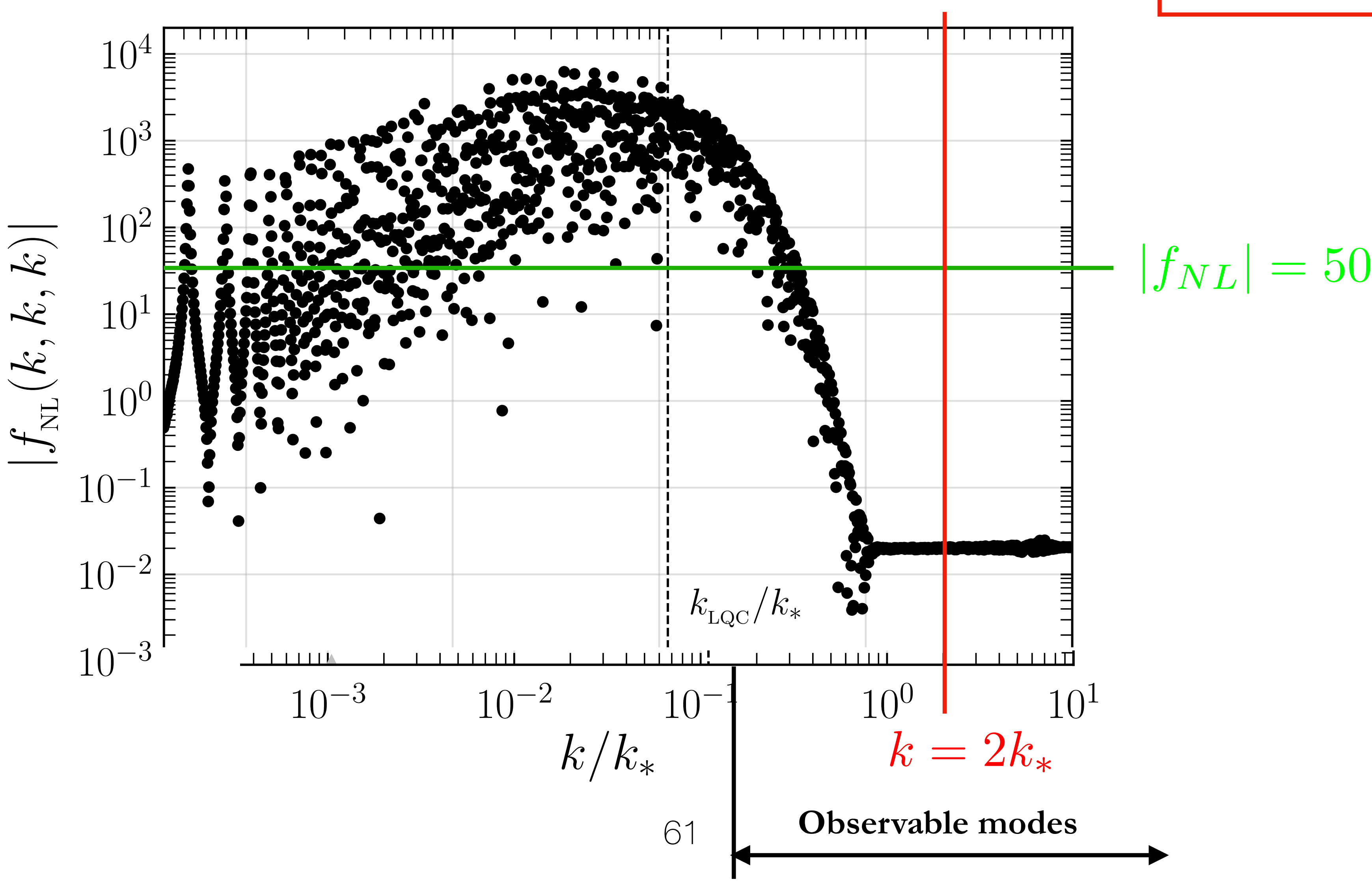
It is expected the oscillations will attenuate the effects of f_{NL} in the CMB and actual constrains may be weaker

Results from Plank imply

Upper bound: $|f_{NL}| \lesssim 50$ for $\ell \lesssim 5$ (or equivalently, $k \lesssim 2k_*$)

Ok with observations

$\phi(t_B) = 7.62\, M_{P\ell}$

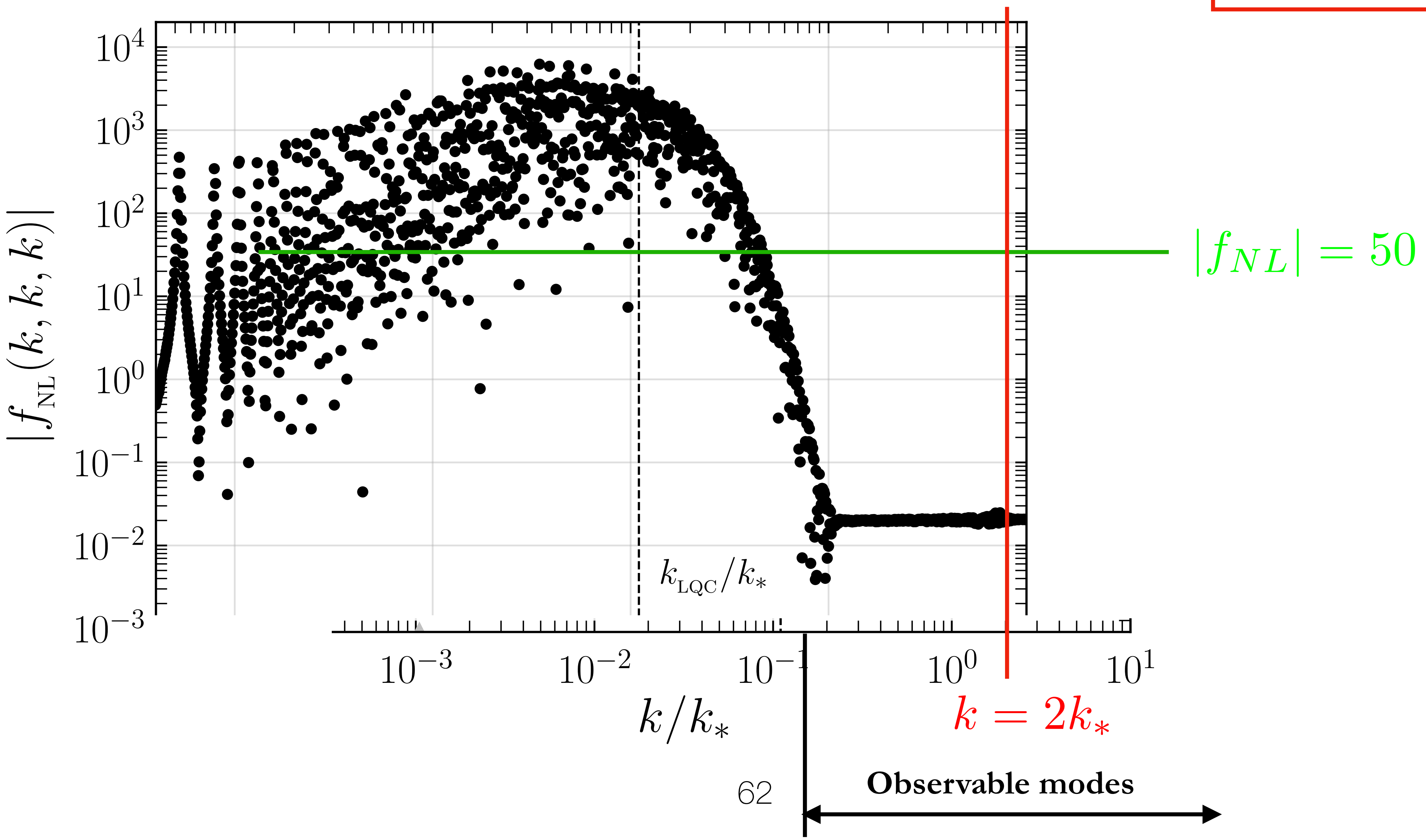


Results from Plank imply

Upper bound: $|f_{NL}| \lesssim 50$ for $\ell \lesssim 5$ (or equivalently, $k \lesssim 2k_*$)

Ok with observations

$\phi(t_B) = 7.82\, M_{P\ell}$

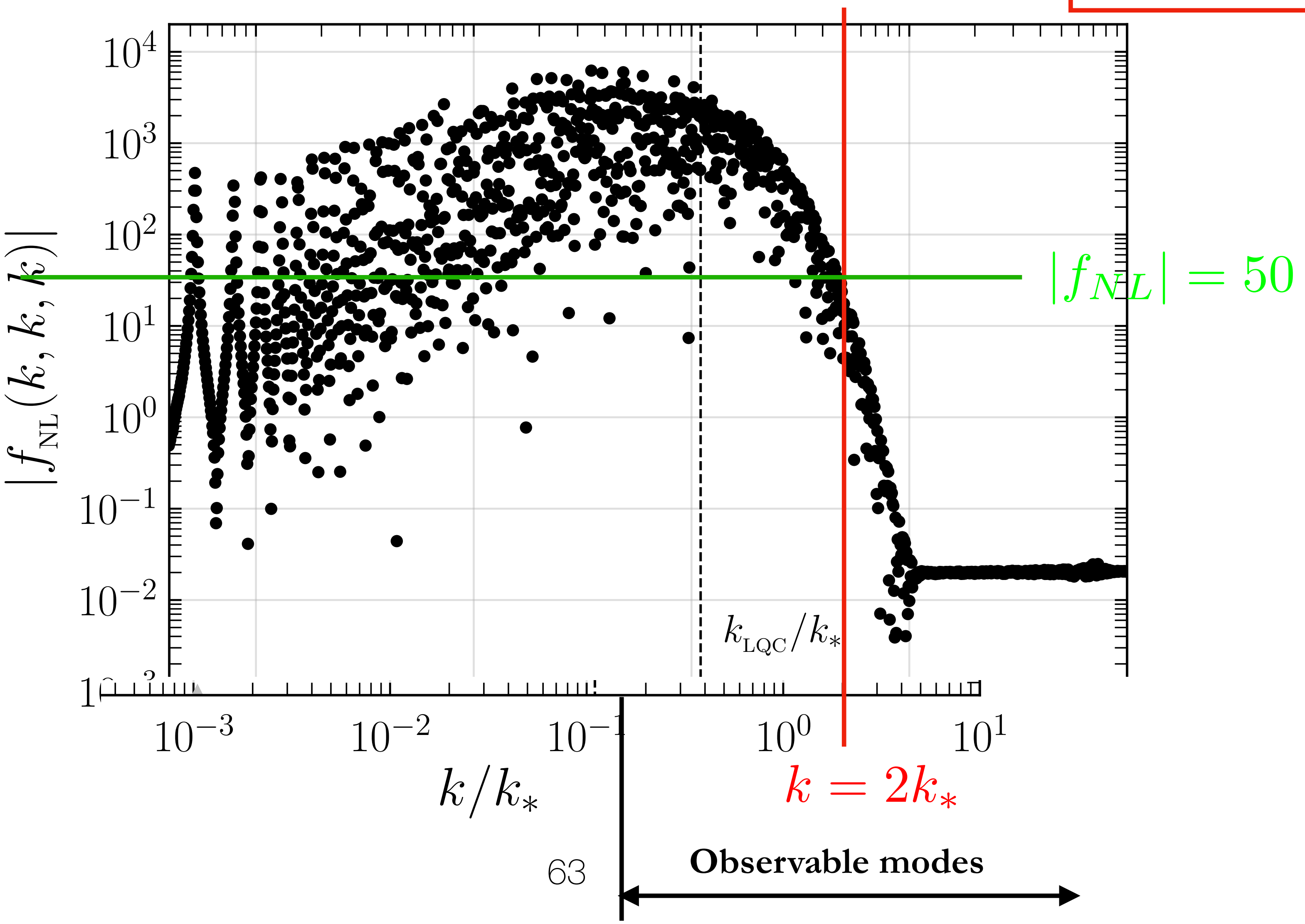


Results from Plank imply

Upper bound: $|f_{NL}| \lesssim 50$ for $\ell \lesssim 5$ (or equivalently, $k \lesssim 2k_*$)

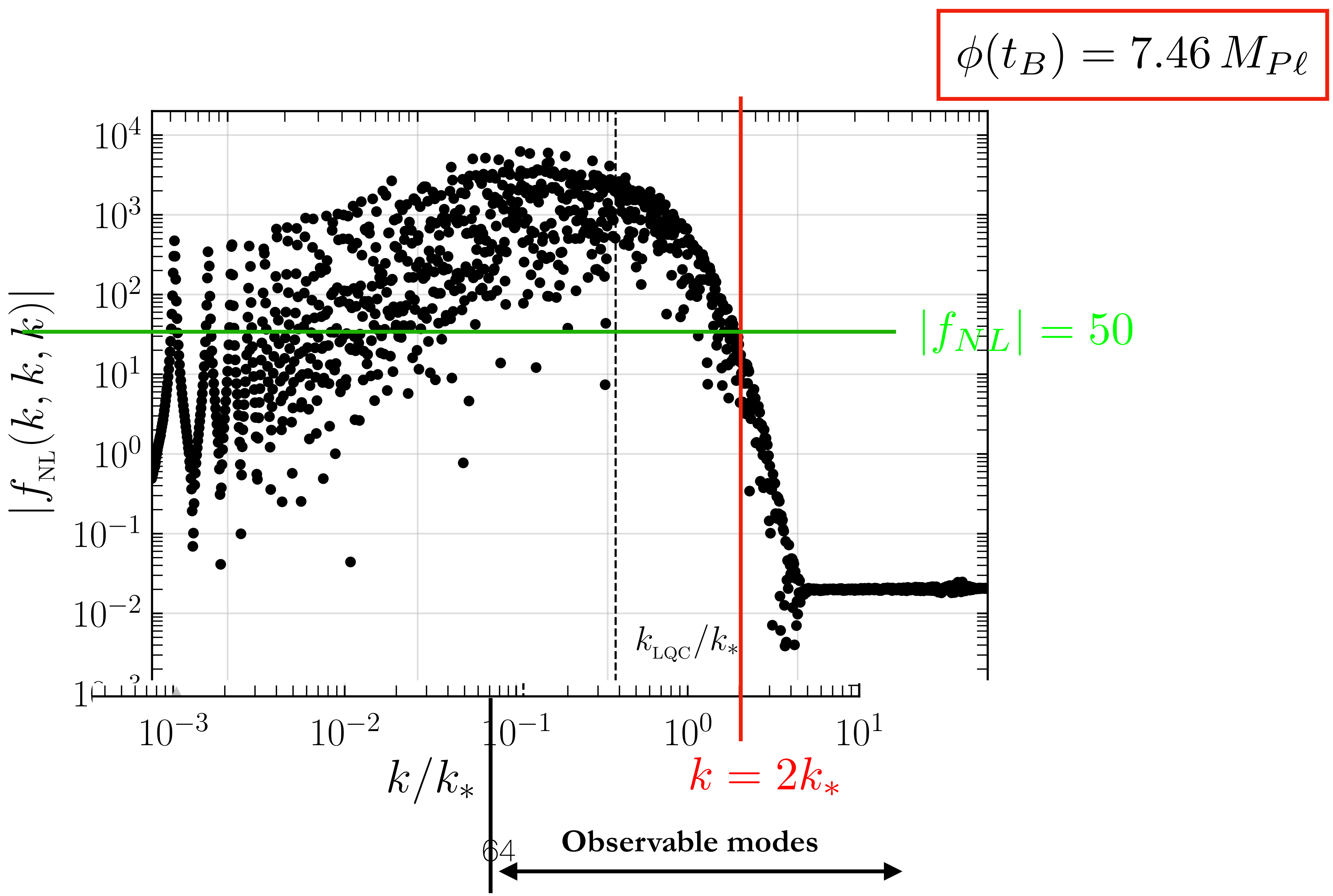
This values corresponds to Minimum $\phi(t_B)$

$\phi(t_B) = 7.46\, M_{P\ell}$

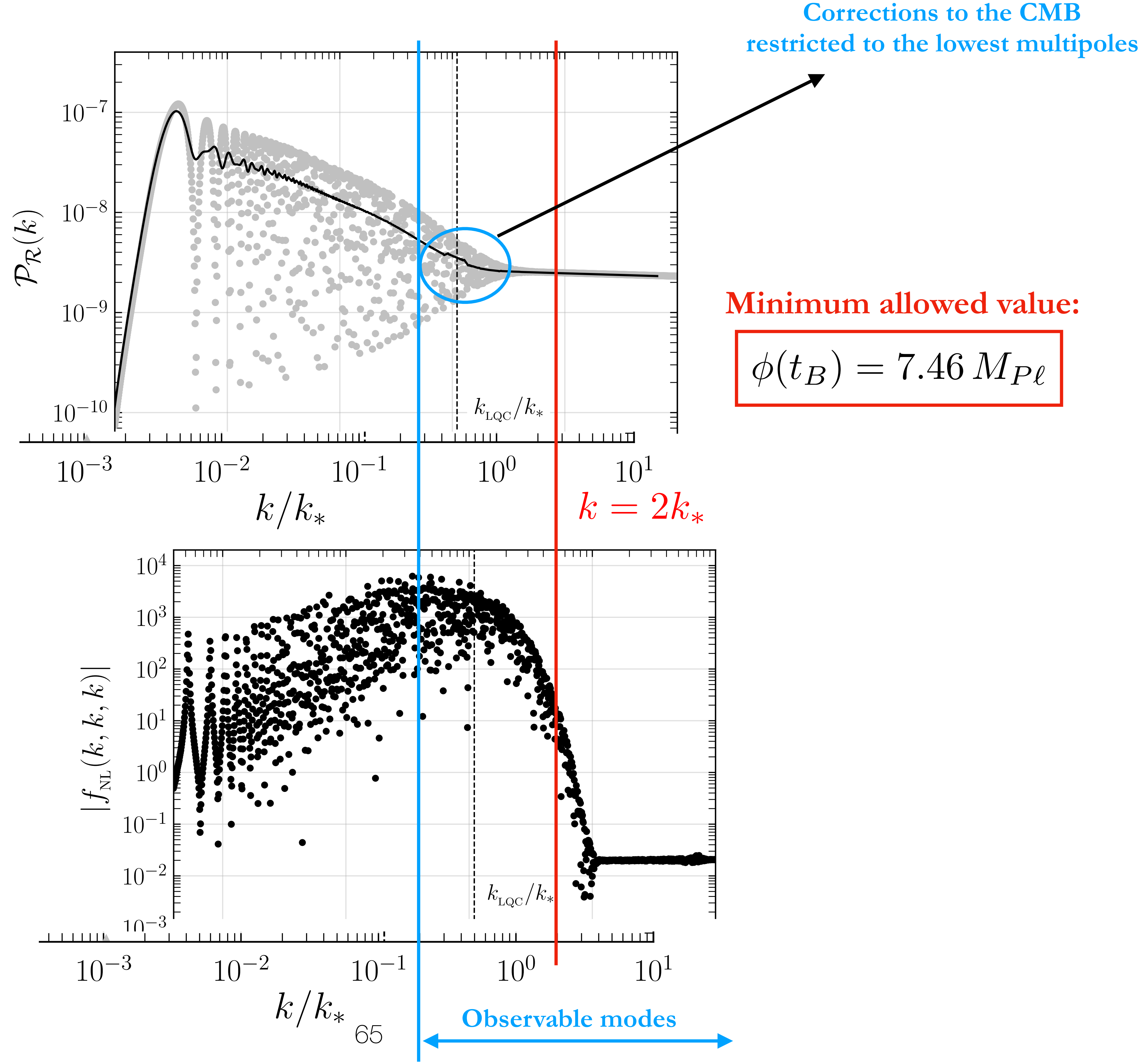


Conclussions:

- **Minimum allowed value** $\phi(t_B) = 7.46\, M_{P\ell}$
- **For values larger than** $\phi(t_B) = 7.82\, M_{P\ell}$ **scales modified by the bounce are redshifted to super-Hablle scale**



Non-Gaussianity vs Power spectrum



Conclussions:

In the worse-case scenario we just considered, the LQC corrections on the power spectrum must be restricted to $k \lesssim 1/2k_* \quad (\ell \lesssim 20)$

However, I expect the oscillatory character of f_{NL}**to weaken this constraints**

How much exactly: difficult to answer. Work in progress

Additional Details

7. Summary

Summary

(1) We have computed, for the first time, non-Gaussianity from LQC. This provides a new dimension in the study of phenomenological aspects of LQC. Also, it makes previous results more robust and complete.

(2) The LQC pre-inflationary dynamics makes the state of perturbations to be excited and non-Gaussian at the onset of inflation, relative to the Bunch-Davis vacuum

(3) The bounce introduces a new scale on the problem k_{LQC}

(4) The amplitude of f_{NL} **is enhanced for IR wave-numbers relative to** k_{LQC}

$$|f_{\rm NL}(k_1,k_2,k_3)| \propto e^{-\alpha\,(k_1+k_2+k_3)/k_{\rm LQC}}, \text{ with } \alpha \approx 0.65$$

(4) Perturbation theory remains under control

(5) $f_{NL}(k_1, k_2, k_3)$ **is highly oscillatory. This attenuates its observational effects.**

Still, the average value is significantly larger than in standard inflation.

(6) The LQC-non-Gaussianity has a particular "shape" that would help us to identify it in observations (e.g. Large Scale Structure)

(7) Observational upper bounds on non-Gaussianity impact the allowed region in the parameter space