The Gravitational Origin of the Weak Interaction's Chirality

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In Collaboration
with
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References

- S. Alexandrov
- S.A, A. Marciano, R. Altair
- A. Ashtekar
- K. Krasnov
- J. Plebanski
- Jacobson and Smolin
- S. Speziale
- Capovilla and Smolin
- G. Lisi
- L. Friedel

Some Questions

- Why is the Weak Interaction Maximally parity violating?
- Why is the standard model Chiral?
- Like gravity the weak force interacts universally with all fermions

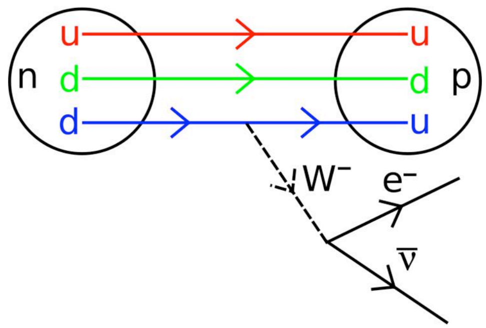
A Lesson on Chiral Gauge Theory

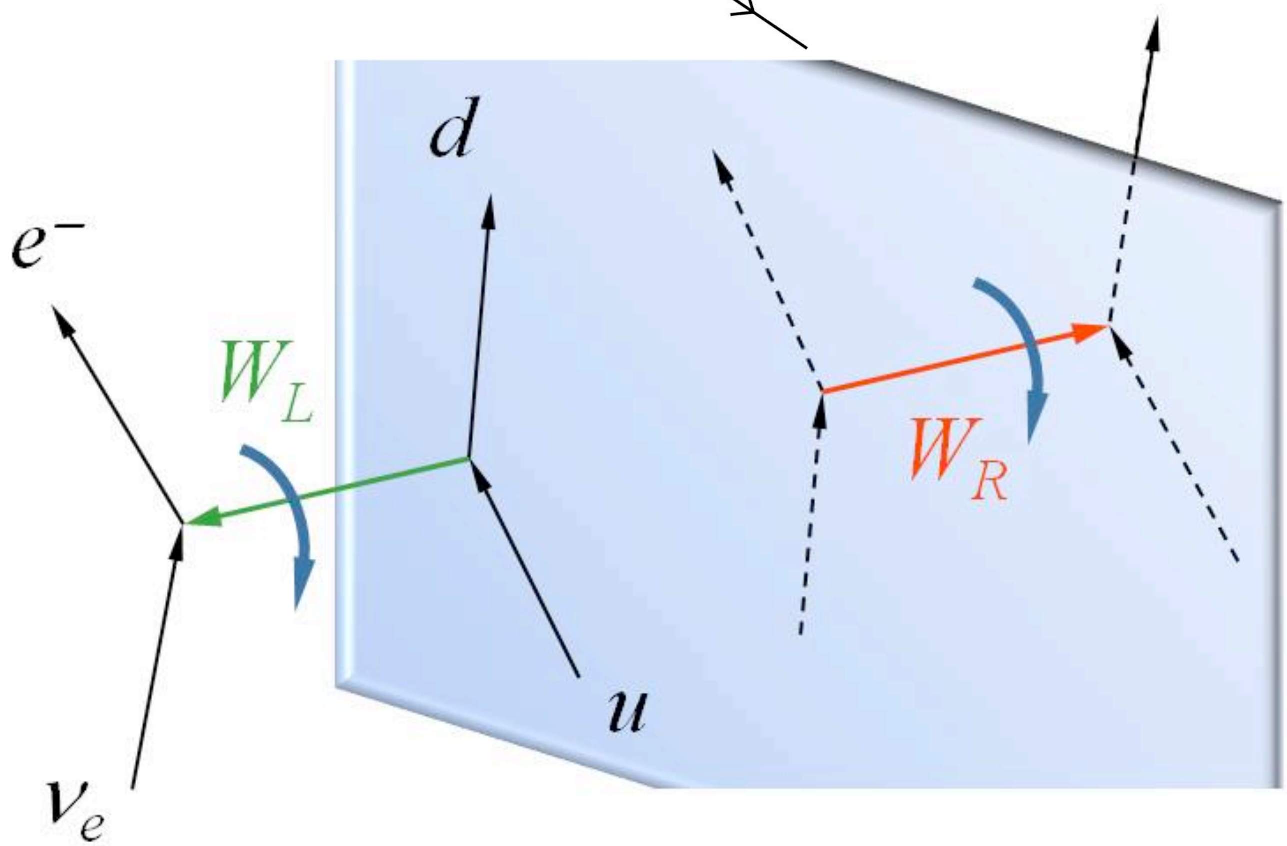
$$\mathcal{L} = -\frac{1}{4}\mathrm{F}^{\mathrm{a}}_{\mu\nu}\mathrm{F}^{\mathrm{a}\mu\nu} + \mathrm{ig}_1\mathrm{A}^{\mathrm{a}}_{\mathrm{L}}\sigma_\mu \mathrm{J}^\mu_{\mathrm{L}} + \mathrm{ig}_2\mathrm{A}^{\mathrm{a}}_{\mathrm{R}}\sigma_\mu \mathrm{J}^\mu_{\mathrm{R}}$$

But Parity Violating if couplings are not the same.

Maximal Parity violation if right handed gauge field is missing.

Beta Decay





The Lesson

Parity violation can emerge from a Chiral and Parity Symmetric Mother Theory.

But what could it be....

General Relativity and BF Theory

Consider the Following Action

$$I^{BF} = \int B^i \wedge F_i + \frac{\Lambda}{2} B^i \wedge B_i.$$

$$F^i = -\Lambda B^i, \qquad \mathcal{D} \wedge B^i = 0$$

Ansatz: $B_{AB} = e_A^{\ A'} \wedge e_{BA'}$

$$I^{JSS} = \int \epsilon_{abcd} \left(e^a \wedge e^b \wedge F^{+cd} + \frac{\Lambda}{2} e^a \wedge e^b \wedge e^c \wedge e^d \right)$$

Homework:

Using $$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

$$e \wedge e \wedge e \wedge e \simeq dete$$

Show that

$$I^{JSS} = \int \epsilon_{abcd} \left(e^a \wedge e^b \wedge F^{+cd} + \frac{\Lambda}{2} e^a \wedge e^b \wedge e^c \wedge e^d \right)$$

Is equivalent to Einstein-Hilbert with a Cosmological Constant

Isogravity

Nesti, Percacci J.Phys (07), Alexander hep-th 0706.4481

- Idea: View Gravity as a gauge theory with a connection valued in

$$SO(3,1;C) = \mathrm{SL}(2,\mathbb{C})_L \times \mathrm{SL}(2,\mathbb{C})_R$$

- This is realized by the fact that the Complex Ashtekar-Sen variable is Chiral.

Mechanism

- Treat one SL(2,C) as the connection for gravity (Jacobson, Smolin '84)
- The other SL(2,C) connection as the weak interaction
- Mother Theory is parity invariant
- Q: Is there a parity violating sector (solutions) and what are they?

Mother Theory

Mother Theory:
Extended Plebanski

Krasnov, Smolin

$$S = \int \frac{1}{8\pi G}\left\{\varepsilon_{abcd}\, B^{ab}\wedge F^{cd} - \frac{1}{2}\Psi_{abcd}\, B^{ab}\wedge B^{cd}\right\} + \left(\frac{\Lambda}{16\pi G} - \frac{g^2}{2}\Psi^2_{abcd}\right)\varepsilon_{efgh}\, B^{ef}\wedge B^{gh} + \frac{\alpha}{2}\varepsilon_{abcd}\, F^{ab}\wedge F^{cd}$$

- The Theory is Parity Symmetric, which was shown to be a Bi-Metric Theory with a ghost (instability)
- We found a new Parity violating sector that is stable and has a self-consistent perturbative expansion.

Rewrite Action with Spinorial Indices

$A, B = 0, 1$ Left Connection

$A', B' = 0', 1'$ Right Connection

$$A^{ab} = A^{AA'BB'} = \varepsilon^{AB} A^{A'B'} + A^{AB} \varepsilon^{A'B'}$$

$$\begin{aligned} S \;=\; & \int \frac{\imath}{4\pi G} \Big\{ B^{AB} \wedge F_{AB} - B^{A'B'} \wedge F_{A'B'} + \frac{\lambda}{6G} (B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'}) \\ & - \frac{1}{2} \Psi_{ABCD} B^{(AB} \wedge B^{CD)} + \frac{1}{2} \Psi_{A'B'C'D'} B^{(A'B'} \wedge B^{C'D')} - \Psi_{A'B'AB} B^{A'B'} \wedge B^{AB} \Big\} \\ & + \frac{\imath g^2}{2} (\Psi^2_{ABCD} + \Psi^2_{A'B'C'D'} + \Psi^2_{ABA'B'})(B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'}) \,, \end{aligned}$$

Equations of Motion

$$
\begin{aligned}
F_{AB} &= \Psi_{ABCD}\, B^{CD} + \Psi_{ABA'B'}\, B^{A'B'} - \left(\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2\right) B_{AB}\,, \\
F_{A'B'} &= \Psi_{A'B'C'D'}\, B^{C'D'} - \Psi_{A'B'AB}\, B^{AB} + \left(-\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2\right)\, B_{A'B'}
\end{aligned}
$$

$$
\begin{aligned}
\Psi_{ABCD} &= \frac{1}{8\pi G g^2 W}\, B_{(AB} \wedge B_{CD)} \\
\Psi_{A'B'C'D'} &= -\frac{1}{8\pi G g^2 W}\, B_{(A'B'} \wedge B_{C'D')} \\
\Psi_{ABA'B'} &= \frac{1}{4\pi G g^2 W}\, B_{AB} \wedge B_{A'B'}
\end{aligned}
$$

$$\mathcal{D} \wedge B_{AB} = \mathcal{D}' \wedge B_{A'B'} = 0$$

Symmetric Solution

- Speziale showed that this theory is a bi-metric theory with 8 DOF:
  two spin-2 fields, one ghost scalar
  (8=2+5+1)

Can be obtained by expanding B in g

$$B_{AB} = B^{(0)}_{AB} + g^2 b_{AB}\,,$$
$$B_{A'B'} = B^{(0)}_{A'B'} + g^2 b_{A'B'}$$

And solving EOM order by order in g

Parity Breaking Solution

This phase instead has one graviton
and triplet of SU(2) gauge fields

- When coupling constant g is small.
- The Right Handed Field Strength is dominated by the Cosmological Constant

$$\lambda\, g^2 = \xi.$$

Do we get a self
consistent solution?

$$F_{A'B'} \approx -B^{CD} \frac{B_{(A'B'} \wedge B_{CD)}}{4\pi G g^2 W} - \frac{\lambda}{3G} B_{A'B'} + O(g^2)$$

$$W = 24\,\imath\, e + O(g^2)$$

$$B_{A'B'} = -\pi G g^2 \left(\delta_\xi \mathbb{1} + \gamma_\xi \star\right) F_{A'B'} + g^6 b_{A'B'}$$

$$\delta_\xi = \left(\frac{1}{16} + \frac{\xi}{3}\right) \frac{1}{\left(\left(\frac{1}{16} + \frac{\xi}{3}\right)^2 - \left(\frac{3}{128}\right)^2\right)} \qquad \text{and} \qquad \gamma_\xi = -\frac{3\,\delta_\xi}{128\left(\frac{1}{16} + \frac{\xi}{3}\right)}$$

The Action II

$$S = \int \frac{\imath}{4\pi G}\left\{B^{AB}\wedge F_{AB} - B^{A'B'}\wedge F_{A'B'} + \frac{\lambda}{6G}(B_{AB}\wedge B^{AB} - B_{A'B'}\wedge B^{A'B'})\right\}$$
$$+\frac{81\,\imath}{128\pi^2 G^2 g^2 W}\Big((B_{AB}\wedge B_{CD})^2 + (B_{A'B'}\wedge B_{C'D'})^2 - 4(B_{AB}\wedge B_{A'B'})^2\Big)\,.$$

$$B^{AB} = \Sigma^{AB} + g^2 b^{AB}$$

$$S = S^{(0)}(e^{AA'}, A_{AB}, A_{A'B'}) + S^{(1)}(b_{AB}, b_{A'B'}, e^{AA'}, A_{AB}, A_{A'B'})$$

We arrive at the leading order action

$$
\begin{aligned}
S^{(0)} \quad = \quad & \int \frac{\imath}{4\pi G}\,\Sigma^{AB} \wedge F_{AB} + \frac{\lambda}{12\pi G^2}\,e \\
& -\frac{e}{4g_{YM}^2} F_{\mu\nu}^{A'B'} F_{A'B'\rho\sigma} g^{\mu\rho} g^{\nu\sigma} - \imath\,\Theta\,F^{A'B'} \wedge F_{A'B'} \\
& +\frac{9G^2}{(16\pi)^2\lambda^2 e}\left(F_{(A'B'} \wedge F_{C'D')}\right)^2 .
\end{aligned}
$$

$$
-\frac{1}{4g_{YM}^2} = g^2\left[\delta_\xi\,\gamma_\xi\left(\xi\frac{\pi^2}{3} - \frac{1}{64} - 74\right) + \gamma_\xi\right]
$$

while the Θ angle is

$$
\Theta = g^2\left[(\delta_\xi^2 + \gamma_\xi^2)\left(\xi\frac{\pi^2}{6} - \frac{1}{128} - 37\right) + \delta_\xi\right]
$$

Leptonic Coupling

Spin index

Isospin index

SU(2)

SU(2)

$$\psi_L{}^{ab}$$

$$\psi^a \otimes \psi^b = \psi^{ab}$$

$$\psi_L = \begin{pmatrix} \nu_L^1 & e_L^1 \\ \nu_L^2 & e_L^2 \end{pmatrix}$$

$$\nu_L{}^a = \psi_L{}^{a1}, \qquad e_L{}^a = \psi_L{}^{a2}$$

Matter Coupling

$$S_L^{\mathrm{Dirac}} = \int B^{AB} \wedge \rho_A \wedge (\mathcal{D}\lambda)_B + \tau_{ABC} \wedge B^{(AB} \wedge \rho^{C)}$$

Variation w.r.t Lagrange Multiplier

$$\Sigma^{(AB} \wedge \rho^{C)} = 0 \Longrightarrow \rho^A = e^{AA'} \bar{\lambda}_{A'}$$

$$S_L^{\mathrm{Dirac}} = \int \bar{\lambda}_{A'} e_A^{A'} \wedge \Sigma^{AB} \wedge (\mathcal{D}\lambda)_B$$

Matter Coupling II

By Symmetry Right Handed Fermions:

$$S_R^{\text{Dirac}} = \int B^{A'B'} \wedge \rho'_A \wedge (\mathcal{D}\lambda)'_B + \tau_{A'B'C'} \wedge B^{(A'B'} \wedge \rho^{C')}$$

Unlike the unprimed-Leptons

$$\Sigma^{(AB} \wedge \rho^{C)} = 0$$

The primed fermions obey

$$F^{(A'B'} \wedge \rho^{C')} = 0$$

Which has no simple general solution

Reality Conditions

Initially we regard all fields as complex (for the lorentzian case), and then specify reality conditions which are to be imposed on the solutions of the equations of motion.

Step I

We differentiate the right and left two forms as

$$B^{A'B'} = B^{L\,i}\,\sigma_i^{A'B'}, \quad B^{AB} = B^{R\,i}\,\sigma_i^{AB}$$

Step II

We then use these to define the left and right Urbantke metrics

$$\tilde{g}^R_{ab} = B^{Ri}_{ac} B^{Rj}_{bd} B^{Rk}_{ef} \varepsilon_{ijk} \epsilon^{bdef}\,,$$

$$\tilde{g}^L_{ab} = B^{Li}_{ac} B^{Lj}_{bd} B^{Lk}_{ef} \varepsilon_{ijk} \epsilon^{bdef}\,.$$

In either case the correct reality conditions are

$$\tilde{g}^L_{ab} = (\tilde{g}^L_{ab})^*\,,$$

$$\tilde{g}^R_{ab} = (\tilde{g}^R_{ab})^*\,.$$

In the symmetric case this tells us that both left and right handed metrics are real, whereas in the asymmetric solution we learn that $\tilde{g}^R_{ab}$ is real and the Yang-Mills connection ω^i_a is real and hence in $SU(2)$.

These can be implemented by adding these reality conditions to the action so they become equations of motion which arise by varying new Lagrange multipliers $\lambda^{ab}_{L,R}$:

$$
\begin{aligned}
S^{wrc} \; = \; & \int \frac{\imath}{4\pi G} \Big\{ B^{AB} \wedge F_{AB} - B^{A'B'} \wedge F_{A'B'} + \frac{\lambda}{6G}(B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'}) \\
& -\frac{1}{2}\Psi_{ABCD} B^{(AB} \wedge B^{CD)} + \frac{1}{2}\Psi_{A'B'C'D'} B^{(A'B'} \wedge B^{C'D')} - \Psi_{A'B'AB} B^{A'B'} \wedge B^{AB} \Big\} \\
& + \frac{\imath g^2}{2}(\Psi^2_{ABCD} + \Psi^2_{A'B'C'D'} + \Psi^2_{ABA'B'})(B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'}) \\
& + \lambda^{ab}_R \Big(\tilde{g}^R_{ab} - (\tilde{g}^R_{ab})^*\Big) + \lambda^{ab}_L \Big(\tilde{g}^L_{ab} - (\tilde{g}^L_{ab})^*\Big)
\end{aligned}
$$

EOM for B is modified

$$
F_{AB} = \Psi_{ABCD}\, B^{CD} + \Psi_{ABA'B'}\, B^{A'B'} - (\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2)\, B_{AB} + 4\pi \imath\, G\, \lambda^{ef}_R \,\frac{\delta \tilde{g}^R_{ef}}{\delta B^{AB}}\, .
$$

But the new term vanishes because the equation of motion for B^{AB*} yields

$$
\lambda^{ef}_R \frac{\delta \tilde{g}^{R*}_{ef}}{\delta B^{AB*}} = 0\, ,
$$

which implies that λ^{ab}_R vanishes. Meanwhile, variation of λ^{ab}_R enforces the reality of $\tilde{g}^R_{ef}$

New Prediction

Left handed spinor transforms like
a weak doublet scalar, what entity carries this quantum number?

Right handed spinor transforms like a weak singlet spin 1/2 particle, what is this?

These two particles transform into each other under parity

New Interactions Dark Hypercharge

$$
\begin{aligned}
S_{(0)}^{U(1)_{\mathbb{C}}} \;=\;& \frac{e}{4g_{YM}^2}(f_{\mu\nu}f_{\rho\sigma} + f'_{\mu\nu}f'_{\rho\sigma})g^{\mu\rho}g^{\nu\sigma} + \Theta(f\wedge f + f'\wedge f') + \\
&+\frac{9\,g^2\,G^2}{256\,\xi^2 e}\Big((\tilde{f}\wedge\tilde{f})^2 + (\tilde{f}'\wedge\tilde{f}')^2 + (\tilde{f}\wedge\tilde{f}')^2 - 4(\tilde{f}\wedge F_{AB})^2 + \\
&-4(\tilde{f}\wedge F_{A'B'})^2 - 4(\tilde{f}'\wedge F_{AB})^2 - 4(\tilde{f}'\wedge F_{A'B'})^2\Big)\,,
\end{aligned}
$$

Novel Features

- the two $U(1)$ factors have the same Yang-Mills coupling constant as the $SU(2)_L$ factor, so there is coupling constant unification;
- however they will couple differently to matter
- there is a universal four point coupling of vector potentials of the form $(F \wedge F)^2$ which has a universal coupling

Conclusion

- Chirality and Parity Violation in EW Theory arises from its other gravitational "hand"
- Quantum Numbers of Higgs and Sterile neutrino arise naturally.
- We expect new predictions for upcoming LHC (or existing LHC experiments)
- The Cosmological constant plays an important role in parity violation.
- What are the cosmological consequences of this modified gravity theory?
- What is the indentity of the Auxillary fields? (S.A, E. Livine, A. Marciano)