Loop Quantum Gravity: Frequently Asked Questions

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- Origin of this Talk:

Organizers of the 11th Marcel Grossmann Conference in Berlin asked me to summarize some recent advances in LQG and address some concerns raised by ‘outsiders’. I accepted after some hesitation because everyone I consulted said it is important to have our view-point recorded in an official setting. I received a lot of input from colleagues in Canada, France, Germany, Mexico and the US. It was the least pleasant talk I have given but the exercise was useful. Jorge suggested that I give an expanded version of the FAQs part as an ILQG seminar since very few us were at MG11.

- Goal:

To get feed back from our community to sharpen and improve our responses to FAQs. As in any large and healthy community, different people have somewhat different viewpoints. But I hope this talk will serve as a vehicle to develop a broad consensus on the successes and challenges of LQG. I will stop at several points during the talk and ask for comments and questions so we can have an active dialog.

1. Quantum Corrections

- Shouldn't Einstein's equations get quantum corrections?

Yes. And they do receive quantum corrections in LQG.
Consider QED. There one begins with the classical Maxwell-Dirac action and then proceeds with quantization. One does not argue that the classical action must be modified because of quantum corrections. Yet, the effective action —which is meant to incorporate all quantum effects— has all sorts of additional terms representing quantum corrections.

The viewpoint is similar with Einstein-matter actions in LQG. The classical theory one 'quantizes' is general relativity (or supergravity). But Einstein equations do receive quantum corrections. In fact, one would expect the effective action to exhibit Planck scale *non-localities*.

We already know the leading corrections in quantum cosmology. The Friedmann equations $(\dot{a}/a)^2 = 8\pi G/3$ is replaced by $(\dot{a}/a)^2 = (8\pi G/3)[1 - \rho_{\rm matter}/\rho_{\rm crit}]$ where $\rho_{\rm crit} = 3/16\pi^2\gamma^3 G^2\hbar \approx 0.8\rho_{\rm Pl}$. The precise form of this quantum correction leads to profound departures from general relativity in the Planck regime.

2. Ambiguities in the Hamiltonian Constraint

- Isn't there a large number of ambiguities in dynamics of LQG?

Yes. Indeed, these have been well known to the LQG community for quite some time, pointed out in many reviews (e.g. by AA & Lewandowski). This is precisely the incompleteness of the current status of LQG.

Some of these correspond to factor ordering ambiguities which are present also in ordinary quantum mechanics. They are partially reduced by the requirement that the constraint should be self-adjoint so one can pass to the physical Hilbert space by group averaging. In quantum cosmology, explicit calculations have shown that the remaining factor ordering freedom does not change the qualitative predictions of the theory —-only some minor details change. Therefore there is reason to hope that the situation would be similar in the full theory.

There is a more significant ambiguity associated with the choice of representation j associated with the new edge that is added to the Hamiltonian constraint. This does change the constraint qualitatively. In quantum cosmology, one gets a higher order difference equation (Vandersloot). But Perez has given arguments to the effect that we should only use the fundamental representation $j = 1/2$.

Finally there are ambiguities associated with the choice of triangulations and treatment of the 'non-Euclidean part of the constraint'. At present these are not controlled and require more work. In my view, fresh approaches involving gauge fixing of the Gauss and Diffeomorphism constraints, derivations from spin-foams and discretization schemes a la Gambini-Pullin have to be pursued aggressively.

• In the current approach, the constraint algebra can not be verified in quantum theory. Is this not a fatal drawback?

No. Logically there is nothing wrong: i) first imposing the Gauss constraint; ii) defining the Diff constraint only the Hilbert space of gauge invariant states and solving it; and iii) then defining the Hamiltonian constraint on the Hilbert space of gauge and Diff invariant states and then solving it. If this procedure leads to a theory with a rich semi-classical sector, it would be a viable physical theory. (2+1 theory)
However, in practice, to resolve ambiguities and have a better chance at a viable theory, requiring satisfaction of the constraint algebra could be very helpful. But this is not essential logically.

• Aren't these ambiguities just a reflection of the ambiguities associated with perturbative non-renormalizability of Einstein gravity?

No! The two have entirely different origins: Incompleteness of our current understanding *versus* Inadequacy of the Gaussian fixed point.

⋆ 2+1 gravity is also power counting renormalizable but exactly and unambiguously soluble in LQG. The Madrid group has compared and contrasted the perturbative and non-perturbative treatments for 2+1 gravity coupled to a scalar field. This needs to be more widely understood.

⋆ There exist perturbatively non-renormalizable but exactly soluble models, e.g., GN(3). It admits a non-Gaussian fixed point (NGFP). Initially it was thought that the correlation functions will not be tempered distributions, i.e., will be worse behaved than those in perturbative field theories, reflecting the perturbative non-renormalizability. This turned out not to be the case!

⋆ By now there is significant evidence that Einstein gravity also admits a NGFP (Reuter, Luscher, Buonanno, Saureressig, Percacci, Perini, ...). Furthermore there are some qualitative similarities between their findings and spin-foams/LQG. In particular, the effective space-time dimension at the Planck scale appears to be two.

3. Assessing Progress

- If a definite Hamiltonian constraint has not emerged yet, could one not say that there has been little progress since the Wheeler-DeWitt geometrodynamics?

It is clarifying to compare the situation in string theory. There, the perturbation series is known to diverge —furthermore it does so uncontrollably because it is not even Borel summable. So, to any physical question —such as the graviton-graviton scattering amplitude— the full answer in perturbative string theory is infinite. Now, one may say that this also happens in QED. But there we know the theory is incomplete and we should not trust its predictions beyond (at most) first 137 terms in perturbation theory. But a theory which claims to be complete can not take that refuge. So, the standard belief is that we should not trust perturbation theory. Non perturbative effects are crucial.
This was realized almost two decades ago but we still only have a skeleton of the candidate, non-perturbative M theory. So, the analogous question in string theory would be: could one not say that there has been little progress since supergravity?

I think that in both cases there is considerable incompleteness and diversity of ideas. But also considerable progress: Consistent frameworks have emerged and special cases have been well-understood.

- LQG Examples:

i) A strong uniqueness theorem of the kinematical framework has emerged;
ii) A good understanding of the geometry of quantum horizons in equilibrium and a statistical mechanical derivation of entropy of black holes;
iii) Spin-foam models have led to a fertile approach to obtaining the graviton propagator, effective low energy theories *and* probing non-perturbative aspects of YM theories ; and,
iv) Dynamical ideas *in detail* in mini-superspaces and found physically desirable results, including singularity resolution.

4. Spin foams, BH Entropy and LQC

• Much of the work in spin-foams is carried out in the Riemannian context which has no simple relation to the physical Lorentzian sector. Why is it interesting?

• Because it addresses long standing problems in a background independent fashion. Interesting mathematical physics like ADS/CFT which assumes unphysical boundary conditions. Furthermore, unlike ADS/CFT, the Riemannian spin-foams directly suggest strategies for the physically interesting sector. Formally, things go through at a level of rigor often assumed in particle physics.
Other criticisms relating to spin foams were beautifully addressed by Laurent Friedel in
http://www.math.columbia.edu/ voit/wordpress/?p=330
see two January 23rd entries under < L says: > towards the end.

• In the black hole entropy calculation, what is the justification of assuming the Boltzmann statistics for punctures?

Misconception! No such assumption. One just counts the states of the quantum horizon geometry. Difference between a systematic derivation *versus* heuristic re-interpretation of the calculation.

- Isn't LQC too restrictive because of the huge symmetry reduction?

⋆ Absolutely! Inclusion of inhomogeneities is crucial. Focal point of significant current research.

However:
⋆ In classical general relativity, the BKL conjecture has received considerable support. Says that near space-like singularities spatial derivatives become negligible; homogeneity better and better approximation. So, homogeneous situations may provide an excellent handle over quantum geometry near space-like singularities.

⋆ Good models to provide intuition and guidance. For example, recent detailed analytical/numerical studies have provided new insights for constructing the physical sector of the theory, forced us to abandon naive dynamics, and revealed interesting physics in the Planck domain. Similarity with checking ADS/CFT in symmetric situations (e.g. the Penrose limit).

5. General Question

- Are LQC and string theory two versions of the same theory *or* are they mutually incompatible?

Personal answer: Neither! Certainly not like the Pauli (operator) and Schrödinger (wave-function) treatments of the Hydrogen atom. Both LQC and String theory are very incomplete and they have very different basic assumptions. However, the final theory may well have elements of both: Background independence and quantum geometry ideas from LQG and unification strategies and perturbative techniques from string theory.

Misconceptions can arise because not only do the two approaches have different starting points but they use very different languages making translations non-trivial. But this is variety is also very good. For, as Feynman advised us:

> *"It is very important that we do not all follow the same fashion... It's necessary to increase the amount of variety .... the only way to do it is to implore you few guys to take a risk ..."*