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# On matter fields in loop quantum gravity

## revisiting r-Fock representations & shadow states


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# Loop Quantum Gravity + Matter

- Gravity and matter are intertwined
- Quantum matter fields need (eventually) quantum gravity
  - Could the converse statement be also true?
- Could quantum gravity restrict the matter content?
- Emergent regime of quantum matter fields on fixed classical background
  - Treatment of symmetries
  - Coarse-graining of gravity coupled to matter

# Loop Quantum Gravity + Matter

| Scalar field | Gauge field | Fermions |
|---|---|---|
| Point holonomies<br>$h_v \ , \ \pi_\Sigma$ | Holonomies<br>$h_e \ , \ E_S$ | Half-densities<br>$\theta_v \ , \ \overline{\theta}_v$ |
| Cylindrical functions<br>(vertex sets / graphs + colors) | | |
| Hilbert space + quantum dynamics | | |

What are the physically relevant (classes of) states?

# Quantum states for gravity + matter

## Complexifier coherent states
[Thiemann, Sahlmann, Giesel, ...]

- Complexifier C
- $\hat{g}_e(m) := e^{-\hat{C}_\Gamma} h_e e^{\hat{C}_\Gamma}$
- $\hat{g}_e(m)\Psi_{\Gamma,m} = g_e(m)\Psi_{\Gamma,m}$

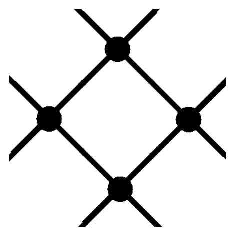
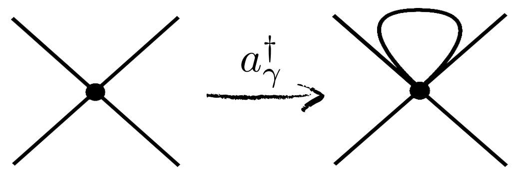
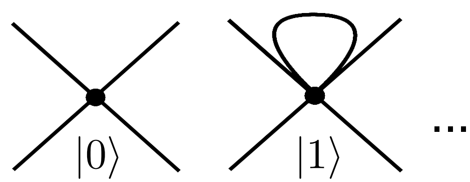
## Graph coherent states
[M.A.]

- Graph change
- Annihilation op. $\hat{a}_\gamma$
- $\hat{a}_\gamma \Psi^z_{\{\Gamma(o)\}} = z\Psi^z_{\{\Gamma(o)\}}$

## r-Fock shadow states
[Varadarajan, Ashtekar, Lewandowski, Thiemann, Sahlmann]

- r-Fock representations
- Fock states mapped to Cyl$^*$
- Projecting states in Cyl$^*$ onto separable sub-Hilbert spaces

# Quantum states for gravity + matter

| Complexifier Coherent St. | Graph Coherent. St. | r-Fock shadow states |
|---|---|---|
| $\hat{C}_\Gamma \propto \sum_{e\in\Gamma} X^i_e X^i_e$<br><br>$\Psi_m = \prod_{e\in\Gamma} \psi_{g_e(m)}$<br>$\psi_{g_e(m)} = \sum_\rho d_\rho e^{-\alpha^2 \lambda_\rho} \chi_\rho(g_e h_e^{-1})$ | <br>$\mathcal{V}_\gamma a_\gamma = \mathrm{Tr}^{(\nu)}\left[h_\gamma\right]^\dagger$<br>$a^\dagger_\gamma \mathcal{V}_\gamma = \mathrm{Tr}^{(\nu)}\left[h_\gamma\right]$<br>$\mathcal{V}_\gamma := (a_\gamma a^\dagger_\gamma)^{-1/2}$<br>$[a_\gamma, a^\dagger_\gamma] = \mathbb{1}$<br><br>$\hat{a}_\gamma \Psi^z_{\{\Gamma^{(o)}\}} = z \Psi^z_{\{\Gamma^{(o)}\}}$<br>$\vert z\rangle = e^{z a^\dagger_\gamma}\vert 0\rangle = \sum_n \frac{z^n}{\sqrt{n!}}\vert n\rangle$ | • Introduce the U(1) smeared holonomy-electric field algebra $(h^r_e, E(x))$<br>• Quantize this algebra à la Fock $(\hat{h}^r_e, \hat{E}(x))$ : r-Fock representation<br>• Correspondence: $(\hat{h}^r_e, \hat{E}(x)) \leftrightarrow (h_e, E^r)$<br>• Identify Fock coherent states as elements in Cyl*<br>$\mathcal{Z}^r_F = \sum_{\Gamma,\vec{n}} F_\Gamma[f_r, \vec{n}]\langle \mathcal{N}_{\Gamma,\vec{n}}\vert$<br>• Projections onto $\mathcal{H}_\Gamma$ |

# From Q.G.+M. to qu. fields on a fixed background

How to connect the low energy physics to the loop quantum gravity framework?

How to relate the Fock quantization of matter fields and the loop quantization?

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**R-Fock representations**

M. Varadarajan [PRD '00, '01, '02] – ***U(1)***$^n$ gauge theory

- Original motivation:
  understanding the relation between quantum states of linearized gravity and states in LQG.

- What is it:
  - r-Fock reps. are reps. for matter fields propagating on Minkowski spacetime, which connect the standard Fock reps. to the background independent loop reps.
  - measurements at a given "scale": $\exists$ r-Fock rep. $\equiv$ Fock rep.

- Role:
  - Provide new measures for the loop states.
  - Provide mappings of Fock states to the Loop space.

# r-Fock representation for abelian gauge theory

$$\{A_a(x), E^b(y)\} = \frac{1}{q}\delta_a^b \delta^{(3)}(x,y)$$

Smearing function
(Schwartz)

$$\lim_{r\to 0} f_r(x-y) = \delta^{(3)}(x,y)$$

rep. = Fock rep.

***HA$_r$*:** $(h^r, E)$

$$A_a^r(x) := \int_{R^3} d^3y f_r(x-y) A_a(y)$$

$$h_\gamma^r(A) := \exp\left[i\int_\gamma ds\ \dot\gamma^a(s) A_a^r(\gamma(s))\right]$$

"smeared holonomy"

Isomorphic

***HA*:** $(h, E_r)$

$$h_\gamma(A) := \exp\left[i\int_\gamma ds\ \dot\gamma^a(s) A_a(\gamma(s))\right]$$

$$E_r^a(x) := \int_{R^3} d^3y f_r(x-y) E^a(\vec{y})$$

The Fock rep. of ***HA$_r$*** induces the r-Fock rep. of ***HA***

# r-Fock representation for abelian gauge theory

- Fock vacuum expectation values:

r-Fock vacuum

$$\langle 0|\hat{h}^r_\gamma|0\rangle = \exp\left[-\int \frac{d^3k}{4q^2|k|}|\tilde{X}^a_{\gamma,r}(k)|^2\right] =: \langle 0_r|\hat{h}_\gamma|0_r\rangle$$

$X^a_{\gamma,r}(x) := \int_\gamma ds\dot{\gamma}^a(s)f_r(x-\gamma(s))$

r-Fock rep. = cyclic rep. generated from the r-Fock vacuum

- r-Fock measure & AL-measure:

$$G_{IJ} := \int \frac{d^3k}{|k|}\ \tilde{X}^a_{e_I,r}(k)\ \overline{\tilde{X}^a_{e_J,r}(k)}$$

$$d\mu^r_{U(1)} = \left(\sum_{\Gamma,\vec{n}} \exp\left[-\frac{1}{4q^2}\sum_{I,J} n_I n_J G_{IJ}\right]\ \overline{\mathcal{N}_{\Gamma,\vec{n}}}\right) d\mu^o_{U(1)}$$

- Fock states mapped to Cyl$^*$, in particular the vacuum st. & canonical coherent states:

$$\mathcal{Z}^r_F = \sum_{\Gamma,\vec{n}} \exp\left[-\frac{1}{q^2}\sum_I n_I \int d^3k\ \overline{\tilde{X}^a_{e_I,r}(k)} Z_a(k)\right] \exp\left[-\frac{1}{q^2}\sum_{I,J} n_I n_J G_{IJ}\right] \langle \mathcal{N}_{\Gamma,\vec{n}}|$$

# r-Fock states and shadows in abelian gauge theory

- r-Fock states:

$$\mathcal{Z}_F^r = \sum_{\Gamma,\vec{n}} \exp\left[-\sum_I n_I \int d^3k\ \overline{\tilde{X}^a_{e_I,r}(k)} Z_a(k)\right] \exp\left[-\sum_{I,J} n_I n_J G_{IJ}\right] \langle \mathcal{N}_{\Gamma,\vec{n}}|$$

  - Elements of Cyl$^*$
  - Non normalizable
  - Encode Minkowskian geometry
  - Non local spatial correlations in the coefficients

- Shadow states = projections of Fock states on separable sub-Hilbert spaces:
  exp. : fixed graph, dynamically super-selected sector, …

$$|\mathcal{Z}_F^r(\Gamma)\rangle = \sum_{\vec{n}} \exp\left[-\sum_I n_I \int d^3k\ \overline{\tilde{X}^a_{e_I,r}(k)} Z_a(k)\right] \exp\left[-\sum_{I,J} n_I n_J G_{IJ}\right] |\mathcal{N}_{\Gamma,\vec{n}}\rangle$$

  - The set of shadows on all possible graphs captures the full information in the r-Fock state
  - Elements of $\mathcal{H}_\Gamma$ : allow the analysis of the semi-classical properties of r-Fock states

# Beyond abelian gauge theory

- Shadow states - operator generalization to non-Abelian case: [A. Ashtekar, J. Lewandowski, CQG '01.]

$$|\mathcal{Z}^r_\Gamma\rangle = \sum_{\vec{n}} e^{-\frac{1}{q^2}\sum_I n_I \mathcal{Z}^r_I} e^{-\frac{1}{q^2}\sum_{I,K} n_I n_K G_{IK}} |\mathcal{N}_{\Gamma,\vec{n}}\rangle = \sum_{\vec{n}} e^{-\frac{1}{q^2}\sum_I \hat{E}_I \mathcal{Z}^r_I} e^{-\frac{1}{q^2}\sum_{I,K} \hat{E}_I \hat{E}_K G_{IK}} |\mathcal{N}_{\Gamma,\vec{n}}\rangle$$

$$\longrightarrow \qquad SU(2): \quad |\mathcal{Z}^r_\Gamma\rangle = \sum_{\vec{j}} e^{-\sum_I \hat{J}^i_I \mathcal{Z}^r_{i,I}} e^{-\sum_{I,K} \hat{J}^i_I \hat{J}^i_K G_{IK}} |\Psi_{\Gamma,\vec{j}}\rangle$$

  - Not gauge invariant: group averaging

- Shadow states v.s. complexifier coherent states: [T. Thiemann CQG '02]
  - In the abelian case: shadow states are complexifier coherent states
  - Non-abelian generalization fails

- r-Fock representation for scalar field: [A. Ashtekar, J. Lewandowski, H. Sahlmann, CQG '03.]

- r-Fock representation for fermionic field (work in progress)

- r-Fock measures for SU(N) gauge theory [M.A., J. Lewandowski '22]

- Defining the measure in the case of U(1):

$$\int d\mu^r_{U(1)}\ h_\gamma(A) := \langle 0|\hat{h}^r_\gamma|0\rangle \qquad \Rightarrow \qquad \int_{\bar{\mathcal{A}}/\bar{\mathcal{G}}} d\mu^r_{U(1)}\ \Psi(A) := \langle 0|\psi(\hat{h}^r_{\gamma_1},\ldots,\hat{h}^r_{\gamma_K})|0\rangle$$

consequence of Mandelstam identities for U(1): every U(1) cylindrical function can be expressed as a linear combination of ***Wilson loops***.

# SU(N) gauge theories

- Defining the measure in the case of U(1):

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- Defining the measure in the case of SU(N):
  - We "cannot" use smeared holonomies: gauge transformations are peculiar.
  - ***Mandelstam identities*** for SU(N) suggest that the natural generalization is in terms of Wilson loops:

$$\int d\mu^r_{SU(N)}\ W^J_{\gamma_1}(A)\ldots W^J_{\gamma_{N-1}}(A) := \langle\Omega|\hat{\mathcal{W}}^{r,J}_{\gamma_1}\ldots\hat{\mathcal{W}}^{r,J}_{\gamma_{N-1}}|\Omega\rangle$$

Vacuum of the interacting theory

$$\hat{\mathcal{W}}^{r,J}_\gamma := \mathrm{Tr}\left[\hat{h}^{r,J}_\gamma\right]$$

# r-Fock measures for SU(N) gauge theories

"Poor man's version":

$$\langle\Omega|\hat{\mathcal{W}}^{r,J}_{\gamma_1}\dots\hat{\mathcal{W}}^{r,J}_{\gamma_{N-1}}|\Omega\rangle \longrightarrow \langle 0|\hat{W}^{r,J}_{\gamma_1}\dots\hat{W}^{r,J}_{\gamma_{N-1}}|0\rangle$$

Vacuum of the interacting theory

Free theory vacuum

The r-Wilson loop operator:

$$\hat{W}^{r,J}_{\gamma} := \sum_{n=0}^{\infty}\mathrm{Tr}\left[\prod_{m=1}^{n}\tau^{J}_{i_m}\right]\mathcal{P}\!\!\int_{\gamma} ds_1\dots ds_n\prod_{m=1}^{n}\int\frac{d^3k_m}{q\sqrt{2|k_m|}}\tilde{X}^{a_m}_{\gamma,r}(s_m,k_m)\left(c^{i_m\dagger}_{a_m}(k_m)+c^{i_m}_{a_m}(-k_m)\right)$$

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Define the r-Fock measure via:

$$\int d\mu^{r}_{SU(N)}\, W^{J}_{\gamma_1}(A)\dots W^{J}_{\gamma_{N-1}}(A) := \langle 0|\hat{W}^{r,J}_{\gamma_1}\dots\hat{W}^{r,J}_{\gamma_{N-1}}|0\rangle$$

# r-Fock measures for SU(N) gauge theories

- Linear functional:

$j_o$ – fundamental rep.

$$\Phi^r_F : \mathcal{HA} \longrightarrow \mathbb{C}$$

$$\Phi^r_F\left[\sum_{i=1}^{M} a_i W^{j_o}_{\gamma^i_1}\dots W^{j_o}_{\gamma^i_{N-1}}\right] := \sum_{i=1}^{M} a_i \langle 0|\hat{W}^{r,j_o}_{\gamma^i_1}\dots\hat{W}^{r,j_o}_{\gamma^i_{N-1}}|0\rangle$$

**- Definiteness** : *convergence* of the expansion of the expectation value;

**- Positivity** : *Mandelstam identities* for the smeared Wilson loop operators;

- **Existence of an induced measure on $\overline{A}/\overline{G}$ :** continuity w.r.t. the C$^*$-norm on $\overline{A}/\overline{G}$

$$\left|\Phi_F\left[\sum_{i=1}^{M} a_i W^{r,j_o}_{\gamma_i}\right]\right| \leq \sup_{A\in\bar{\mathcal{A}}/\bar{\mathcal{G}}}\left|\sum_{i=1}^{M} a_i W^{j_o}_{\gamma_i}(A)\right|$$

***the smearing :*** $A \in \mathcal{S}^* \longrightarrow A^r \in \mathcal{S}^* \cap \bar{\mathcal{A}}$

# r-Fock measures for SU(N) gauge theories

- Mapping between measures:

$$\boxed{d\mu^r_{SU(N)} = \left( \sum_{\Gamma} \sum_{\{j,\iota\}} \Phi^r_F \left[ \Psi_{\Gamma,\{j,\iota\}} \right] \overline{\Psi_{\Gamma,\{j,\iota\}}} \right) d\mu^o_{SU(N)}}$$

lift to a gauge invariant measure on $\overline{A}$ via group averaging.

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SU(2) example:

$$\Phi^r_F \left[ W^{1/2}_{\gamma^i} \right] = \sum_{n=0}^{\infty} \frac{1}{2^n n!} \mathcal{P} \int_{\gamma} ds_1 \ldots ds_{2n} \underbrace{\sum_{\sigma}}_{\text{Permutations}} \Upsilon^{(1/2)}_{\sigma(2n)} \left( \prod_{m=1}^{n} \int \frac{d^3 k}{2q^2 |k|} \tilde{X}^a_{\gamma,r}(s_{\sigma(2m-1)}, k) \tilde{X}^a_{\gamma,r}(s_{\sigma(2m)}, -k) \right)$$

$$\Upsilon^{(1/2)}_{\sigma(2n)} := \delta^{i_{\sigma(1)} i_{\sigma(2)}} \ldots \delta^{i_{\sigma(2n-1)} i_{\sigma(2n)}} \mathrm{Tr} \left[ \prod_{m=1}^{2n} \tau_{i_m} \right]$$

# Summary & outlook

Fock states as shadow states :

- Matter states encoding Minkowski geometry;
- Non-local correlations;
- Graphs superposition could be restricted by the dynamics;

To explore:

- Non-locality (entanglement) & semi-classical prop. of shadow states;
- Extensions beyond the abelian case
- Effective dynamics for the shadow states as approximate physical states;
- Role in the construction of a continuum limit for LQG;

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Thank you!