Some views on Loop Quantum Cosmology

Aurélien Barrau
UJF/CNRS Grenoble, France

Some works in collaboration with
M. Bojowald, B. Bolliet, T. Cailleteau, J. Grain, L. Linsefors, F. Vidotto

A great time for quantum cosmology !

As pointed out by by Wilczek and Krauss, the recent detection of cosmological B-modes (if confirmed) shows that gravity should be treated as a quantum field.

The first approach we have been following : anomaly freedom

$$\begin{aligned}
\{\vec{D}[\vec{M}_1], \vec{D}[\vec{M}_2]\} &= -\vec{D}[\mathcal{L}_{\vec{M}_2}\vec{M}_1]\,, \\
\{C[N], \vec{D}[\vec{M}]\} &= -C[\mathcal{L}_{\vec{M}}N]\,, \\
\{C[N_1], C[N_2]\} &= \vec{D}[N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1]
\end{aligned}$$

When quantum corrections are inserted in the equations of gravity, in particular in cosmological perturbation equations, it is never clear whether the delicate consistency conditions summarized in the first-class nature of the constraint algebra remain intact. Especially background-independent frameworks cannot directly rely on standard covariance arguments because their notion of space-time, encoded in previous equations, is supposed to emerge in some way from solutions to their equations.

When gauge fixing before quantization is checked to be self-consistent, this attitude is legitimate. But the resulting dynamics and physical predictions can, in general, be quantitatively different from the theory quantized without fixing the gauge. Whenever available, the latter should be preferred because it implements the full system. This is especially true for gravity (dynamics is part of the gauge system. It seems more natural to quantize gauge transformations and the dynamics at the same time and not fix one part (the gauge) in order to derive the second part (the dynamics). If this is ignored, the gauge could be fixed according to transformations that will be subsequently modified.).

The method followed is to quantize the constraints without classical specifications of gauge or observables.

The associated effective viewpoint is (i) to take the corrections suggested by operator definitions in some approach to quantum gravity,
(ii) to parametrize them so as to allow for sufficient freedom to encompass the ambiguities and unknowns in quantum operators,
(iii) to insert them in the classical constraints and
(iv) to compute their algebra under Poisson brackets.

For example

Smeared constraints for GR :

$$\begin{aligned}\mathcal{C}_1 &= G[N^i] = \frac{1}{2\kappa}\int_\Sigma d^3x\; N^i C_i,\\ \mathcal{C}_2 &= D[N^a] = \frac{1}{2\kappa}\int_\Sigma d^3x\; N^a C_a,\\ \mathcal{C}_3 &= S[N] = \frac{1}{2\kappa}\int_\Sigma d^3x\; NC,\end{aligned}$$

$$\{G[N^i] + D[N^a] + S[N], G[M^i] + D[M^a] + S[M]\} \approx 0.$$

$$\{\mathcal{C}_I, \mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K.$$

← First class algebra. However, when going to the quantum version anomalies usually appear.

$$\{\mathcal{C}^Q_I, \mathcal{C}^Q_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}^Q_K + \mathcal{A}_{IJ}.$$

Where are we in this approach ?

☺ **Good points :**

- successfully done for scalar, vector and tensor modes with holonomy corrections.

- the mu-bar scheme is recovered.

- shown to be *consistent.*

- successfully done for scalar, vector and tensor modes with inverse-volume corrections.

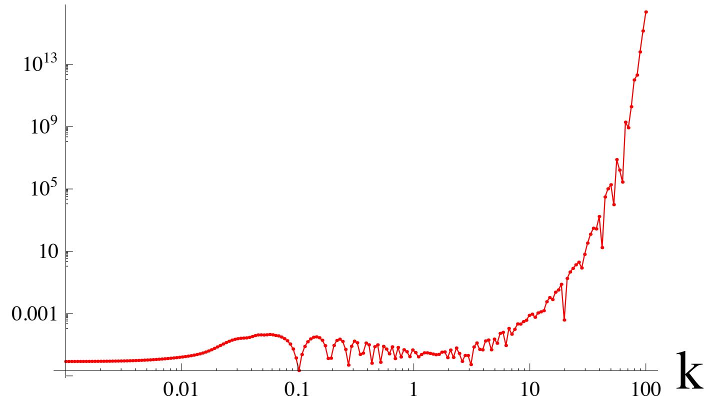
- successfully done for holonomy + inverse-volume corrections.

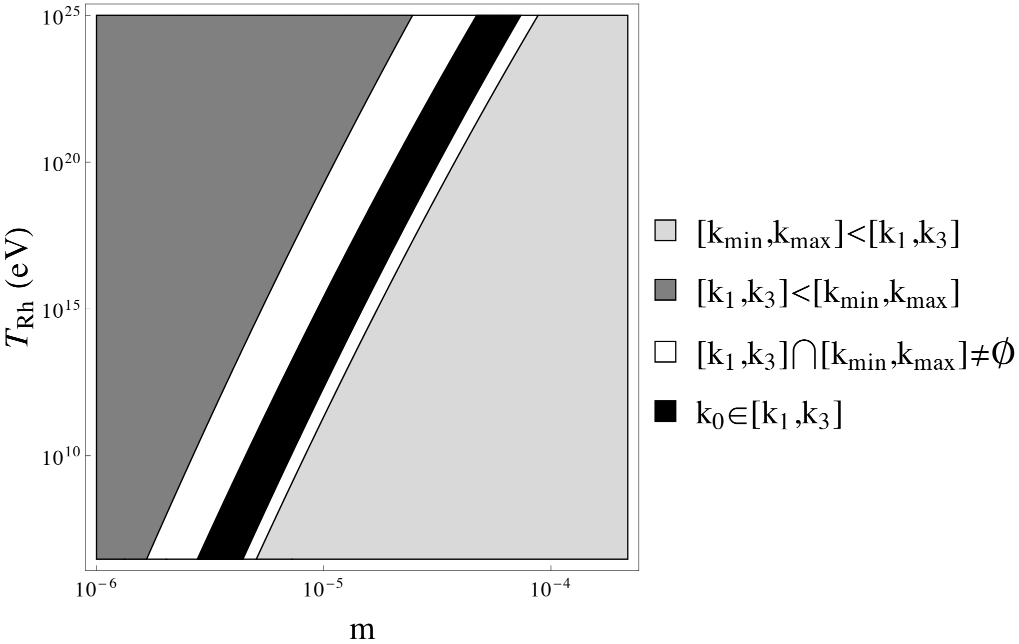
$$\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} = 0,$$
$$\{H_{tot}[N], D_{tot}[N^a]\} = -H_{tot}[\delta N^a \partial_a \delta N],$$
$$\{H_{tot}[N_1], H_{tot}[N_2]\} = D_{tot}\left[\Omega \frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right]$$

$$v''_{\mathrm{S(T)}} - \mathbf{\Omega}\, \nabla^2 v_{\mathrm{S(T)}} - \frac{z''_{\mathrm{S(T)}}}{z_{\mathrm{S(T)}}} v_{\mathrm{S(T)}} = 0$$

- associated phenomenology

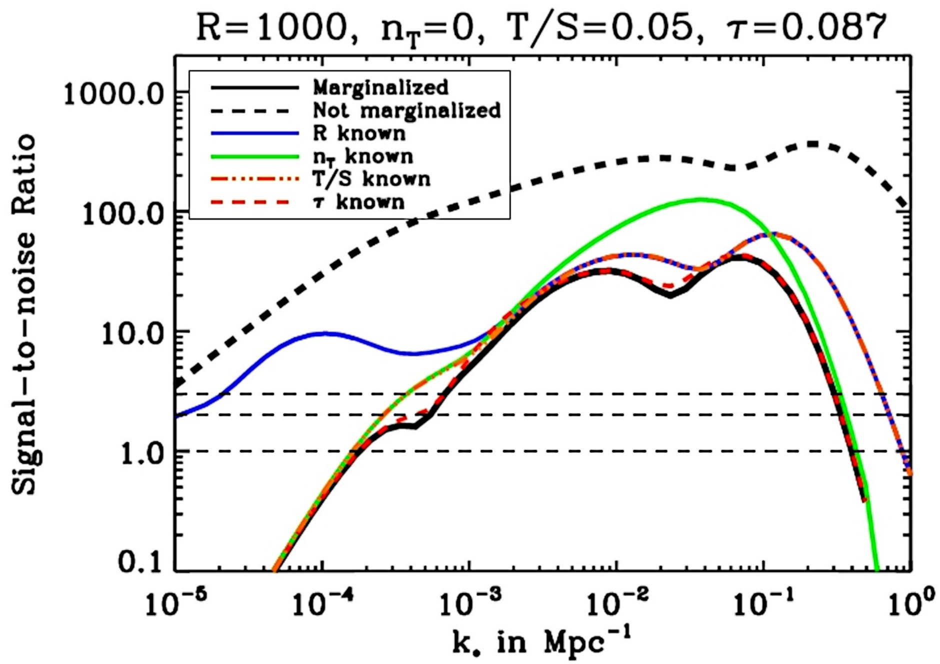
$\mathcal{P}_T$





10^{25}
10^{20}
10^{15}
10^{10}
T_{Rh} (eV)
10^{-6}
10^{-5}
10^{-4}
m
$[k_{min},k_{max}]<[k_1,k_3]$
$[k_1,k_3]<[k_{min},k_{max}]$
$[k_1,k_3]\bigcap[k_{min},k_{max}]\neq\emptyset$
$k_0\in[k_1,k_3]$

→



R=1000, n_T=0, T/S=0.05, τ=0.087
Marginalized
Not marginalized
R known
n_T known
T/S known
τ known
Signal-to-noise Ratio
1000.0
100.0
10.0
1.0
0.1
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
k_* in Mpc^{-1}

- **I think the approach *is* consistent and makes sense.** (In particular, the Hojman–Kuchar–Teitelboim theorem, which shows uniqueness of the classical dynamics for second-order field equations under the assumption of a classical space-time structure, does not underline any inconsistency in the deformed-algebra approach, as both the constraints and the algebra are simultaneously deformed in a consistent way. In addition, the phase space considered is truncated due to the way perturbations are handled.)

But ☹ (Bad point) :

- **How much does this capture from LQG ?**

→ In my opinion, not much. Maybe this is the only weak point but it is far from being a detail !!

- **A comparison with Agullo, Ashtekar & Nelson results is in progress**

→ At this stage, I believe that the Agullo, Ashtekar & Nelson approach is more reliable and closer to the full theory

If the anomaly-free approach is however taken seriously an intriguing consequence is the change of signature.

Equations of motion for modes become elliptic rather than hyperbolic differential equations. This change of signature can be seen from the fact that the usual C,C Poisson Bracket with $\beta = -1$ is obtained for 4-dimensional Euclidean space. (But there is no piece of classical Euclidean space as $\beta = -1$ is found only for one maximum-curvature slice. One should therefore not expect an effective Hamiltonian constraint to resembles the classical Euclidean one.)

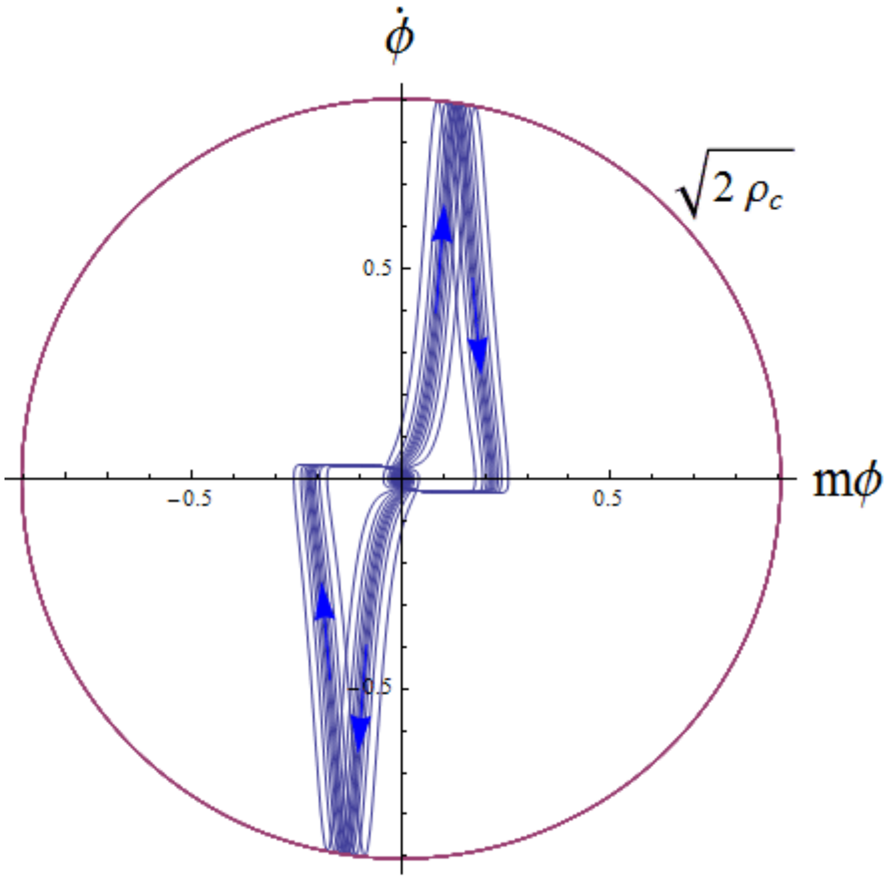
Signature change could be an intriguing consequence of effective holonomy modifications, which had been overlooked until spherically symmetric inhomogeneity and cosmological perturbations were studied in an anomaly-free way. Indeed, without inhomogeneity, one cannot determine the signature because (i) one cannot see the relative sign between temporal and spatial derivatives and (ii) the relation Poisson Bracket trivially equals zero in homogeneous models. Nevertheless, signature change is not a consequence of inhomogeneity, the latter rather being used as a test field.

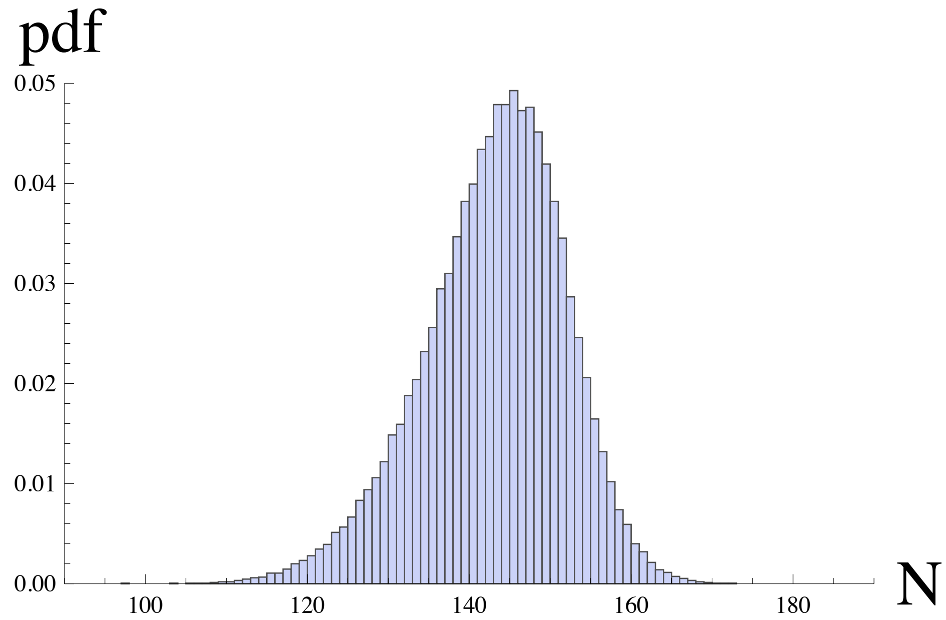
A change of sign in front of the Laplacian of the Mukhanov–Sasaki equations is not a novelty in cosmology. The same effect occurs, for instance, in higher-order gravity where the Gauss–Bonnet curvature invariant is non-minimally coupled with a scalar field. In that case, this change of sign is simply interpreted as a classical instability. Within the same model, one can have further modifications to the Mukhanov–Sasaki equations which may introduce ghost and tachyon instabilities as well as superluminal propagation. Although all of these features are problematic and can be avoided by a restriction of the parameter space, the nature of the space-time wherein perturbation modes propagate remains purely classical and Lorentzian. (The theory is of higher-curvature type and does not lead to deformations of the constraint algebra.)
Here, however, the change in the perturbation equations is a direct consequence of the deformation of the constraint algebra of gravity and, hence, of a deformation of the classical space-time structure.

→ Associated phenomenology is in progress. Fixes a preferred "time" to set initial conditions.

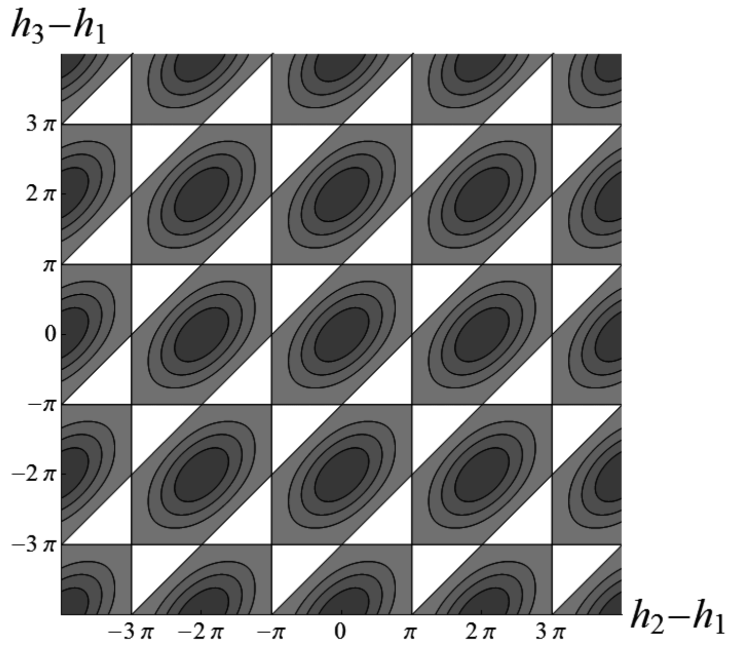
Indirect probes

This as been pioneered By Ashtekar and Sloan. Might be the only probe usable in practice. Recently revisited :



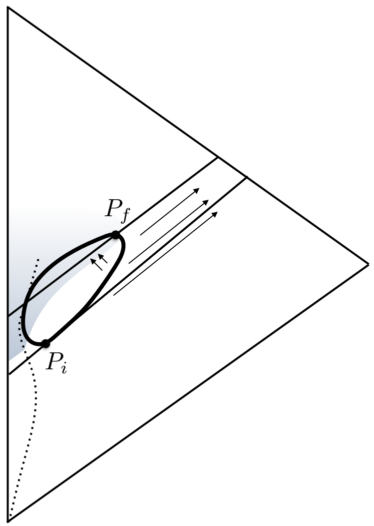


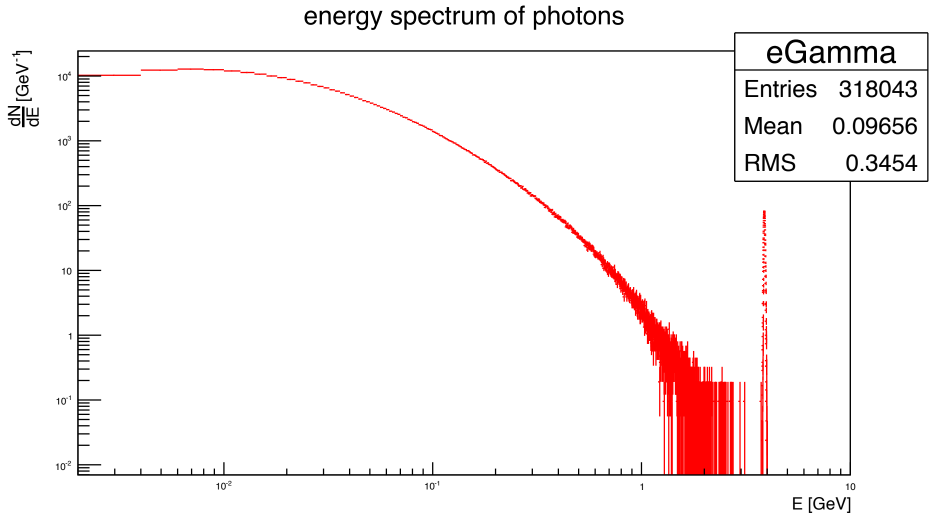
A major issue here is anisotropies. Param et al. have obtained important results. We reach slightly different conclusions. This requires further investigations. Basically : more shear leads to less inflation.



Crazy ideas

- De Sitter contraction phase
- Planck stars





A possible question to Ivan.

Your approach is in my opinion the best one. However, one can wonder if there is a fully consistent space-time structure in this framework as, instead of fixing the gauge, you solve classical constraints for gauge-invariant modes before implementing loop modifications and quantizing the deparametrized Hamiltonian. In deparametrized models, the evolution generator is a combination of some terms in the constraints. If the former is modified, it is questionable to assume that the constraints and perturbative observables are still classical. The resulting equations may be formally consistent, but just as with gauge-fixed modifications, a corresponding space-time picture might not exist.

Although shown to be mathematically equivalent to some quantum field theory on a quantum geometry (a classical-type metric with quantum-corrected coefficients), doesn't the "dressed metric" approach assume, in some sense implicitly, a standard space-time structure ? The validity of this assumption rests on an undeformed constraint algebra, whose very existence can be questioned in the presence of the holonomy modifications used: As shown by the derivation of anomaly-free modified constraints, the algebra of constraints is generically deformed by quantum-geometry corrections.