Asymptotics of spin foam models with an Immirzi parameter

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History

4d BF 3d QG

- Ponzano, Regge 1968
- Witten 1989
- Ooguri 1990
- JWB, Crane 1996 4d QG??
- Barbieri 1997
- Engle, Pereira, Rovelli, Livine, Freidel, Krasnov 2007 γ

Q tetrahedron

Spin foam model

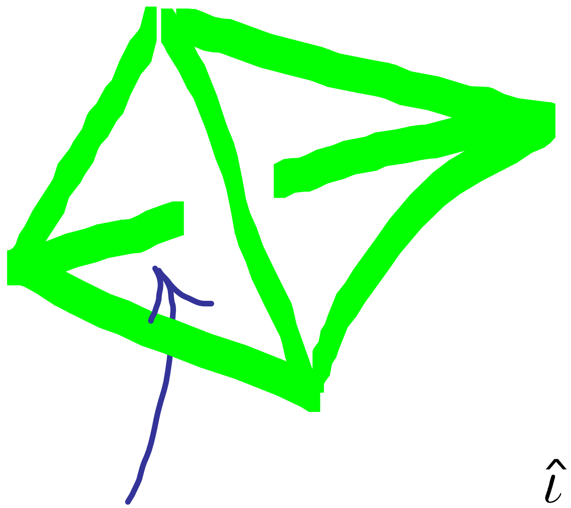
triangulated manifold

$$\mathcal{Z}(\mathcal{T}) = \sum_{\iota,\rho} \prod_{\Delta_2} f_2(\rho) \prod_{\Delta_3} f_3(\rho,\iota) \prod_{\Delta_4} f_4(\rho,\iota). \qquad (1)$$

ρ : representation

ι : intertwiner

Quantum tetrahedron



k = area

$$\hat{\iota} \in \mathsf{Hom}_{\mathrm{SU}(2)}(\mathbb{C}, \bigotimes_{i=1}^{4} V_{k_i}) \tag{2}$$

EPRL intertwiner ι

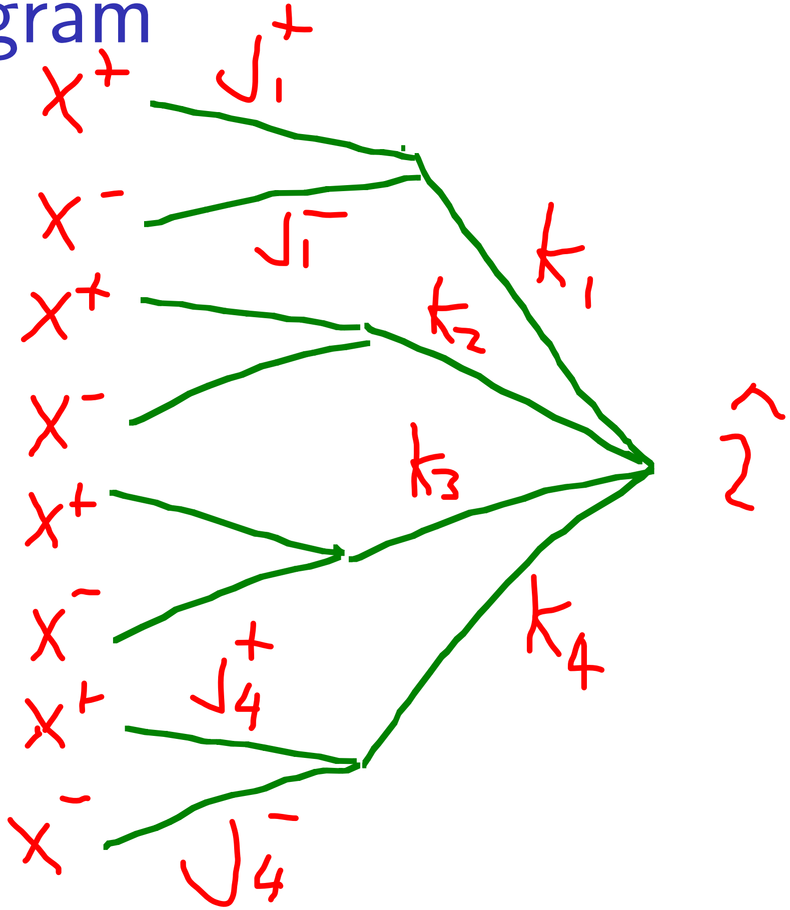
(j^-, j^+) : Spin(4)

$$\phi : \mathrm{Hom}_{\mathrm{SU}(2)}(\mathbb{C}, \bigotimes_{i=1}^{4} V_{k_i}) \to \mathrm{Hom}_{\mathrm{Spin}(4)}(\mathbb{C}, \bigotimes_{i=1}^{4} V_{(j_i^-, j_i^+)}) \tag{3}$$
$$\hat{\iota} \mapsto \phi(\hat{\iota}) = \iota$$

$$j^{\pm} = \frac{1}{2}|1 \pm \gamma|\, k \tag{4}$$

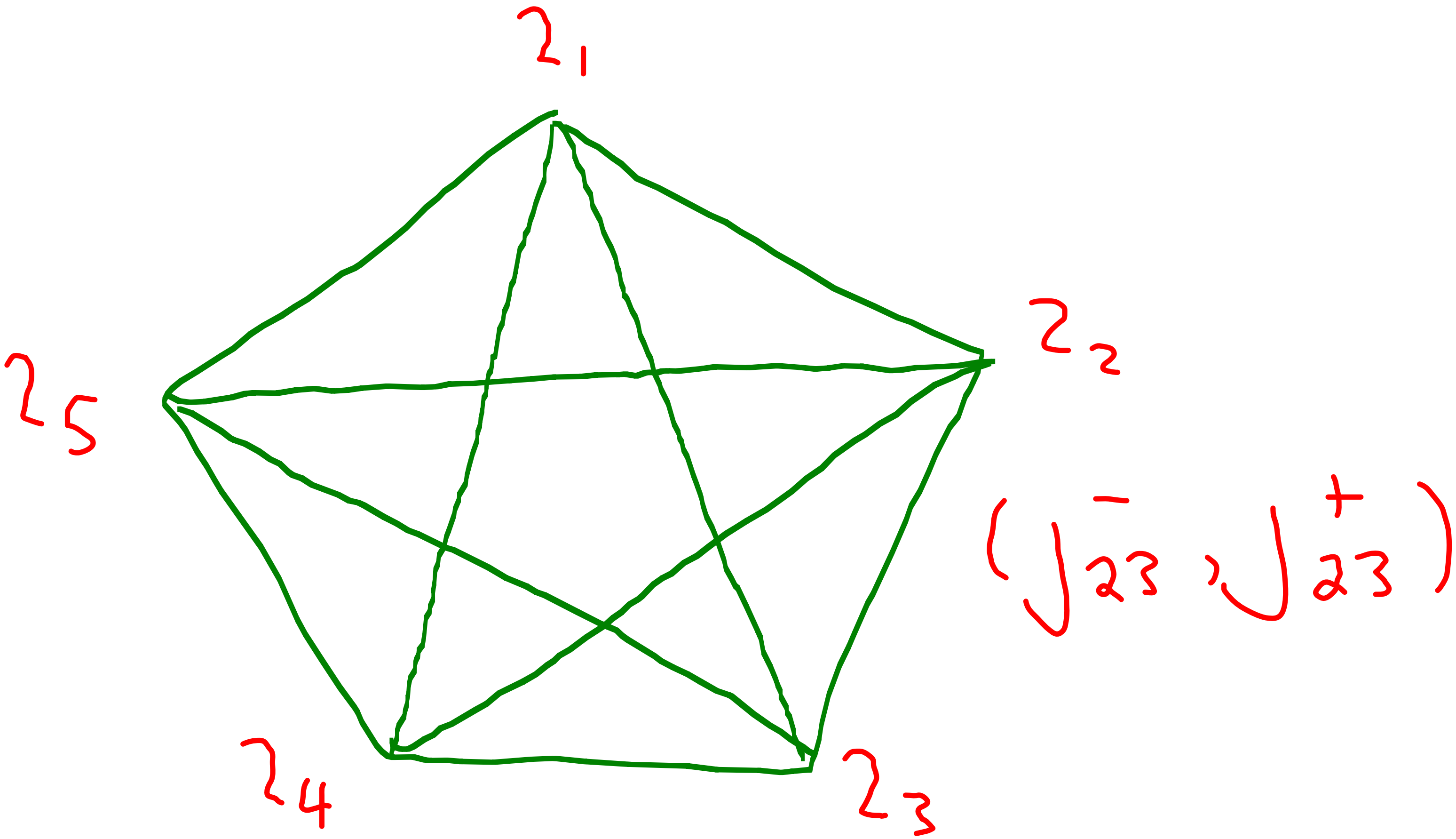
Intertwiner diagram

$$\int_{Spin(4)} dX^{+} dX^{-}$$



$$= 2$$

4-simplex amplitude



Asymptotic problem

$$k_{ab} \longrightarrow \lambda k_{ab} \tag{5}$$

$\lambda_a \rightarrow ?$

Coherent states for $\mathrm{SU}(2)$

Spin k

$n \in S^2$

Lie alg

$$(\mathbf{L}.\mathbf{n})\,\alpha = ik\,\alpha \tag{6}$$

Bra-ket :

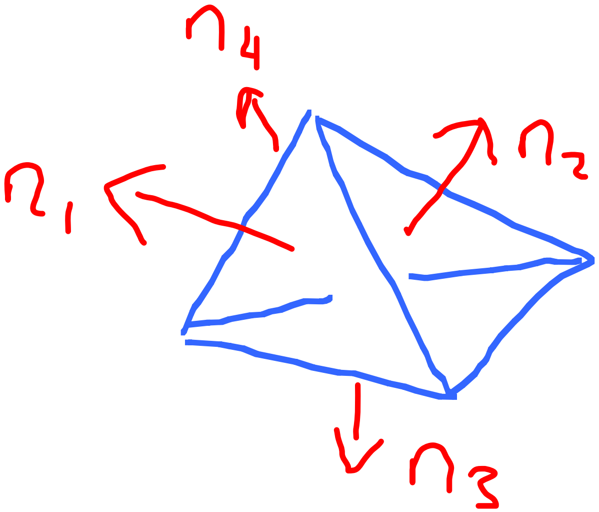
$$\alpha = |k, \mathbf{n}, \theta\rangle \tag{7}$$

Coherent tetrahedron (Livine-Speziale)

$$\hat{\iota}^{k_1 k_2 k_3 k_4}(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4) = \int_{\mathrm{SU}(2)} dh \bigotimes_{i=1}^{4} h|k_i, \mathbf{n}_i, \theta_i\rangle \qquad (8)$$

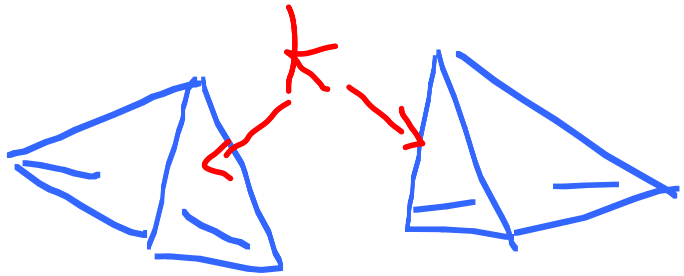
Quantization commutes with reduction (Guillemin-Sternberg)

$$k_1\mathbf{n}_1 + k_2\mathbf{n}_2 + k_3\mathbf{n}_3 + k_4\mathbf{n}_4 = 0 \qquad (9)$$



metric orientation

Boundary state for a closed 3-manifold Σ



area geometry

$$\psi(k, \mathbf{n}) = \bigotimes_{\text{tetrahedra}} \hat{\iota}^{k_1 k_2 k_3 k_4}(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4) \qquad (10)$$

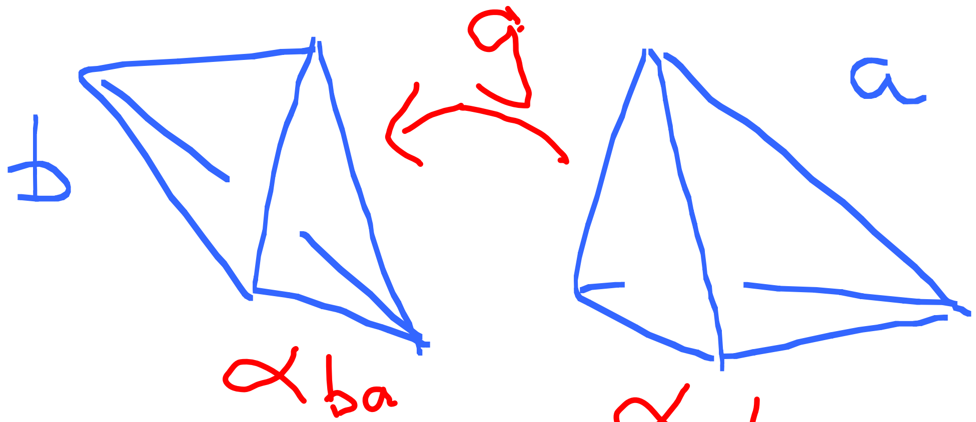
- Boundary data $\mathcal{B}_\Sigma = \{k, \mathbf{n}\}$

ψ

$k \to \infty$

Regge state

$g \in SU(2)$



- Regge geometry $\subset$ area geometry
- Canonical choice of phase

$$\alpha_{ba} = g_{ab} J \alpha_{ab} \tag{11}$$

- J = standard antilinear map for $\mathrm{SU}(2)$.

Main conjecture: Regge dominates

Asymptotic formula

dirvectors

(b^-, b^+) (b^+, b^-)

$$f_4(\psi_\lambda) \sim (-1)^{\chi'} \left(\frac{2\pi}{\lambda}\right)^{12} \left[2N^{\gamma}_{+-} \cos\left(\lambda\gamma \sum_{a<b} k_{ab}\Theta_{ab}\right) \right.$$

(b^+, b^+)

$$+ N^{\gamma}_{++} \exp\left(i\lambda \sum_{a<b} k_{ab}\Theta_{ab}\right)$$

(b^-, b^-)

$$\left. + N^{\gamma}_{--} \exp\left(-i\lambda \sum_{a<b} k_{ab}\Theta_{ab}\right)\right] \quad (12)$$

Proof

- Write f_4 as an integral, using coherent states.
- Write integral as $e^{\lambda S}$
- Use stationary phase for $\lambda \to \infty$
- Classify critical points of S
- Interpret critical points as 4-simplex geometries
- Use Regge state condition to cancel arbitrary phases

c.f 1997

Outlook

- Special cases: $\gamma = 0$, $\gamma \to \infty$
- Numerical checks
- Lorentzian version
- Gluing properties, Pachner moves, renormalisation