

# Asymptotics of spin foam models with an Immirzi parameter

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Hellmann

# History

4d BF

3d QG

- ▶ Ponzano, Regge 1968
- ▶ Witten 1989
- ▶ Ooguri 1990
- ▶ JWB, Crane 1996
- ▶ Barbieri 1997
- ▶ Engle, Pereira, Rovelli, Livine, Freidel, Krasnov 2007

4d QG??

Q tetrahedron

γ

# Spin foam model

triangulated manifold

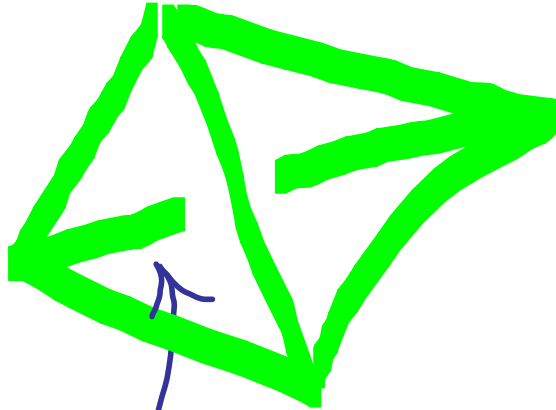


$$Z(T) = \sum_{\iota, \rho} \prod_{\Delta_2} f_2(\rho) \prod_{\Delta_3} f_3(\rho, \iota) \prod_{\Delta_4} f_4(\rho, \iota). \quad (1)$$

$\rho$  : representation

$\iota$  : intertwiner

# Quantum tetrahedron



$k = \text{area}$

$$\hat{t} \in \text{Hom}_{\text{SU}(2)}(\mathbb{C}, \bigotimes_{i=1}^4 V_{k_i}) \quad (2)$$

## EPRL intertwiner $\iota$

$$(j^-, j^+) : \text{Spin}(4)$$

$$\phi : \text{Hom}_{\text{SU}(2)}(\mathbb{C}, \bigotimes_{i=1}^4 V_{k_i}) \rightarrow \text{Hom}_{\text{Spin}(4)}(\mathbb{C}, \bigotimes_{i=1}^4 V_{(j_i^-, j_i^+)}) \quad (3)$$

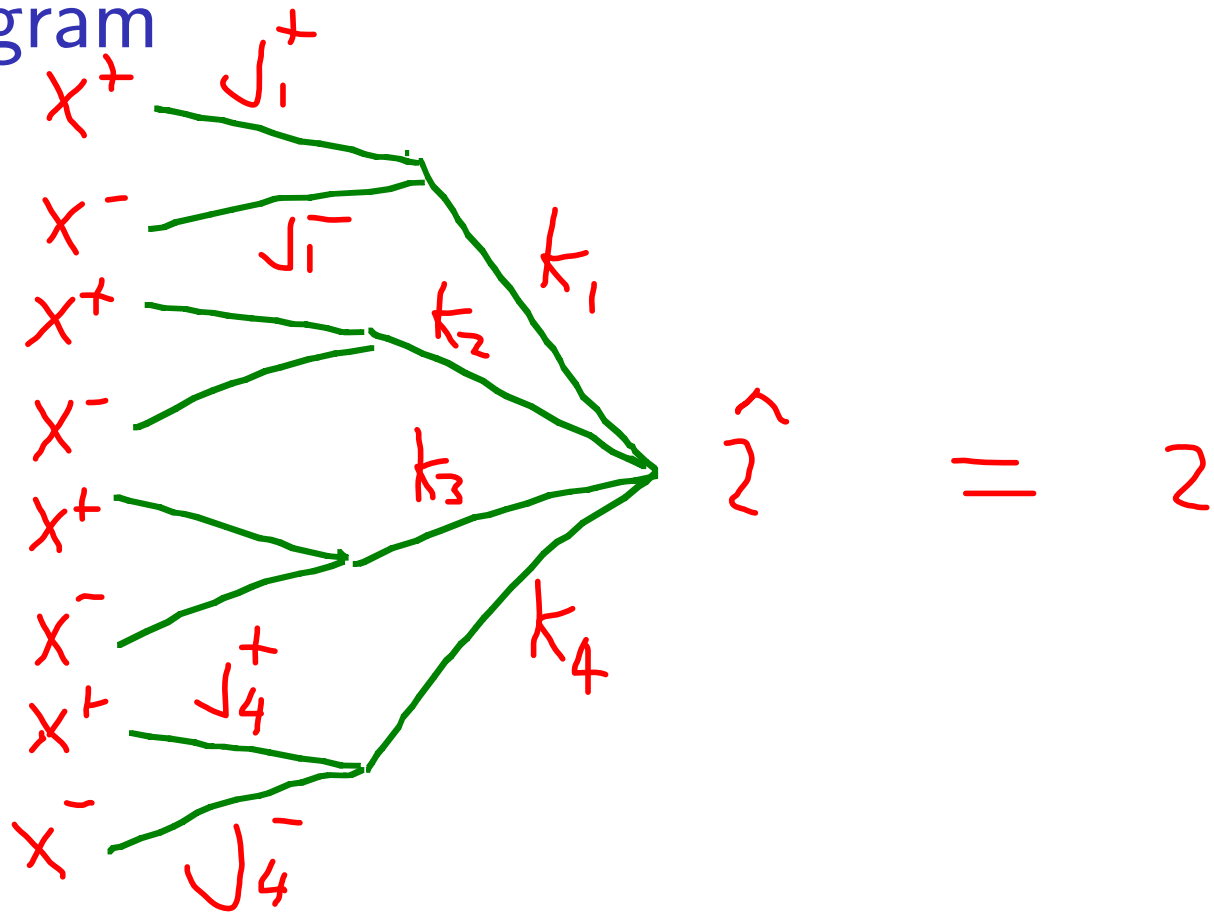
$$\hat{\iota} \mapsto \phi(\hat{\iota}) = \iota$$

$$j^\pm = \frac{1}{2} |1 \pm \gamma| k \quad (4)$$

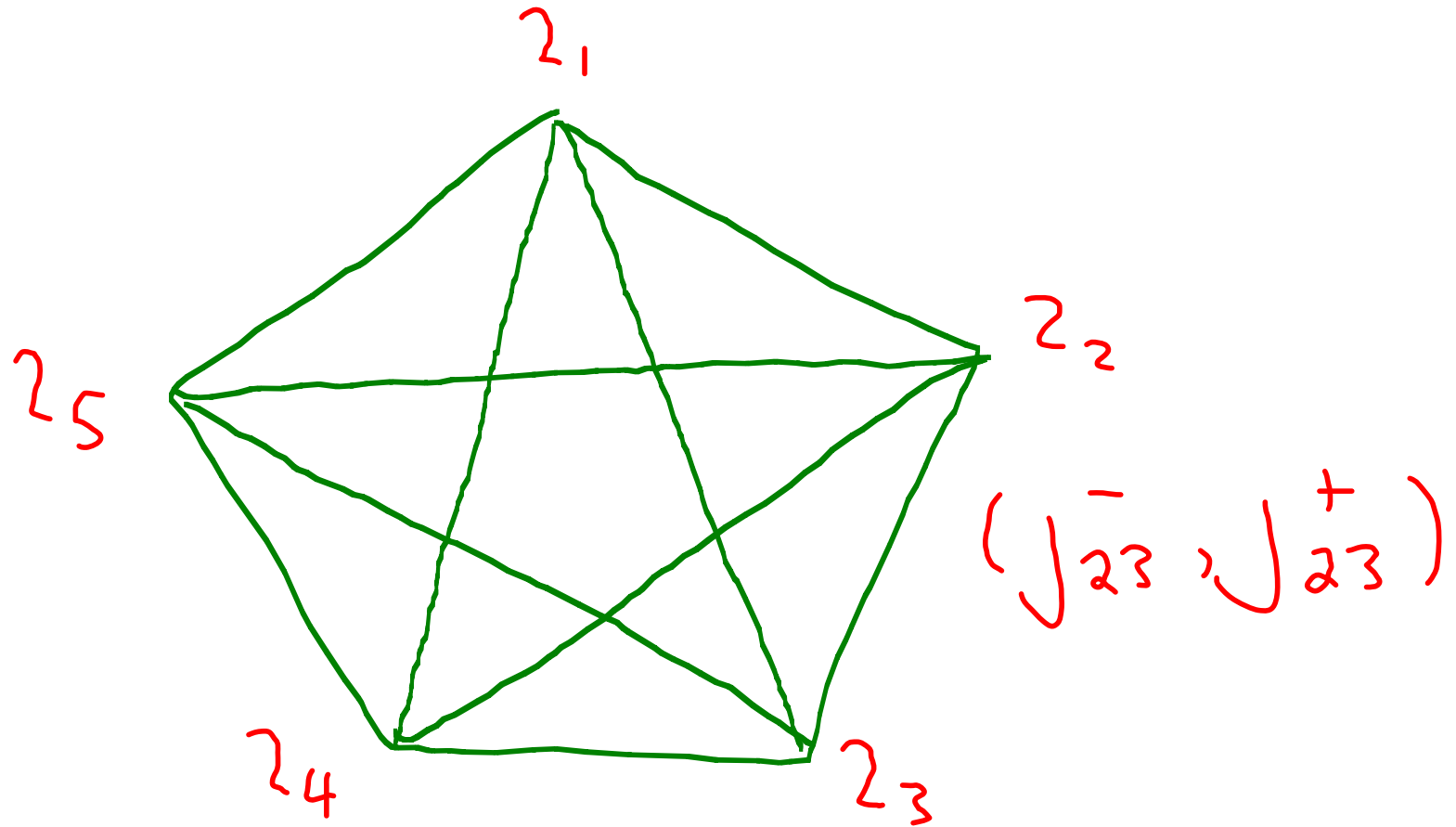
# Intertwiner diagram

$$\int dX^+ dX^-$$

Spin(4)



# 4-simplex amplitude



# Asymptotic problem

$$k_{ab} \rightarrow \lambda k_{ab} \quad (5)$$

$$\lambda_a \rightarrow ?$$



## Coherent states for SU(2)

Spin  $k$   
 $n \in S^2$

Lie alg

$$(\mathbf{L} \cdot \mathbf{n}) \alpha = ik \alpha \quad (6)$$

Bra-ket :

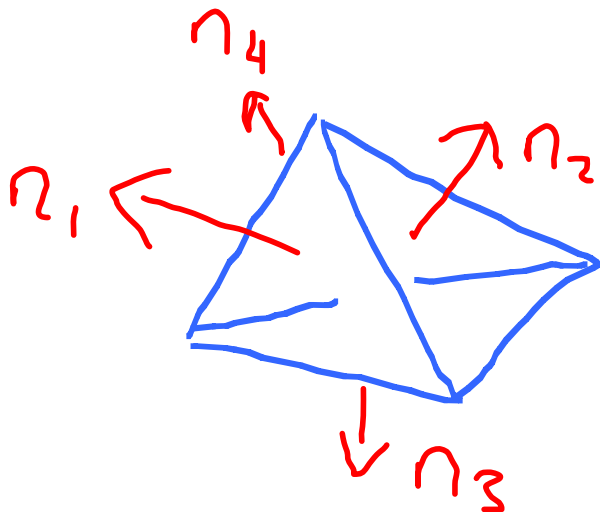
$$\alpha = |k, \mathbf{n}, \theta\rangle \quad (7)$$

# Coherent tetrahedron (Livine-Speziale)

$$\hat{L}^{k_1 k_2 k_3 k_4}(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4) = \int_{\text{SU}(2)} dh \bigotimes_{i=1}^4 h |k_i, \mathbf{n}_i, \theta_i\rangle \quad (8)$$

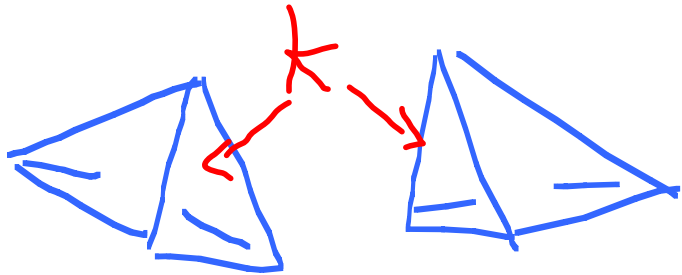
Quantization commutes with reduction (Guillemin-Sternberg)

$$k_1 \mathbf{n}_1 + k_2 \mathbf{n}_2 + k_3 \mathbf{n}_3 + k_4 \mathbf{n}_4 = 0 \quad (9)$$



metri + orientation

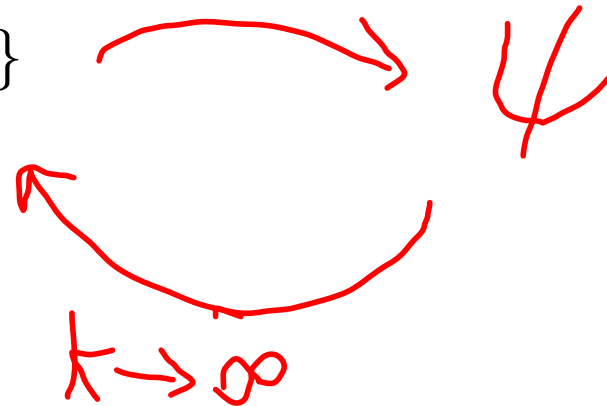
# Boundary state for a closed 3-manifold $\Sigma$



area geometry

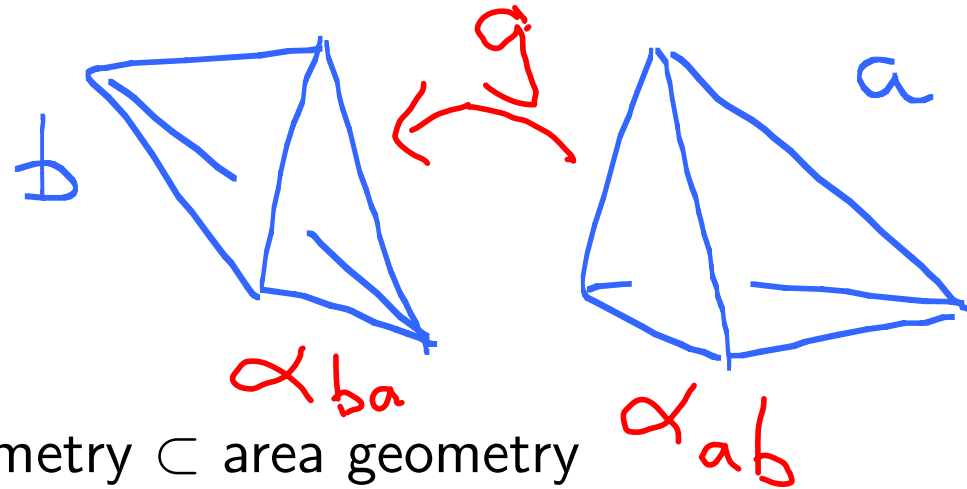
$$\psi(k, \mathbf{n}) = \bigotimes_{\text{tetrahedra}} \hat{t}^{k_1 k_2 k_3 k_4}(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4) \quad (10)$$

- ▶ Boundary data  $\mathcal{B}_\Sigma = \{k, \mathbf{n}\}$



## Regge state

$$g \in \text{SU}(2)$$



- ▶ Regge geometry  $\subset$  area geometry
- ▶ Canonical choice of phase

$$\alpha_{ba} = g_{ab} J \alpha_{ab} \quad (11)$$

- ▶  $J$  = standard antilinear map for  $\text{SU}(2)$ .

Main conjecture: Regge dominates

# Asymptotic formula

divectors

$(b^-, b^+)$   $(b^+, b^-)$

$$f_4(\psi_\lambda) \sim (-1)^{\chi'} \left(\frac{2\pi}{\lambda}\right)^{12} \left[ 2N_{+-}^\gamma \cos \left( \lambda \gamma \sum_{a < b} k_{ab} \Theta_{ab} \right) \right.$$

$$+ N_{+++}^\gamma \exp \left( i \lambda \sum_{a < b} k_{ab} \Theta_{ab} \right)$$

$(b^+, b^+)$

$$+ N_{---}^\gamma \exp \left( -i \lambda \sum_{a < b} k_{ab} \Theta_{ab} \right) \left. \right] \quad (12)$$

$(b^-, b^-)$

# Proof

- ▶ Write  $f_4$  as an integral, using coherent states.
- ▶ Write integral as  $e^{\lambda S}$
- ▶ Use stationary phase for  $\lambda \rightarrow \infty$
- ▶ Classify critical points of  $S$
- ▶ Interpret critical points as 4-simplex geometries
- ▶ Use Regge state condition to cancel arbitrary phases

c.s.  
1997

# Outlook

- ▶ Special cases:  $\gamma = 0$ ,  $\gamma \rightarrow \infty$
- ▶ Numerical checks
- ▶ Lorentzian version
- ▶ Gluing properties, Pachner moves, renormalisation