

# The cosmological constant (non)-problem

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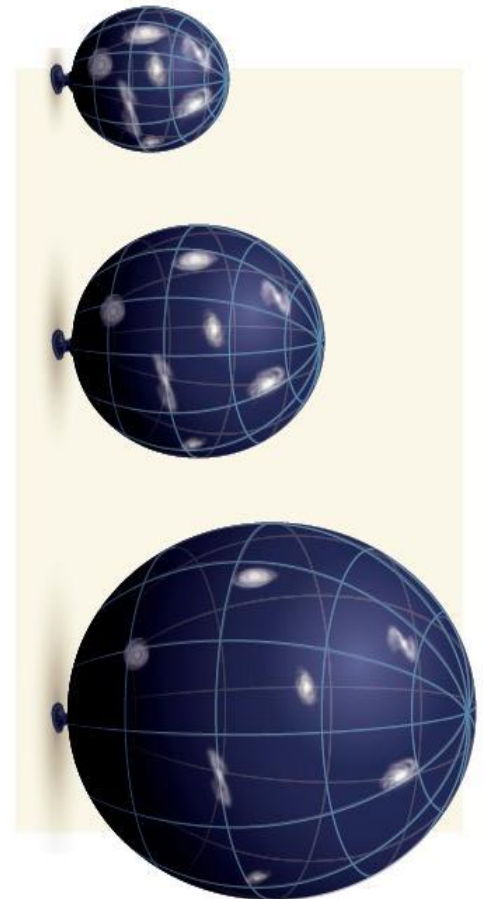
talk based on:

E.B., C. Rovelli, ``*Why all these prejudices against a constant?*'' [arXiv:1002.3966](https://arxiv.org/abs/1002.3966) [astro-ph.CO]

E.B., C. Rovelli & R. Kolb, ``*Is dark energy really a mystery?*'' [Nature 466](https://doi.org/10.1038/466321a), 321-322 (2010)

# Outline

- **The universe is expanding. Observations indicate that its expansion is speeding up.**
- To account for this acceleration, a mysterious substance – *dark energy* – is often invoked. The underlying physics is unknown. This is often presented as a great mystery, 75% of the content of the Universe.
- A simple explanation is to hand: the cosmic acceleration is predicted and simply described by General Relativity (GR) with a **positive cosmological constant  $\Lambda$** .
- The standard model of cosmology ( $\Lambda$ CDM) assumes the presence of  $\Lambda$  and provides the best account of the present observational data [\[Lahav-Liddle, pdg 2010\]](#).
- Three objections to  $\Lambda$  are commonly presented, and nourish the 'mystery':
  - Einstein's greatest blunder
  - The problem of cosmic coincidence
  - The problem of QFT vacuum energy
- We argue that there is confusion, historical or conceptual, in each of these counter-arguments to  $\Lambda$ .



# Part 1.

## $\Lambda$ , Einstein's greatest blunder ?

*"Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life."*

George Gamow, *My World Line* (Viking, 1970) p44.

# Einstein's greatest blunder

- [1916](#), Einstein introduces  $\Lambda$  in GR eqs for generality
- [1917](#),  $\Lambda$  fine-tuned to have a *static Universe*
- [1929](#), Hubble, observational evidence of the expansion of the Universe

*Story often told:*

Einstein, and after him the relativity community, rejected  $\Lambda$  as it spoils the beauty of GR just to account for apparent staticity

*The true blunder:*

Einstein missed the prediction of the cosmic expansion – before its discovery – failing to see that

- without  $\Lambda$
- with a generic value of  $\Lambda$
- even with fine-tuned  $\Lambda$  (because of instability)

the Universe is not static in GR

- Cosmological constant: integral part of GR

$$S[g] = \frac{1}{16\pi G_N} \int (R(g) - 2\Lambda) \sqrt{-g} d^4x$$

- Two fundamental constants

$G_N$  = strength of the gravitational interaction

$\Lambda$  = 'zero-point' curvature of space-time

to be determined experimentally



[ Einstein and Hubble at Mt. Wilson in 1931, Caltech Archives ]

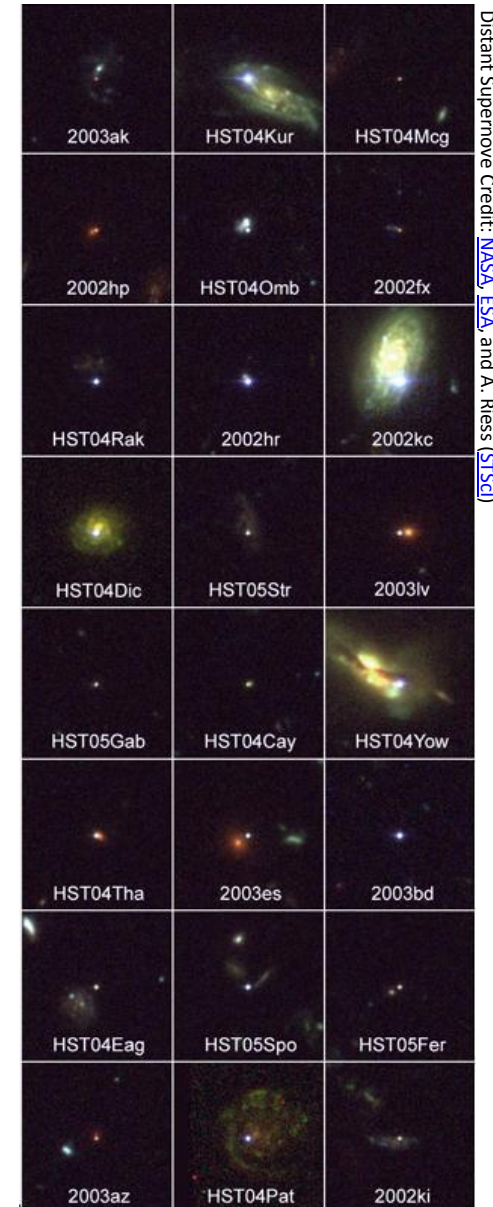
# Cosmic acceleration: the discovery (1998)

Two breakthroughs enabled discovery of cosmic acceleration:

- i. the demonstration that type Ia supernovae (SNe Ia) are standardizable candles [ [Phillips 1993](#) ].
- ii. the deployment of large mosaic CCD cameras on 4m class telescopes, enabling the systematic search of large areas of sky, containing  $\sim 1000$  galaxies, for these rare events.

By comparing images taken weeks apart, the discovery of SNe at redshifts  $z \sim 0.5$  could be 'scheduled' on a statistical basis.

- In mid-1990s, two independent teams took advantage of these breakthroughs ( the [Supernova Cosmology Project](#) and the [High-z SN Search](#) ) to measure the SN Hubble diagram to much larger distances than was previously possible.
- Both teams found that distant SNe are  $\sim 0.25$  mag dimmer than they would be in a decelerating Universe, indicating that **the expansion has been speeding up for the past 5 Gyr**
- When analyzed assuming a Universe with matter and cosmological constant  $\Lambda$ , their results provided evidence for  $\Lambda > 0$  at greater than 99% confidence.



Distant Supernovae Credit: NASA, ESA, and A. Riess (STScI)

# Cosmic acceleration: theory

- Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

- Def. density parameters

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} \quad , \quad \Omega_{\text{matter}} = \frac{8\pi G_N \rho}{3H^2}$$

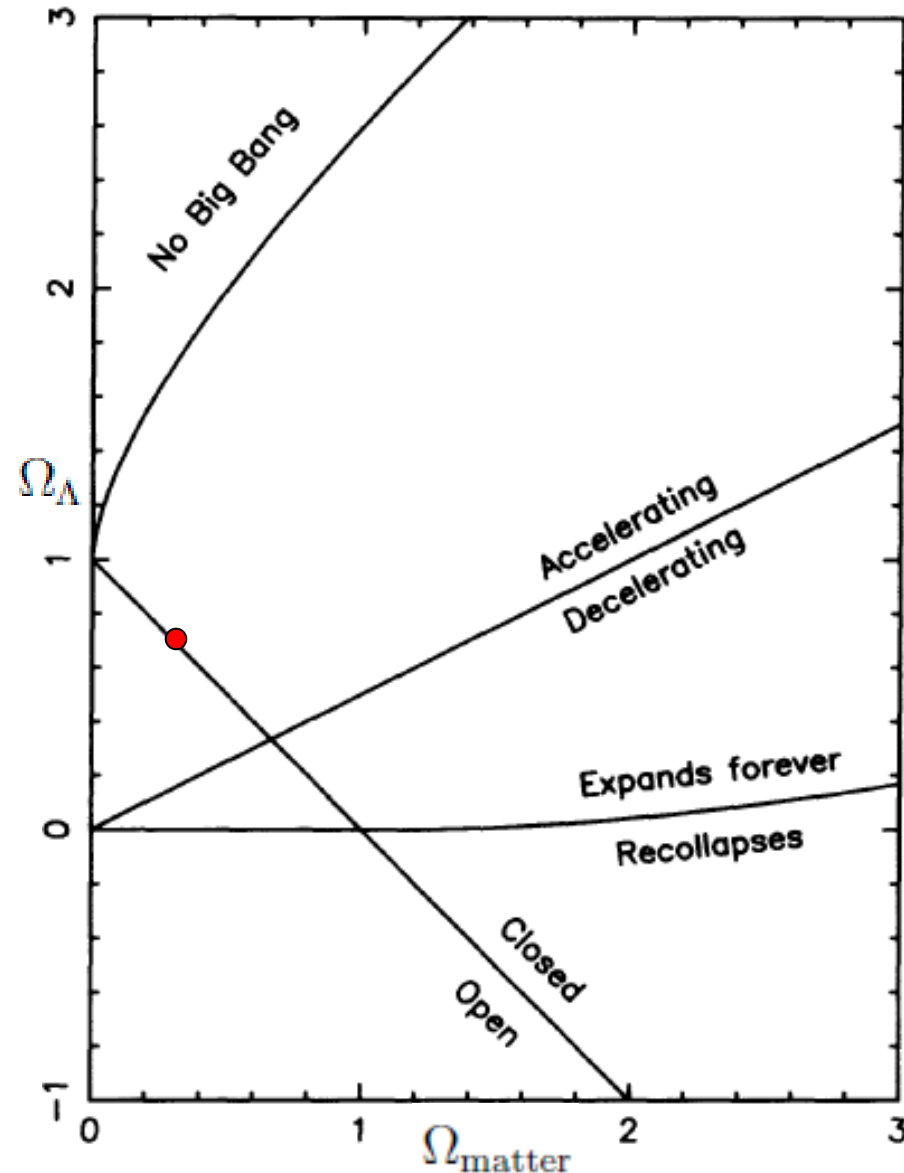
- Friedmann eqn again

$$\Omega_\Lambda + \Omega_{\text{matter}} = 1 + \frac{k}{a^2 H^2}$$

- Acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

- To have cosmic acceleration  $\ddot{a} > 0$ 
  - either positive  $\Lambda$
  - or admit that matter today can have an exotic eqn of state  $\rho + 3P < 0$



[ in the diagram: pressureless matter today assumed, [Liddle 2003](#) ]

# Cosmic acceleration: observational status

- Standard model of cosmology ( $\Lambda$ CDM)  
**Lambda-Cold Dark Matter model**
- Observed values of density params today

$$\Omega_{\Lambda} \simeq 0.757 \pm 0.021 \quad , \quad \Omega_{\text{matter}} \simeq 0.246 \pm 0.028$$

- Hubble constant

$$H_0 \simeq 72 \pm 5 \text{ km s}^{-1} \text{Mpc}^{-1} \approx (13.8 \text{ Gyr})^{-1}$$

- Deceleration parameter

$$q_0 \equiv -\frac{1}{H_0^2} \frac{\ddot{a}_0}{a_0} \simeq -0.64 \pm 0.03$$

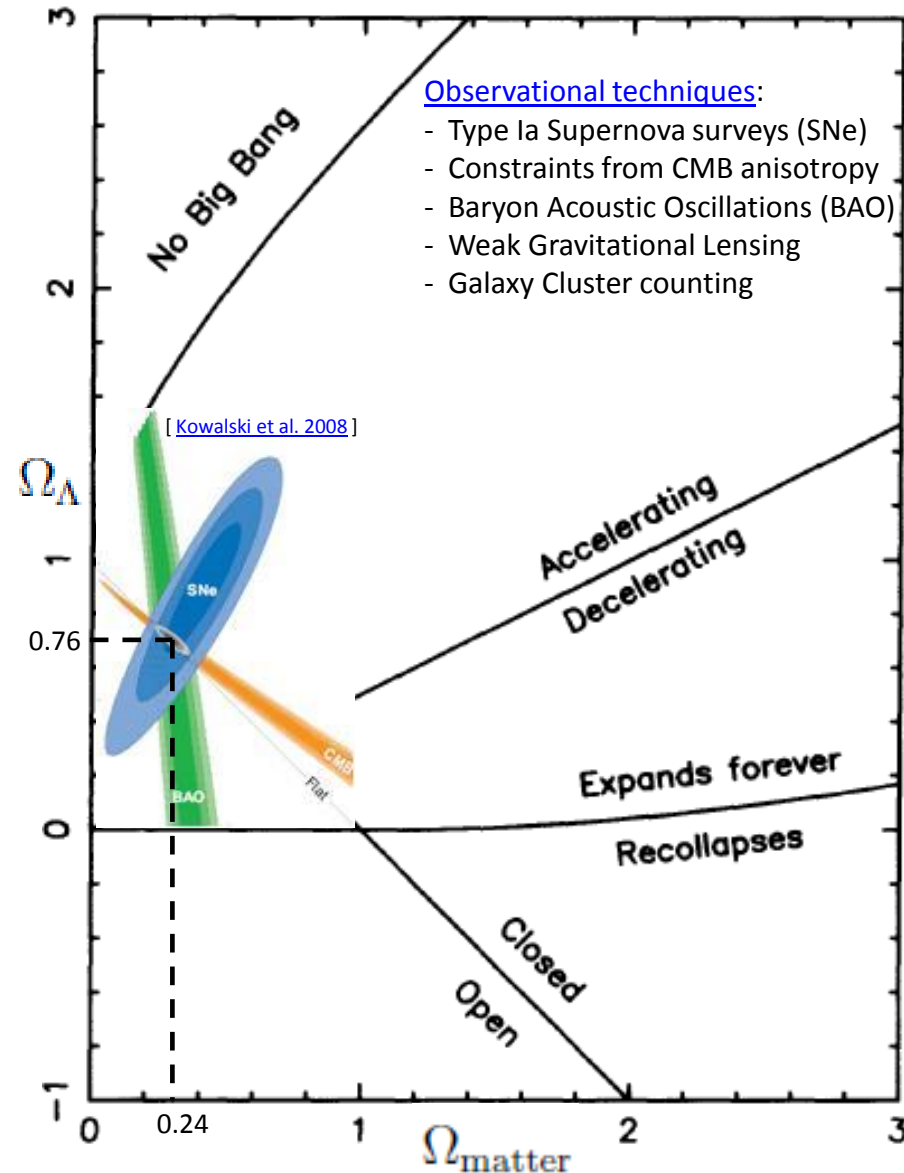
- Radius of curvature  $\frac{k}{a_0^2 H_0^2} \simeq 0.003 \pm 0.010$   
(much larger than Hubble radius)

- Success of Einstein's General Relativity with**  $\Lambda \approx (10^{26} \text{ m})^{-2}$

- Exciting mystery in the matter sector

$$\Omega_{\text{matter}} = \Omega_{\text{known matter}} + \Omega_{\text{cold dark matter}}$$

$$\Omega_{\text{baryon}} \simeq 0.042 \pm 0.002 \quad , \quad \Omega_{\text{cold dark matter}} \simeq 0.204 \pm 0.028$$

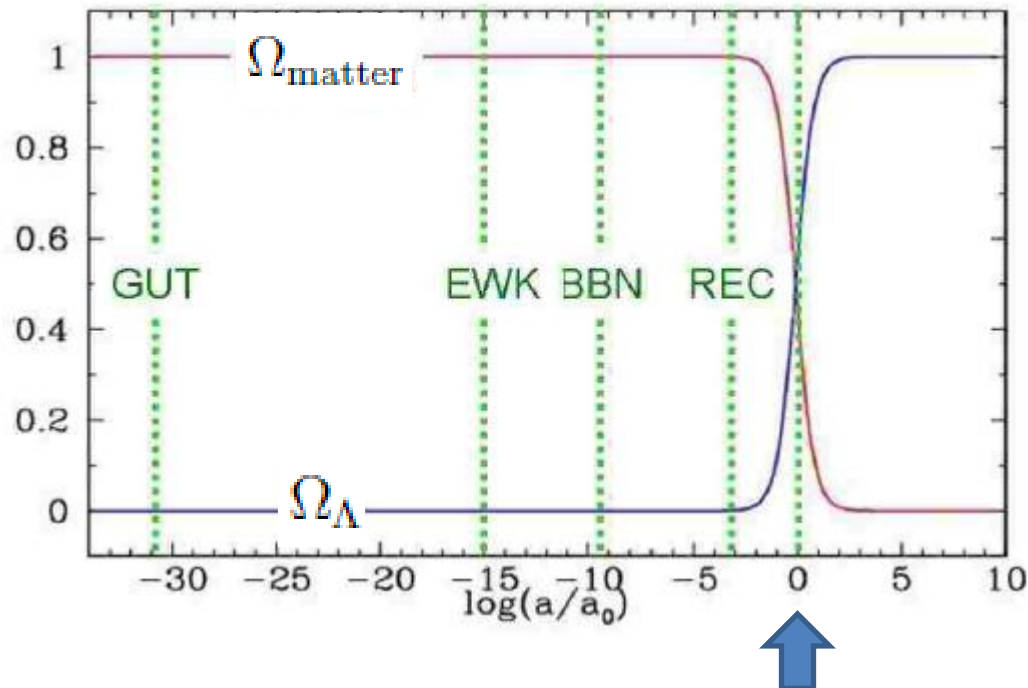




## Part 2.

Cosmic coincidence problem ?

Why  $\Omega_{\text{matter}} \sim \Omega_{\Lambda}$  now?





# The cosmic coincidence problem

- Standard 'coincidence' argument against the cosmological constant scenario :

*data indicate that we happen to live in a very brief phase of cosmic history when  $\Omega_{\text{matter}} \sim \Omega_{\Lambda}$*

- unlikely coincidence
- clashes with the cosmological principle

- Matter density *today*

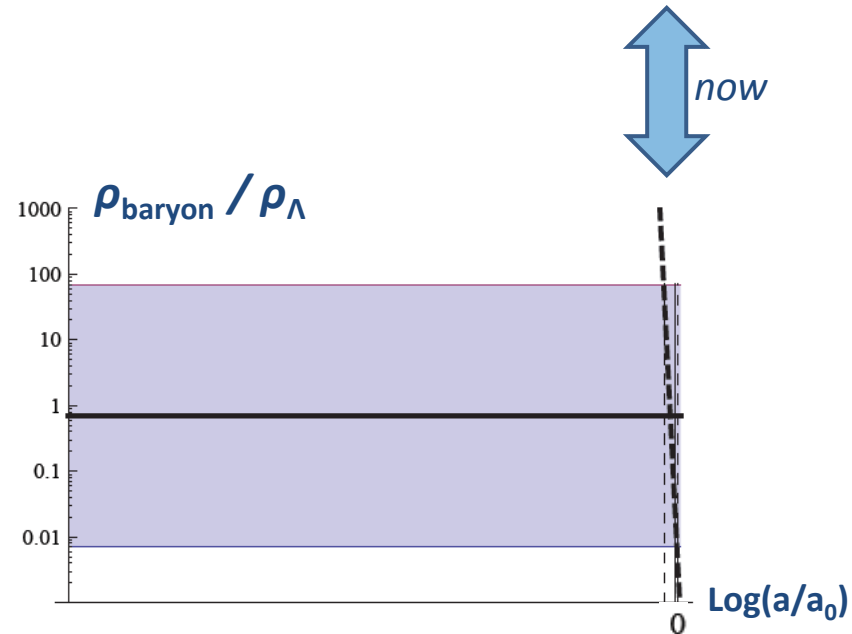
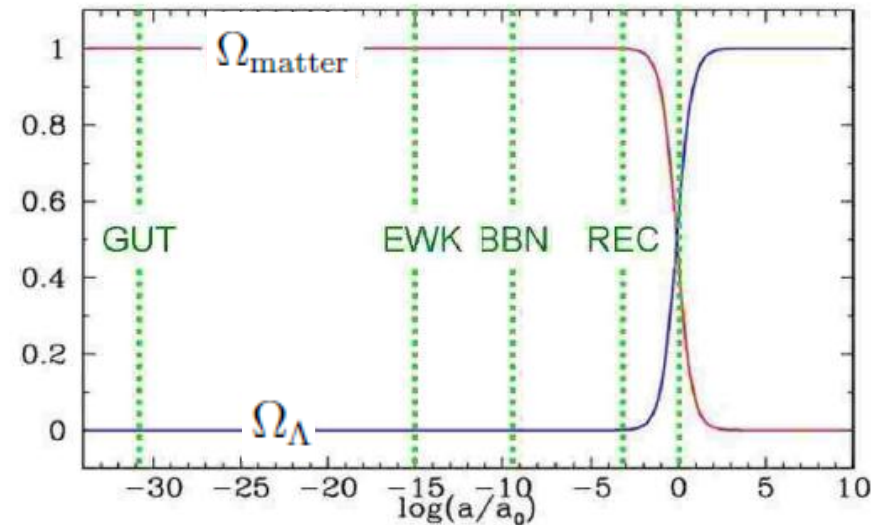
$$\rho_{\text{matter}} \simeq 10^{11} M_{\odot} / \text{Mpc}^3 \sim 10^{-23} \text{ g/m}^3$$

scales in time as  $a^{-3}$

- Cosmological const. = *constant* energy-density

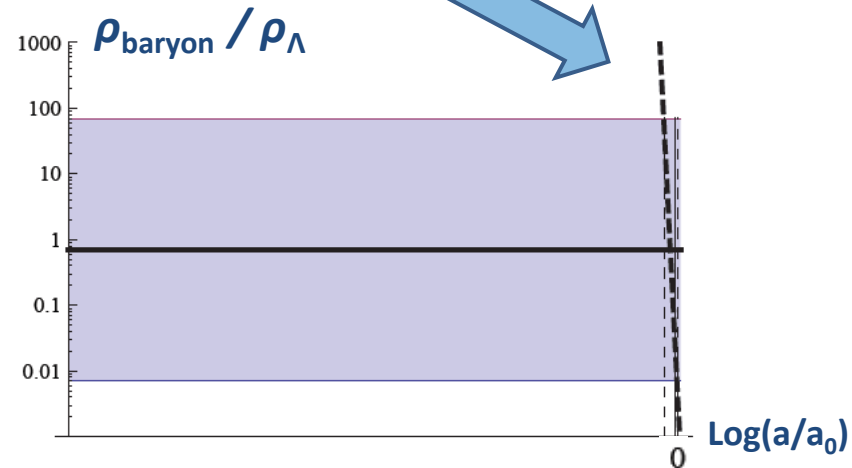
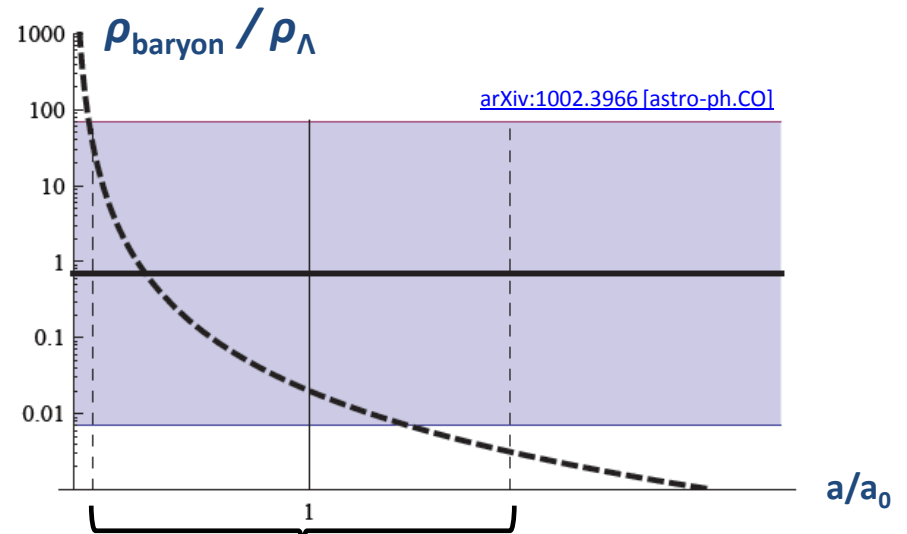
$$\rho_{\Lambda} \equiv \frac{\Lambda}{8\pi G_N} \simeq 10^{-23} \text{ g/m}^3$$

- Argument used to argue that the acceleration is in fact due to a fluid – *dark energy* – with density varying with time and with exotic eqn of state  $P = w(a) \rho$  ,  $w(a_0) < -1/3$



# Cosmic coincidence problem ?

- Unlikely coincidence ? Very brief phase?  
*Probability arguments are tricky and should be handled with care*
- Why should we assume equiprobability on a logarithmic scale?
- *Equiprobability in proper-time* (or in the scale factor  $a$ ) is more reasonable
- When this is done, the short phase lasts for half the life of the Universe  $t_H$
- Problem put back into perspective:
  - not a fine-tuned coincidence
  - issue of orders of magnitudes:  
why do we happen to live in an age of the Universe that is not many orders of magnitude smaller or larger?
- The cosmological principle cannot be applied uncritically: quite reasonable that we can live only during those few Gyr [Weinberg 1989, sec. V]



## Part 3.

The problem of QFT vacuum energies.

Do vacuum energies gravitate?



# QFT vacuum energy $\rho_{\text{vac}}$ and $\Lambda$

- In Quantum Field Theory (QFT), the **vacuum energy density** diverges quartically with the physical UV cut-off  $M_{UV}$

$$\rho_{\text{vac}} = \int_{|\vec{p}| < M_{UV}} \frac{d^3 \vec{p}}{(2\pi \hbar)^3} \frac{1}{2} \hbar \omega(\vec{p}) \sim \hbar^{-3} M_{UV}^4, \quad \langle 0 | T_{\mu\nu} | 0 \rangle = -\rho_{\text{vac}} g_{\mu\nu}$$

- Severe problem if it contributes to the effective cosmological constant:**

$$\Lambda_{\text{eff}} = \Lambda + 8\pi G_N \rho_{\text{vac}}$$

the hypothetical contribution is enormously larger than the observed value of  $\Lambda$

$$\Lambda_{\text{obs}} \approx (10^{26} \text{ m})^{-2}, \quad 8\pi G_N \rho_{\text{Pl}} \approx (10^{-35} \text{ m})^{-2}$$

- Open puzzle of QFT in the presence of gravity.**

The problem pertains to QFT, not to GR with  $\Lambda$ . It is a mistake to identify  $\Lambda$  with the vacuum energy. The problem does not affect the  $\Lambda$ CDM model.

- $\rho_{\text{vac}}$ , is it real? can it be measured? does it gravitate?
- Discussion 1:
  - Casimir effect
  - Vaporization heat in solids
  - Loop corrections to electrostatic energy in nuclei
- Discussion 2: Effective Field Theory, Naturalness, and subtraction

# Casimir Effect and Vacuum energy $\rho_{\text{vac}}$

- Casimir effect: often presented as an argument for the reality of vacuum energies

$$E(a) = \hbar^{-3} M_{UV}^4 V - \frac{\pi^2 \hbar}{720 a^4} V$$

$$= E_0 + \Delta E$$

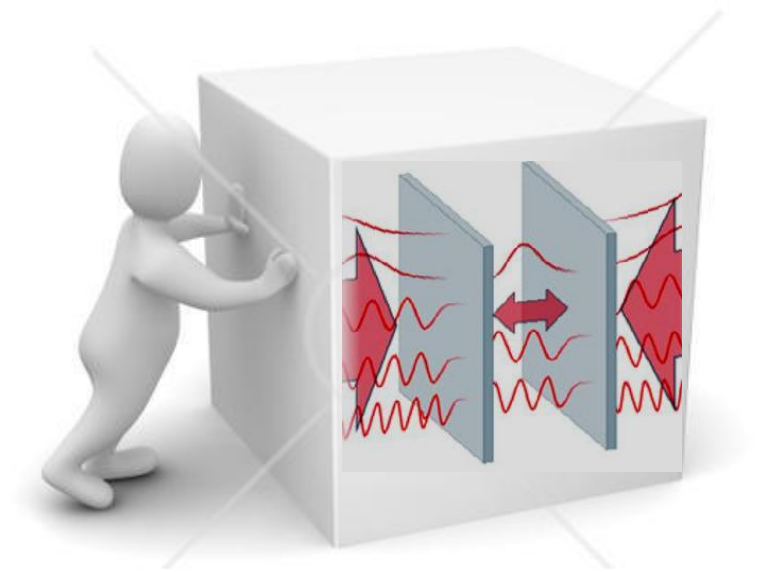
- In fact it probes only the dependence of  $\Delta E$  on external parameters = boundary conditions

$$F = -\frac{d}{da} \Delta E$$

- Physical argument : the term  $\hbar^{-3} M_{UV}^4 V$  does not contribute to the inertial mass of the box, and therefore does not gravitate
- Massless scalar field is a  $S^3$  Universe of radius  $a$  [Ford 1976]  
Static FRW background. Renormalized energy-momentum tensor, vacuum state, depends on  $a$

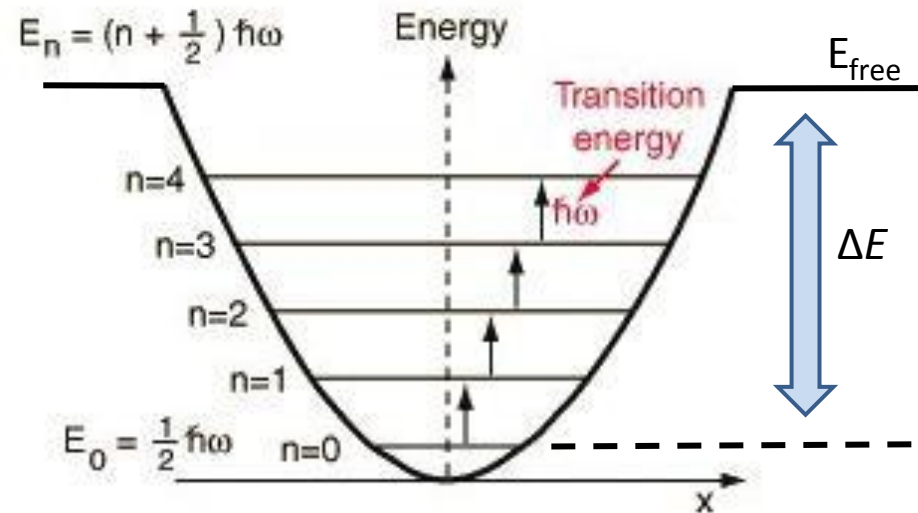
$$\rho(a) = \frac{\hbar}{480\pi^2 a^4} \quad , \quad P(a) = \frac{\hbar}{1440\pi^2 a^4}$$

- It does not behave as a cosmological constant,  **$P(a) = 1/3 \rho(a)$**



# $\frac{1}{2} \hbar \omega$ and the heat of vaporization in solids

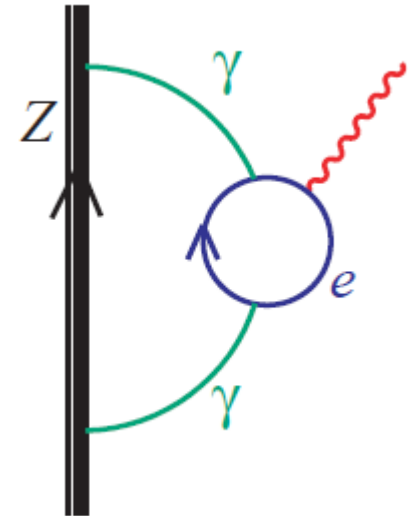
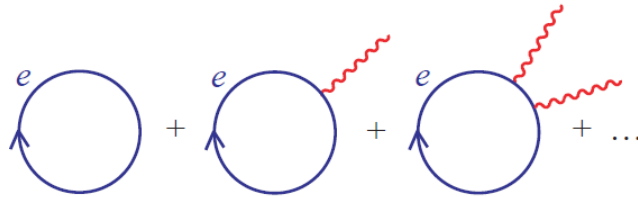
- 1911 , Nernst at 1st Solvay conference:  
*do the zero-point energy really exist?*  
is there a physical process that allows to measure the  $\frac{1}{2} \hbar \omega$  ?
- 1913, Stern, Born- VonKarman,  
in order to explain experimental data  
on the heat of complete vaporization of  
monoatomic solids, zero-point energy  
needs to be taken into account
- Notice that the process is only sensitive  
to the ionization energy,  
$$\Delta E = E_{\text{free}} - \frac{1}{2} \hbar \omega ,$$
  
not to the zero-point energy.



# Loop corrections to the *mass* of a nucleus

- [[Polchinski 2006](#)]: we know that in some situations, loop corrections do gravitate. E.g., electrostatic contribution to the mass of a nucleus.

Why the analogous loops should not gravitate in a vacuum environment?



- Electrostatic contribution to the mass of a nucleus:  $\Delta E = \frac{\alpha}{4\pi} \ln(m_e R_{\text{nuc}}) M_{\text{nuc}}$

for Aluminium  $\frac{\Delta E}{M_{\text{nuc}}} \simeq 10^{-3}$  , for Platinum  $\frac{\Delta E}{M_{\text{nuc}}} \simeq 3 \times 10^{-3}$

- From [[Dicke et al. 1964](#)], we know that Aluminium and Platinum have the same ratio of gravitational to inertial mass to one part in  $10^{12}$
- Notice: only the Log part of the loop correction contributes



# A cosmological constant problem in QFT ?

- Why the mass of the electron is about 0.5 MeV and not much larger?

why is  $\rho_\Lambda \equiv \frac{\Lambda}{8\pi G_N} \simeq 10^{-23} \text{ g/m}^3 \simeq (10^{-3} \text{ eV})^4$  and not much larger?

- The statement often heard, that QFT gives the *wrong* prediction for  $\Lambda$ , is not correct. It simply gives no prediction.
- Standard QFT allows to describe how coupling constants renormalize with the scale, not to *predict* their value
- To be compared to the current paradigm in the [hep-ph community](#) :  
**Effective Field Theory = QFT + naturalness principle**
- Fruitful point of view:
  - *physical cut-off* = scale of new physics
  - low-energy dimensionful constants have the scale of the cut-off ( unless they are protected by a symmetry )
- *Natural* value of the vacuum energy density (in flat space EFT)  
 $\rho_\Lambda \sim M_{UV}^4$  unless a symmetry protects it to 0
- *Does it gravitate?*

# Ode to Effective Field Theory reasoning

[ [Burgess 1998](#) ]  
[ [Polchinski 1992](#) ]

- Quadratic (and quartic?) divergences in QFT are interesting as they can be used to estimate the scale of new physics

- [ [Das et al 1967](#) ] Electromagnetic pion mass difference, 1-loop QED quadratic divergence  $\Rightarrow$  prediction of the mass of the  $\rho$
- [ [GIM1970](#) ] Quadratic divergent contribution to the mass difference of  $K_L$  and  $K_S$   $\Rightarrow$  prediction of the *charm*
- [ [201?](#) ] 1-loop quadratic divergence of the mass of the Higgs  $\Rightarrow$  new physics at the TeV
- *Quartic divergence of vacuum energy*  $\Rightarrow$  ???



$$m_{\pi^0} \simeq 145 \text{ MeV}$$

$$m_{\pi^\pm} \simeq 150 \text{ MeV}$$

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3}{16\pi^2} \alpha M_{UV}^2$$

$$M_{UV} \sim m_\rho \simeq 770 \text{ MeV}$$

- Dangerous to extend flat space arguments to curved space without the due care
- Notice that: for the EFT of gravitons + SM matter on flat Minkowski space, the term of dim 4 (cosmological constant) decouples from gravitons
- [ [Maggiore 2010](#) ]: EFT in FRW Universe. Vacuum energy with subtraction as for the ADM mass. Effect  $M_{UV}^2 H^2$  compatible with present constraints. It does not behave as a cosmological constant.

# General Relativity with a cosmological constant $\Lambda$

- Empirical adequacy:

The Lambda-Cold Dark Matter ( $\Lambda$ CDM) model is today the standard model in cosmology and is "almost universally accepted by cosmologists as the best description of the present data" [\[Lahav-Liddle, 'The particle data book' 2010\]](#)

- Theoretical consistency:

- $\Lambda$  is a completely natural part of General Relativity
- It is no-more no-less mysterious than any of the other numerous constants in fundamental theories
- There are open issues (e.g. why QFT vacuum energies do not contribute to  $\Lambda$ , what happens during phase transitions) but these pertain to other domains of physics, not to the problem of the *physical reason for the accelerated expansion of the Universe*

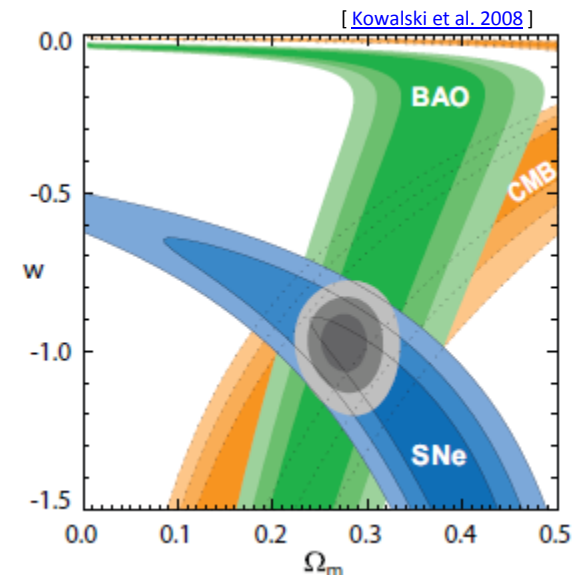
- It is important to **keep testing the  $\Lambda$  scenario**, and continue to explore alternative ideas. E.g. parameterizations

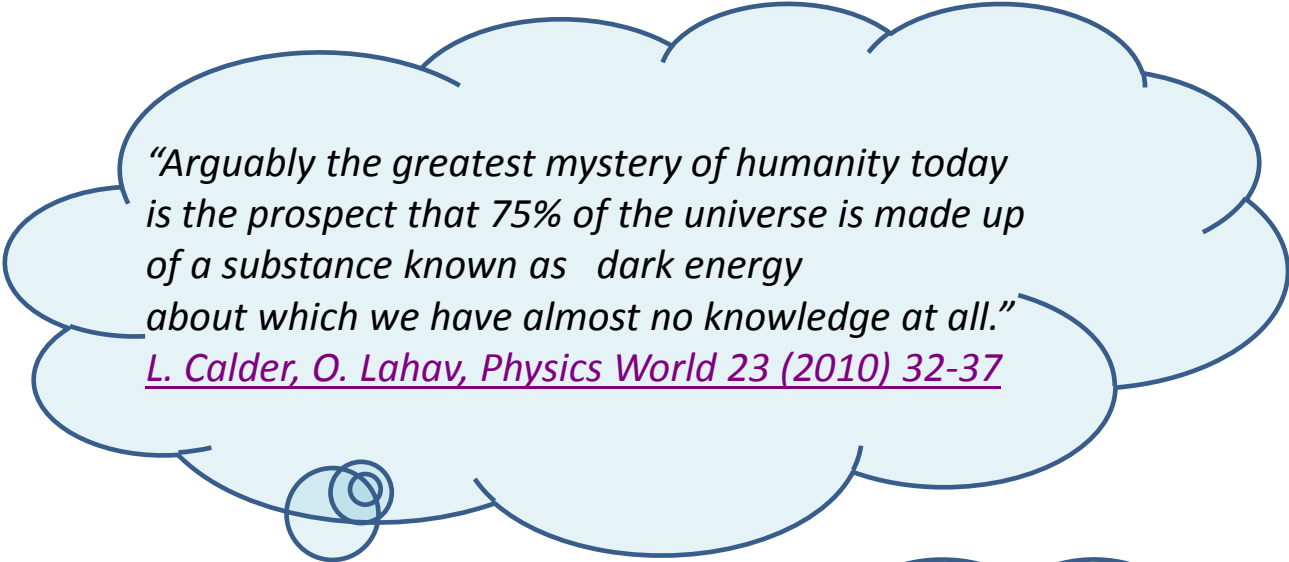
$$w(a) = w_0 + (1 \pm \frac{a}{a_0}) w_1$$

$$P = w(a) \rho$$

$$w_0 \simeq -1 \pm 0.2, \quad w_1 \simeq 0 \pm 1$$

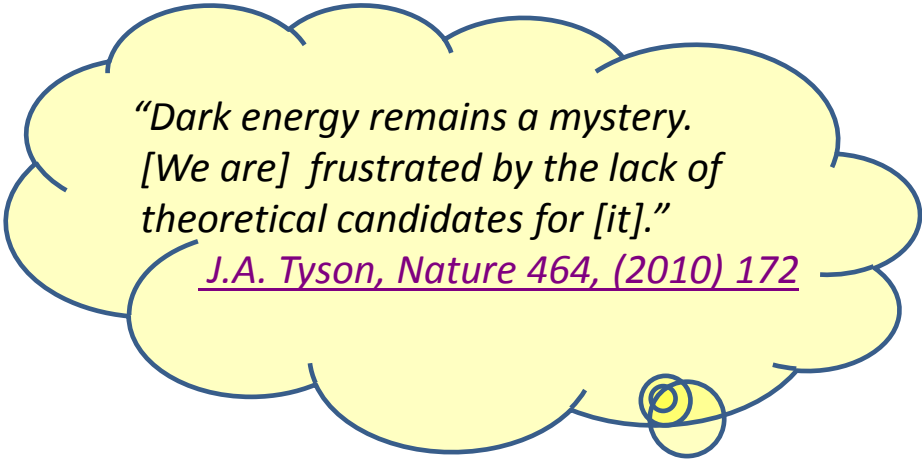
⇒ see [\[Kolb et al., The Dark Energy Task Force\]](#)





*"Arguably the greatest mystery of humanity today is the prospect that 75% of the universe is made up of a substance known as dark energy about which we have almost no knowledge at all."*

*L. Calder, O. Lahav, Physics World 23 (2010) 32-37*



*"Dark energy remains a mystery. [We are] frustrated by the lack of theoretical candidates for [it]."*

*J.A. Tyson, Nature 464, (2010) 172*

**This is exaggerated.**

# Summary

- We must keep testing the  $\Lambda$ CDM scenario and continue explore alternatives ideas.
- But to say "lack of theoretical candidates" or "great mystery" for the cosmic acceleration, is badly misleading.
- It is especially wrong to talk about  
"a substance"

It is like saying that the centrifugal force that pushes out from a merry-go-round is the effect of a "mysterious substance".

- The acceleration in the cosmic expansion is **well-understood and well-accounted for, by current fundamental physical theory.**
- It is not due to a vacuum energy or a mysterious substance: it is a long distance repulsive force due to the **intrinsic dynamics of geometry**.
- *No great mystery here!*

