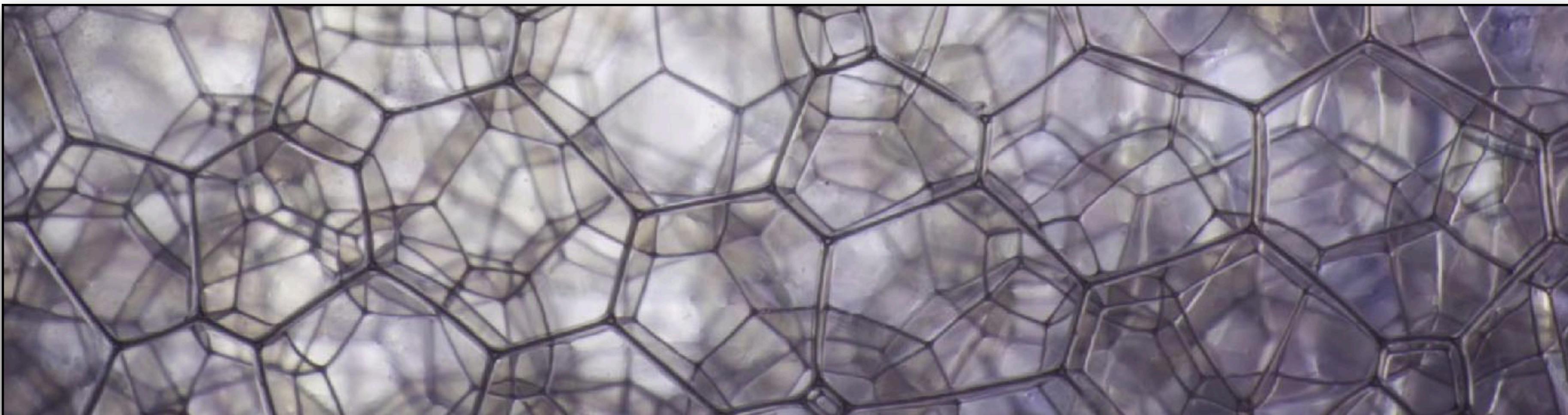


Entanglement in loop quantum gravity

Eugenio Bianchi
Penn State, Institute for Gravitation and the Cosmos



[soap foam - microphotography by Pyanek]

International Loop Quantum Gravity Seminar
Tuesday, Nov 7th — 2017

<http://relativity.phys.lsu.edu/ilqgs/>

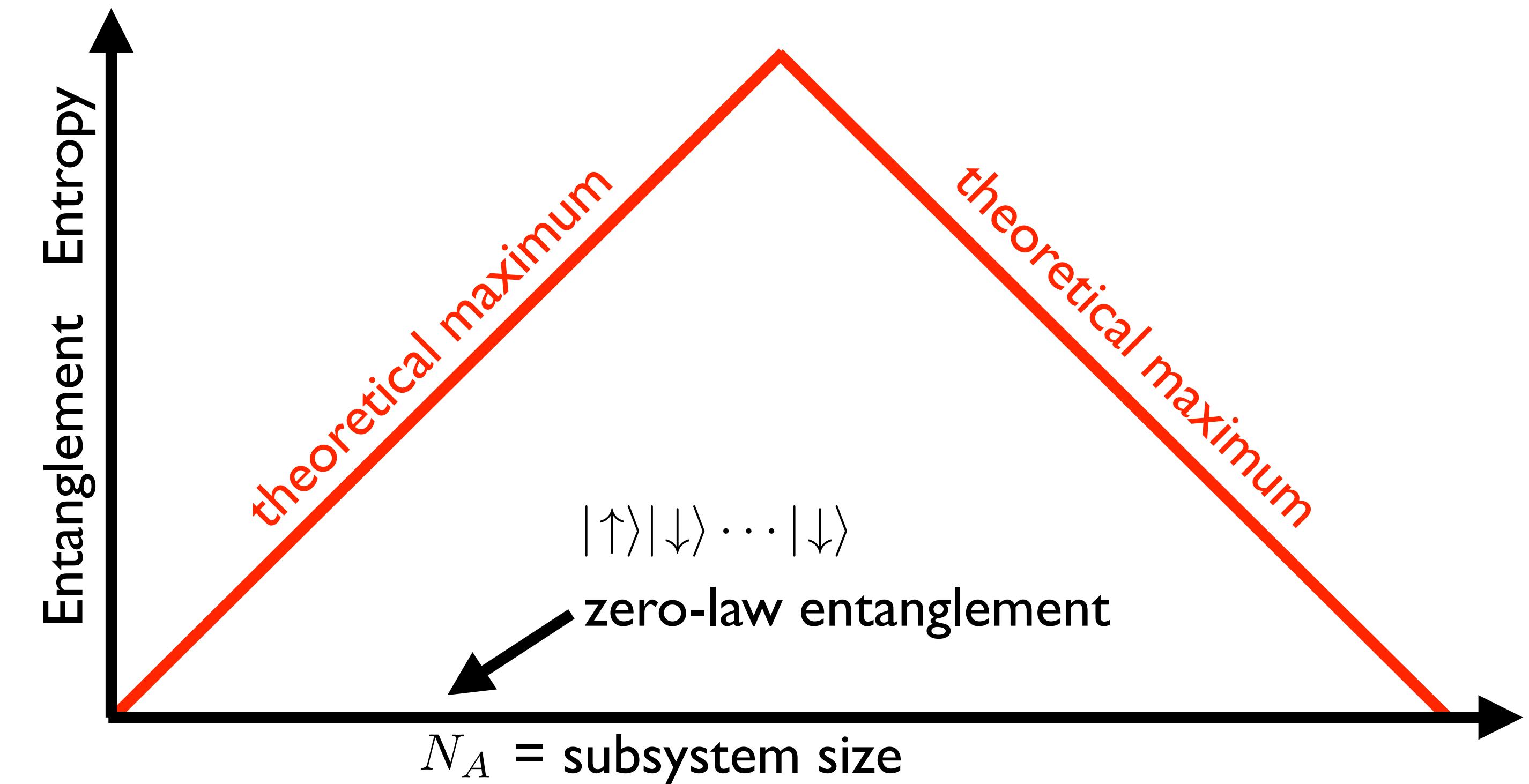
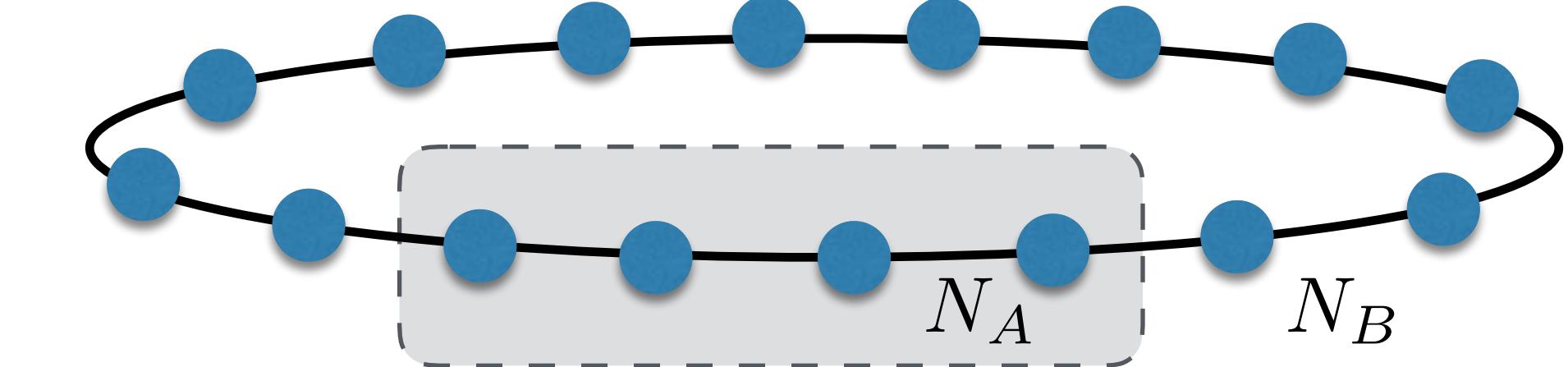
Entanglement as a probe of locality - e.g. 1d fermionic chain

Hilbert space: 2^N dimensional $\mathcal{H} = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2$

Geometric subsystem $\rho_A = \text{Tr}_B(|\psi\rangle\langle\psi|)$

Entanglement entropy $S_A(|\psi\rangle) = -\text{Tr}_A(\rho_A \log \rho_A)$

- 1) Factorized basis states $|\uparrow\rangle|\downarrow\rangle \cdots |\downarrow\rangle$
zero law



Entanglement as a probe of locality - e.g. 1d fermionic chain

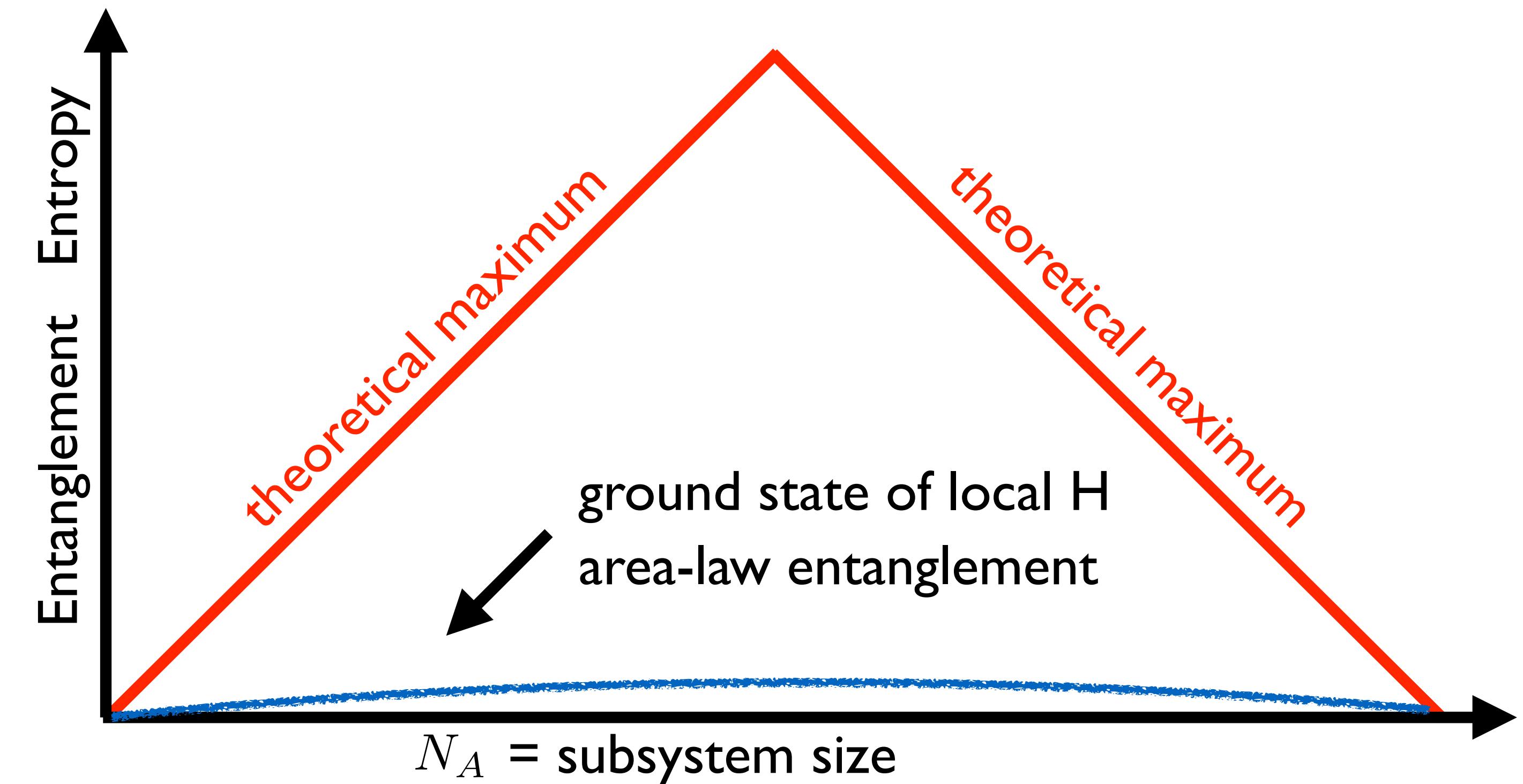
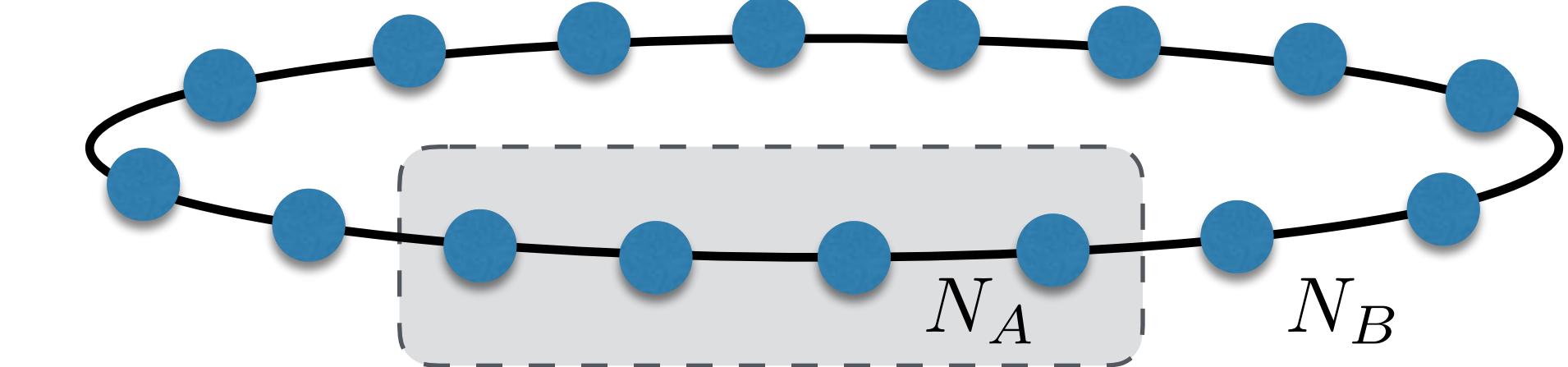
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zero law

2) Ground state of a local Hamiltonian
area law



[Sorkin (10th GRG) 1985]
[Srednicki, PRL 1993]

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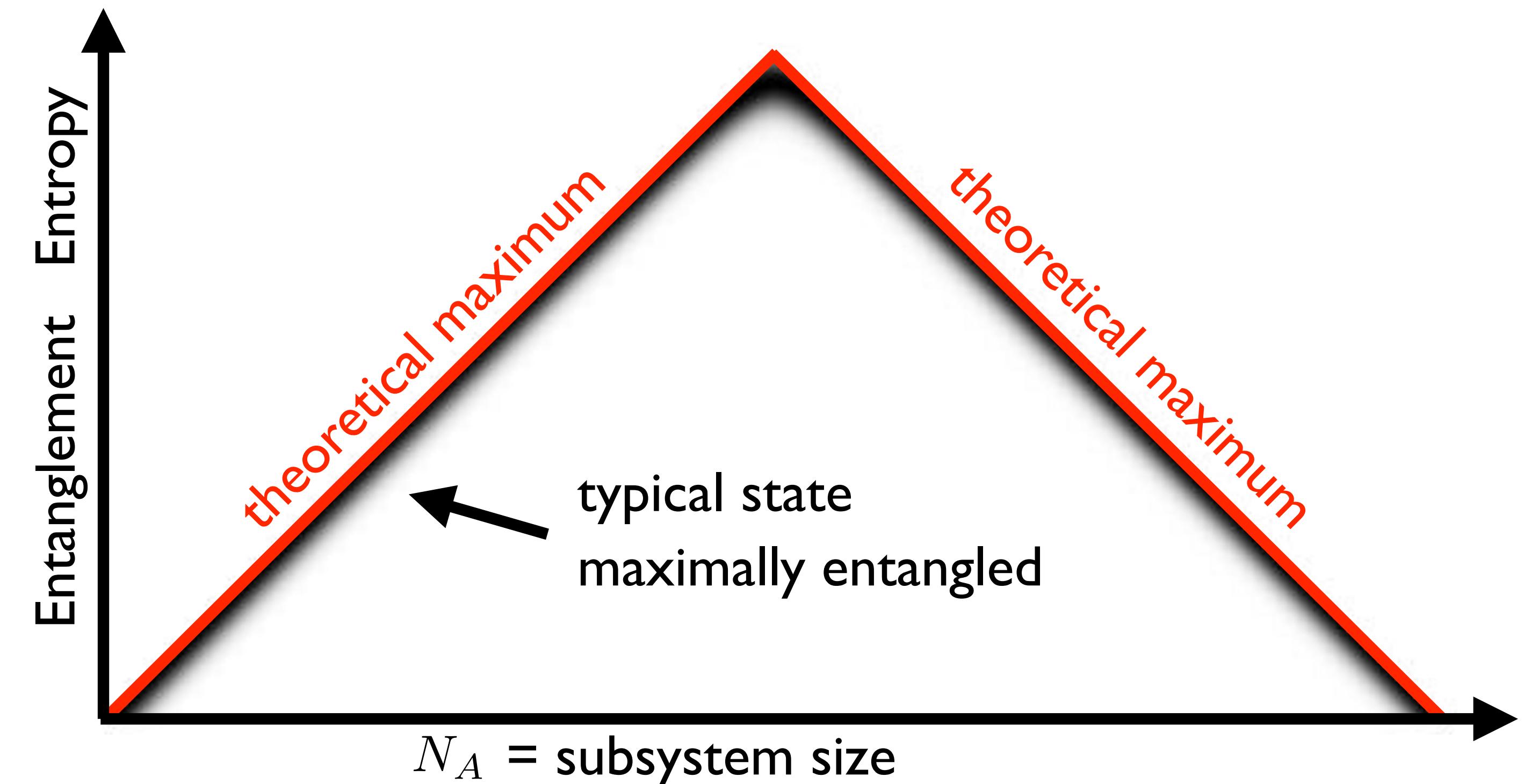
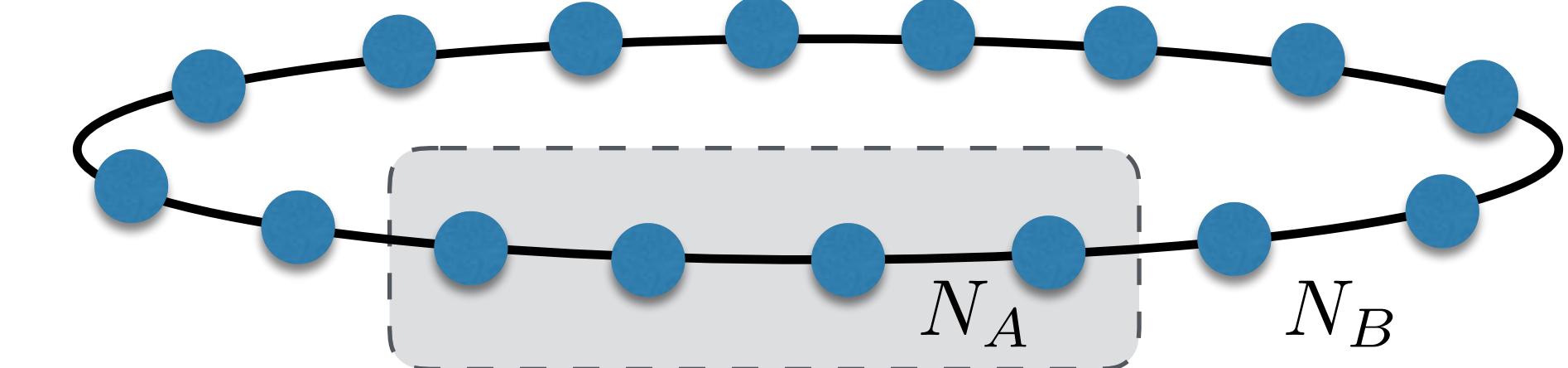
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volume law - maximally entangled



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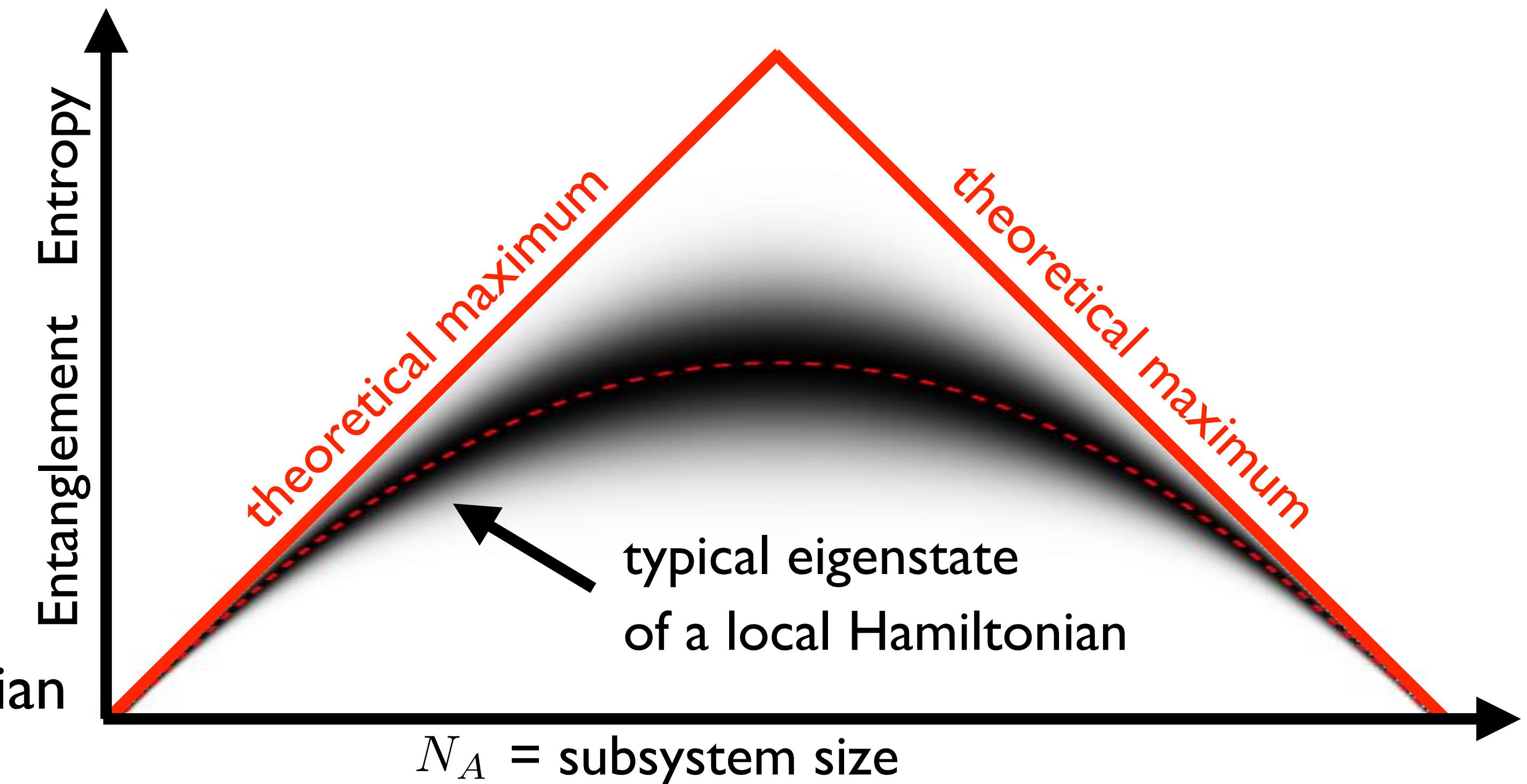
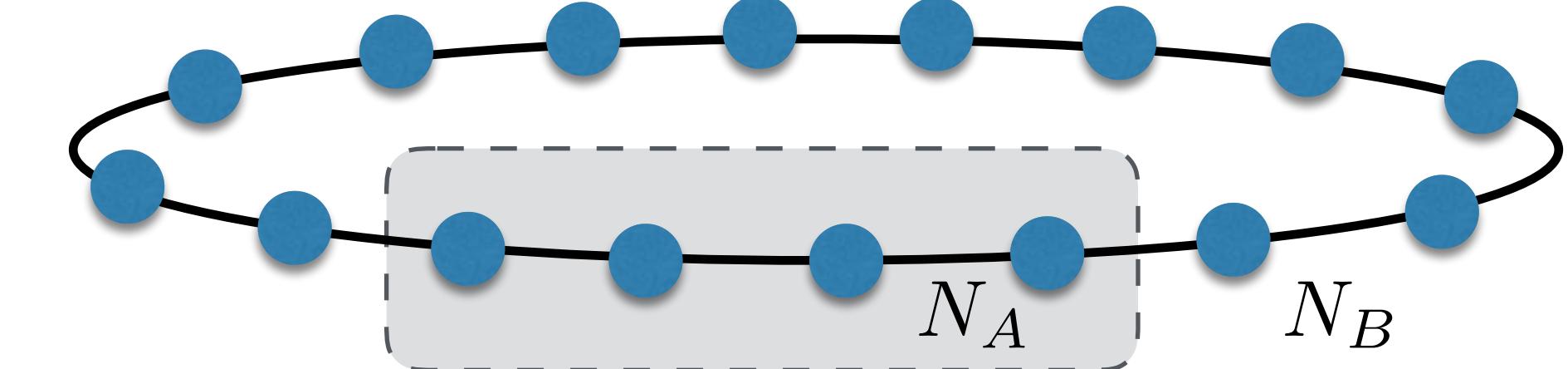
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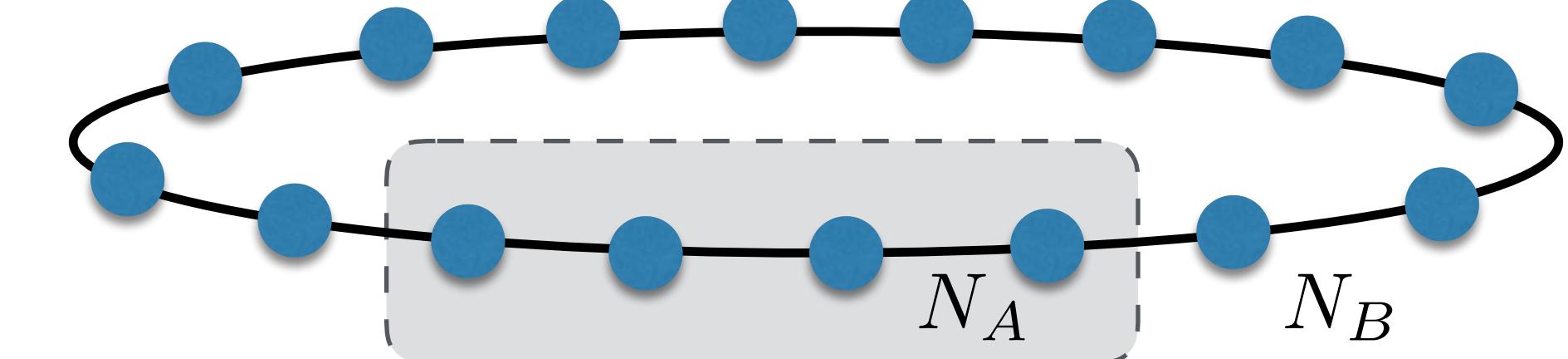


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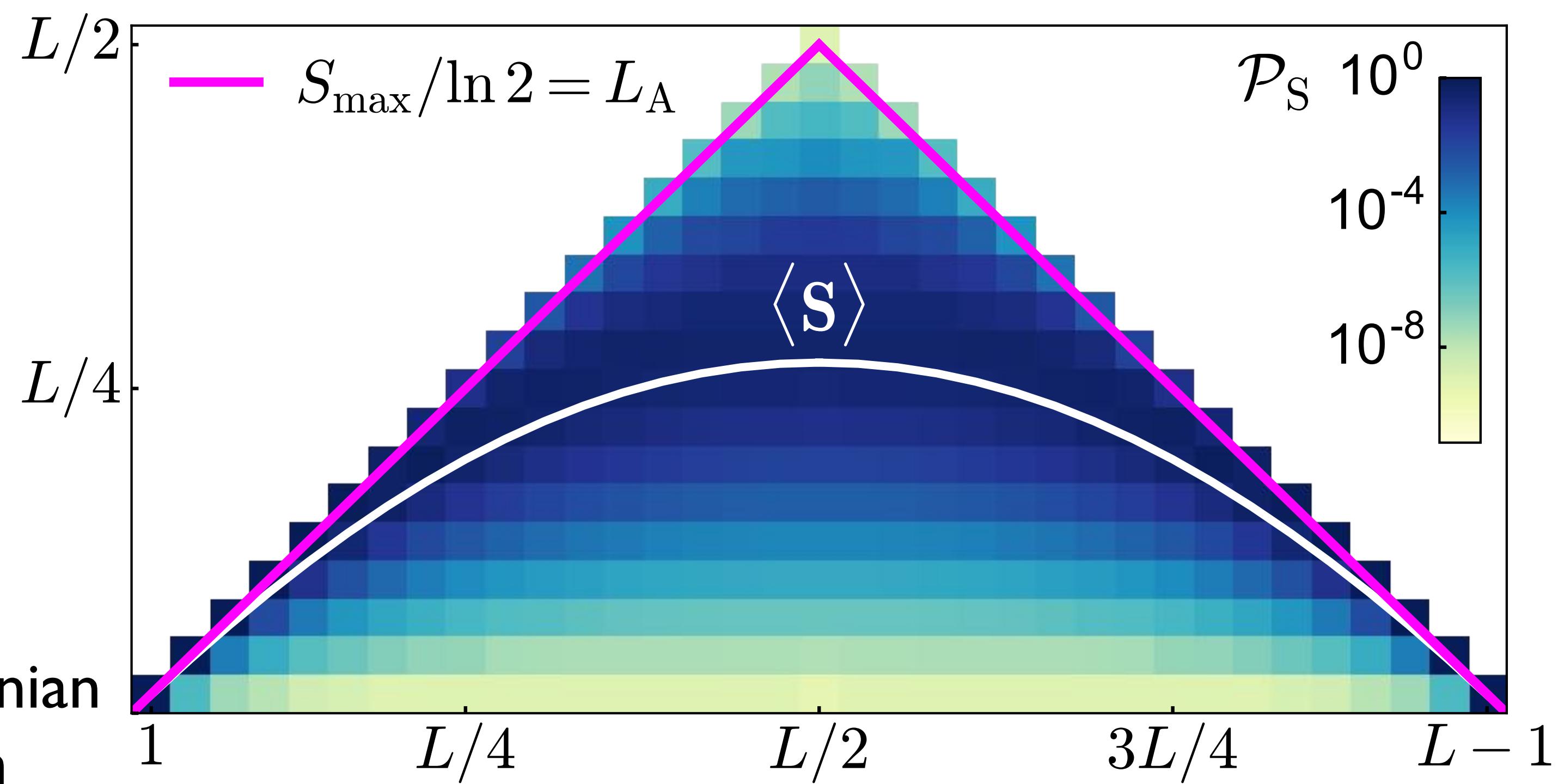


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Building space from entanglement

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Defining entanglement entropy in loop quantum gravity

Entanglement entropy

$$S_R(|\psi\rangle) = -\text{Tr}(\rho \log \rho)$$

[Ohyu-Petz book 1993]

characterizes the statistical fluctuations in a sub-algebra of observables

$$\mathcal{A}_R \subset \mathcal{A}$$

Two extreme choices of subalgebra:

a) Determine the algebra of Dirac observables of LQG,
then consider a subalgebra



difficult to use

b) Enlarge the Hilbert space of LQG to a bosonic Fock space,
then consider a bosonic subalgebra

[EB-Hackl-Yokomizo 2015]



*useful for
building space*

Other choices:

- In lattice gauge theory, trivial center sub-algebra

[Casini-Huerta-Rosalba 2013]

- Adding d.o.f. (on the boundary), electric center subalgebra

[Donnelly 2012] [Donnelly-Freidel 2016] [Anza-Chirco 2016]

- Intertwiner subalgebra (at fixed spin)

[Livine-Feller 2017]

[Chirco-Mele-Oriti-Vitale et al 2017] [Delocalp-Dittrich-Riello 2017]

- ...

Bosonic formulation of LQG on a graph

[also known as the *twistorial* formulation]

- Two oscillators per end-point of a link

spin from oscillators $|j, m\rangle = \frac{(a^{0\dagger})^{j+m}}{\sqrt{(j+m)!}} \frac{(a^{1\dagger})^{j-m}}{\sqrt{(j-m)!}} |0\rangle$

[Schwinger 1952]

- Hilbert space of LQG and the bosonic Hilbert space

$$L^2(SU(2)^L / SU(2)^N) \subset \mathcal{H}_{\text{bosonic}}$$

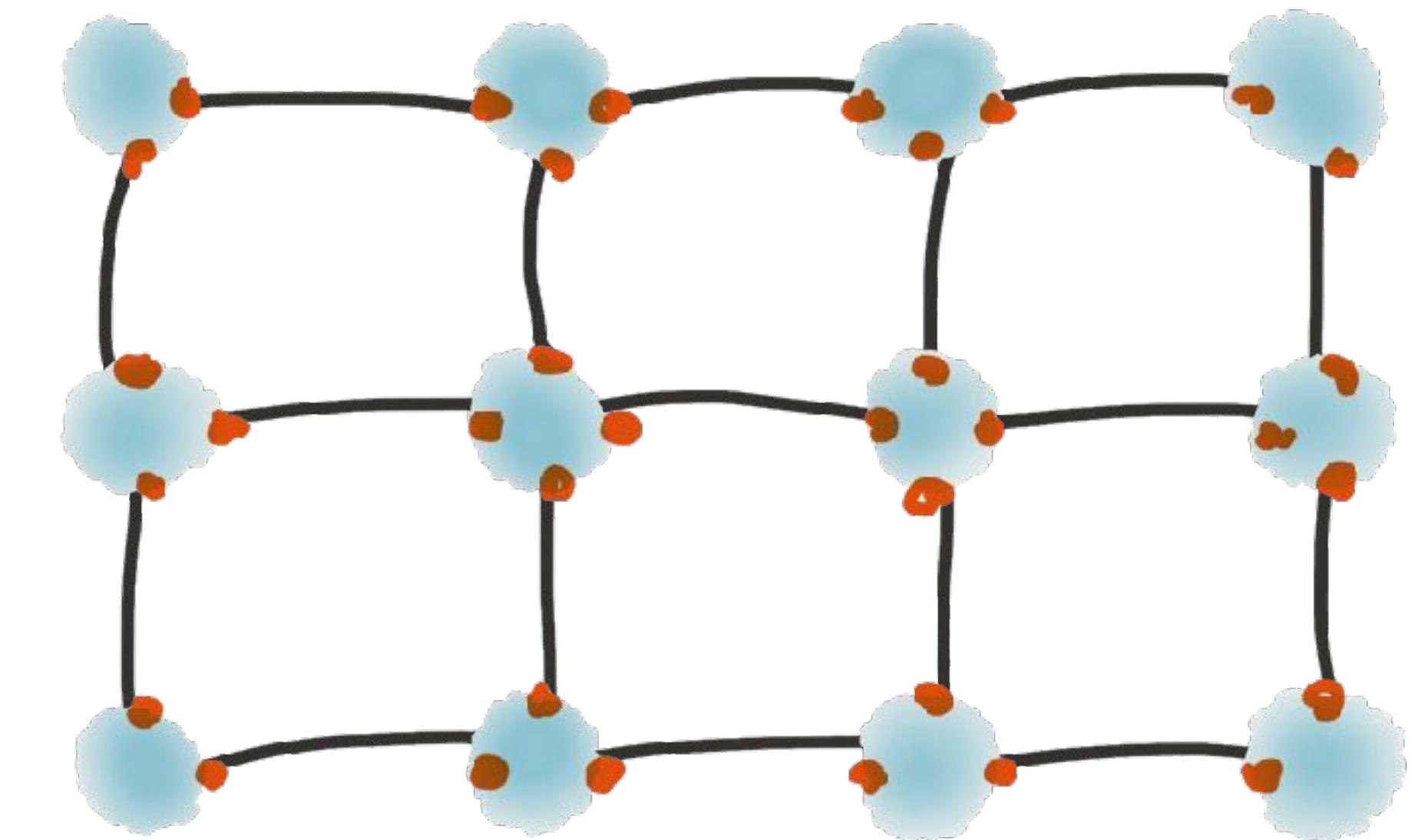
$$|\psi\rangle = \sum_{n_i=1}^{\infty} c_{n_1 \dots n_{4L}} |n_1, \dots, n_{4L}\rangle$$

[Girelli-Livine 2005] [Freidel-Speziale 2010]
[Livine-Tambornino 2011] [Wieland 2011]

[EB-Guglielmon-Hackl-Yokomizo 2016]

- The bosonic Hilbert space factorizes over nodes: easy to define and compute the entanglement entropy

- Geometric operators in a region R of the graph generate a subalgebra $\mathcal{A}_R^{\text{LQG}} \subset \mathcal{A}_R^{\text{bosonic}}$



Entanglement entropy of a bosonic subalgebra A

- Spin-network state $|\Gamma, j_l, i_n\rangle$

factorized over nodes

no correlations, zero entanglement entropy in A

- Coherent states $P|z\rangle = P e^{z_A^i a_i^{A\dagger}} |0\rangle$

not factorized over nodes *only because of the projector P*

exponential fall off of correlations

area law from Planckian correlations only

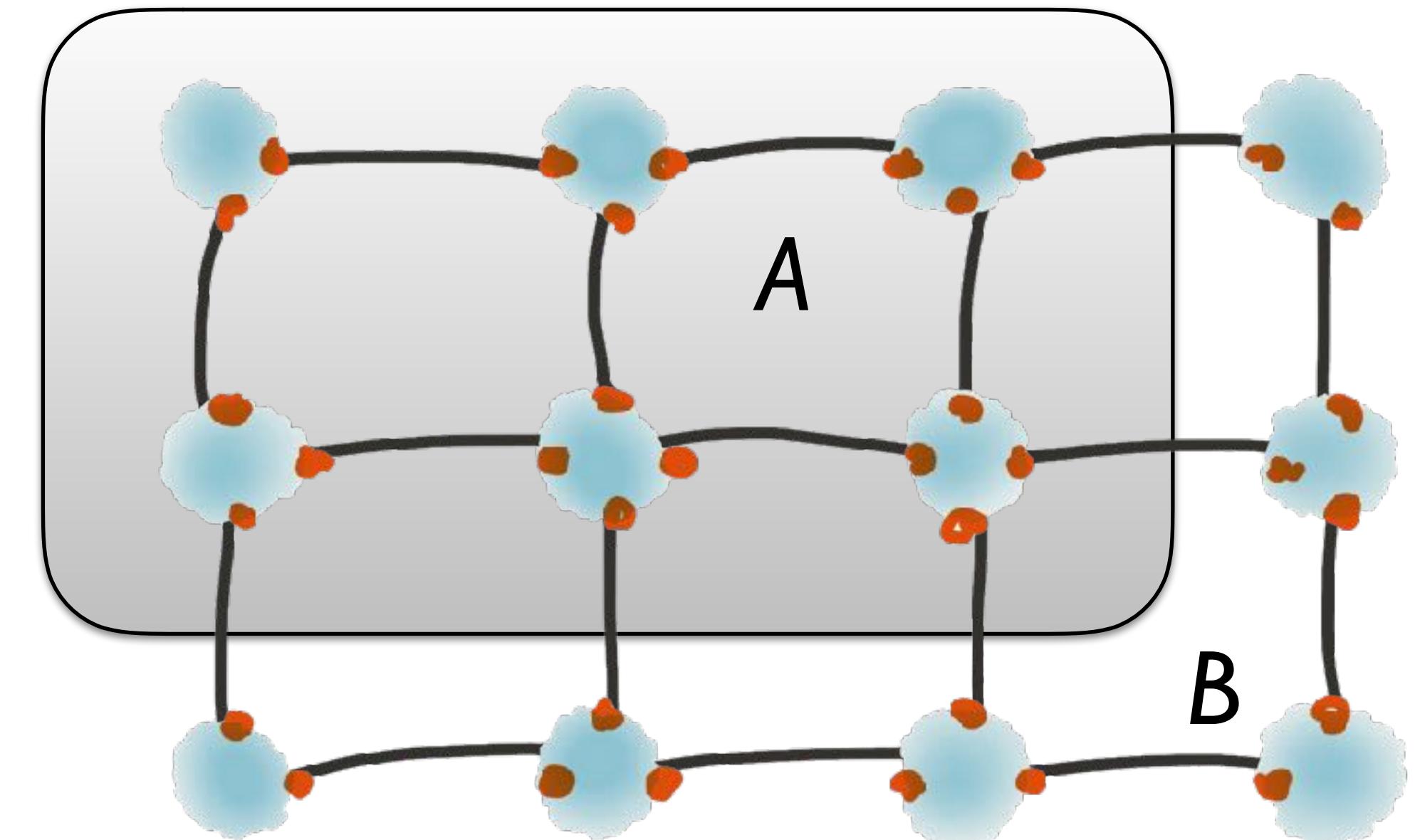
- Squeezed states $P|\gamma\rangle = P e^{\gamma_{AB}^{ij} a_i^{A\dagger} a_j^{B\dagger}} |0\rangle$

not factorized over nodes because of the projector P and because of off-diag. terms in γ_{AB}^{ij}

long-range correlations from γ_{AB}^{ij}

efficient parametrization of a corner of the Hilbert space characterized by correlations

zero-law, area-law, volume-law entanglement entropy depending on γ_{AB}^{ij}

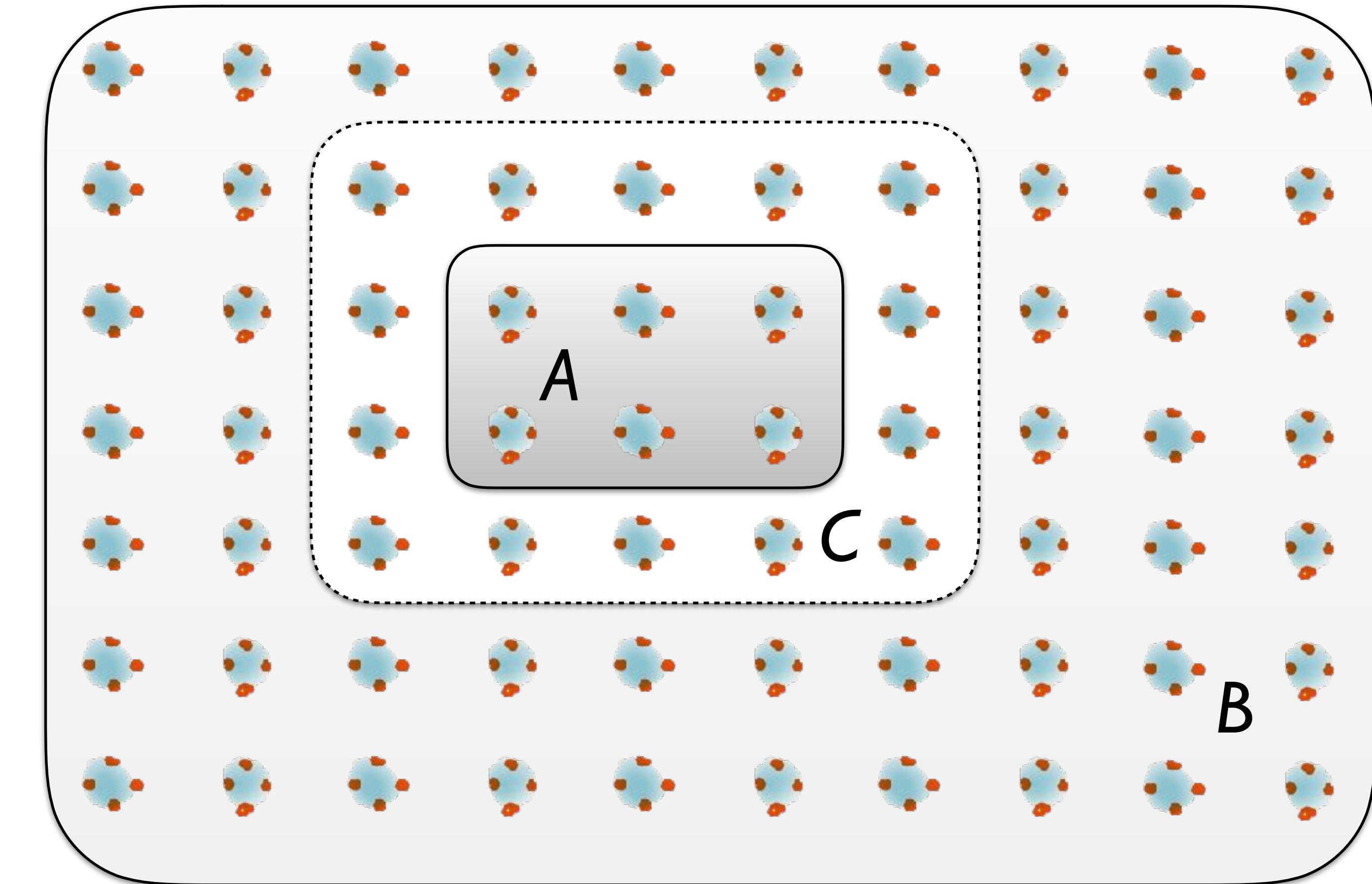


Area law: formulation relevant for semiclassical physics and QFT

Entanglement between region A and B
with buffer zone C

$$\begin{aligned} S_{\text{geom}} &= \frac{1}{2} S(\rho_{AB} | \rho_A \otimes \rho_B) \\ &= \frac{1}{2} \text{Tr}(\rho_{AB} \log \rho_{AB} - \rho_{AB} \log(\rho_A \otimes \rho_B)) \\ &\sim \text{Area}(\partial A) \end{aligned}$$

Area law for squeezed state with long-range
correlations encoded in γ_{AB}^{ij}



The area-law is non generic in loop quantum gravity, property of semiclassical states
The entanglement entropy provides a probe of the architecture of a quantum spacetime

Entanglement and the architecture of a spacetime geometry

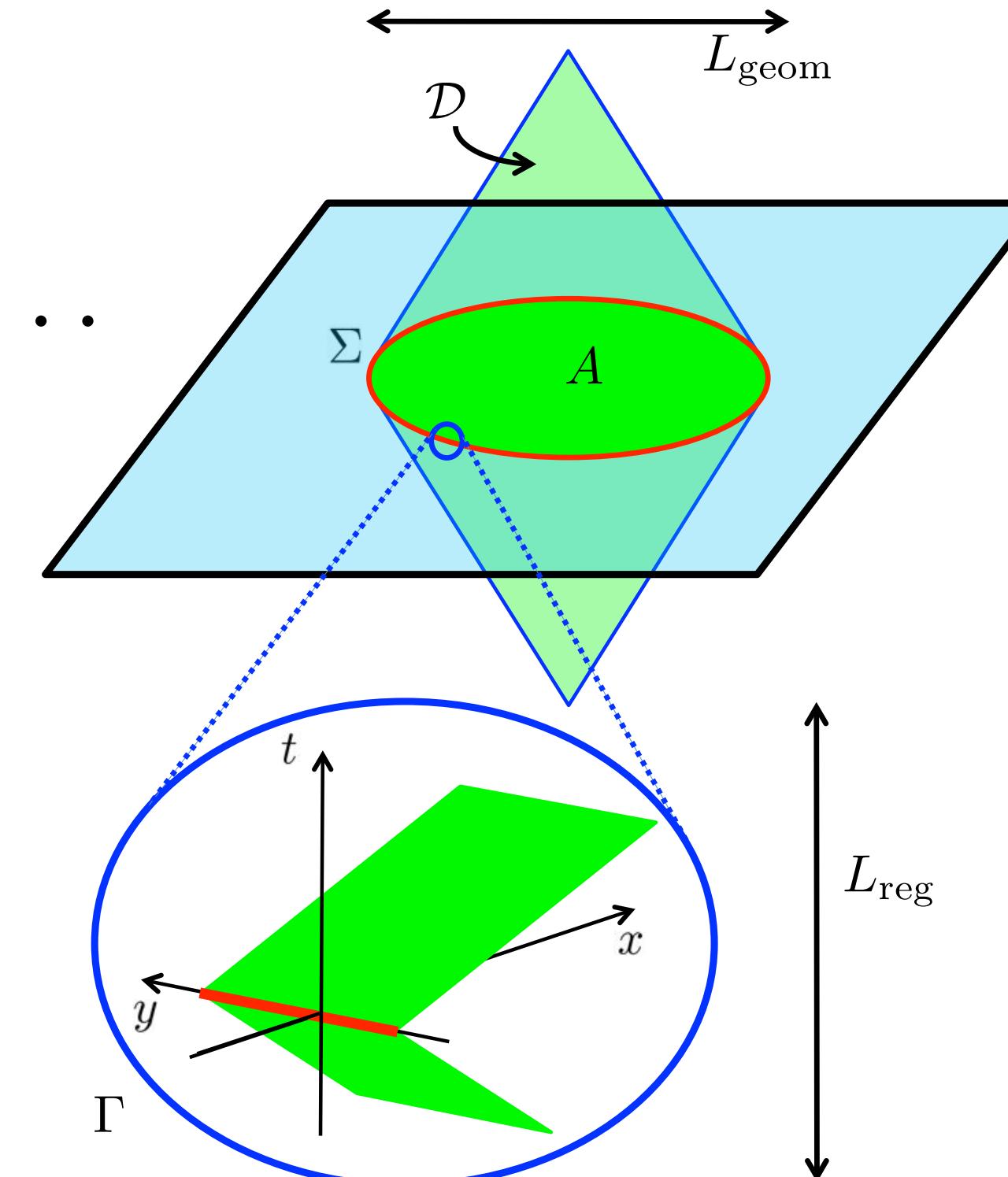
- Entanglement entropy as a probe of the architecture of spacetime

Area-law not generic, property of semiclassical states

[EB-Myers 2012]

“On the Architecture of Spacetime Geometry”

$$S_A(|0\rangle) = 2\pi \frac{\text{Area}(\partial A)}{L_{\text{Planck}}^2} + \dots$$



Arguments from: Black hole thermodynamics (Bekenstein, Hawking, Sorkin,...)

Holography and AdS/CFT (Maldacena,.. Van Raamsdonk,.. Ryu, Takayanagi,...)

Entanglement equilibrium (Jacobson)

Loop quantum gravity (EB)

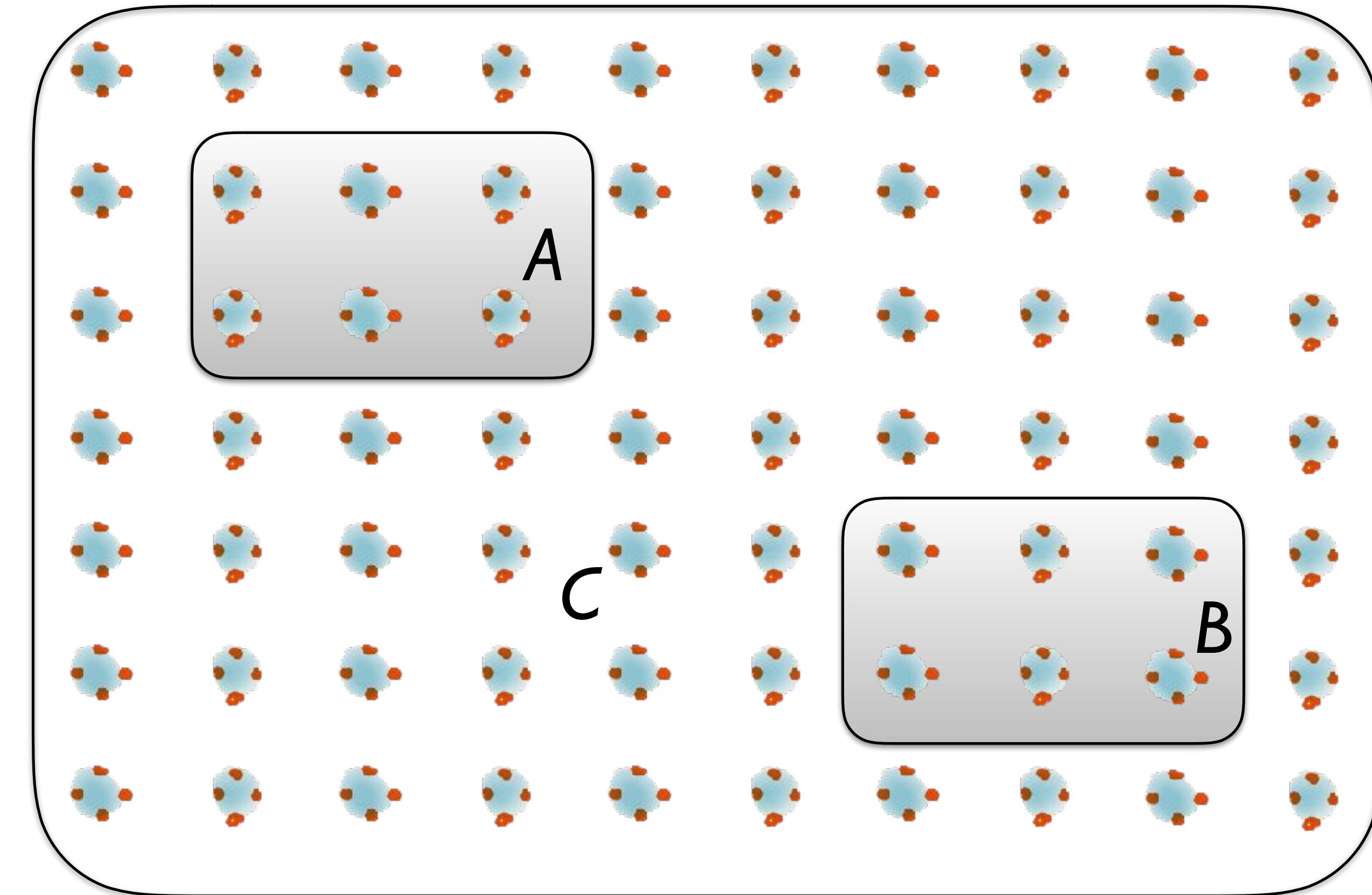
Long-range correlations and the bosonic mutual information

- Macroscopic observables in region A and B
- Correlations bounded by relative entropy of A, B

$$\frac{(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle)^2}{2 \|\mathcal{O}_A\|^2 \|\mathcal{O}_B\|^2} \leq I(A, B)$$

where

$$I(A, B) \equiv S(\rho_{AB} | \rho_A \otimes \rho_B) = S_A + S_B - S_{AB}$$



The bosonic formulation is useful because it allows us to define and compute the mutual information $I(A, B)$

This quantity bounds from above the correlations of all LQG-geometric observables in A and B

Building space from entanglement

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Gluing quantum polyhedra with entanglement

- Fluctuations of nearby quantum shapes are in general uncorrelated: twisted geometry

[Dittrich-Speziale 2008] [EB 2008]
[Freidel-Speziale 2010]
[EB-Dona-Speziale 2010]
[Dona-Fanizza-Sarno-Speziale 2017]

- Saturating uniformly the short-ranged relative entropy

$$\frac{\left(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle \right)^2}{2 \|\mathcal{O}_A\|^2 \|\mathcal{O}_B\|^2} \leq I(A, B)$$

where

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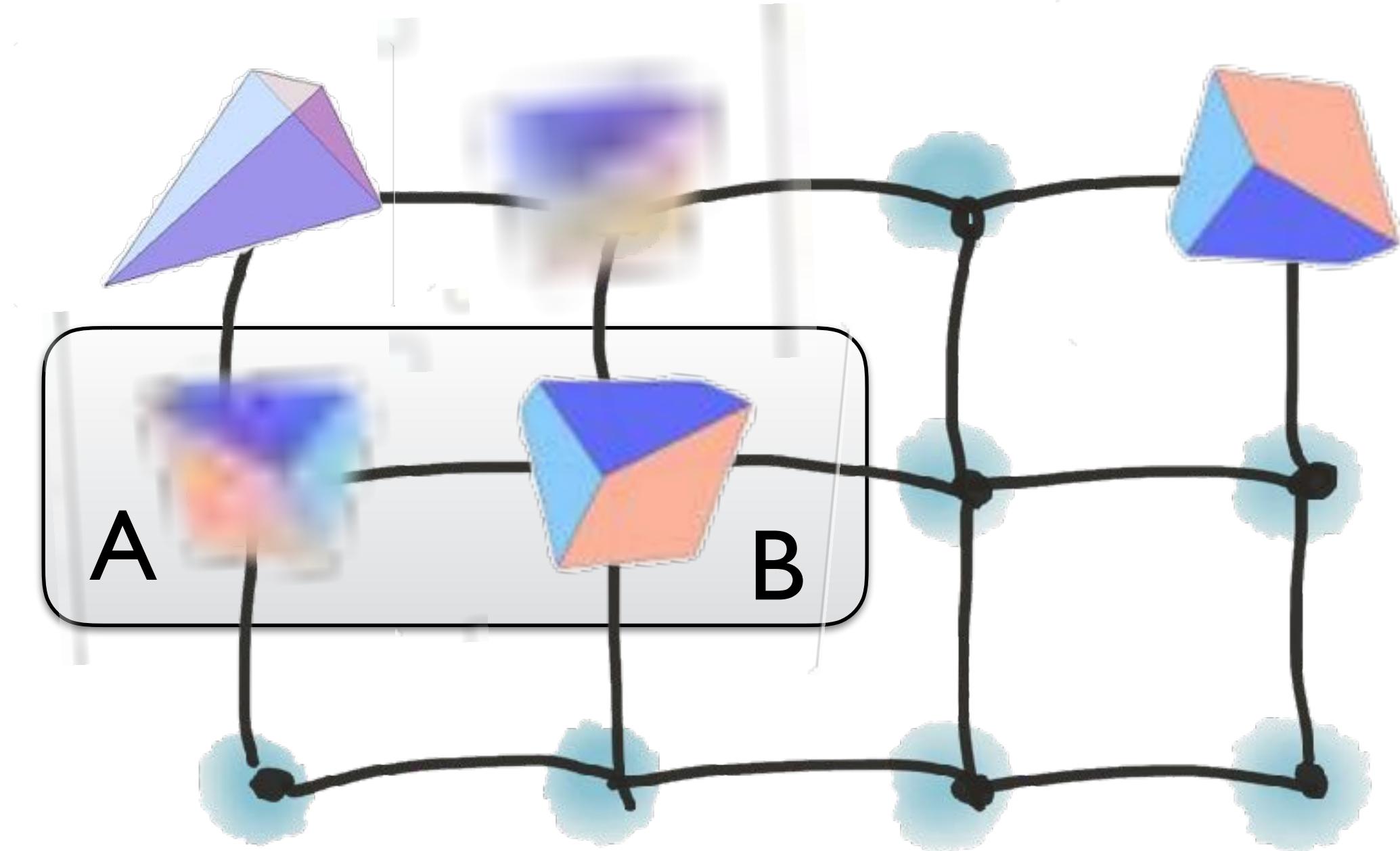
correlates fluctuations of the quantum geometry

State with

$$\max_{\langle A, B \rangle} \sum I(A, B)$$

Glued geometry from entanglement

[EB-Baytas-Yokomizo, to appear]



Building space from entanglement

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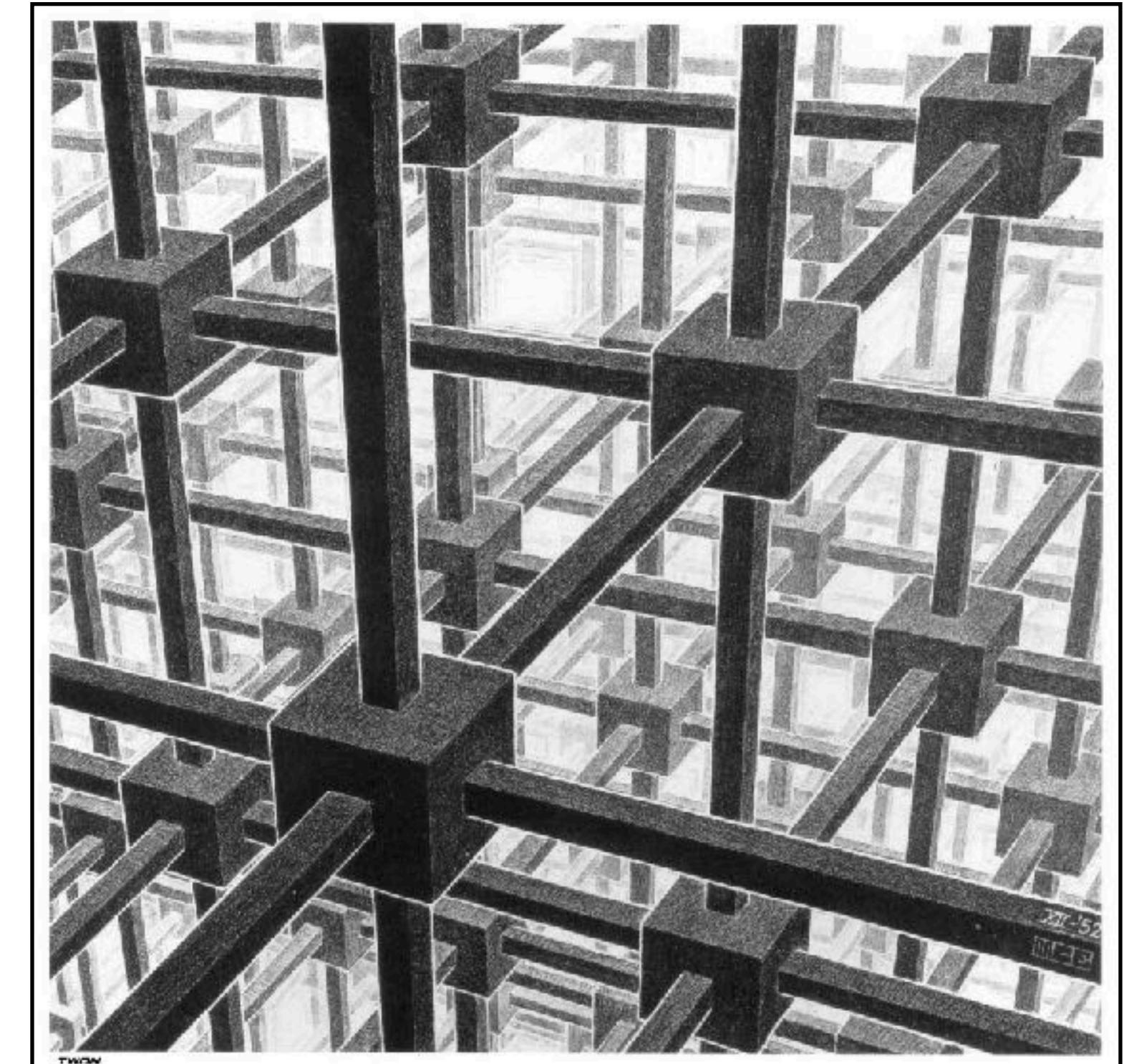
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Lorentz invariance in LQG

- Discrete spectra are not incompatible with Lorentz invariance [Rovelli-Speziale 2002]
- Lorentz invariant state in LQG ?
 - 1) Minkowski geometry as expectation value
 - 2) Lorentz-invariant 2-point correlation functions, 3-point...
- Homogeneous and isotropic states in LQG ? similarly (1), (2)

Strategy: double-scaling encoded in the state

- use squeezed states defined in terms of 1- and 2-point correlations
- graph, e.g. cubic lattice with N nodes
- choose the diagonal entries of the squeezing matrix γ_{AB}^{ij} to fix the expectation value of the spin $\langle j \rangle$
- choose the off-diagonal entries of γ_{AB}^{ij} to fix the correlation function $\mathcal{C} = \langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle$ at a lattice distance n_0
- the correlation function can be expressed in terms of the physical length $\ell \sim n_0 \sqrt{\langle j \rangle}$
- take the limit of the squeezed state $|\gamma\rangle$ such that $\langle j \rangle \rightarrow 0$, $n_0 \rightarrow \infty$ with $\mathcal{C}(\ell)$ fixed
- the limit can be studied at fixed physical volume $V \sim N (\sqrt{\langle j \rangle})^3$, with symmetries imposed on $\mathcal{C}(\ell)$



Escher 1953

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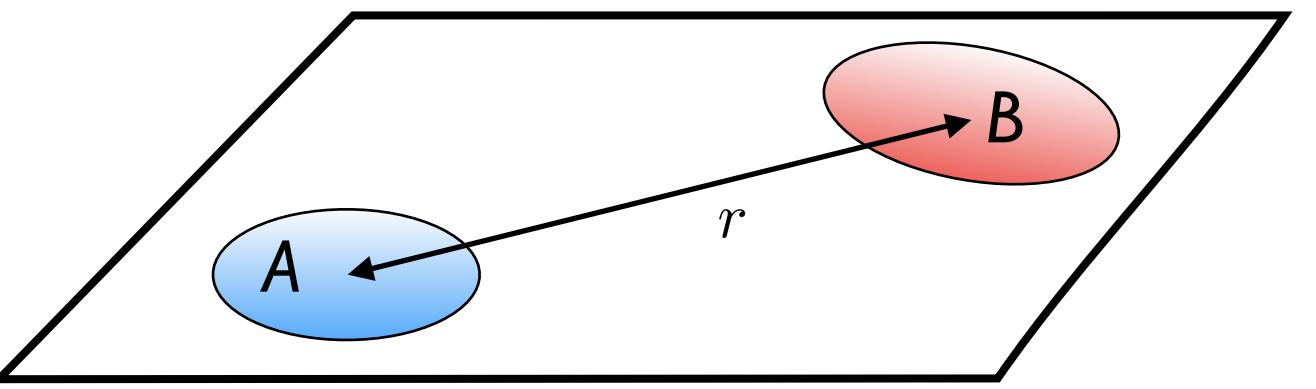
Correlations at space-like separation

- In quantum field theory

Fock space $\mathcal{F} = \mathbb{C} \oplus \mathcal{H} \oplus S(\mathcal{H} \otimes \mathcal{H}) \oplus \dots$

contains

- ~~(i) states with no space-like correlations~~
- { (ii) states with specific short-ranged correlations (e.g. Minkowski vacuum)
crucial ingredient for quantum origin of cosmological perturbations



- In loop quantum gravity

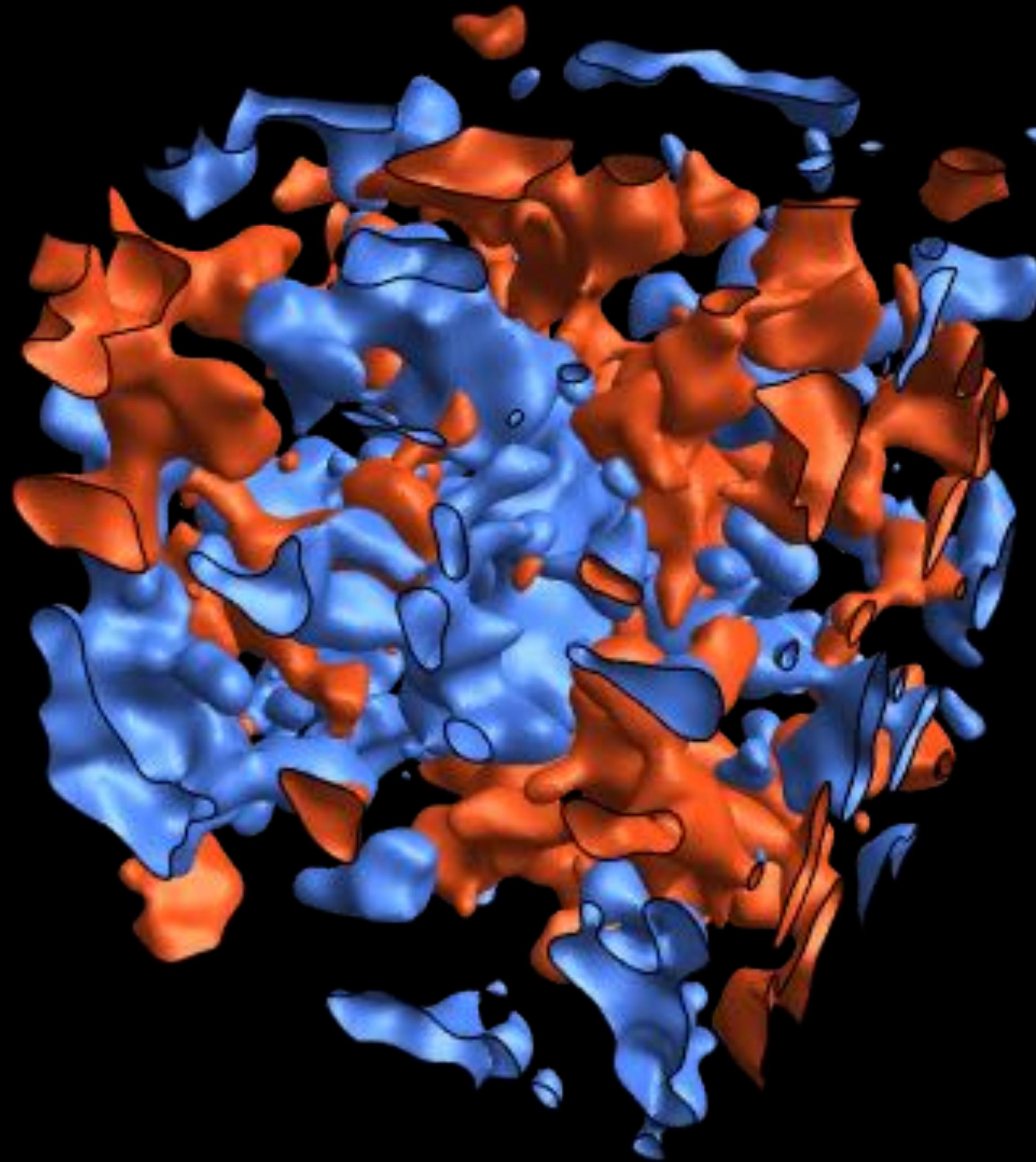
Hilbert space $\mathcal{H}_\Gamma = L^2(SU(2)^L / SU(2)^N)$

contains

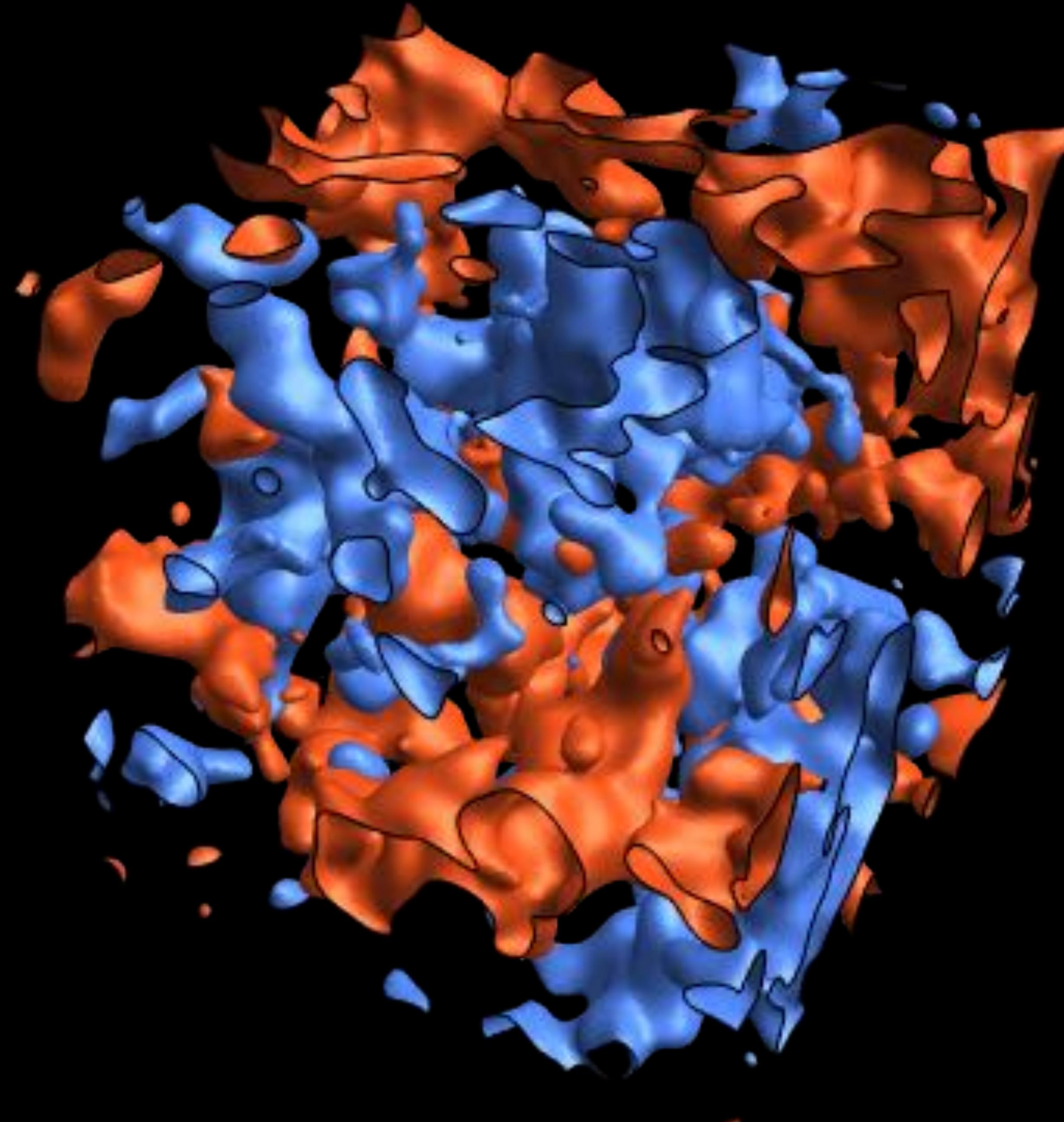
- { (i) states with no space-like correlations (e.g. spin-networks)
(ii) states with long-range space-like correlations (e.g. squeezed vacua)

The Vacuum

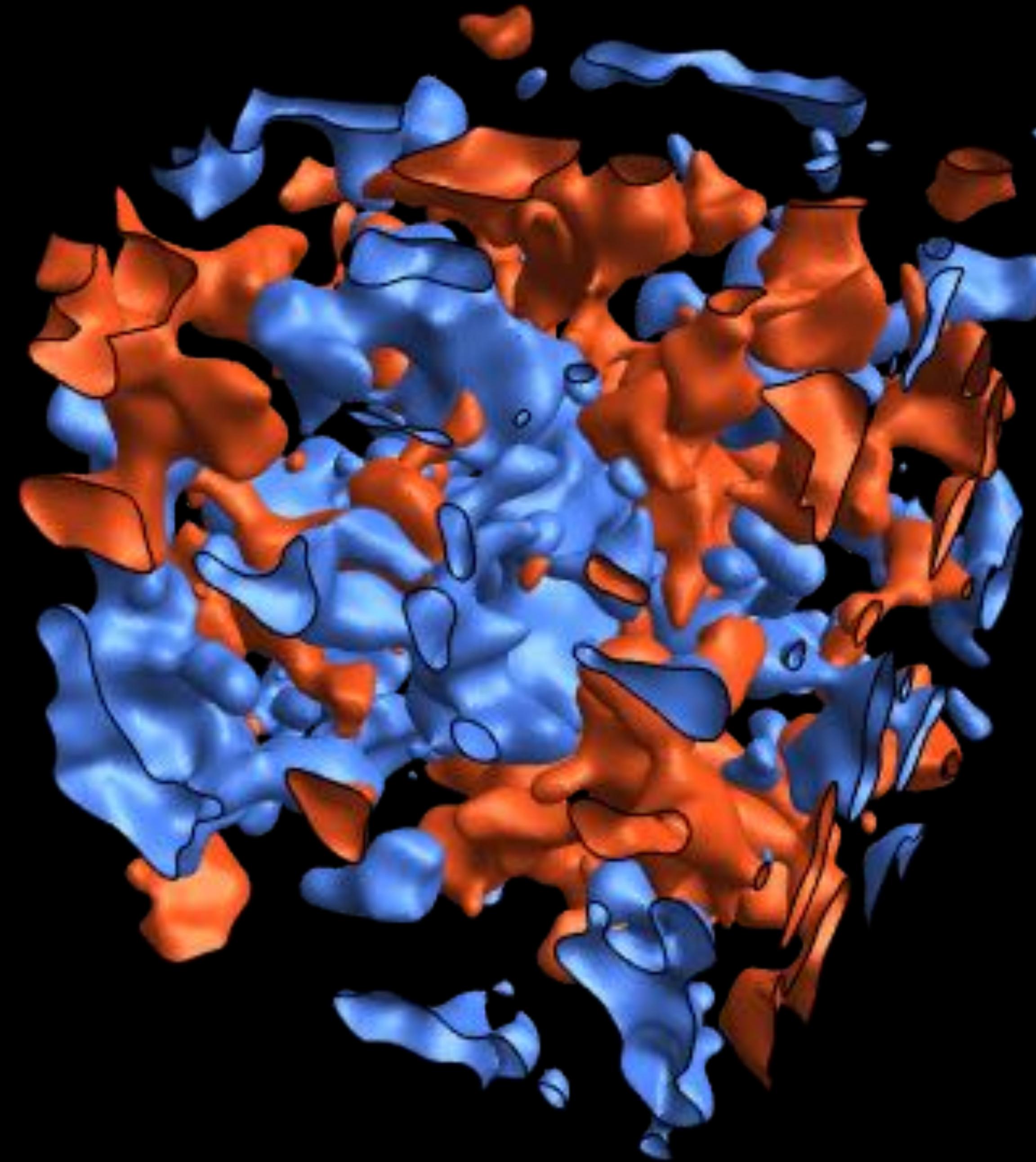
The Vacuum State of a Quantum Field



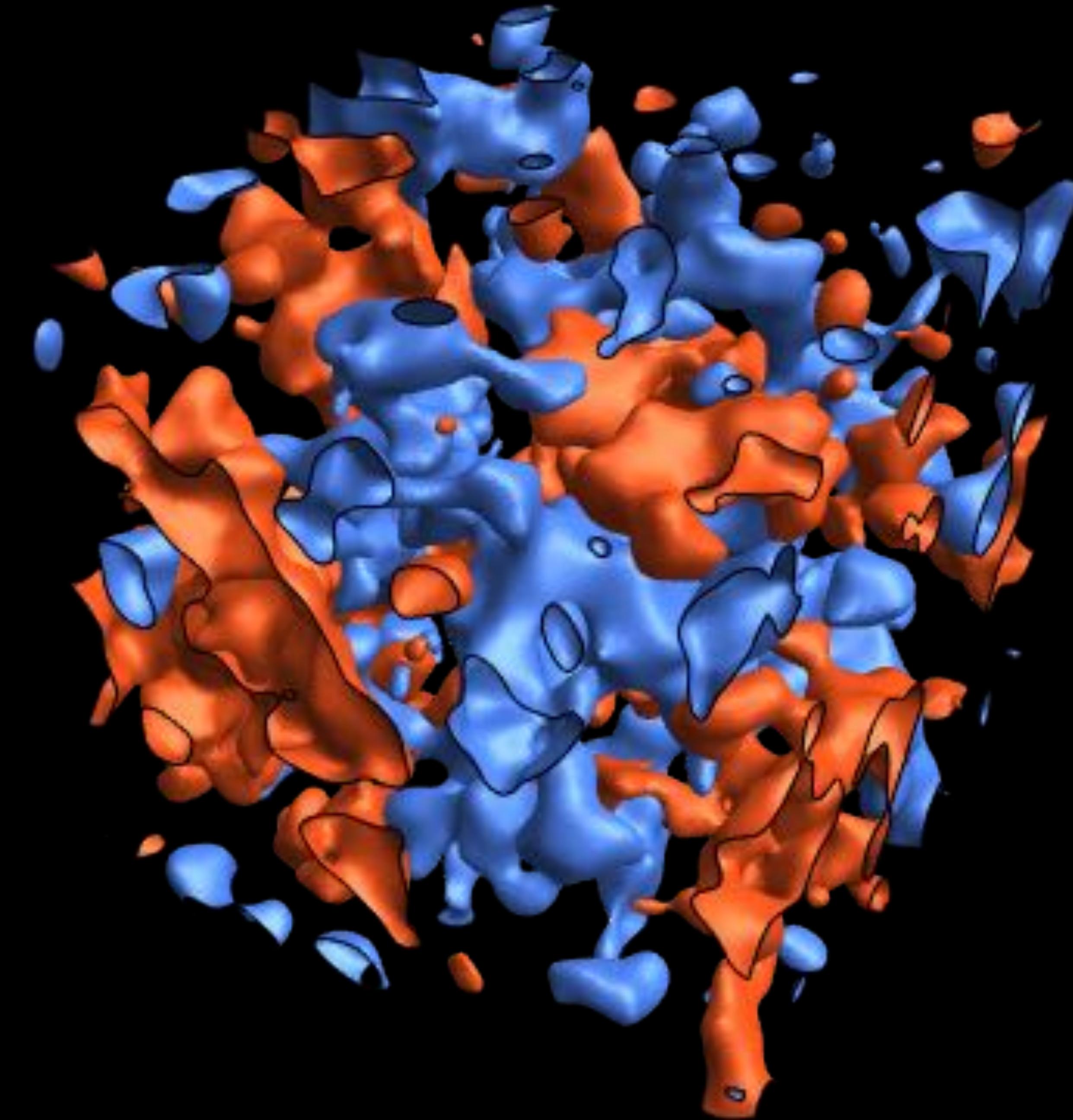
The vacuum state of a quantum field is highly entangled



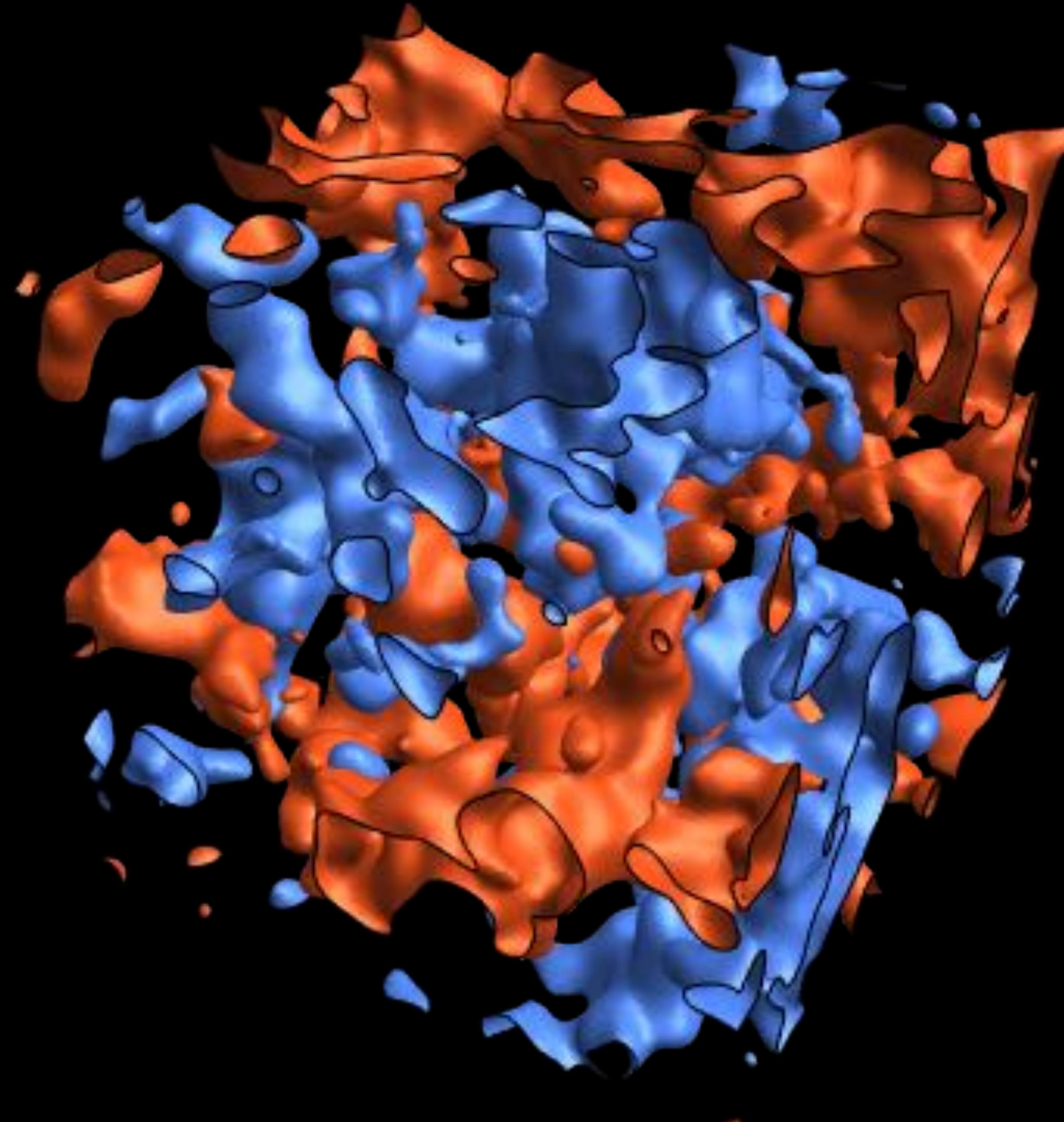
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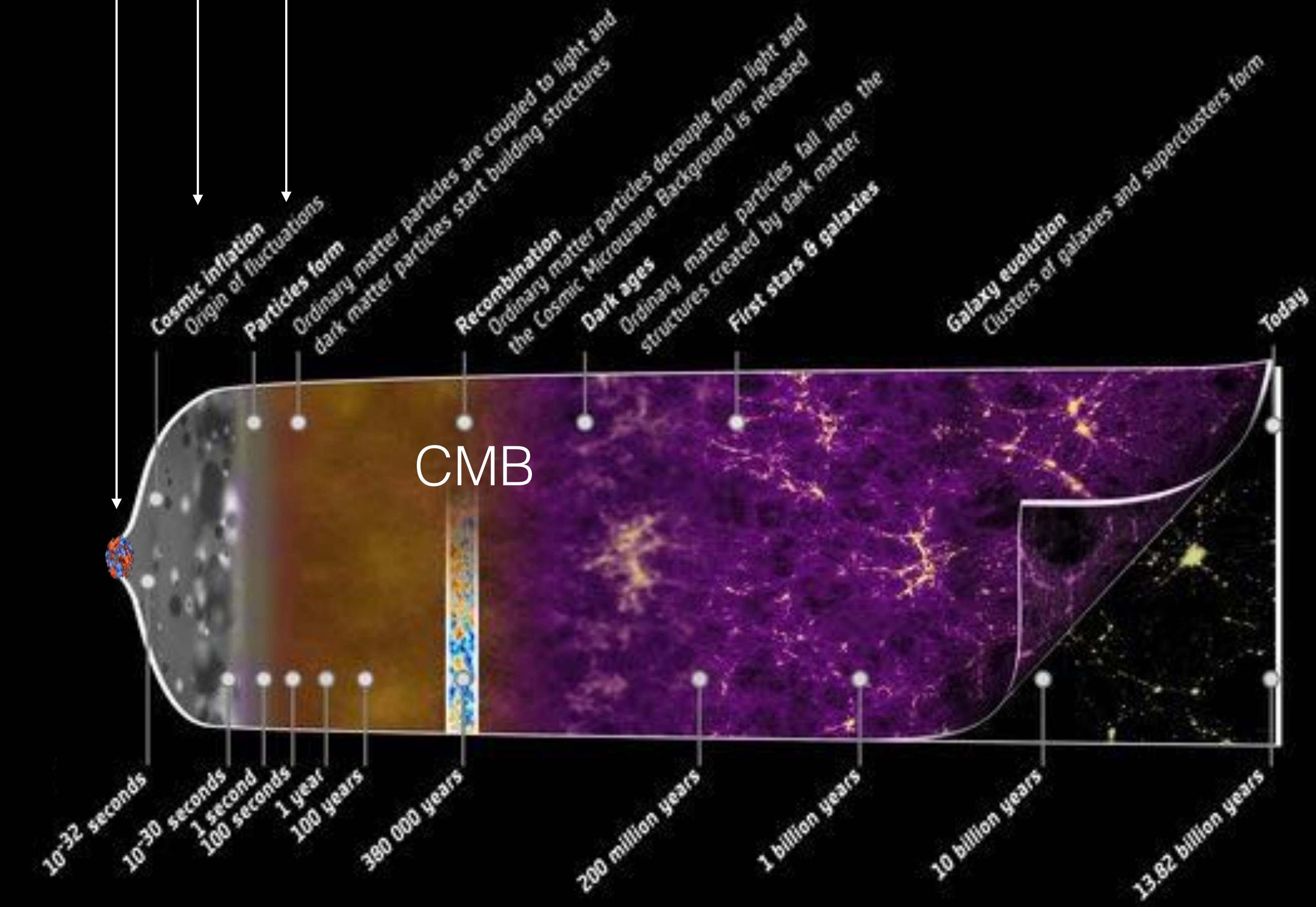
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Planck Scale

Inflation

Hot Big Bang



The Vacuum State of a Quantum Field

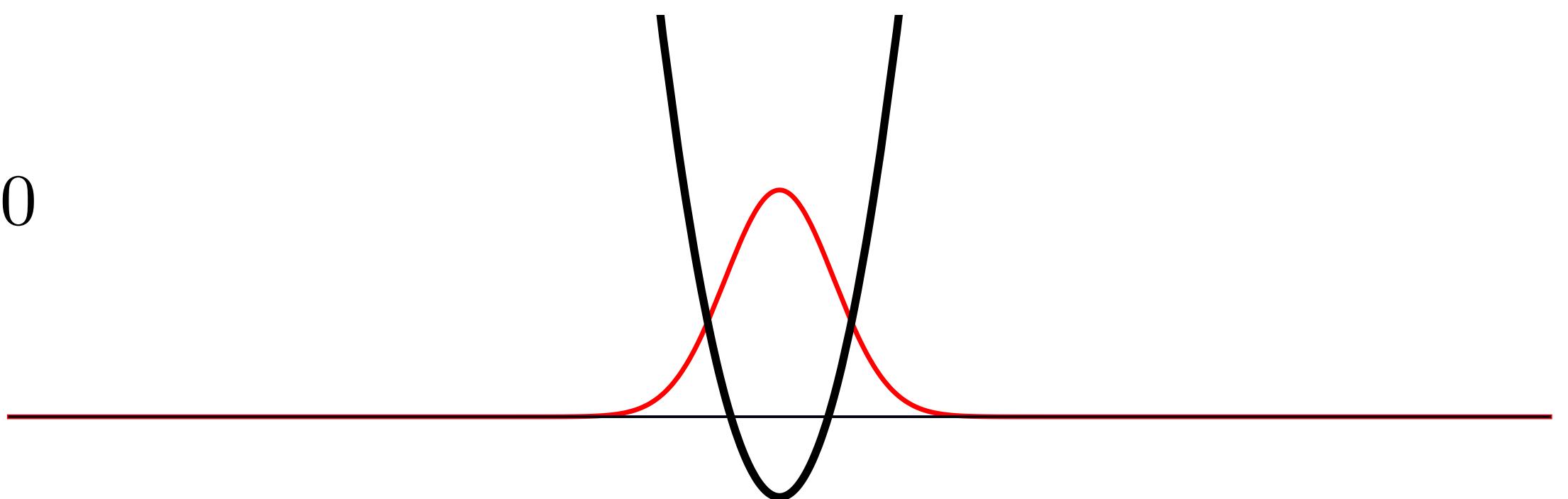
No particles

$$a(\vec{k}) |0\rangle = 0$$

Vanishing expectation value

$$\langle 0 | \varphi(\vec{x}) | 0 \rangle = 0$$

but non-vanishing fluctuations



Uncorrelated momenta

$$\langle 0 | \varphi(\vec{k}) \varphi(\vec{k}') | 0 \rangle = P(|\vec{k}|) (2\pi)^3 \delta(\vec{k} + \vec{k}')$$

with power spectrum

$$P(k) = \frac{1}{2k}$$

Non-vanishing correlations at space-like separation

$$\langle 0 | \varphi(\vec{x}) \varphi(\vec{y}) | 0 \rangle = \int_0^\infty \frac{k^3 P(k)}{2\pi^2} \frac{\sin(k |\vec{x} - \vec{y}|)}{k |\vec{x} - \vec{y}|} \frac{dk}{k} = \frac{1}{(2\pi)^2} \frac{1}{|\vec{x} - \vec{y}|^2}$$

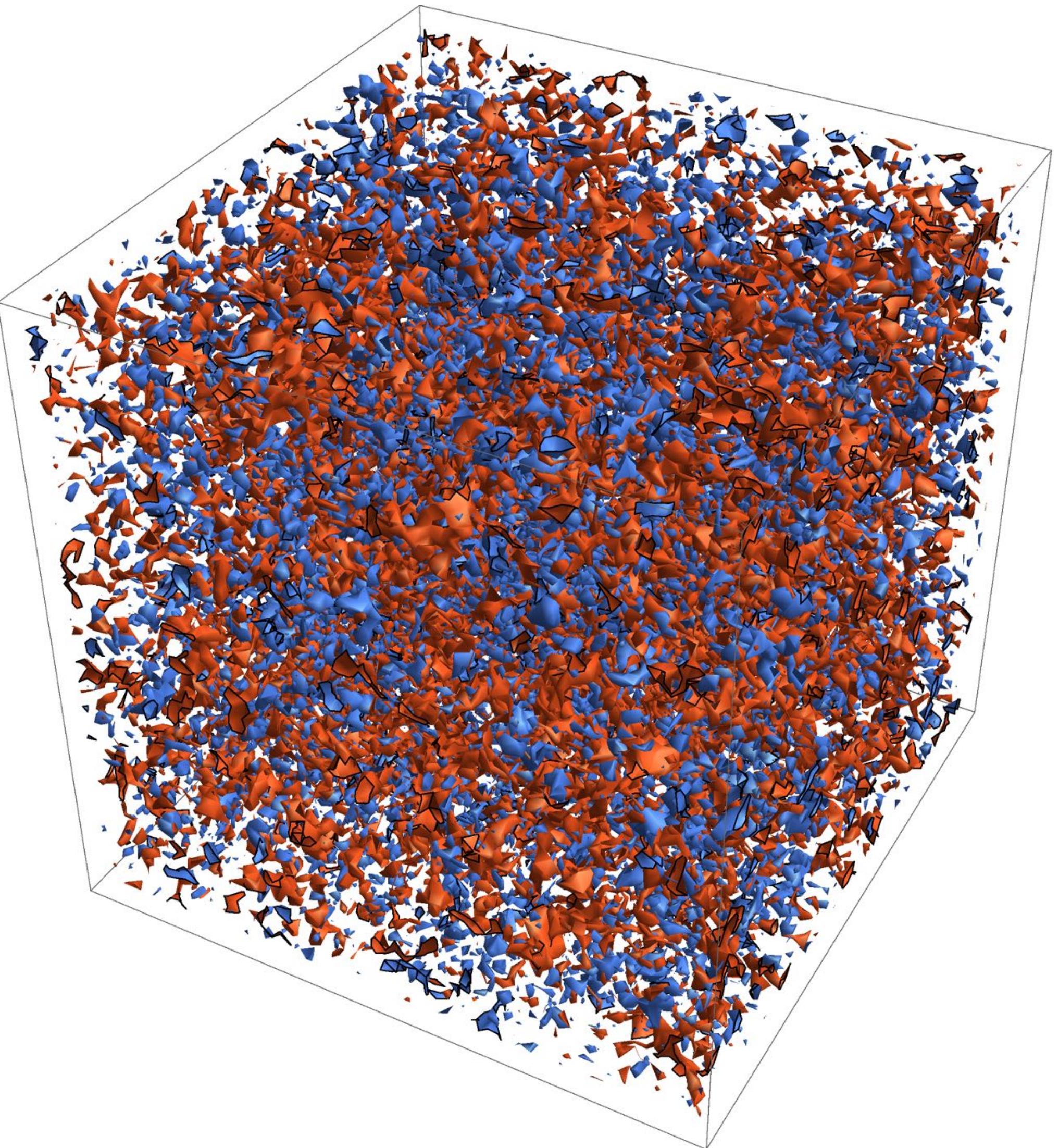
Fluctuations of the field averaged over a region of size R

$$(\Delta \varphi_R)^2 \equiv \langle 0 | \varphi_R \varphi_R | 0 \rangle - (\langle 0 | \varphi_R | 0 \rangle)^2 \sim \frac{1}{R^2}$$

The vacuum of a quantum field after inflation

Minkowski

$$P(k) = \frac{1}{2k}$$



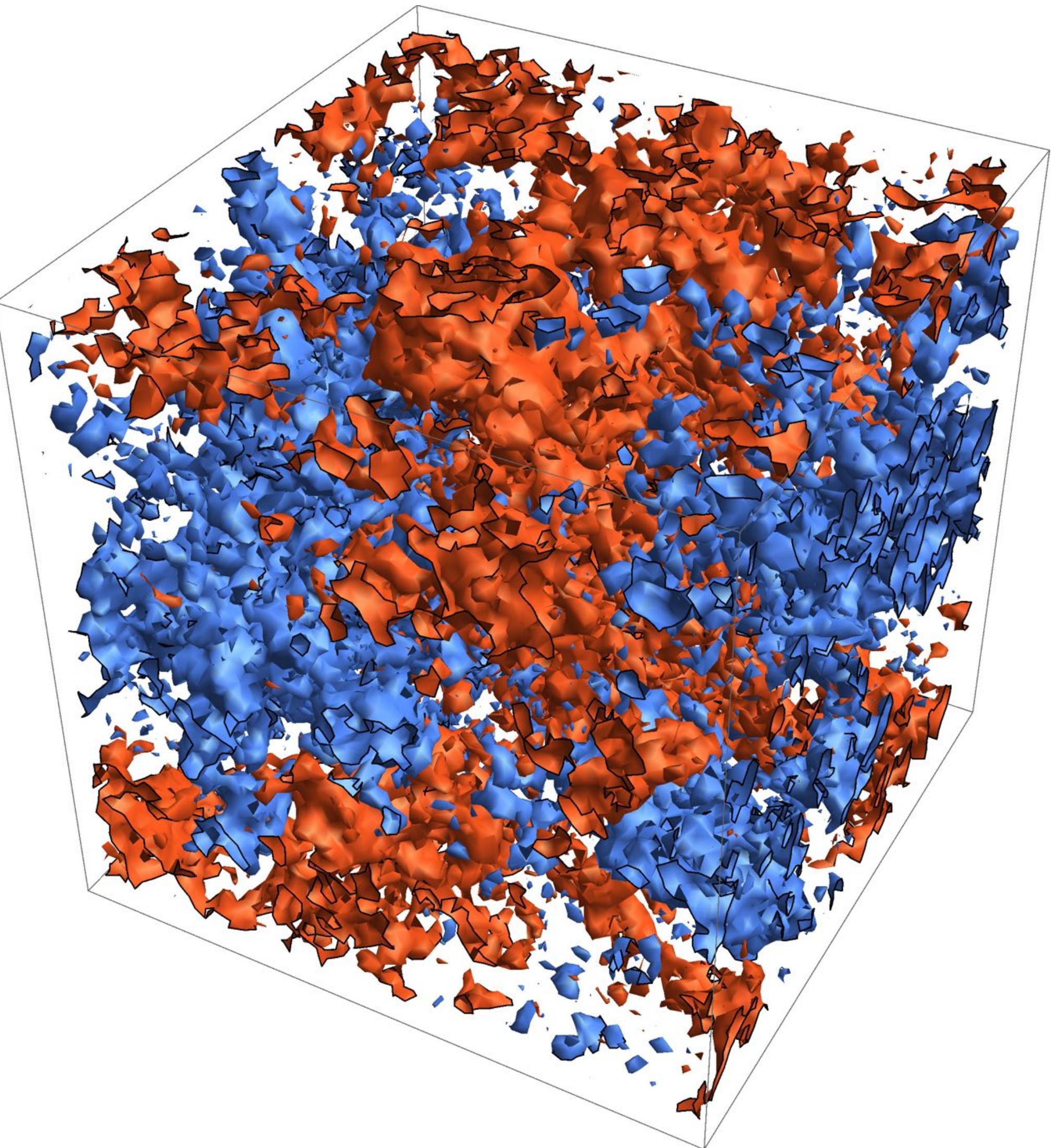
The vacuum of a quantum field after inflation

Minkowski

$$P(k) = \frac{1}{2k}$$

de Sitter

$$P(k) = \frac{1}{2k} e^{-2H_0 t} + \frac{H_0^2}{2 k^3}$$



The vacuum of a quantum field after inflation

Mukhanov-Chibisov (1981)

Minkowski

$$P(k) = \frac{1}{2k}$$

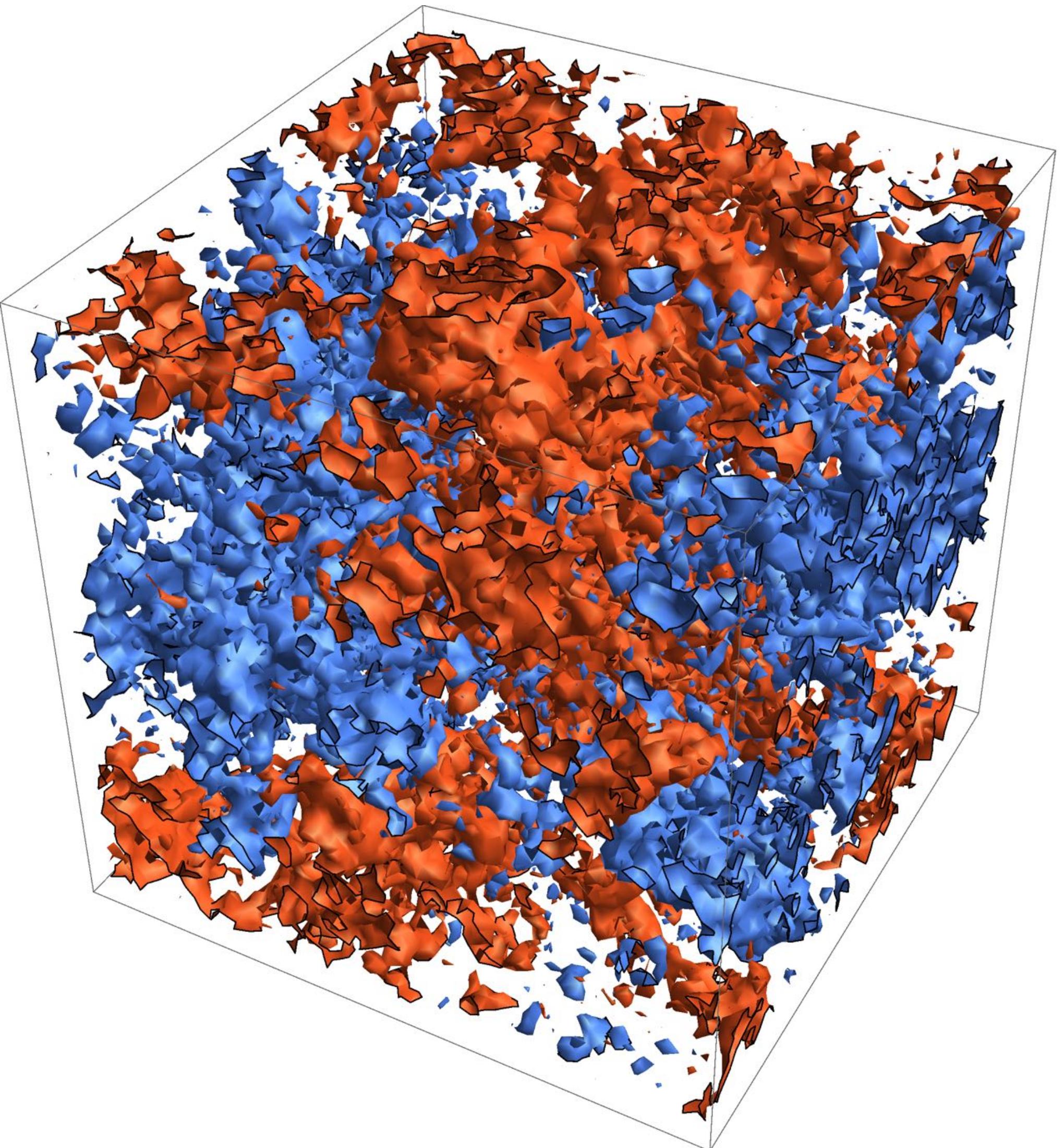
de Sitter

$$P(k) = \frac{1}{2k} e^{-2H_0 t} + \frac{H_0^2}{2 k^3}$$

Inflation (quasi-de Sitter)

$$P_s(k) \approx \frac{2\pi^2 A_s}{k^3} \left(\frac{k}{k_*} \right)^{n_s - 1}$$

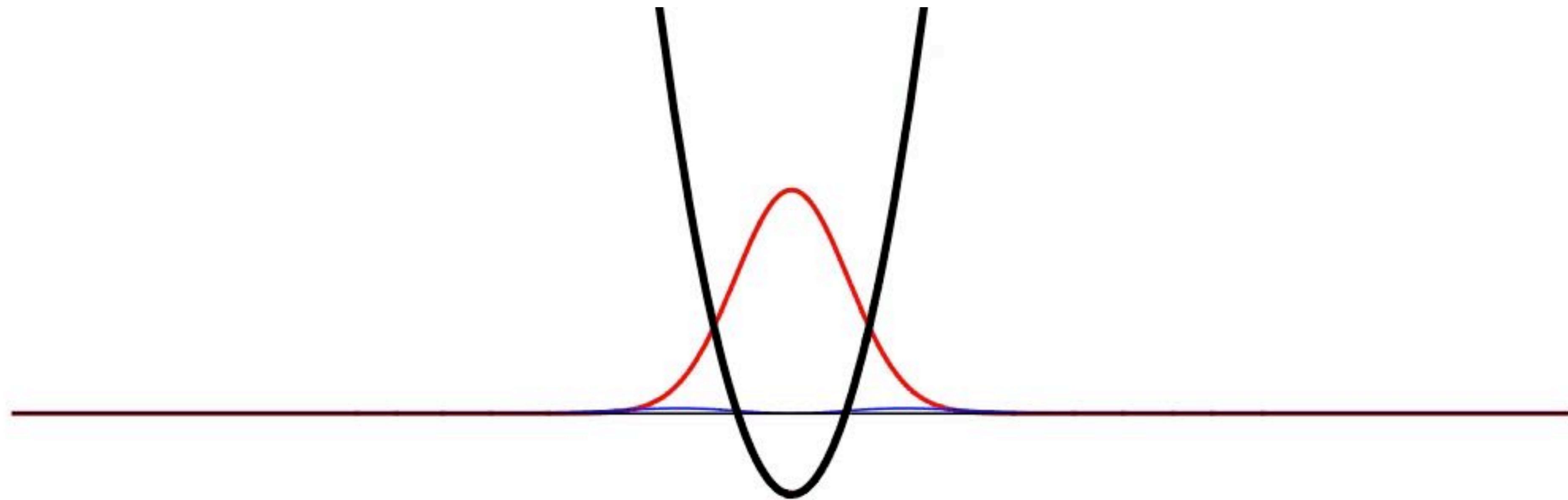
with $A_s \sim \frac{G \hbar H_*^2}{\varepsilon_*}$



Planck 2015

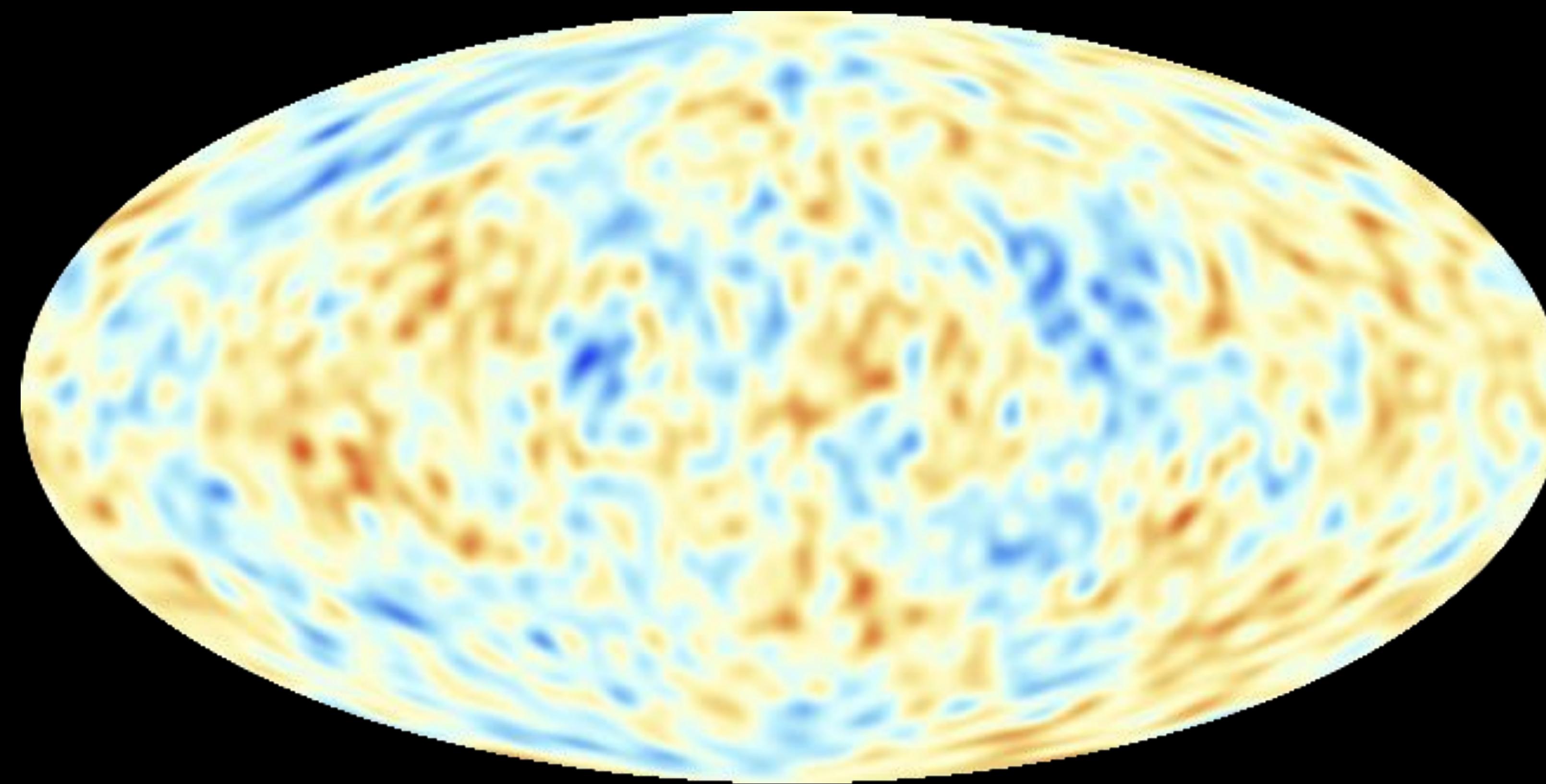
$$A_s(k_*) = 2.47 \times 10^{-9}$$
$$n_s(k_*) = 0.96$$

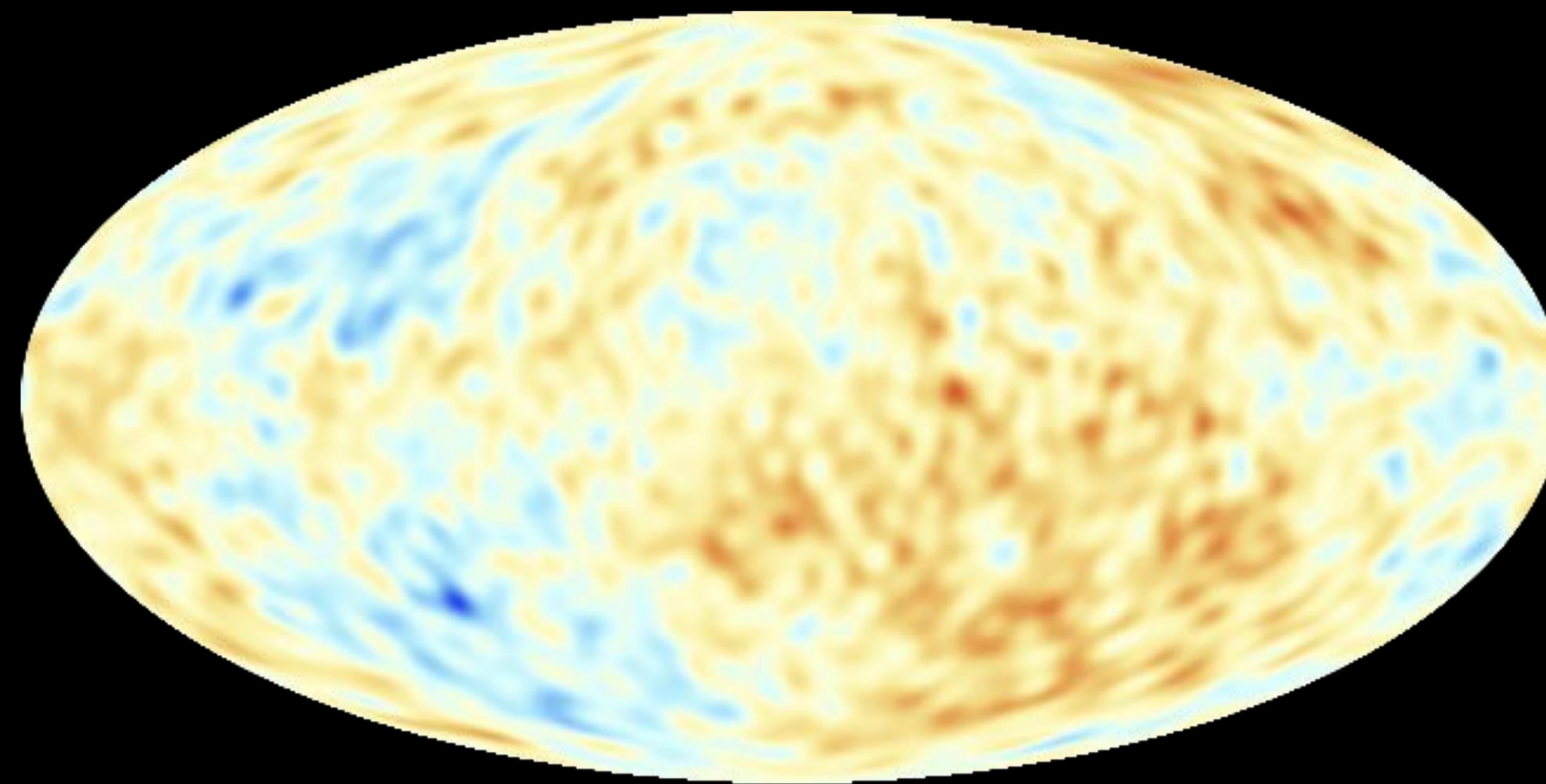
Mechanism: amplification of vacuum fluctuations by instabilities

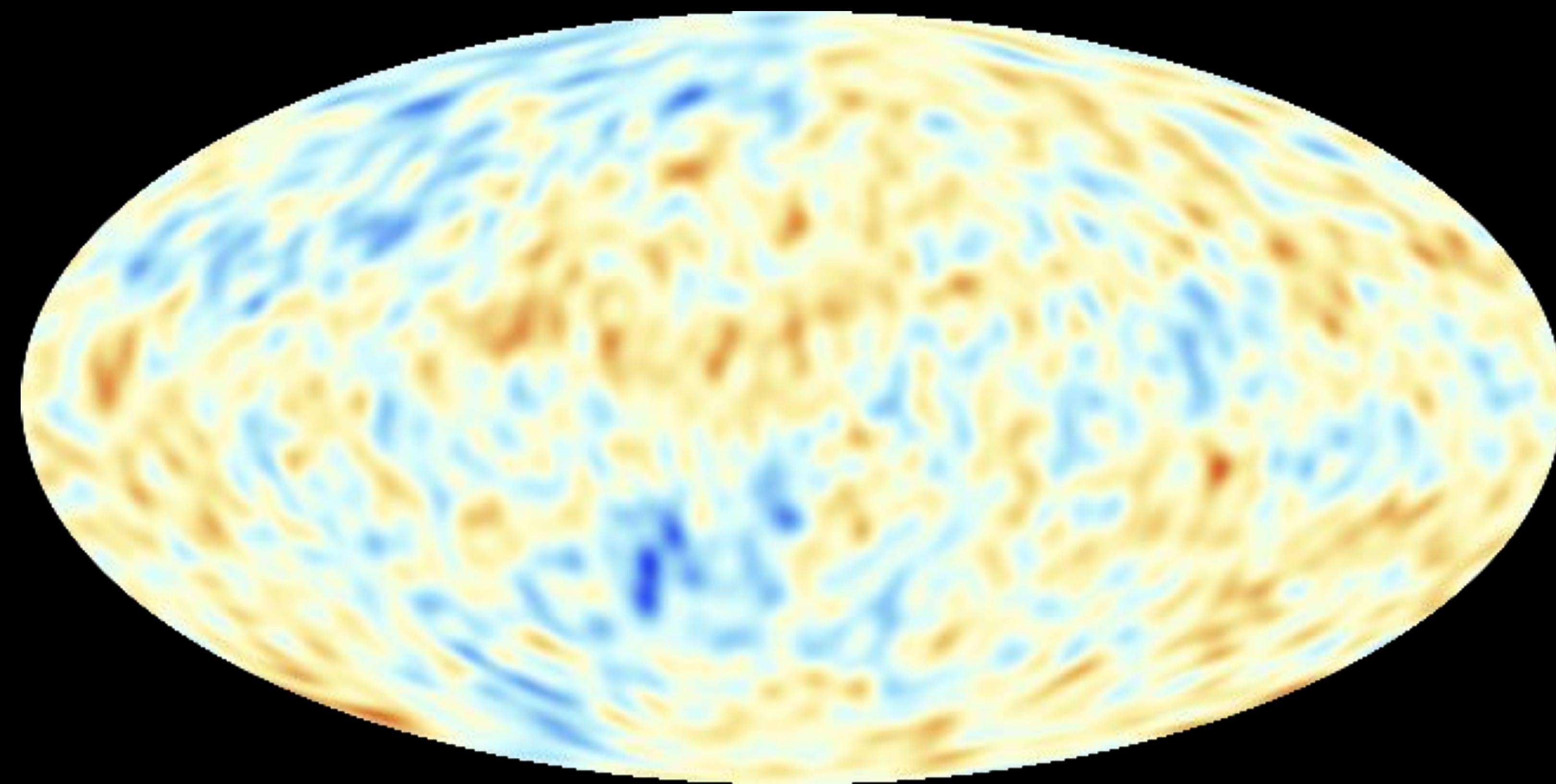


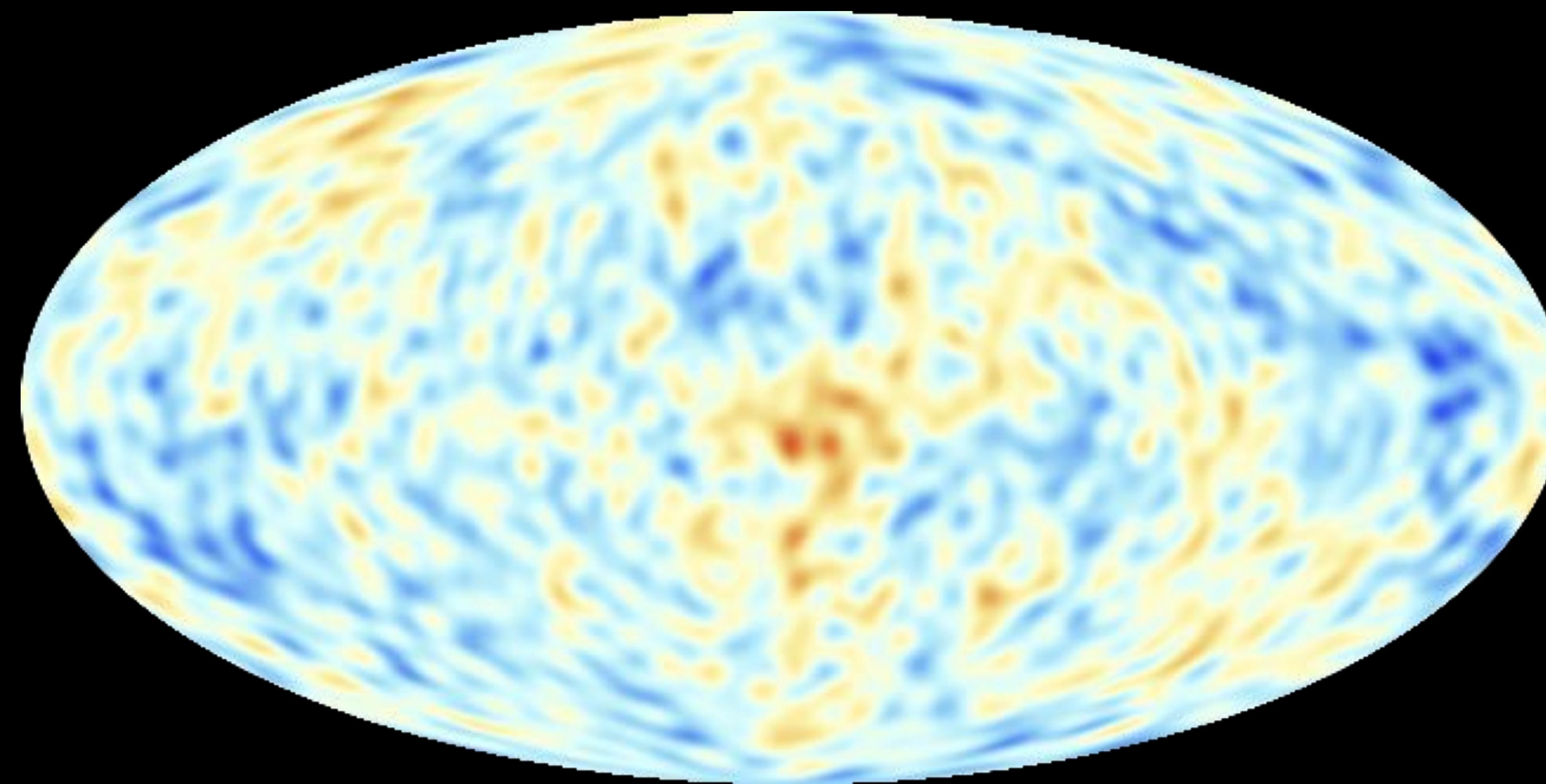
Harmonic oscillator with
time-dependent frequency

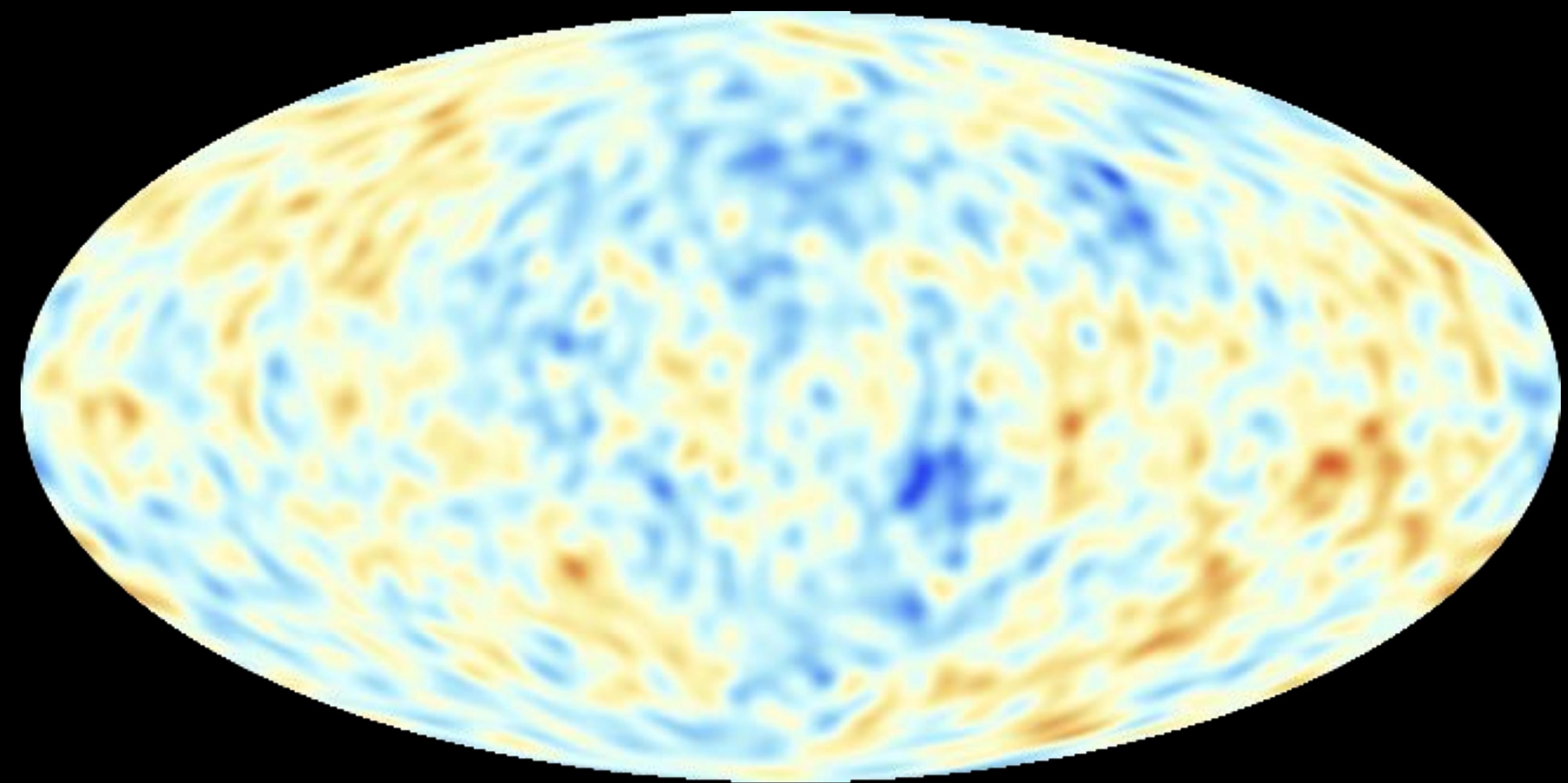
$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}(k^2 - f(t))q^2$$

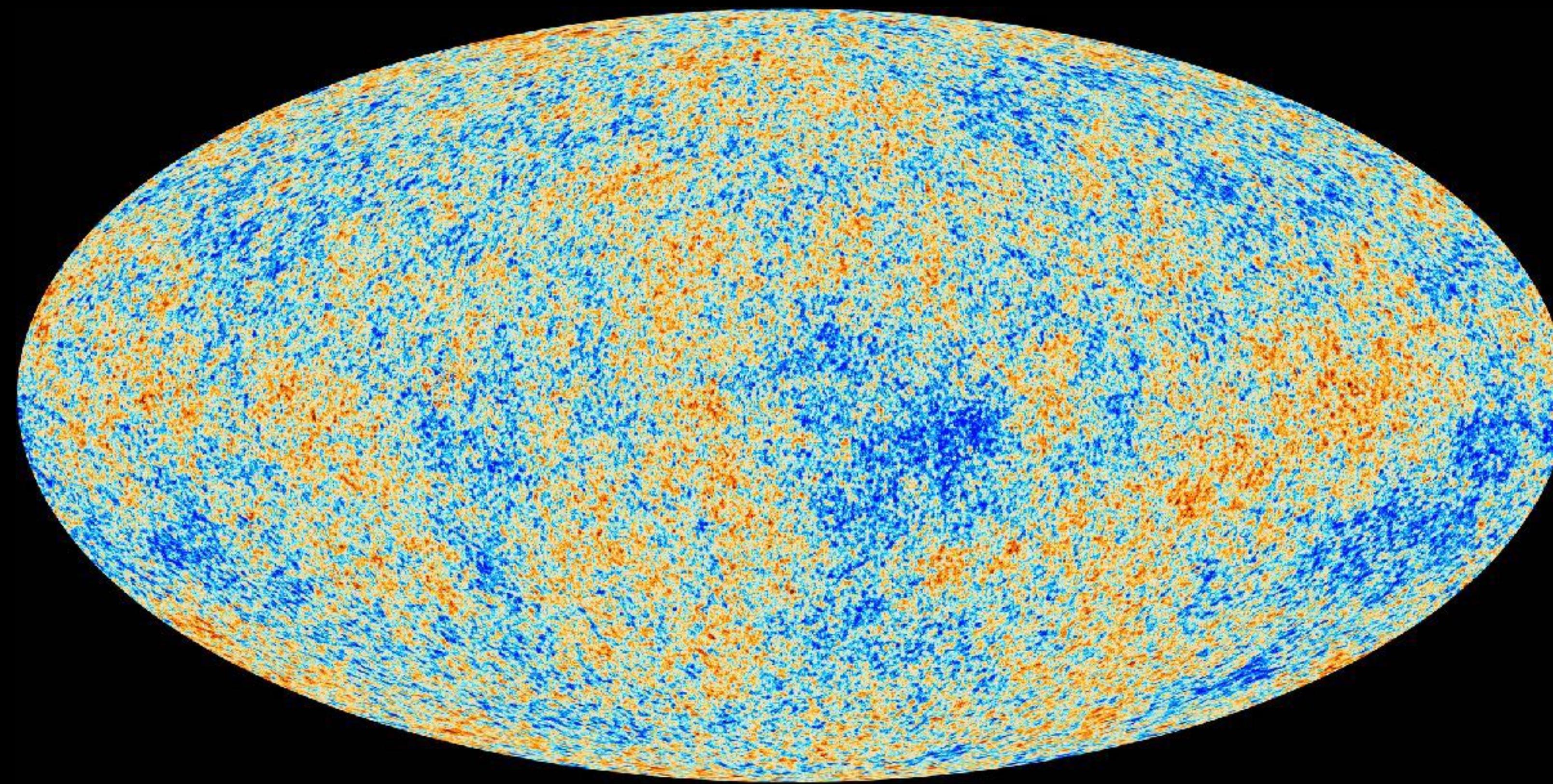




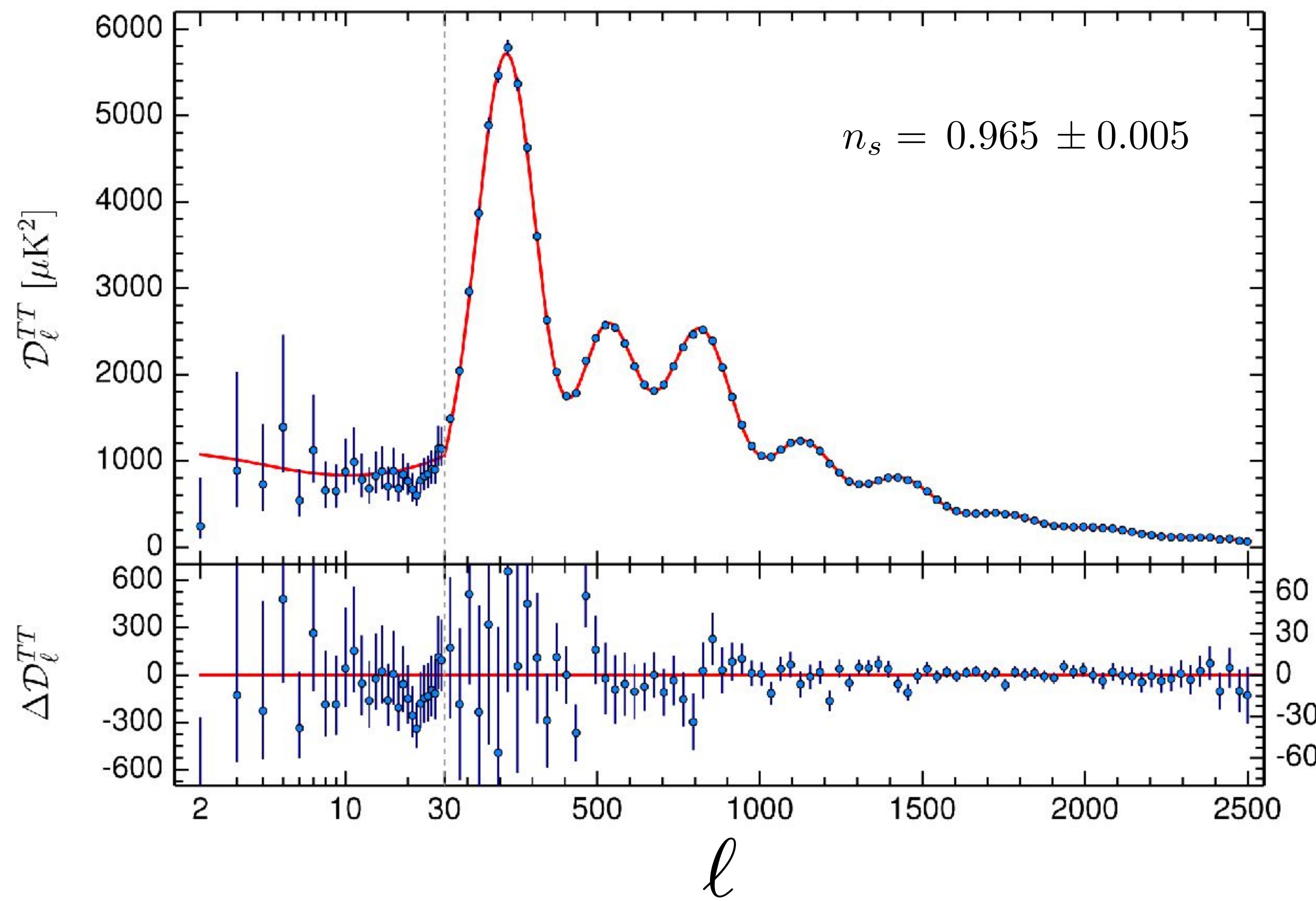








The anisotropies of the Cosmic Microwave Background
as observed by Planck



Planck Collaboration, arxiv.org/abs/1502.02114
``Planck 2015 results. XX. Constraints on inflation''

A distinguishing feature of loop quantum gravity:

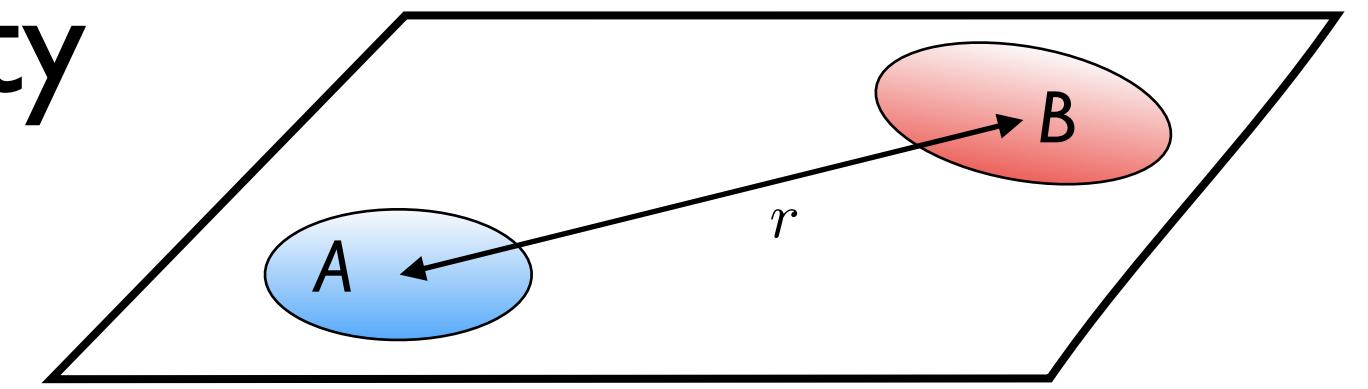
→ existence of states with no correlation at space-like separation

Scenario

→ uncorrelated initial state and its phenomenological imprints

Emergence of space-like correlations in loop quantum gravity

States with *no* space-like correlations: allowed in quantum gravity



BKL conjecture (Belinsky-Khalatnikov-Lifshitz 1970)

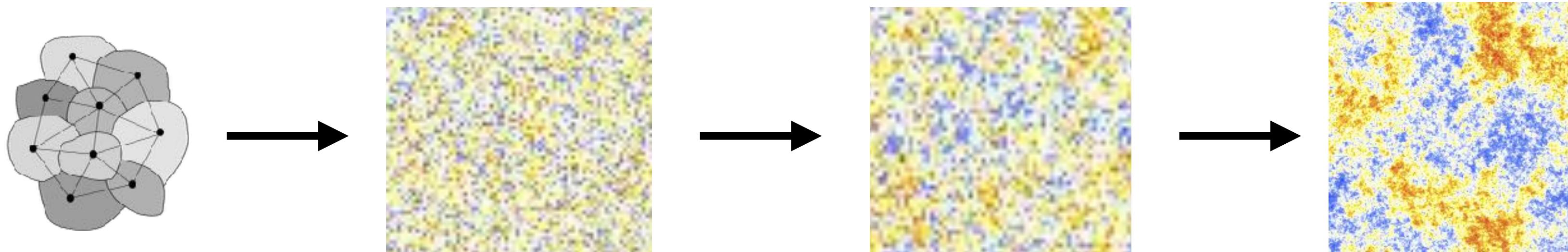
In classical General Relativity, the spatial coupling of degrees of freedom is suppressed in the approach to a space-like singularity

Quantum BKL conjecture (E.B.-Hackl-Yokomizo 2015)

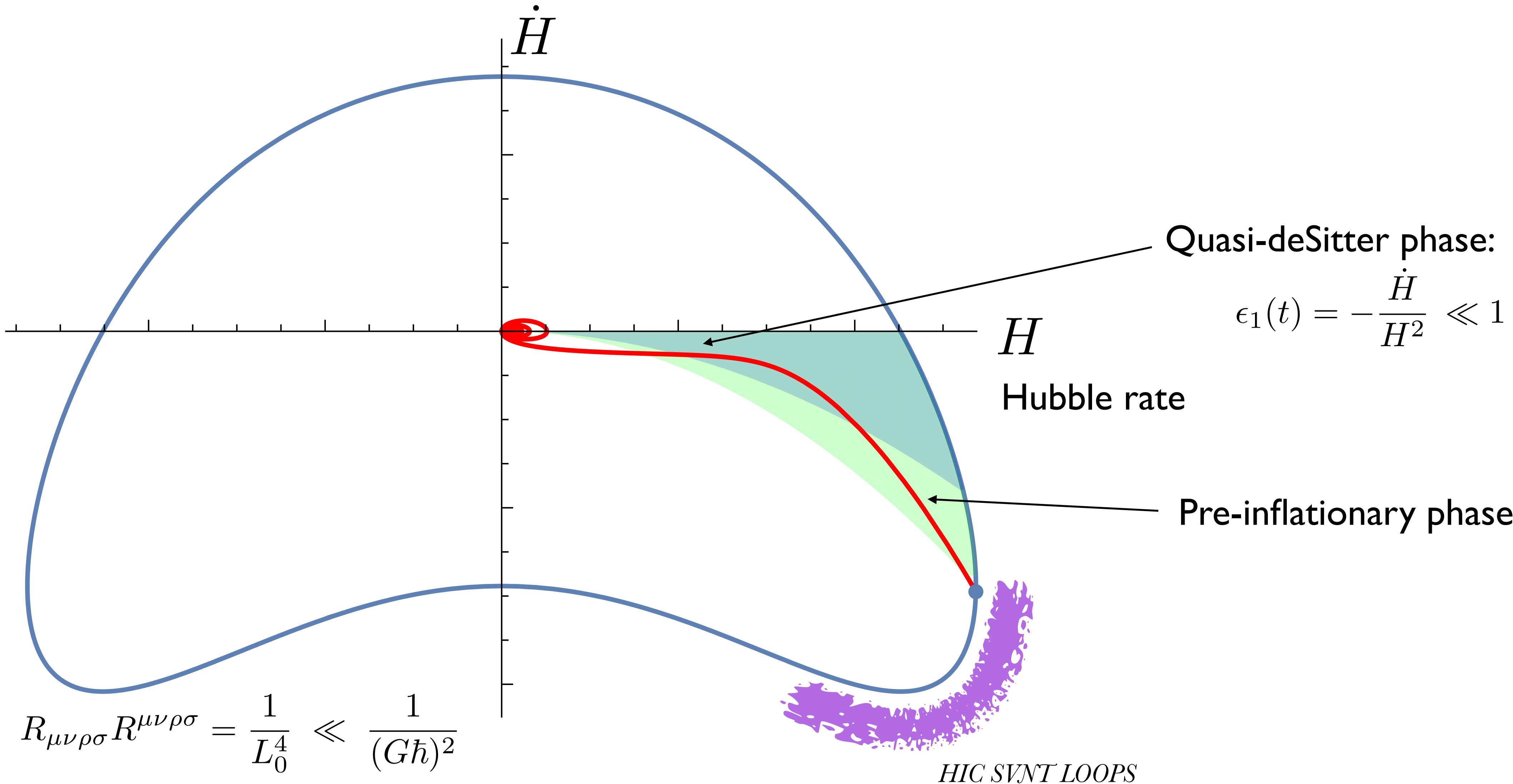
In quantum gravity, correlations between spatially separated degrees of freedom are suppressed in the approach to a Planck curvature phase

$$\left\{ \begin{array}{l} \hat{H} \Psi[g_{ij}(x), \varphi(x)] = 0 \\ \lim_{a \rightarrow 0} \Psi[a, \phi, \delta g_{ij}(x), \delta \varphi(x)] = \prod_{\vec{x}} \psi(\phi, \delta g_{ij}(x), \delta \varphi(x)) \end{array} \right.$$

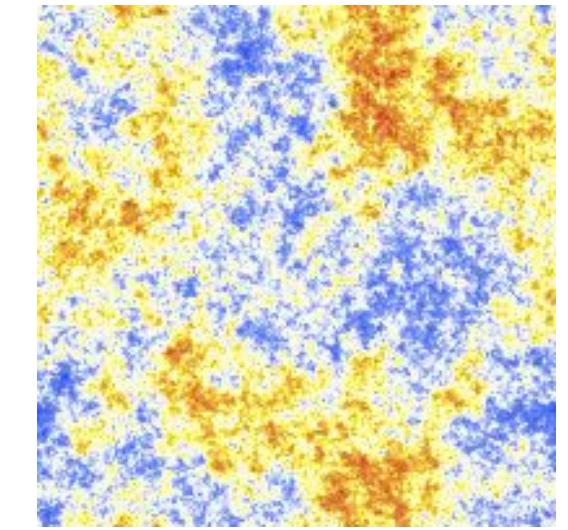
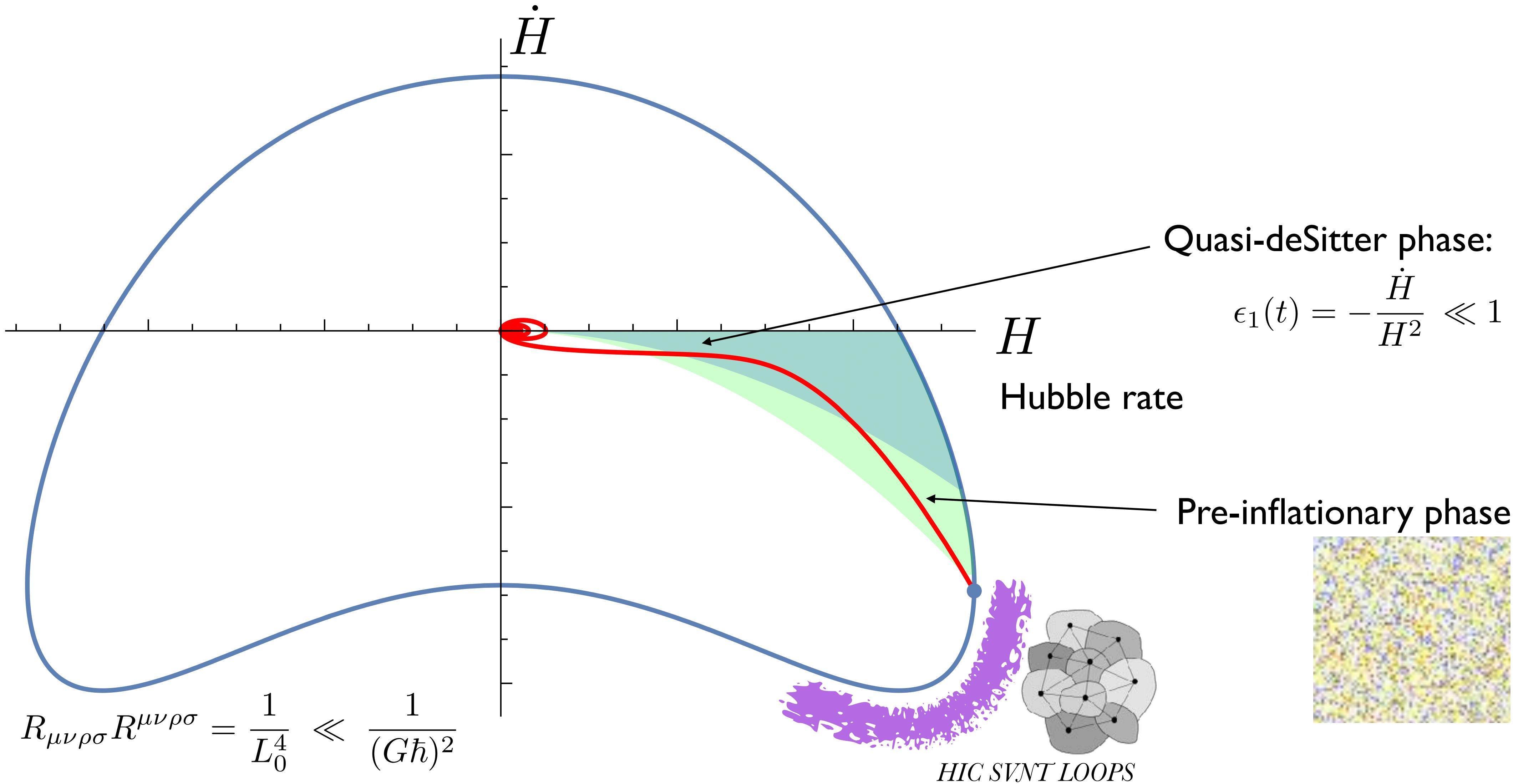
Scenario: the correlations present at the beginning of slow-roll inflation are produced in a pre-inflationary phase when the LQG-to-QFT transition takes place



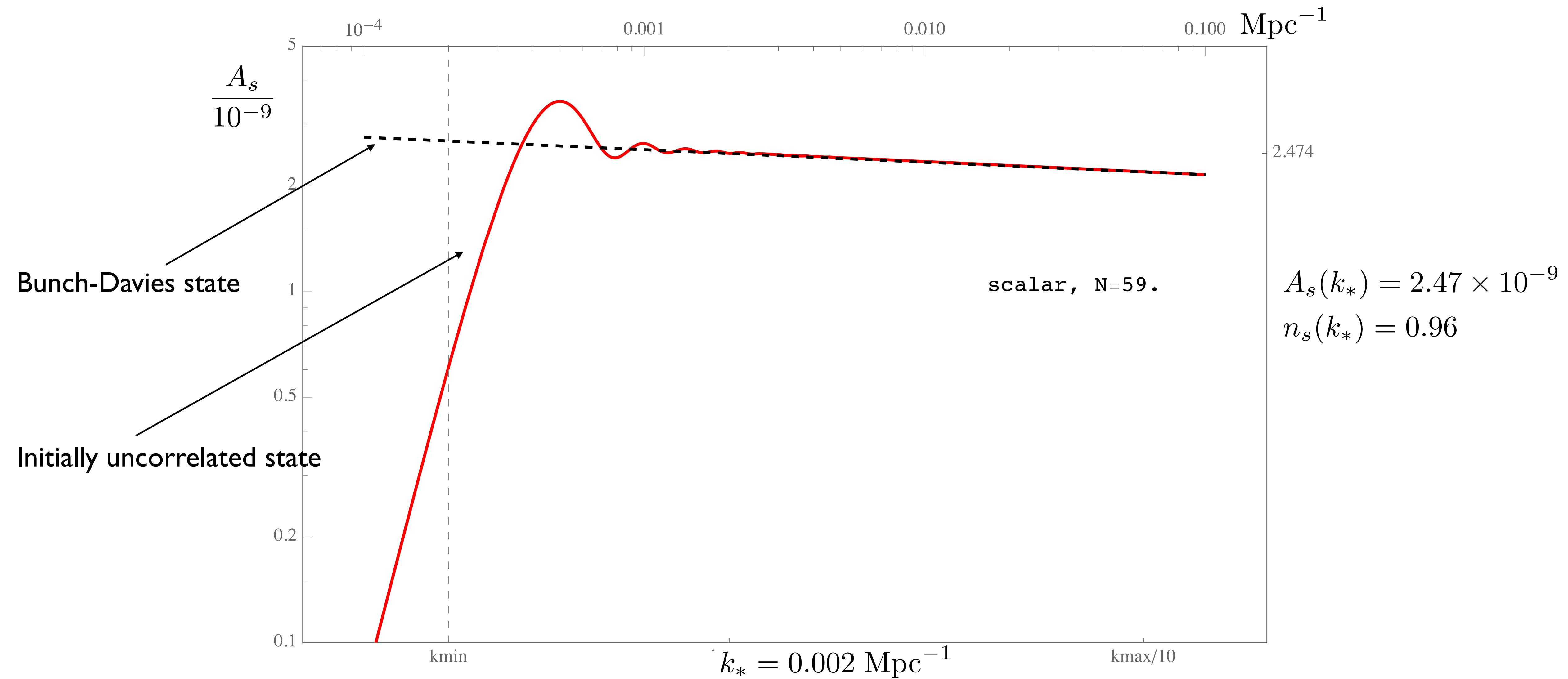
Background dynamics and pre-inflationary initial conditions



Perturbations and pre-inflationary initial conditions



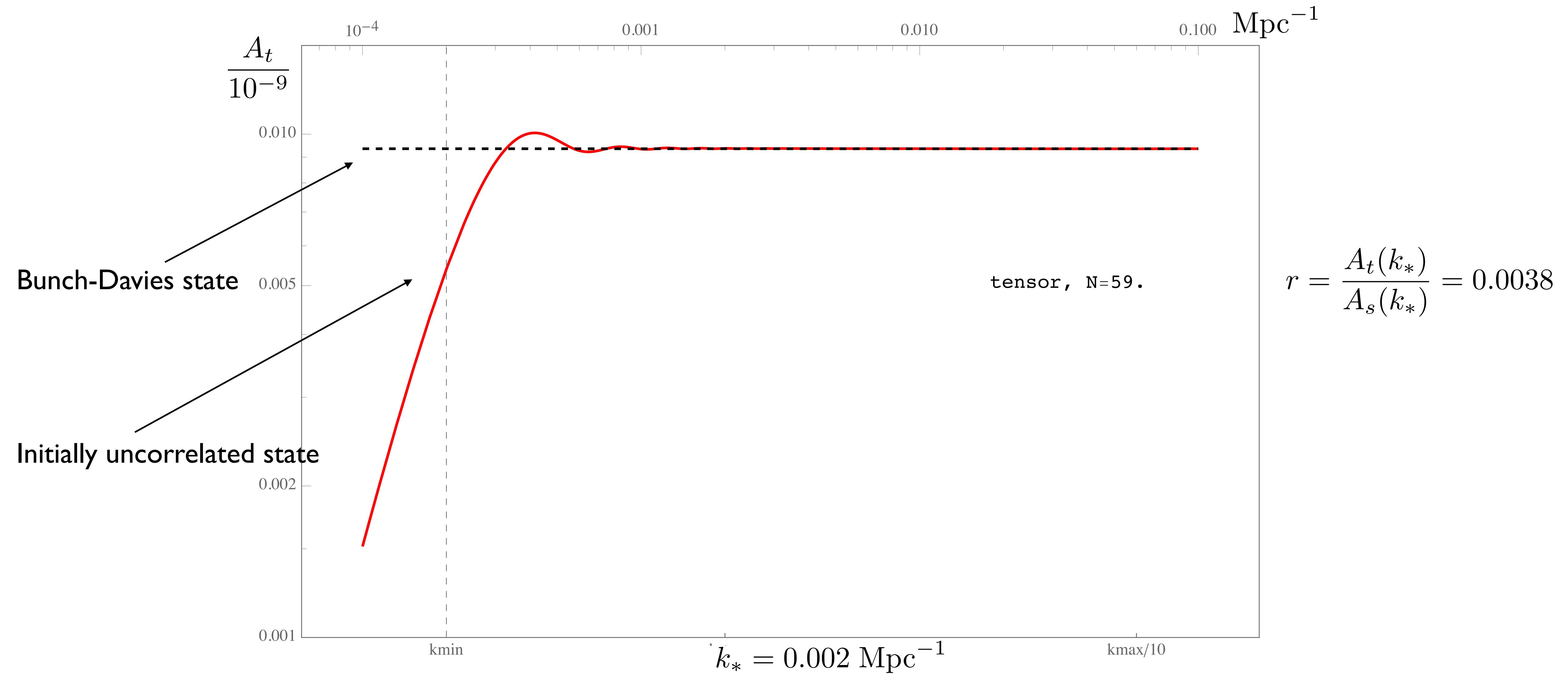
Scalar power spectrum with LQG-to-QFT initial conditions



Power suppression at large angular scales

E.B.-Fernandez 2017

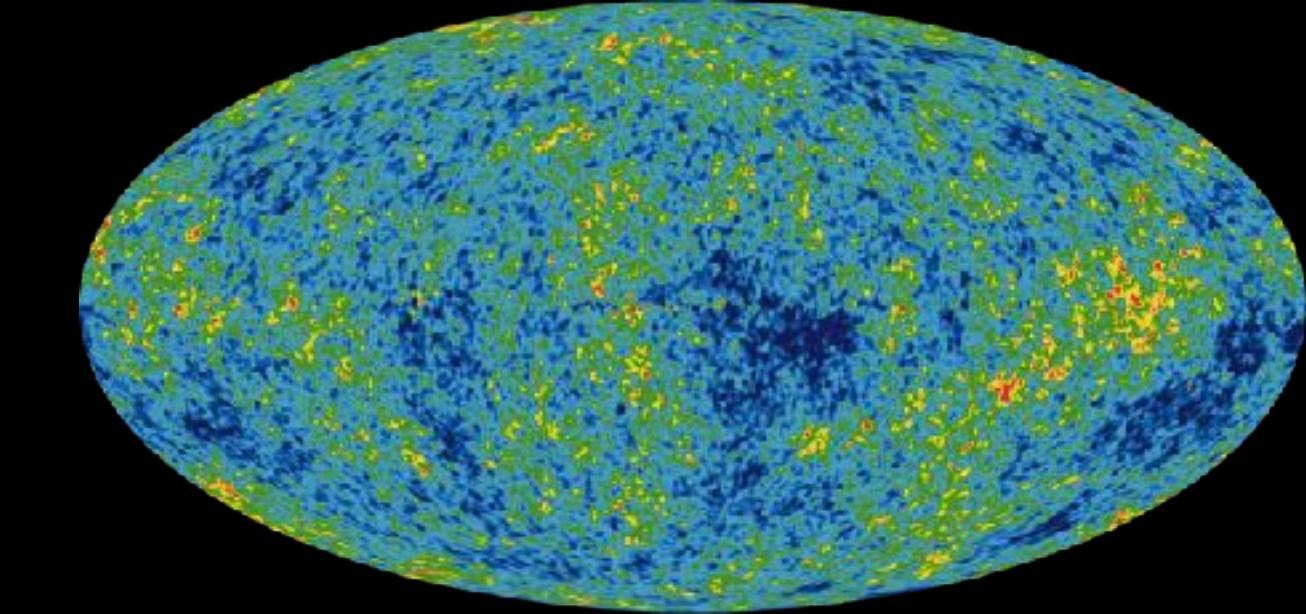
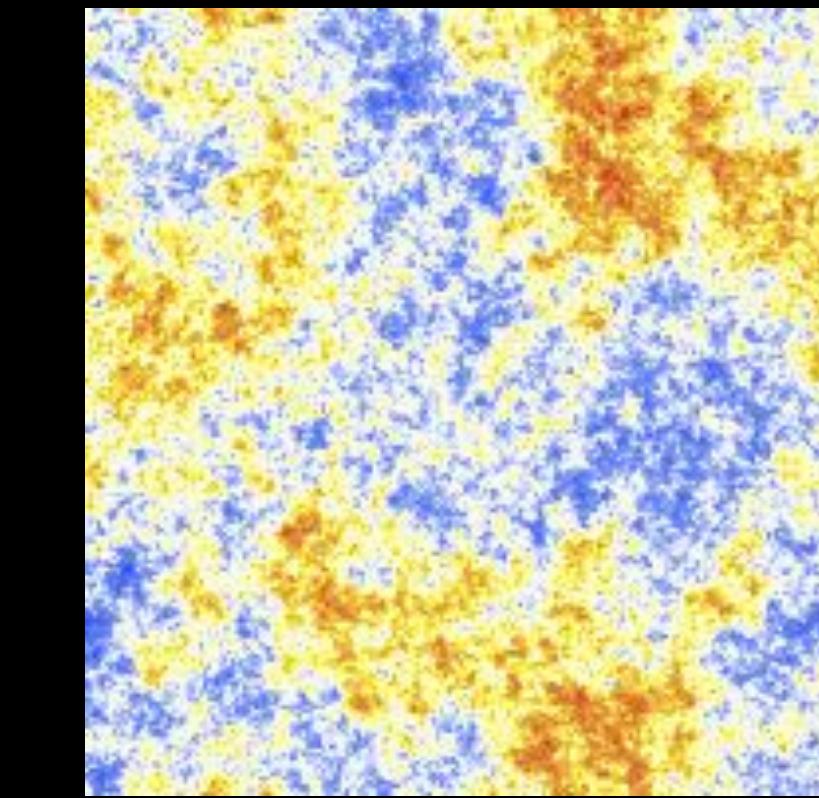
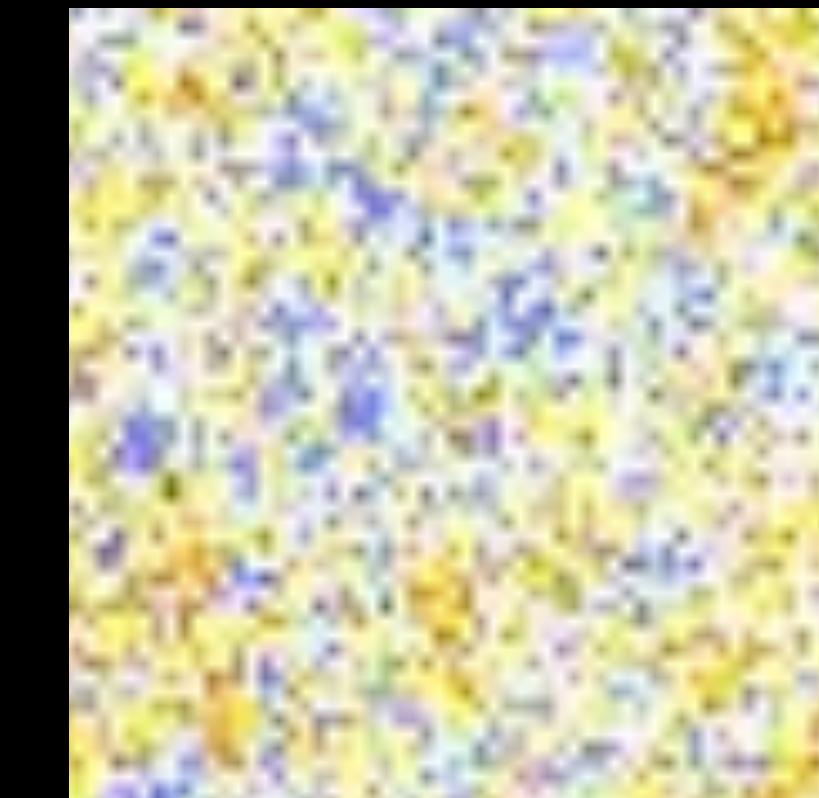
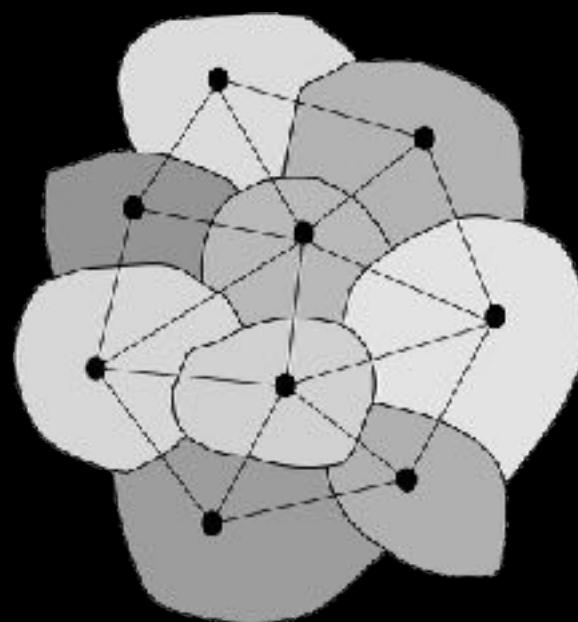
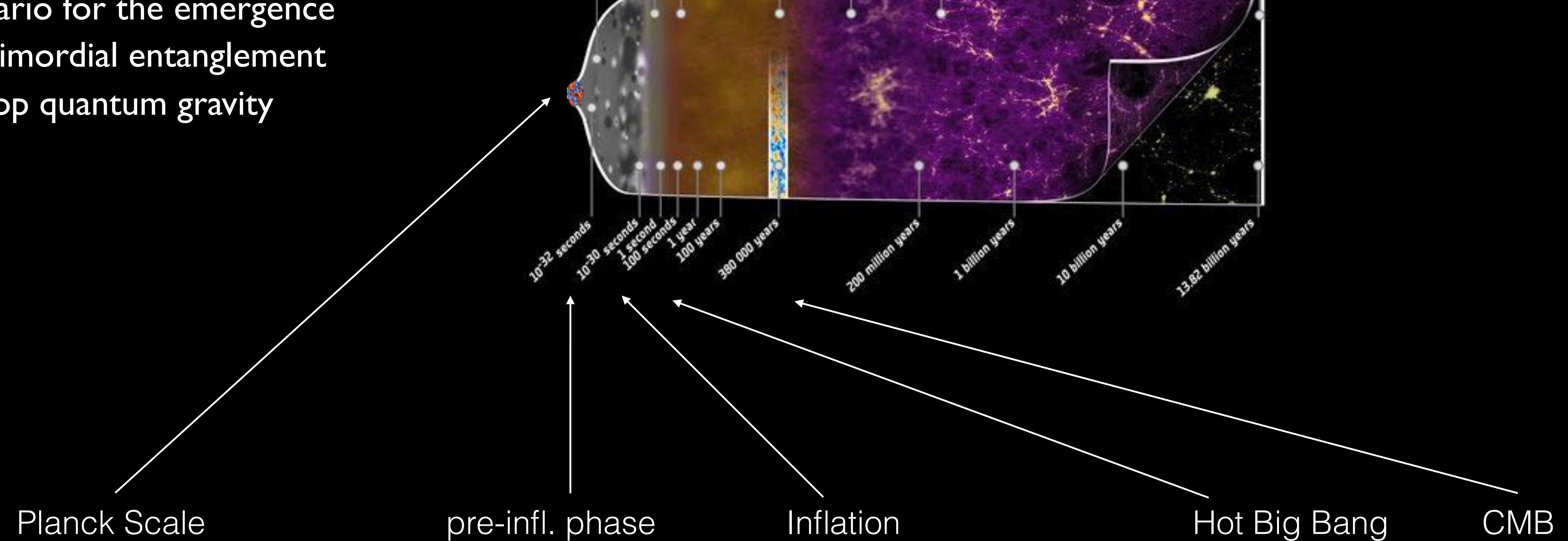
Tensor power spectrum with LQG-to-QFT initial conditions



Power suppression at large angular scales

E.B.-Fernandez 2017

Scenario for the emergence of primordial entanglement in loop quantum gravity



Inflation and spinfoams

- Effective spinfoam action

$$S[e^I, \omega^{IJ}, r, \lambda^{IJ}] = \int \left((1 + 2\alpha r) B_{IJ} \wedge F^{IJ} - \frac{\alpha r^2}{1 + \gamma^2} \frac{1}{4!} \epsilon_{IJKL} B^{IJ} \wedge B^{KL} + B_{IJ} \wedge \nabla \lambda^{IJ} \right)$$

where $B_{IJ} = \frac{1}{8\pi G} \left(\frac{1}{2} \epsilon_{IJKL} e^K \wedge e^L - \frac{1}{\gamma} e_I \wedge e_J \right)$ γ = Barbero-Immirzi parameter

r = 0-form, effective Ricci scalar at a coarse-graining scale

α = coupling constant
dimensions of Area

- It provides an embedding in spinfoams of the Starobinsky model (1979)

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + \alpha R^2) \quad \rightarrow \quad \mathcal{G}_{\mu\nu} + \alpha \mathcal{H}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Friedman eq: $H^2 + 6\alpha (6H^2 \dot{H} - \dot{H}^2 + 2H \ddot{H}) = 0$

gravity-driven inflation

Primordial spectra from adiabatic vacuum in the quasi de-Sitter phase

$$36 \epsilon_1 \alpha H^2 = 1$$

- Scalar perturbations

$$A_s \equiv \frac{k_*^3 P_s(k_*, t_*)}{2\pi^2} \approx \frac{G\hbar H_*^2}{2\pi \epsilon_{1*}}$$

$$n_s \equiv 1 + k \frac{d}{dk} \log \left(k^3 P_s(k, t_*) \right) \Big|_{k=k_*} \approx 1 - 2\epsilon_{1*} - \epsilon_{2*} \\ \approx 1 - 4\epsilon_{1*}$$

- Tensor perturbations

$$A_t \equiv \frac{k_*^3 P_t(k_*, t_*)}{2\pi^2} \approx \frac{G\hbar H_*^2}{2\pi} 48 \epsilon_{1*}$$

$$n_t \equiv k \frac{d}{dk} \log \left(k^3 P_t(k, t_*) \right) \Big|_{k=k_*} \approx -2\epsilon_{1*} + \epsilon_{2*}$$

$$r \equiv \frac{A_t}{A_s} \approx 48\epsilon_{1*}^2$$

$$N_* = \int_{t_*}^{t_{\text{end}}} H(t) dt = 18 H_*^2 \alpha - \frac{1}{2}$$

$$\left\{ \begin{array}{l} \alpha \approx 3.54 \times 10^{10} G\hbar \\ H_* \approx 1.05 \times 10^{-5} \frac{1}{\sqrt{G\hbar}} \\ r \approx 2.4 \times 10^{-3} \\ N_* \approx 70 \end{array} \right.$$

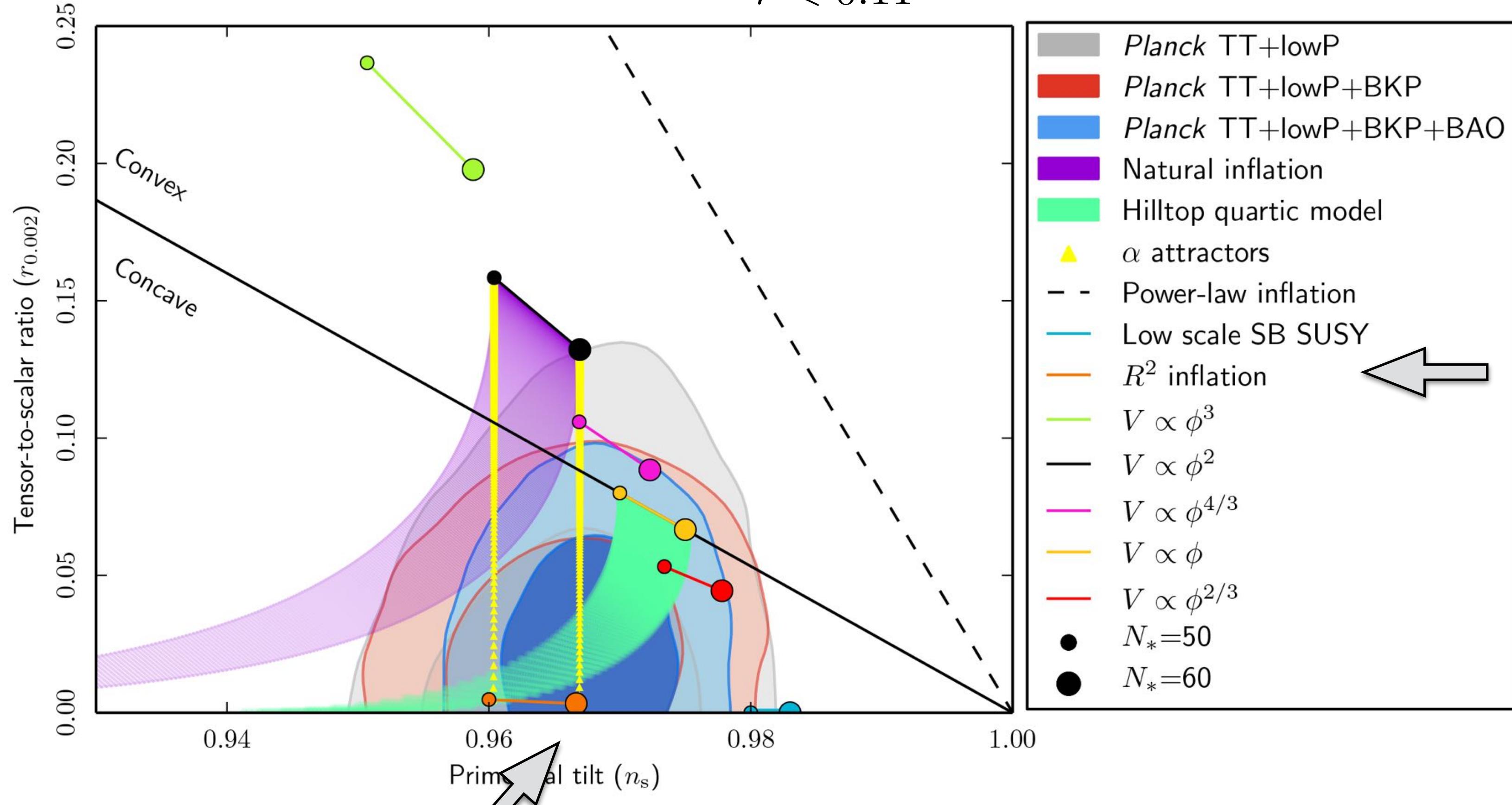
PLANCK 2015

($k_* = 0.002 \text{ Mpc}^{-1}$)

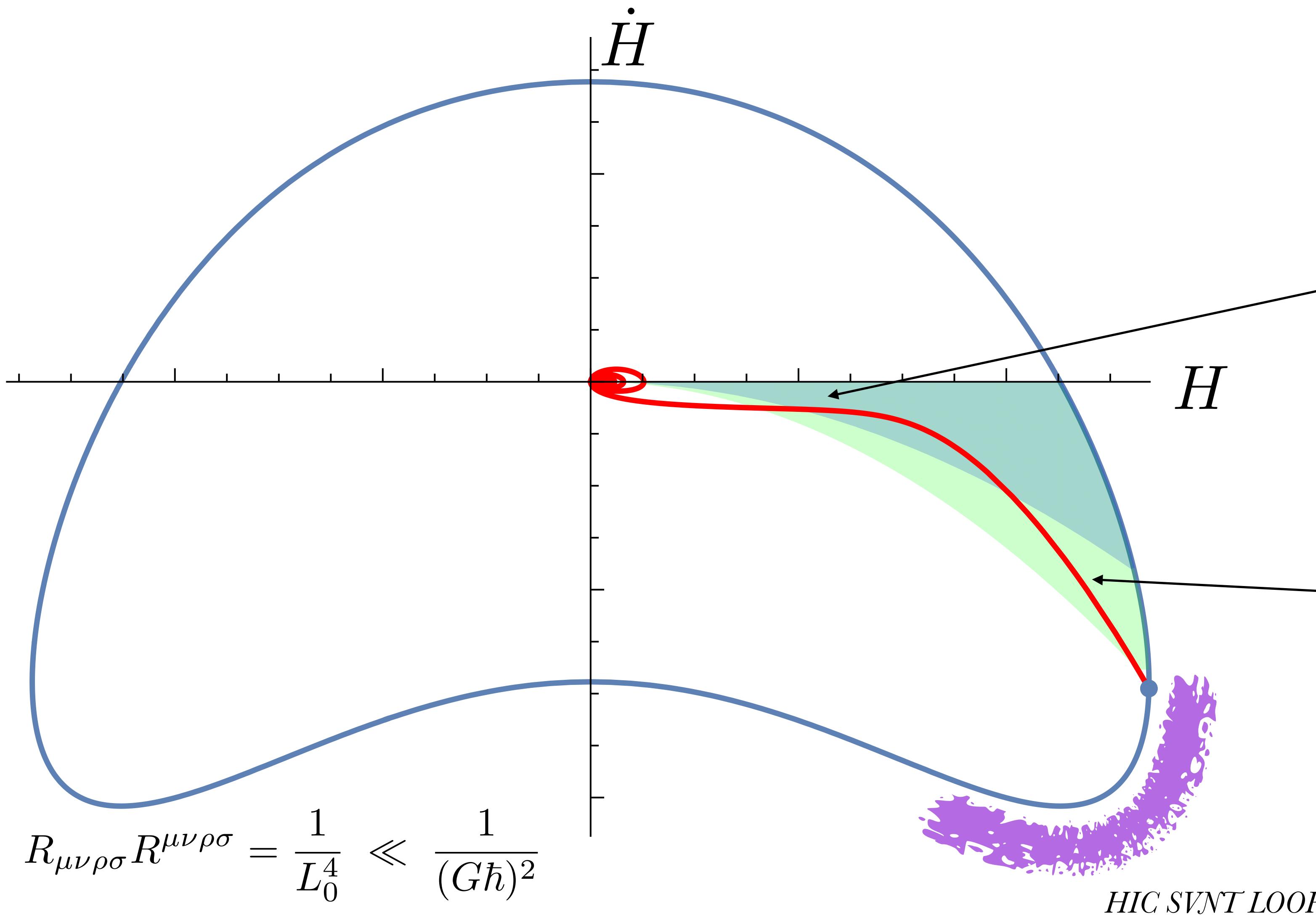
$$A_s = (2.474 \pm 0.116) \times 10^{-9}$$

$$n_s = 0.9645 \pm 0.0062$$

$$r < 0.11$$



Background dynamics and pre-inflationary initial conditions



Friedman eq:

$$H^2 + 6\alpha (6H^2\dot{H} - \dot{H}^2 + 2H\ddot{H}) = 0$$

Quasi-deSitter phase:

$$\epsilon_1(t) = -\frac{\dot{H}}{H^2} \ll 1$$

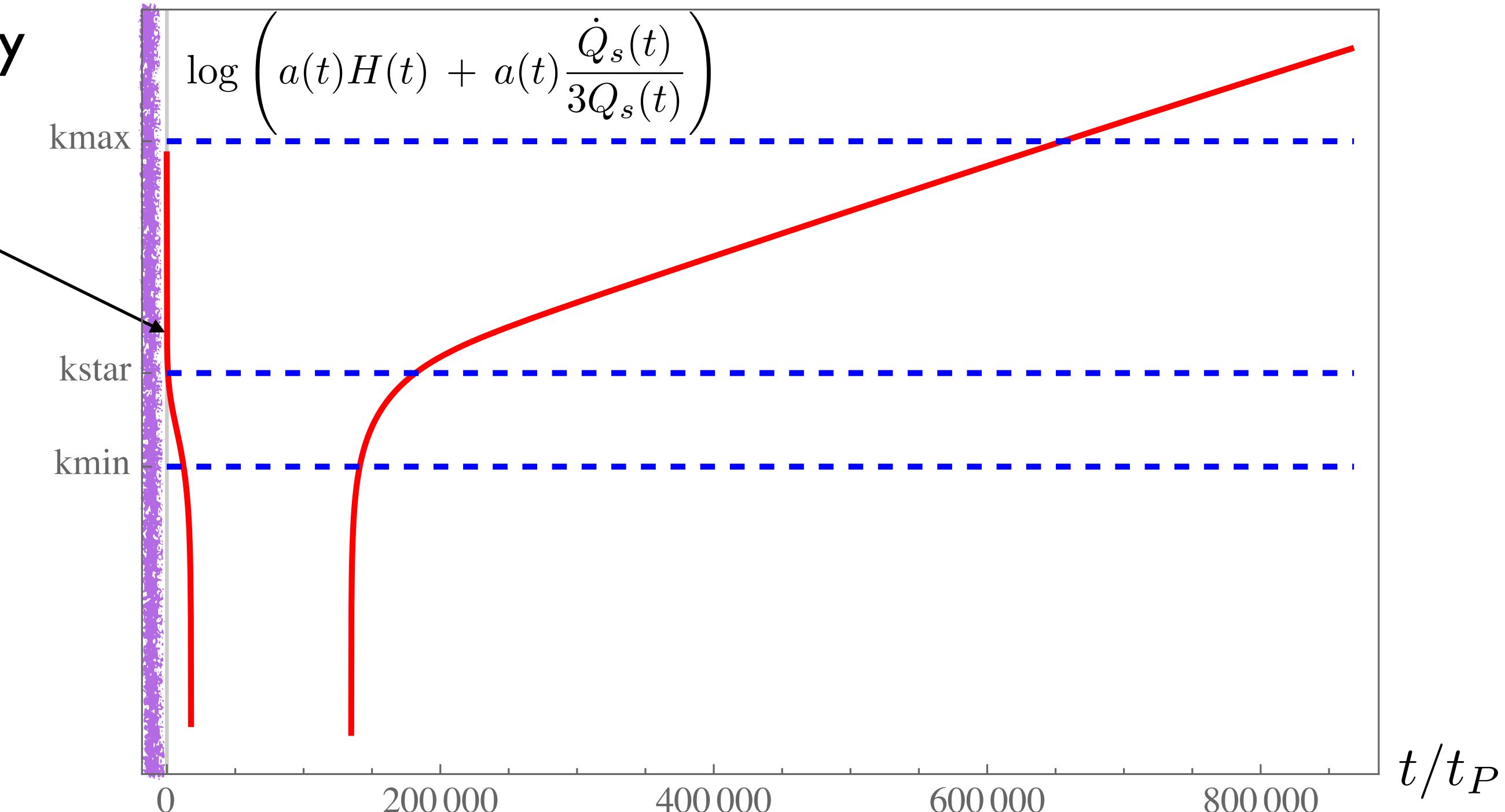
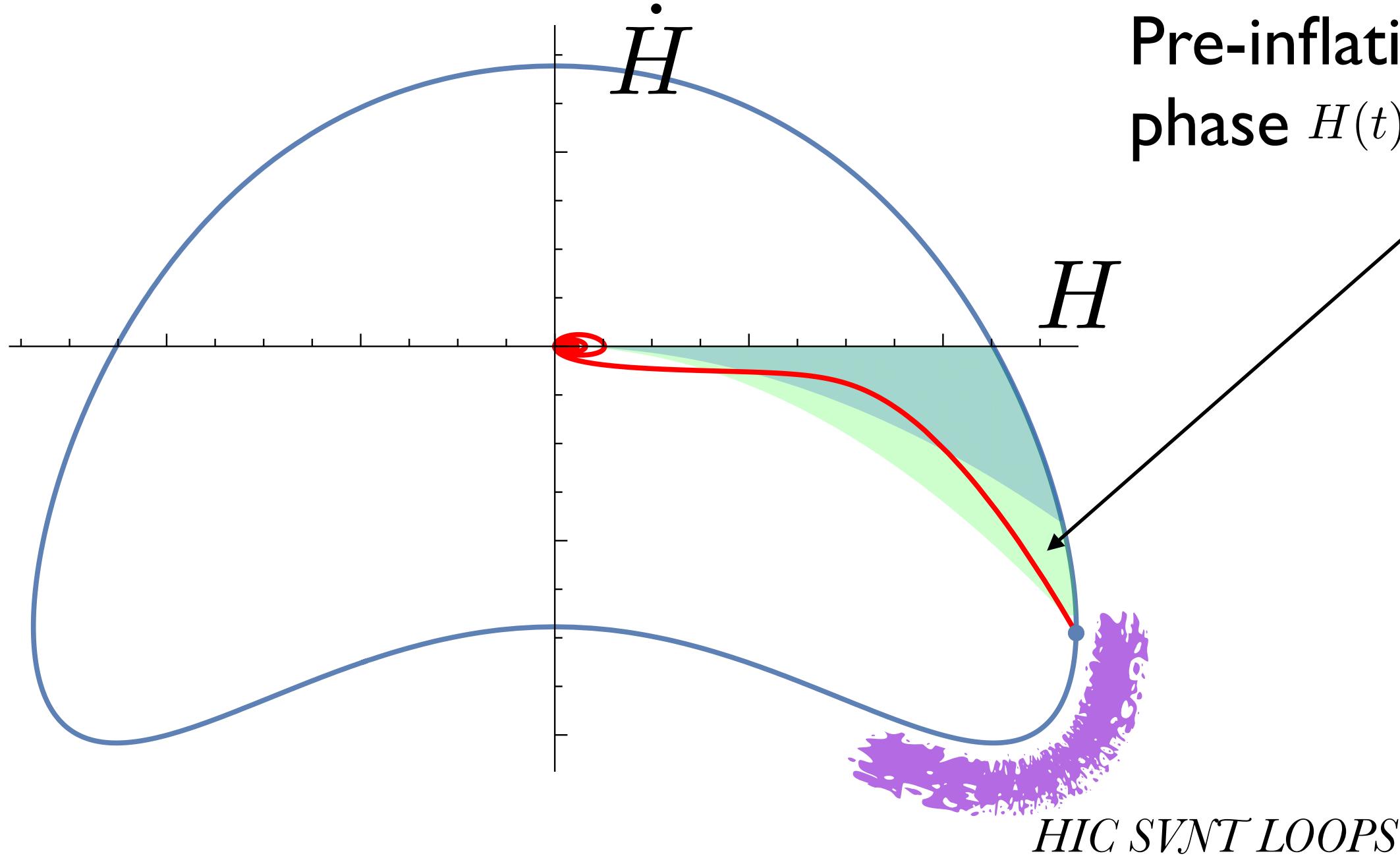
$$\dot{H} = -\frac{1}{36\alpha}$$

Pre-inflationary phase

$$H(t) \approx \frac{1}{2t}$$

HIC SVNT LOOPS

Pre-inflationary initial conditions: scalar and tensor modes



In the pre-inflationary phase
both scalar and tensor perturbations satisfy

$$\ddot{u}(k, t) + \frac{1}{t} \dot{u}(k, t) + \frac{k^2}{H_c t} u(k, t) = 0$$

adiabatic vacuum $u_0(k, t) = \sqrt{\frac{\pi}{2}} \left(J_0(2k \sqrt{t/H_c}) - i Y_0(2k \sqrt{t/H_c}) \right)$

vanishing correlations in the limit $t \rightarrow 0$, Bunch-Davies like correlations produced before the quasi-de Sitter phase