Holographic signatures of resolved cosmological singularities

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based on work in collaboration with Andreas Schäfer, John Schliemann

International Loop Quantum Gravity Seminar

March 21, 2017

Talk in a nutshell

- **Aim:**
  - Use LQG for AdS/CFT:

    $\boxed{\text{Holographic dual of resolved singularity}}$

- **Results:**
  - Proof of principle
    - Necessary computations possible
    - Singularity resolution $\Leftrightarrow$ better behaved dual CFT
  - Many simplifications
    - Allow for analytic computation
    - Otherwise numerics necessary
- **Long term focus:**
  - Strongly coupled finite N gauge theories via holography

Related work: [Freidel '08; Ashtekar, Wilson-Ewing '08; NB, Thiemann, Thurn '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15; Bonzom, Dittirch '15; NB '15; Smolin '16; Han, Hung '16; Chirco, Oriti, Zhang '17, ...]

Plan of the talk

1. Setting and strategy
2. Holographic signatures of cosmological singularities
3. Quantum corrected Kasner-AdS
4. Conclusion

Outline

Gauge / Gravity

Gauge / Gravity in a nutshell

- Conjectured duality between gauge and (quantum) gravity theories
- Dictionary for computations
- Practical importance: Tool for QFT
- Theoretical importance: Quantum gravity model
- Main example: AdS/CFT [Maldacena '97, ...]

Why bother?

- AdS/CFT mostly applied for classical gravity
  - Infinite number of colours N in dual CFT
  - Singularities in classical gravity
- Quantum gravity
  - Singularity resolution
  - Quantum corrections $\leftrightarrow$ finite N

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The Maldacena conjecture

[Maldacena '97; Gubser, Klebanov, Polyakov '98; Witten '98]

Conjectured exact equivalence

Type IIB String Theory on $AdS^5 x S^5$

$\longleftrightarrow$

$\mathcal{N} = 4$ Super Yang-Mills Theory in $4d$

String coupling g_s, String length l_s

't Hooft coupling λ, number of coulors N

1. weak string coupling
   (only tree level)
2. small string length
   (only massless states)

$\downarrow$

1. large number of colours
   (only planar diagrams)
2. large 't Hooft coupling

Well tested low energy equivalence

Type IIB Supergravity asympt. $AdS^5 x S^5$

$g_s \to 0, \quad l_s \to 0$

$\longleftrightarrow$

Planar 4d, $\mathcal{N} = 4$ Super Yang-Mills at strong 't Hooft coupling

$\lambda \to \infty, \quad N \to \infty$

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Working hypothesis 1: effective string theory description

Loop quantized Type IIB Supergravity (asympt. $AdS^5 x S^5$)

$N^2 \propto \frac{R^8}{G_{10}} \propto \frac{R^8}{g_s^2 l_s^8}$ finite, $\quad l_s \to 0$

$\longleftrightarrow$ naively

$\mathcal{N} = 4$ Super Yang-Mills Theory in $4d$ (symmetry broken?)

$\lambda \to \infty$, $\quad N =$ finite

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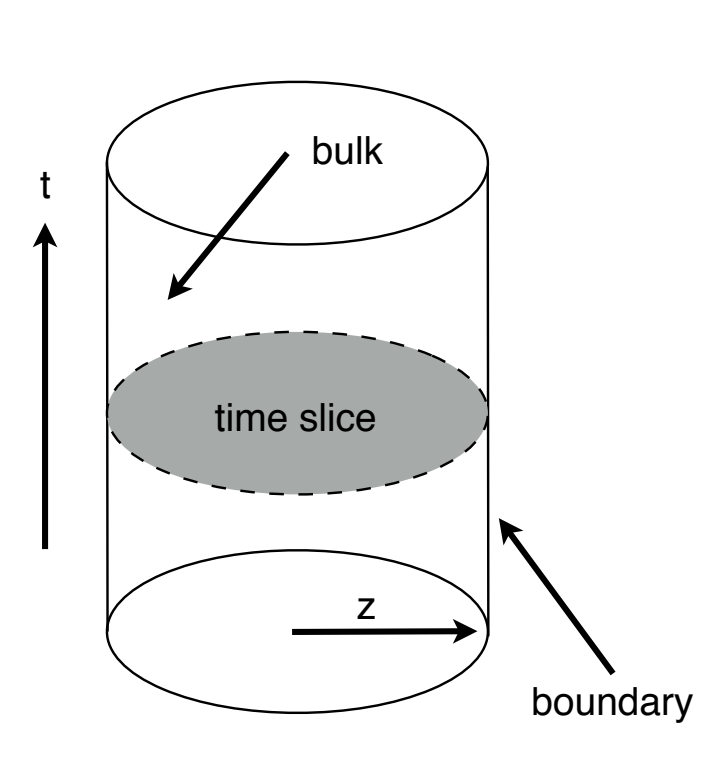
AdS/CFT: geometry and dictionary



Dictionary

CFT (boundary)	Gravity (bulk)
2-point correlator	Green's function / geodesics
Entanglement entropy	Minimal surfaces
Energy-momentum	On-Shell action
...	...

Working hypothesis 2: dictionary

Extends to quantum corrected effective geometries

See also [Ashtekar, Wilson-Ewing '08]

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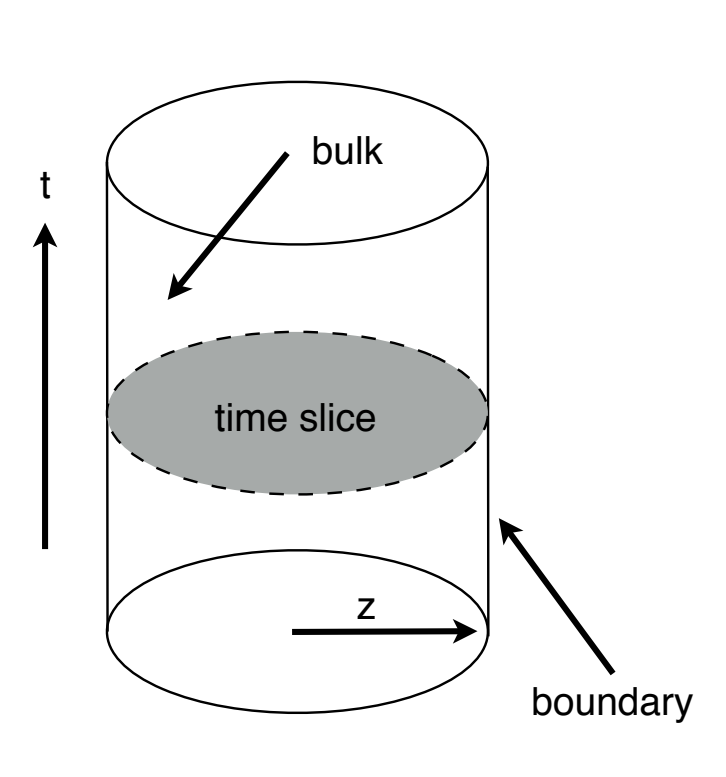

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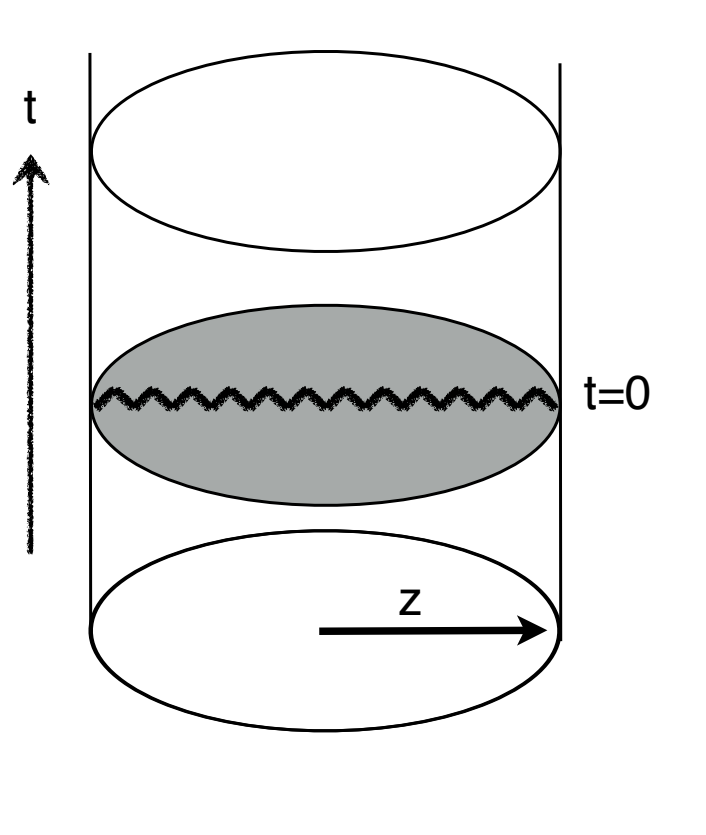
Two-point correlators in Kasner

[Engelhardt, Horowitz '14; Engelhardt, Horowitz, Hertog '15]



Boundary: $ds_4^2(t) = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2, \quad p_i \in \mathbb{R}$

Bulk: $ds_5^2 = \frac{R^2}{z^2}\left(dz^2 + ds_4^2(t)\right)$

Geodesic approximation: (heavy scalar operators)

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle = \exp(-\Delta L_{\text{ren}})$$

Δ: conformal weight of $\mathcal{O}$
L_{ren} : renormalised geodesic length

Caution:

- Geodesics may intersect the singularity
- Multiple geodesics for same boundary endpoints
- Complex solutions

Here: consider only real and classically singular solutions, $p_{i,\text{singular}} < 0$

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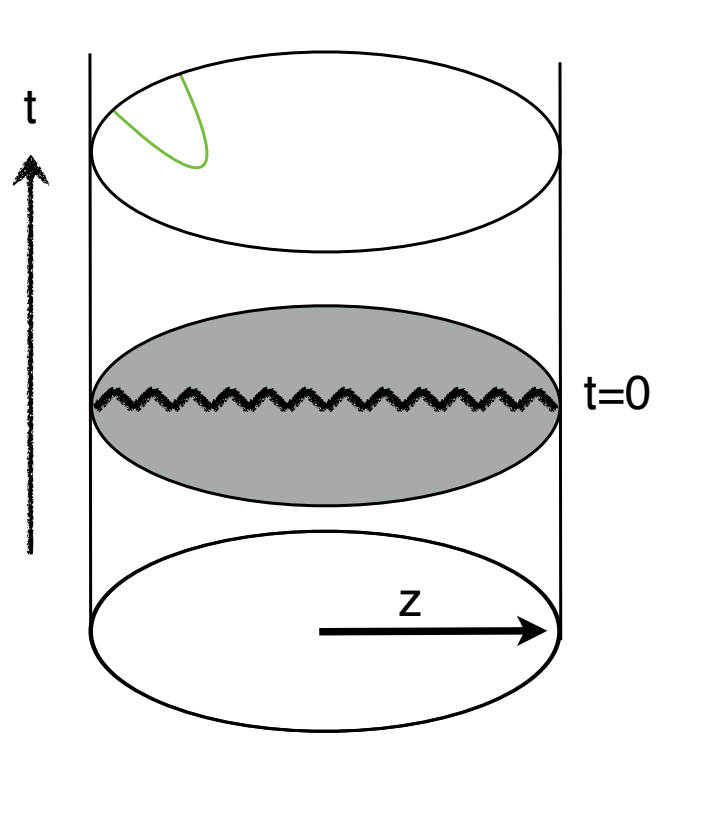

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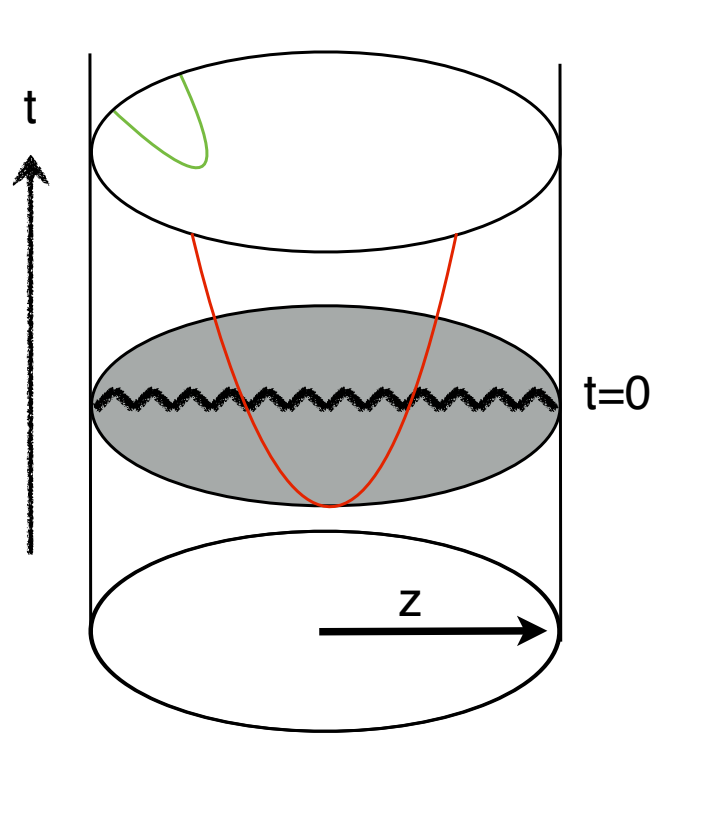

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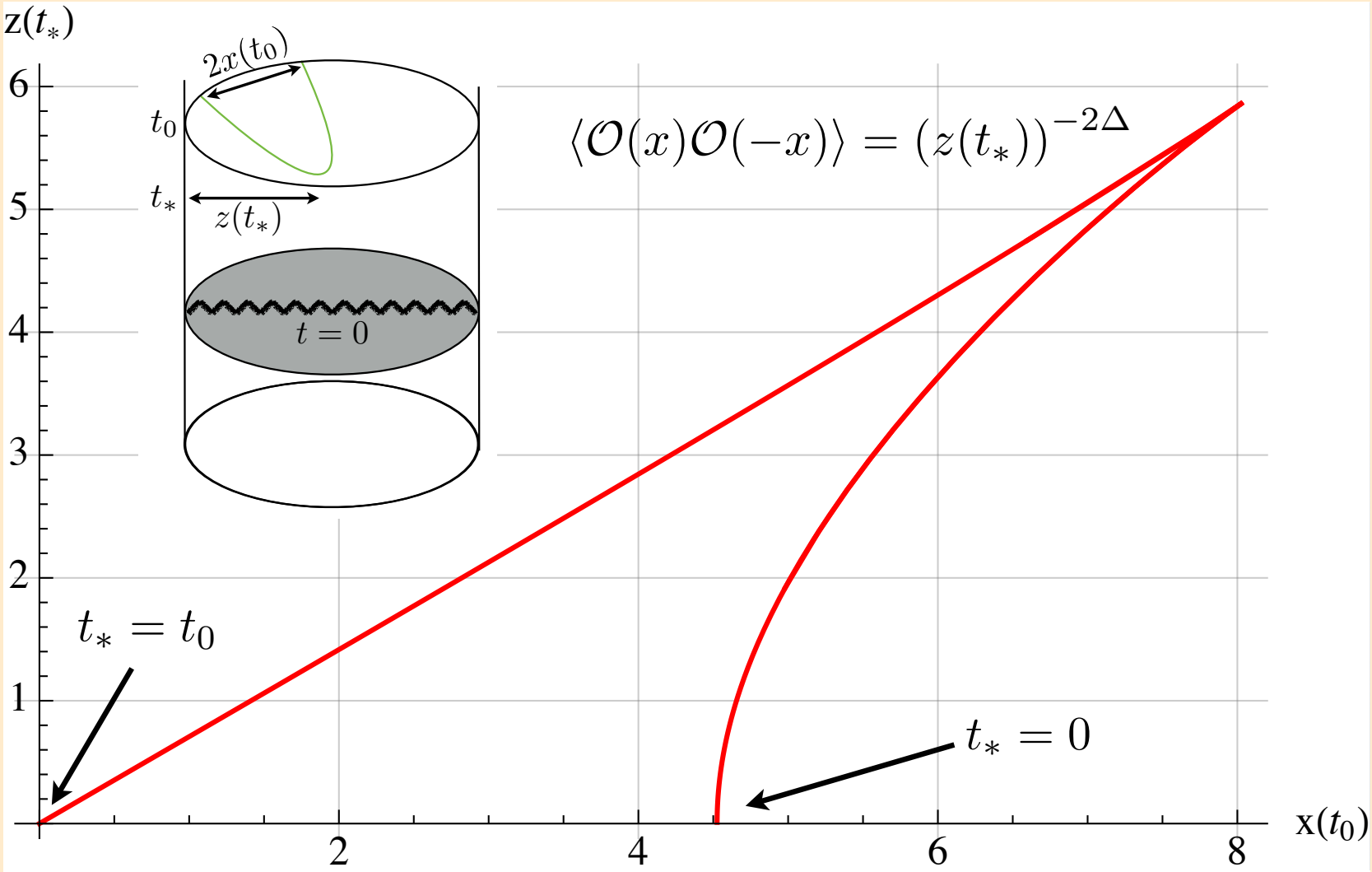

Finite distance pole in two-point correlator!

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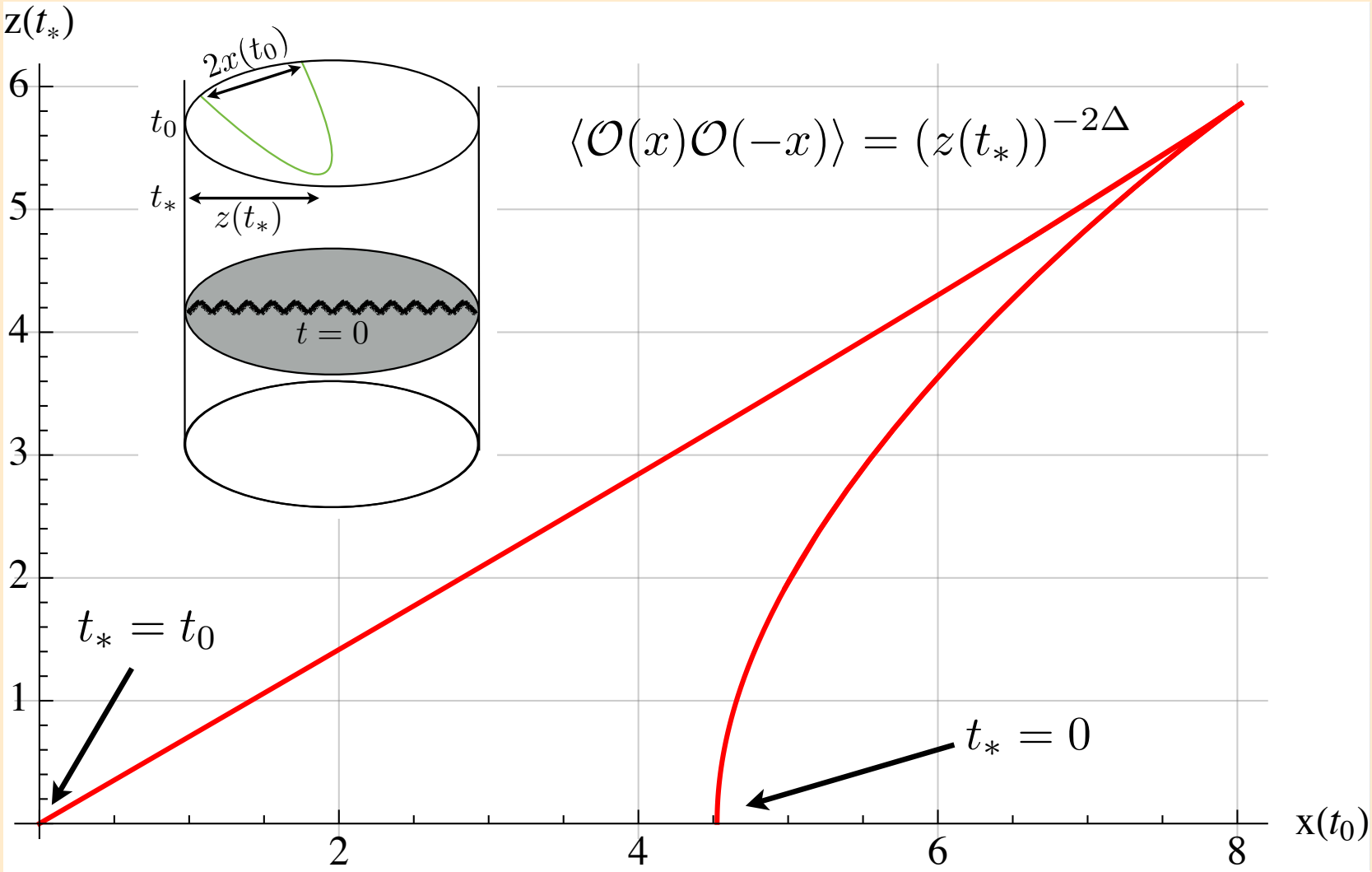

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Outline

Approximating Quantum-Kasner-AdS

- In principle: derive from given model
- Here: guess quantum corrected metric

Classical metric

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- No large curvatures associated with z-direction
- Curvature singularity in t-direction

Idea:

1. z-direction classical
2. effective LQC for the 4d part [Gupt, Singh '12]

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Approximating Quantum-Kasner-AdS

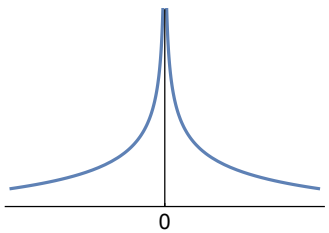
Quantum corrected Kasner from effective LQG [Gupt, Singh '12]

- Singularity resolution
- Kasner transitions (p_i change across the bounce)
- Results obtained numerically

Here: **analytic treatment** $\rightarrow$ need simplification!

1. No Kasner transitions

$$ds_4^2 = -dt^2 + a(t)^2 dx^2 + \ldots, \qquad a(t) = \frac{a_{\text{ext}}}{\lambda^p}\left(t^2 + \lambda^2\right)^{p/2}$$



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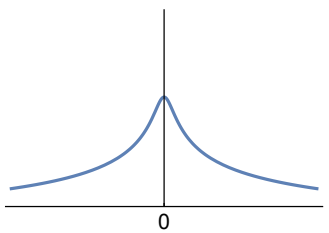

2. 4d Planck scale in bulk, not 5d (more later)

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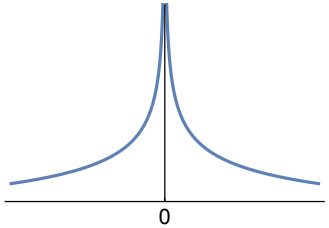

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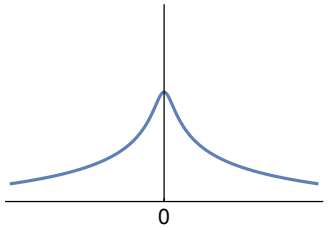

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Solution to geodesic equations

Solutions can be found in closed form:

- Completely in non-affine parametrisation: $z(t), x(t)$
- Partially in affine parametrisation: $z(s)$, $\quad$ s = geodesic length

$$\text{E.g.:}\quad z(t) = \sqrt{z(t_*)^2 - \left(\int_{t_*}^{t} dt' \frac{\tau'^{p/2}}{\sqrt{1-\tau'^{p}}}\right)^2}, \qquad \tau' = \left(\frac{t'^2+\lambda^2}{t_*^2+\lambda^2}\right)$$

$$s(z=\epsilon) \overset{\epsilon \ll 1}{=} 2\log\left(2z(t_*)\right) - 2\log(\epsilon), \qquad \epsilon = \text{boundary regulator}$$

This suffices to compute everything needed analytically!

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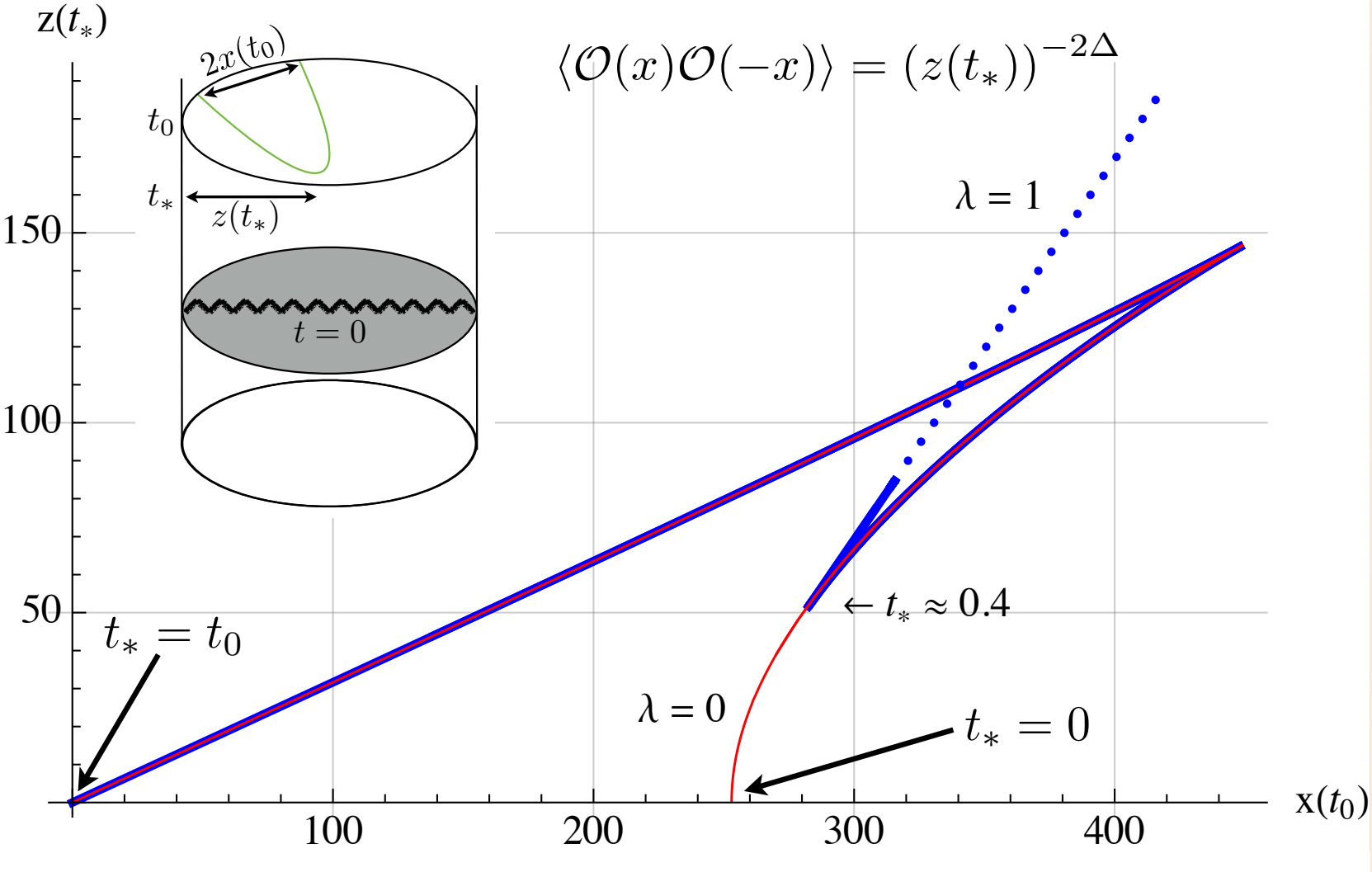

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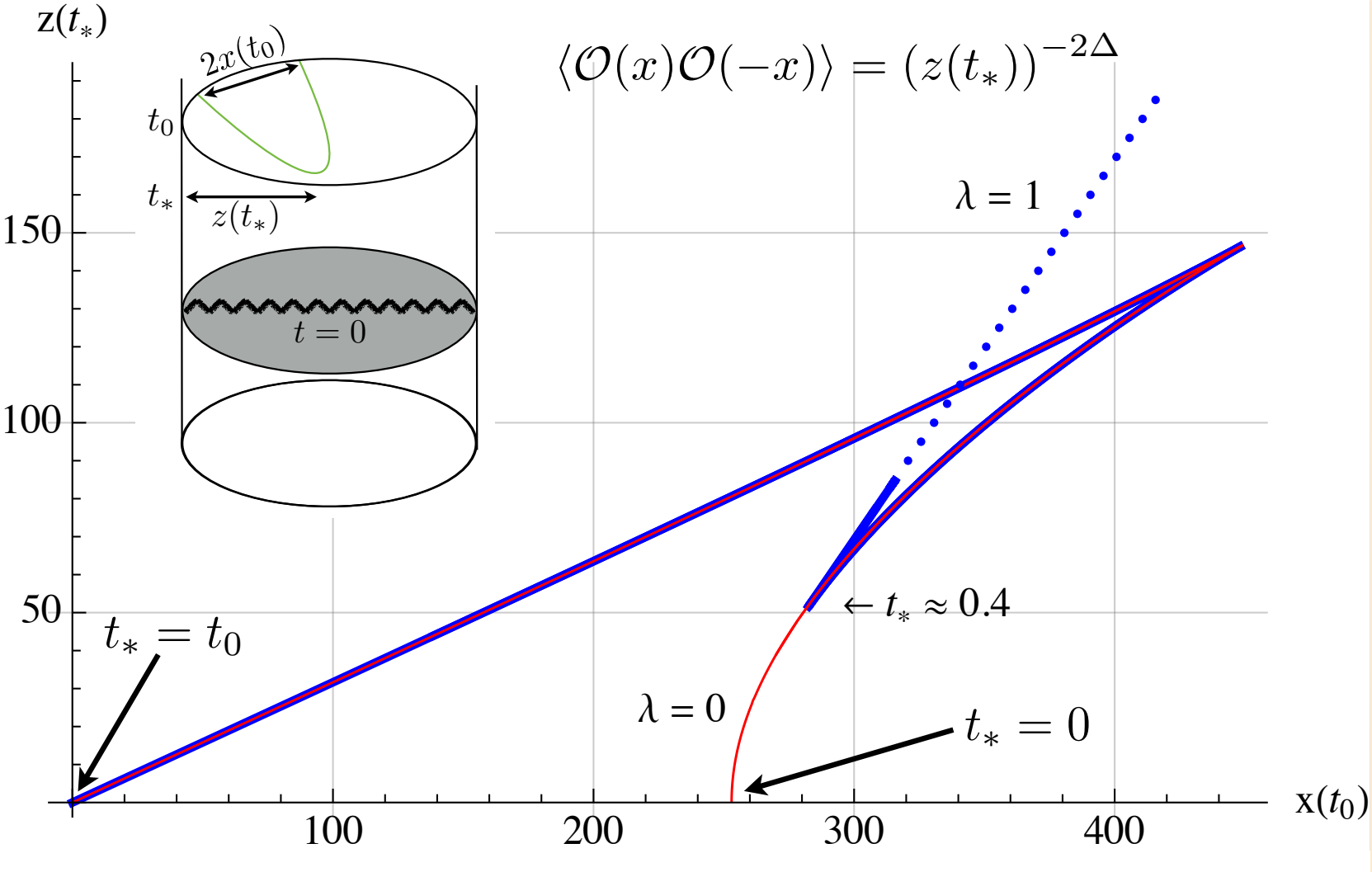

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Asymptotic behaviour

Short distance behaviour

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \xrightarrow{x\to 0} (\mathcal{L}_{\text{bdy}})^{-2\Delta}\,.$$

Typical (short distance) behaviour in conformal field theory

Asymptotic behaviour

Long distance behaviour

- Complex geodesics:

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \xrightarrow{x\to\infty} \propto (\mathcal{L}_{\text{bdy}})^{-\frac{2\Delta}{1-p}}$$

  - $\neq (\mathcal{L}_{\text{bdy}})^{-2\Delta}$ due to Kasner background breaking conformal symmetry

- Real singularity-free geodesic ($p < 0$):

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \xrightarrow{x\to\infty} \propto \lambda^{-2p\Delta} (\mathcal{L}_{\text{bdy}})^{-2\Delta}$$

  - Subdominant to complex contribution
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  - Non-analytic in λ

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4d vs. 5d Planck scale

$$ds_5^2 = \frac{R^2}{z^2}\left(dz^2 + \underbrace{-dt^2 + \frac{a_{\text{ext}}}{\lambda_0^p}\left(t^2+\lambda(z)^2\right)^{p/2} dx^2 + \dots}_{ds_4^2(t,z)}\right)$$

CFT background metric: $ds_4^2(t, z=0)$

4d Planck scale: $\lambda(z) = \lambda_0$	5d Planck scale: $\lambda(z) \propto z$
$-dt^2 + \frac{a_{\text{ext}}^2}{\lambda_0^{2p}}\left(t^2+\lambda_0^2\right)^p dx^2 + \dots$	$-dt^2 + \frac{a_{\text{ext}}^2}{\lambda_0^{2p}} t^{2p} dx^2 + \dots$
Singularity free	**Singular**

Usual AdS/CFT logic: 5d Planck scale

- Bulk quantum gravity from CFT on classical (singular) background
- ds_5^2 not singular at $t = z = 0$ due to $R^{(5)}_{\text{Kretschmann}} = z^4 R^{(4)}_{\text{Kretschmann}} + \dots$

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Conclusion

- **Proof of principle**
  - Merging effective LQG with AdS/CFT possible
- **Many simplifications made**
  - 4d instead of 5d Planck scale
  - No Kasner transitions
- **Future improvements**
  - Fix the above
  - Go beyond Kasner, e.g. black holes ...
  - Go beyond geodesic approximation (Green's functions)
- **Long term goal:**
  - Strongly coupled gauge theory at finite number of colours

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