Black hole entropy from loop quantum gravity:

Generalized theories and higher dimensions

Norbert Bodendorfer

Institute of Theoretical Physics
University of Warsaw

based on work by NB, Thiemann, Thurn [arXiv:1304.2679]
NB, Neiman [arXiv:1304.3025]

International Loop Quantum Gravity Seminar

October 1, 2013
Plan of the talk

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Entropy calculation: Basic ingredients

[Smolin '95; Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Krasnov '97--; ⋯; Engle, Noui, Perez '09--; ⋯]

Isolated horizon boundary of spacetime

Connection variables → boundary degrees of freedom

Idea: Count boundary degrees of freedom in agreement with total area

Important observation for BH entropy from LQG:

\[ S_{\text{BH}} \propto \frac{1}{\gamma} A_H \]

\( \gamma = \) Barbero-Immirzi parameter, \( A_H = \) horizon area

Ingredients on horizon slice \( H \)

- Boundary symplectic structure \( \int_H \delta_1 A^i \wedge \delta_2 A_i \)
- Boundary condition \( F^i(A) = \Sigma^i(E) \)
- Area spectrum \( 8\pi G \gamma \sqrt{j(j + 1)} \)
Higher dimensions: Results in short

Known:

- Isolated horizon (IH) framework extendable to higher dimensions
  [Lewandowski, Pawlowski gr-qc/0410146; Korzynski, Lewandowski, Pawlowski gr-qc/0412108]
  [Ashtekar, Pawlowski, v. d. Broeck gr-qc/0611049; Liko, Booth 0705.1371]
- LQG extendable to higher dimensions
  [NB, Thiemann, Thurn 1106.1103]

New:

- Boundary symplectic structure on IH can be derived
- Boundary condition can be derived
- Quantum theory can be formulated
- State counting problem can be reduced almost to the 3 + 1 dim. one
- Dimension independent log. correction in accordance with Carlip
- Extendable to Lanczos-Lovelock gravity and non-minimal coupling of scalars
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Going to higher dimensions: Canonical variables

3 + 1 dimensions: Bulk variables

- SU(2) connection: $(\gamma) A^a_i$
- Densitized triad: $(\gamma) E^b_j$
- Algebra:
  \[
  \{A^a_i, E^b_j\} = \delta^b_a \delta^j_i
  \]
- Barbero-Immirzi parameter: $\gamma \in \mathbb{R}$
- $E^{ai} E^{bi} = 1/\gamma^2 q^{ab}$
- $A^a_i = \Gamma^a_i + \gamma K^a_i$
- $a, b = 1, 2, 3$
- $i, j = 1, 2, 3$

$D + 1$ dimensions: Bulk variables

- SO($D + 1$) connection: $A_{aIJ}$
- Densitized hybrid vielbein: $\pi^{bKL}$
- Algebra:
  \[
  \{A_{aIJ}, \pi^{bKL}\} = \delta^b_a \delta^{KL}_{IJ}
  \]
- Free parameter: $\beta \in \mathbb{R}$
- $\pi^{aIJ} \pi^{bIJ} = 1/\beta^2 q^{ab}$
- $A_{aIJ} = \Gamma_{aIJ} + \beta 2 n_{[I} K_{a|J]} + \ldots$
- $n^I$ = internal normal
- $a, b = 1, \ldots, D$
- $I, J = 0, \ldots, D$

Holonomies from 1-forms

\[
\int_c A_{aIJ} \tau^{IJ} dx^a
\]

- $c$ = curve, $\tau^{IJ} =$ generators of SO($D + 1$)

Fluxes from $D - 1$ forms

\[
\int_S \pi^{aIJ} n_{IJ} e_{ab_1 \ldots b_{D-1}} dx^{b_1} \wedge \ldots \wedge dx^{b_{D-1}}
\]

- $S = D - 1$ surface, $n_{IJ}$ smearing functions

→ Holonomy-Flux algebra with SO($D + 1$) structure
Going to higher dimensions: Constraints

List of constraints (all first class before using holonomies / fluxes)

- Hamiltonian constraint
- spatial diffeomorphism constraint
- Gauß constraint (solved by using spin networks)
- Simplicity constraint \( \pi^{[IJ}\pi_{b|KL]} = 0 \) (no torsion constraints, gauge unfixing)

→ Most interesting here: Simplicity constraint.

Action of spin network edges:

- Selects “simple” \( SO(D+1) \) representations [Freidel, Krasnov, Puzio hep-th/9901069]
- Labelled by single non-negative integer \( \lambda \) \( (\lambda = 1 \iff j^+ = j^- = 1/2) \)

Action of spin network vertices:

later… important for entropy calculation
Going to higher dimensions: Canonical analysis

Start with Palatini action: $\text{SO}(1, D)$ connection $A_{IJ}$ and $D + 1$-bein $e^I$.

**D+1 split of the Palatini action**

\[
\int \star (e^I \wedge e^J) \wedge F_{IJ}(A) \Leftrightarrow \int_{\mathbb{R}} \int_{\sigma} d^Dx \left( \frac{1}{2} \pi^{aIJ} \dot{A}_{aIJ} - N\mathcal{H} - N^a\mathcal{H}_a - \frac{1}{2} \lambda_{IJ} G^{IJ} - c_{ab}^{\overline{M}} S_{ab}^{\overline{M}} \right)
\]

$\mathcal{H} = \text{Hamiltonian constraint, } \mathcal{H}_a = \text{spatial diffeomorphism constraint, } G^{IJ} = \text{Gauss constraint, } S_{ab}^{\overline{M}} = \text{simplicity constraint}$

$\pi^{aIJ} = \text{momentum of } A_{aIJ}$, $\sigma = \text{spatial slice}$, $N = \text{lapse function}$, $N^a = \text{shift vector}$, $\overline{M} = M_1 \ldots M_{D-3} = \text{multiindex}$

\[n^I e_a^I = 0, \quad \sqrt{q} = \sqrt{\det q_{ab}}, \quad q_{ab} = e_a^I e_b^J \eta_{IJ} = \text{spatial metric, } a = 1, \ldots, D \text{ spatial tensorial indices}\]

\[S_{ab}^{\overline{M}} = \pi^{aIJ} \pi^{bKL} \epsilon_{IJKLM} = 0 \Rightarrow \pi^{aIJ} = 2n^I \sqrt{q} e^a J \quad \text{[Freidel, Krasnov, Puzio hep-th/9901069]}\]

**Apply Dirac’s stability algorithm** [Peldan gr-qc/9305011]

Everything works, but one more constraint appears: $\{\mathcal{H}, S_{ab}^{\overline{M}}\} = D_{ab}^{\overline{M}}$.

Second class partner for the simplicity constraint: $\{S_{ab}^{\overline{M}}, D_{cd}^{\overline{N}}\} = \text{invertible}$

Remove $D_{cd}^{\overline{N}}$ via gauge unfixing. $\to$ Modify Hamiltonian constraint for consistency.

Perform canonical transformation to $\text{SO}(D + 1)$ as gauge group.
Works because ADM phase space of Lorentzian and Euclidean gravity is the same.
Going to higher dimensions: Area operator

The usual construction of the area operator generalizes directly

Idea: area operator \( \sim \sqrt{\text{flux}^2} \) integrated over \((D - 1)\)-surface

Area operator diagonal on edges labelled with \( \lambda \):

Eigenvalues \( 8\pi G \beta \sqrt{\lambda(\lambda + D - 1)}, \quad \lambda \in \mathbb{N} \)

First expectation for entropy:

- Calculation goes through as in 3 + 1 dimensions
- Horizon is modeled as a boundary of the spatial slice
- We count different possibilities to distribute the horizon area
- Area gap ensures finite entropy
- Arguments for statistics of the punctures are the same in higher dimensions
- (Actual calculation of course more detailed...)
“Dimensional” analysis

Boundary condition in $3+1$ dim.: $F^i(A) = \Sigma^i(E)$

- Boundary: Curvature $2$-form $F(A)^i$
- Bulk: $2$-form $\Sigma^i$ build from densitized triad as $\Sigma^i_{ab} = E^{ci} \epsilon_{abc}$

Higher dimensions

- Boundary: Curvature $2$-form $F(A)^{IJ}$
- Bulk: $(D-1)$-form $\pi^{cIJ} \epsilon_{a_1...a_{D-1}c}$ $\rightarrow$ mismatch in tensor structure

Two possibilities:

- Modify form of boundary condition
  $\epsilon \pi^{IJ} \sim (F(A) \wedge \ldots \wedge F(A))^{IJ}$ (with appropriate internal index contraction)

- Use different variables on the horizon (i.e. not a connection)
  $\epsilon \pi^{IJ} = L^{IJ} \epsilon \iff (L^{IJ} \epsilon = \text{internal horizon bi-normal as } (D-1)\text{-form})$
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Geometry of the problem: Two pictures

Show that action principle is well defined using the isolated horizon boundary condition on $\Delta$.

Perform canonical transformations to e.g. Chern-Simons type variables.
Switch gauge group to SO(D+1).
Action principle and covariant canonical framework

Start with the $D + 1$-dimensional Lorentzian Palatini action

$$S = S[A, e] = \int \Sigma^{IJ}(e) \wedge F(A)^{IJ}, \quad \Sigma^{IJ} \sim *(e^I \wedge e^J)$$

No additional boundary contribution on $\Delta$.

Use that $\Delta$ is a non-rotating isolated horizon

$$\Rightarrow \int_{\Delta} \Sigma^{IJ} \wedge \delta A^{IJ} = 0, \quad \Rightarrow \text{usual equations of motion are enforced}$$

Second variation on $\Delta$ reduces to a boundary contribution on $H_2, H_1$:

$$\int_{\Delta} \delta_1 \Sigma^{IJ} \wedge \delta_2 A^{IJ} = \left( \int_{H_2} - \int_{H_1} \right) \text{Boundary symplectic structure}$$

$\rightarrow$ Leads to a boundary contribution in the symplectic structure.
Boundary symplectic structure and boundary condition

**Bulk variables:**

SO($D+1$) connection $A_{aIJ}$, densitized hybrid vielbein $\pi^{bKL}$, $\{A_{aIJ}, \pi^{bKL}\} = \delta^b_a \delta^{KL}_{IJ}$

$n^I = \text{internal normal}$, $\pi^{aIJ} \approx 2/\beta n^I e^a_J$, $s^I = s^a e^I_a = \text{horizon slice normal}$, $\tilde{s}^I = \sqrt{\hbar} s^I$, $\sqrt{\hbar} = \text{area density on } H$

**Boundary symplectic structure**

$3+1$: $\int_H \delta_{[1} e^i \wedge \delta_{2]} e_i \propto \int_H \delta_{[1} A^i \wedge \delta_{2]} A_i$

$\int_H \delta_{[1} \tilde{s}^I \delta_{2]} n_I \propto \frac{A_H}{\chi} \beta \int_H \epsilon^{IJJKLM_2N_2\ldots M_nN_n} \left( \delta_{[1} \Gamma^0_{IJ} \right) \wedge \left( \delta_{2]} \Gamma^0_{KL} \right) \wedge R^0_{M_2N_2} \wedge \ldots \wedge R^0_{M_nN_n}$

$Dn^I = Ds^I = D \tilde{e}^I = 0$, $D = \partial + \Gamma^0$, $R^0_{IJ} = R(\Gamma^0)_{IJ}$, $A_H = \text{horizon area}$, $\chi = \text{Euler characteristic of } H$, $n = (D-1)/2$

**Boundary condition**

$3+1$: $F^i(A) = \Sigma^i(E)$

$\hat{s}_a \pi^{aIJ} \epsilon \propto \frac{2}{\beta} n^I [l \hat{s}^J] \epsilon \propto \frac{A_H}{\chi} \beta \epsilon^{IJKLMNOP_0\ldots K_nL_n} R^0_{K_1L_1} \wedge \ldots \wedge R^0_{K_nL_n}$

**Two choices of boundary variables**

- Connection (Chern-Simons-theory): hard, local DOF for $D > 3$, $D + 1$ even
- Metric variables $n^I$, $\tilde{s}^I$: easier to handle, dimension independent treatment

Norbert Bodendorfer (Warsaw University)  LQG and the Wald entropy formula  Oct. 1, 2013  15 / 29
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Higher dimensions: Problems with Chern-Simons theory

Symplectic structure

\[
\frac{A_H}{\chi \beta} \int_H \epsilon^{IJKLM_2N_2 \ldots M_nN_n} \left( \delta_1 \Gamma^0_{IJ} \right) \wedge \left( \delta_2 \Gamma^0_{KL} \right) \wedge R^0_{M_2N_2} \wedge \ldots \wedge R^0_{M_nN_n}
\]

→ In general very complicated Poisson brackets

Boundary condition: area-related degrees of freedom

\[
\hat{s}_a \pi^{aIJ} \overset{BC}{=} L^{IJ} := 2/\beta n^{[I} \xi^{J]} \propto \epsilon^{IJL_2J_2 \ldots L_nJ_n} R^0_{L_2J_2} \wedge \ldots \wedge R^0_{L_nJ_n}
\]

\[
\left\{ L^{IJ}(x), L^{KL}(y) \right\} = 4 \delta^{(D-1)}(x - y) \delta^L[J \; L^I][K](x)
\]

Same result from bi-normal symplectic structure and Chern-Simons symplectic structure

Other phase space functions in the Chern-Simons theory seem physically irrelevant for the entropy computation. Remove them with stronger boundary condition?

Problem is avoided from the beginning when sticking to bi-normals as variables.
Higher dimensions: Quantization

Smeared binormals: Algebra of fluxes

Use $L_{S}^{IJ} = \int_{S} 2/\beta n^{[I} \tilde{s}^{J]}$ \quad \{L_{S}^{IJ}, L_{S}^{KL}\} = 4 \delta^{[J} L_{S}^{I][K} \rightarrow \text{so}(D + 1) \text{ Lie algebra}$

SU(2) case: [Engle, Noui, Perez, Pranzetti 1006.0634]

Boundary Hilbert space

Product of SO($D + 1$) representation spaces

→ Non-trivial at points where bulk spin network punctures boundary

→ Related to bulk rep. by boundary condition $\hat{s}_{a} \pi^{aIJ} = L^{IJ}$

Off-diagonal horizon simplicity constraints

Non-rotating isolated horizon → Off-diagonal simplicity constraints $L^{[IJ} L_{S_1}^{KL]} S_{2} \approx 0$

$Dk^{I} = 0$ and $L^{IJ} \propto l^{[I} k^{J]}$, $k^{I} \propto (n^{I} + s^{I})$, demand off-diagonal simplicity on two contractible charts

→ Breaks local gauge invariance to global invariance on $H$.

Locally covariant quantization in the context of Chern-Simons theory?
## Higher dimensions: Quantization

### Restrictions on horizon Hilbert space

<table>
<thead>
<tr>
<th>Gauge invariance / tracing:</th>
<th>SO($D + 1$) intertwiner</th>
<th>See also [Rovelli, Krasnov 0905.4916]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk simplicity:</td>
<td>Simple representations of SO($D + 1$) on $H$, label $\lambda \in \mathbb{N}$</td>
<td>[Freidel, Krasnov, Puzio hep-th/9901069]</td>
</tr>
<tr>
<td>Off-diagonal simplicity:</td>
<td>Simple intertwiner</td>
<td>(Intertwining repr. simple)</td>
</tr>
</tbody>
</table>

### 1 to 1 mapping of simple SO($D + 1$) and SU(2) intertwiners

$\rightarrow$ Using dimension formulas from SU(2) counting: 
$$ S = \text{const}(D) \frac{A_H}{\beta} - \frac{3}{2} \log A_H $$

$\rightarrow$ (Up to $\beta$) Same result as Carlip and Solodukhin using CFT methods

[Carlip hep-th/9812013, gr-qc/0005017; Solodukhin hep-th/9812056; log correct.: Kaul and Majumdar gr-qc/0002040]

- Compatible with generalized theories $S \propto S_{Wald}$
- Compatible with analytic continuation of $\beta$ $S = A_H/(4G) + \text{corrections}$

[Fromden, Geiller, Noui, Perez 1212.4060]
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Generalized gravity theories: Wald entropy

\[ S_{\text{Generalized}} = \int \sqrt{-g} \mathcal{L} \]

\[ \mathcal{L} = \mathcal{L} \left( g_{\mu \nu}, R_{\mu \nu \rho \sigma}, \nabla_{\xi_1} R_{\mu \nu \rho \sigma}, \ldots, \nabla_{(\xi_1 \ldots \nabla_{\xi_n})} R_{\mu \nu \rho \sigma}, \psi, \nabla_{\xi_1} \psi, \ldots, \nabla_{(\xi_l \ldots \nabla_{\xi_l})} \psi \right) \]

Entropy from classical first law [Wald gr-qc/9307038]

\[ S_{\text{Wald}} = \frac{1}{4G} \int_{H} \sqrt{h} \left( \frac{-\delta \mathcal{L}}{\delta R_{\mu \nu \rho \sigma}} \epsilon_{\mu \nu} \epsilon_{\rho \sigma} \right) \neq \frac{A_H}{4G} \]

\( \mathcal{L} = \) Lagrangian, \( \sqrt{h} = \) area density on \( H \)
\( \epsilon_{\mu \nu} = 2n_{[\mu} s_{\nu]} = \) horizon slice bi-normal

Here:

Restrict to GR phase space plus standard matter (no higher time derivatives):

- Lanczos-Lovelock gravity plus non-minimally coupled scalars
- Presentation in 3 + 1, works also in higher dimensions
Generalized gravity theories

**Pure GR**

The connection and the momentum both have standard geometric interpretation!

\[ A_{ai} = \Gamma_{ai} + \gamma K_{ai}, \quad 1/\gamma^2 q^{ab} = E^a_i E^b_j, \quad \{ A_{ai}, E^{bj} \} = \delta_a^b \delta^j_i \]

\( \Gamma_{ai} \) = spin connection, \( K_{ai} \) = extrinsic curvature, \( \gamma \) = Barbero-Immirzi parameter, \( q^{ab} \) = spatial metric

**Generalized theory**

The momentum \( P^{ai} \) conjugate to \( A_{ai} = \Gamma_{ai} + \gamma K_{ai} \) is not the densitized triad \( E^{ai} \)!

\[ P^{ai} \propto \frac{\partial \mathcal{L}}{\partial \dot{A}_{ai}} \Rightarrow \{ A_{ai}, P^{bj} \} = \delta_a^b \delta^j_i \]

e.g. non-minimally coupled scalar: \( \mathcal{L} = a(\Phi)R + ... \Rightarrow P^{ai} = a(\Phi)E^{ai} \)

(More discussion on this in the Loops13 talk, available at pirsa.org)
Area → Wald entropy

\[ \sqrt{\left(\hat{s}_a P^a\right)^2} = \text{Wald entropy density on horizon slice } H \]

\[
\hat{s}_a P^a \propto \frac{\partial \mathcal{L}}{\partial R_{\mu \nu \rho \sigma}} \epsilon^{\mu \nu} \epsilon^{\rho \sigma} \times \hat{s}_a E^{ai}
\]

\[
\propto \text{Wald entropy density} \times \text{area density} \times \hat{s}_a E^{ai}
\]

\[ \epsilon^{\mu \nu} = \text{binormal on horizon slice, } \mathcal{L} \text{ undensitized Lagrangian, } \hat{s}_a = \text{horizon slice co-normal} \]

Generalized area operator

Idea: \[ \text{Area} \propto \int \sqrt{|P|^2} \implies \text{Spec(Area)} = \gamma \sqrt{j(j+1)}, \quad j \in \mathbb{N}_0/2 \]

Generalized area density \( \propto \) Wald entropy density

Isolated horizon framework

Calculations as before, just with Wald entropy instead of area

(Horizon connection build from some \((D+1)\)-bein with area density \(\sim\) Wald entropy density)
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Area operator $\rightarrow$ “Wald entropy operator” (on isolated horizon only)

- The only operator from which we know that is has an “easy” spectrum measures Wald entropy. Interpretation of spin networks?
- Twisted geometry interpretation for generalized theories?
  Faces labelled by entropy, not area

Quantization of Wald entropy expected from general arguments

[Bekenstein gr-qc/9710076; Kothawala, Padmanabhan, Sarkar 0807.1481]

Most important ingredient in the horizon theory: area density

Lots of freedom for canonical transformations, different connections, different free parameter on horizon, ...
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Omitted points / further research

Omitted points:

- Ambiguity in the choice of horizon connection variables
  See Loops13 talk available at pirsa.org and [NB, Stottmeister, Thurn 1203.6525, NB, Neiman 1304.3025]

- Polyhedral interpretation
  Generalizing [Bianchi, Dona’, Speziale 1009.3402] to higher dimensions

Further research:

- Issue of obtaining prefactor 1/4 appears also in higher dimensions
  See [Gosh, Frodden, Perez 11--; Frodden, Geiller, Noui, Perez 1212.4060; NB, Stottmeister, Thurn 1203.6525, NB, Neiman 1303.4752, Pranzetti 1305.6714]

- Quantization of higher-dim. Chern-Simons theory on boundary
  More rigorous quantization? Gauge invariance and simplicity constraint?

- Extension to generic isolated horizons
  (Non-rotating condition used only on the covariant side and for Wald entropy)

- Topology corrections
  3 + 1 dim. [Kloster, Brannlund, DeBenedictis: gr-qc/0702036]

- Supergravity
  Isolated horizon: [Liko, Booth 0712.3308]
Outline

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Expectations for higher dimensions
3. Results in higher dimensions: Classical GR
4. Quantization
5. Generalized theories
6. Discussion / Remarks
7. Omitted points / further research
8. Conclusions
Summary

1. **Classical** results:
   - Start: Non-rotating isolated horizon
   - Action principle well defined
   - Boundary symplectic structure
   - Boundary condition

2. **Generalized theory** of gravity:
   - $\sqrt{\text{flux}^2} \propto$ Wald entropy density
   - $\Rightarrow$ Entropy $S \propto S_{\text{Wald}}$

3. **Quantization**
   - Chern-Simons theory description possible, but hard to quantize
   - Largely dimension-independent result from using densitized bi-normals
   - Useful for studying how to impose simplicity constraints

---

Thank you for your attention!

Norbert Bodendorfer (Warsaw University)  LQG and the Wald entropy formula  Oct. 1, 2013
Non-rotating isolated horizon

Definition

A sub-manifold $\Delta$ of $(M, g)$ is said to be a non-expanding horizon (NEH) if

1. $\Delta$ is topologically $\mathbb{R} \times H$ and null.
2. Any null normal $l$ of $\Delta$ has vanishing expansion $\theta_l := h^{\mu\nu} \nabla_\mu l_\nu$
3. All field equations hold at $\Delta$ and $-T^\mu_\nu l_\nu$ is a future-causal vector for any future directed null normal $l$.

Definition

A pair $(\Delta, [l])$, where $\Delta$ is a NEH and $[l]$ an equivalence class of null normals, is said to be a weakly isolated horizon (WIH) if for any $l \in [l]$

4. $\mathcal{L}_l \omega \equiv 0$. ($\nabla_\mu l_\nu \equiv \omega_\mu^l l_\nu$)

Definition

1. A non-rotating isolated horizon (NRIH) is a WIH where to each $l \in [l]$ there is a $k$ with the property “good foliation” (see paper for details), such that
2. $k$ is shear-free with nowhere vanishing spherically symmetric expansion and vanishing Newman - Penrose coefficients $\pi_J \equiv l^\mu m_J^\nu \nabla_\mu k_\nu$ on $\Delta$. 

Norbert Bodendorfer (Warsaw University)  LQG and the Wald entropy formula

Oct. 1, 2013 30 / 29