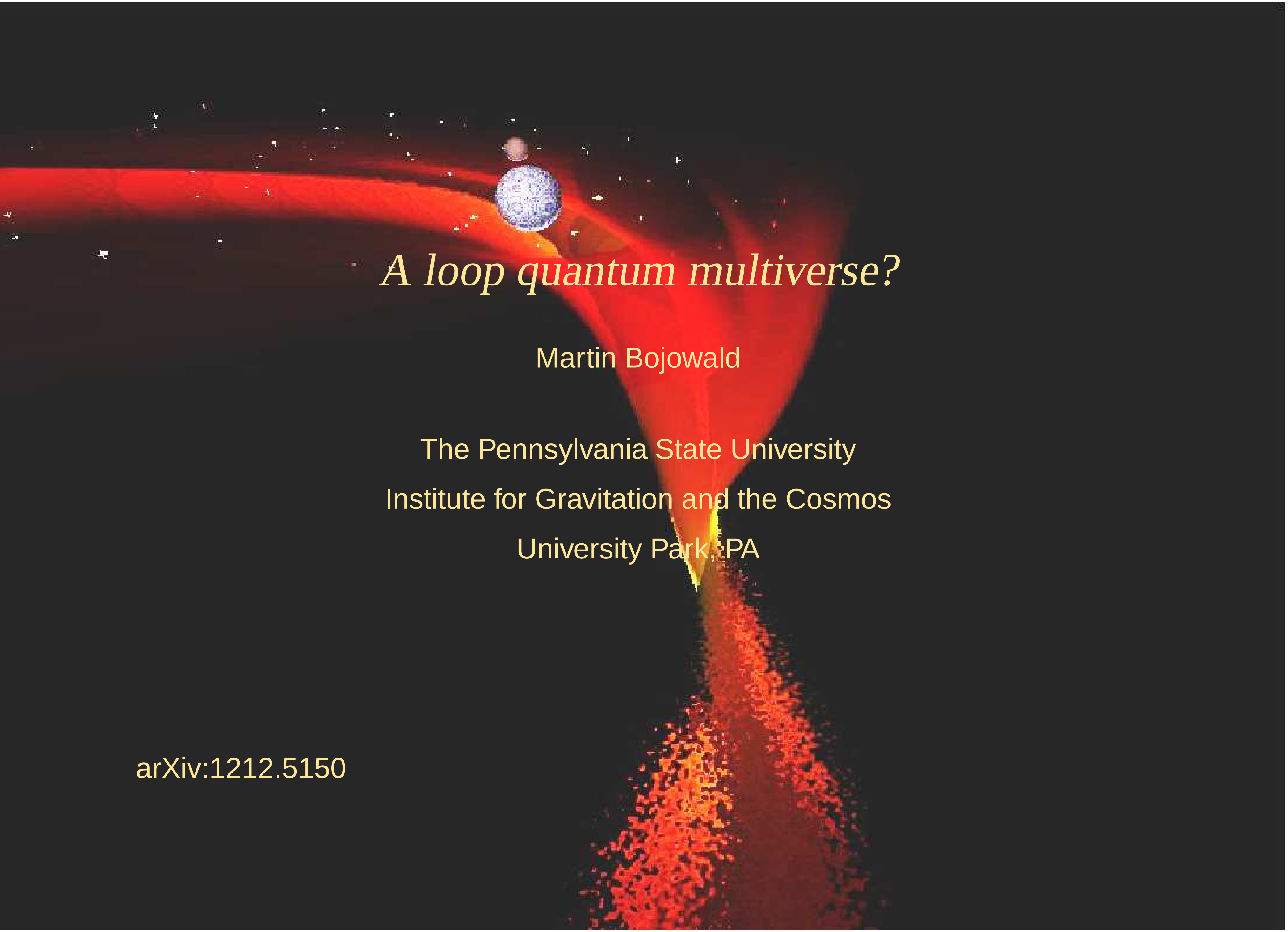

# *A loop quantum multiverse?*


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arXiv:1212.5150

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# Multiverse?

$\longrightarrow$ Inhomogeneity builds up in collapsing universe.

$\longrightarrow$ If space viewed as patchwork of homogeneous pieces, it must be refined in time to maintain good approximation.

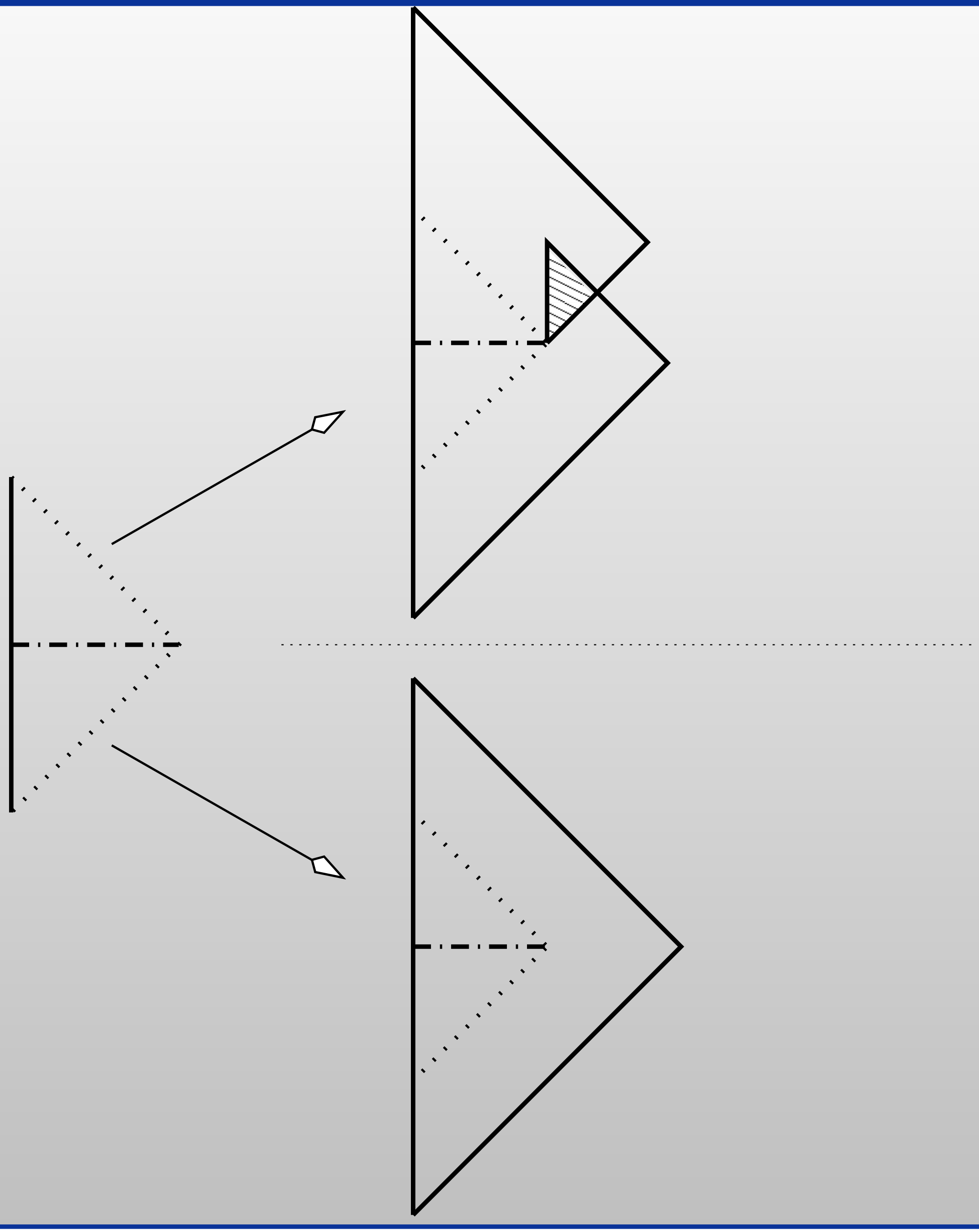
$\longrightarrow$ Assume that homogeneous models are good for patch evolution and bounce.
Patches that reach Planckian density earlier bounce sooner.

$\longrightarrow$ Expanding regions embedded within still contracting space-time. Causally separated?

(Picture similar to bubble nucleation in inflation.)

$\longrightarrow$ Black holes may form and grow.
If singularity resolved, where does it lead to?

## Black-hole singularities



# Inhomogeneity

Singularity resolution in inhomogeneous situations may lead to multiverse picture.

$\longrightarrow$ Loss of predictivity.

$\longrightarrow$ Inflation may not get started if inhomogeneity too large.

Loop quantum cosmology: Bounce models.
$\longrightarrow$ But Planckian regime almost entirely unclear.
$\longrightarrow$ Quantum corrections tricky in inhomogeneous context: strong consistency conditions from covariance.

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# Little bounce, big questions

Bounces in isotropic models simple consequence of modified Friedmann equation

$$\frac{\sin(\ell\mathcal{H})^2}{\ell^2} = \frac{8\pi G}{3}\rho$$

and analogous versions in homogeneous models.
(Some length parameter $\ell$, perhaps Planck length.)

- $\to$ How do quantum corrections (higher curvature) interfere with modification motivated by quantum geometry?
- $\to$ How can modified homogeneous models be embedded within a consistent inhomogeneous space-time structure?

At the heart of quantum gravity:
low-curvature limit and anomaly problem.

# Quantum corrections

Holonomies corrections: higher powers of $(\ell\mathcal{H})^2$, $\ell$ usually assumed to be close to $\ell_{\mathrm{P}}$.

$\longrightarrow$ Evolution of quantum state: Higher-curvature corrections must be added to modified Friedmann equation.
Expected to be of the same order: $\ell_{\mathrm{P}}^2 R \sim \ell_{\mathrm{P}}^2 \mathcal{H}^2$ in isotropic models.

$\longrightarrow$ Bounces confirmed only for kinetic domination in nearly isotropic models: No curvature corrections for free, massless scalar in flat isotropic model (deparameterized).

$\longrightarrow$ Numerical tests include all corrections in a single evolution.
But select specific wave function, usually near-Gaussian.
Amounts to assuming absence of curvature corrections at high density.

# Consistent inhomogeneity

Inclusion of inhomogeneity highly non-trivial:

$\longrightarrow$ Constraints can no longer be modified at will for algebra to remain first class.

$\longrightarrow$ Not clear if modified Friedmann equation can be part of consistent extension to inhomogeneity
(without using gauge fixing or deparameterization).

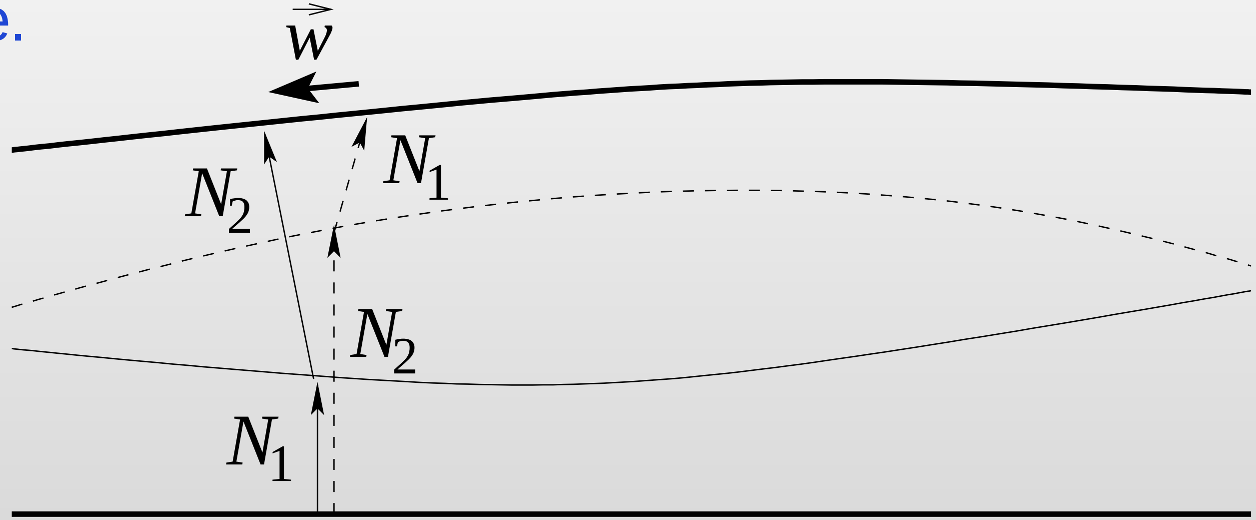
Several (incomplete) consistent versions known:
spherical symmetry and perturbative cosmology.

Imply signature change near Planckian density.

No evolution through bounce.

## General relativity

Non-linear coordinate changes — non-linear deformations of space.




Hypersurface-deformation algebra: $(S(\vec{w}(x)), T(N(x)))$ with

$$
\begin{aligned}
[S(\vec{w}_1), S(\vec{w}_2)] &= S(\mathcal{L}_{\vec{w}_2}\vec{w}_1) \\
[T(N), S(\vec{w})] &= T(\vec{w}\cdot\vec{\nabla}N) \\
[T(N_1), T(N_2)] &= S(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1)
\end{aligned}
$$

Implies Poincaré algebra for linear $N$ and $\vec{w}$ in local coordinates.

# Symmetry and dynamics

- → Hojman, Kuchař, Teitelboim 1974–76:
  Second-order field equations for metric invariant under hypersurface-deformation algebra must equal Einstein's.

- → Dirac 1958:
  Invariance under hypersurface-deformation algebra implies general covariance.

*How does quantum physics affect hypersurface deformations?*

## Quantum corrections

Loop quantum gravity:

$\longrightarrow$ Inverse-triad corrections from quantizing

$$\left\{A_a^i, \int \sqrt{|\det E|}\mathrm{d}^3x\right\} = 2\pi G\epsilon^{ijk}\epsilon_{abc}\frac{E_j^b E_k^c}{\sqrt{|\det E|}}$$

$\longrightarrow$ Higher-order corrections: holonomies for curvature.

$\longrightarrow$ Quantum back-reaction

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# Modified space-time

$\longrightarrow$ Inverse-triad corrections in Hamiltonian

$$\frac{1}{16\pi G}\int \mathrm{d}^3x N\alpha \frac{\epsilon_i{}^{jk}F^i_{ab}E^a_jE^b_k}{\sqrt{|\det E|}} + \cdots$$

$\longrightarrow$ Algebra modified. *Deform but do not violate covariance.*

[with G Hossain, M Kagan, S Shankaranarayanan 2009]

$$\begin{aligned}
\{S(\vec{w}_1), S(\vec{w}_2)\} &= S(\mathcal{L}_{\vec{w}_2}\vec{w}_1)\\
\{T(N), S(\vec{w})\} &= T(\vec{w}\cdot\vec{\nabla}N)\\
\{T(N_1), T(N_2)\} &= S(\alpha^2(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1))
\end{aligned}$$

$\longrightarrow$ Operators in Abelian $2+1$.

[A Henderson, A Laddha, C Tomlin, M Varadarajan 2012]

# Cosmological perturbation equations

[with G Calcagni 2010]

Dynamics of density perturbations $u$, gravitational waves $w$:

$$-u'' + s(\alpha)^2 \Delta u + (\tilde{z}''/\tilde{z})u = 0$$

$$-w'' + \alpha^2 \Delta w + (\tilde{a}''/\tilde{a})w = 0$$

Different speeds $\alpha$ and $s(\alpha)$ for different modes:
characteristic corrections to tensor-to-scalar ratio.

Consistent but non-classical space-time structure:
no invariant line element $\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b$.

# Holonomy corrections

Pointwise holonomy corrections:

$$
\begin{aligned}
\{S(\vec{w}_1), S(\vec{w}_2)\} &= S(\mathcal{L}_{\vec{w}_2}\vec{w}_1) \\
\{T(N), S(\vec{w})\} &= T(\vec{w}\cdot\vec{\nabla}N) \\
\{T(N_1), T(N_2)\} &= S(\beta(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1))
\end{aligned}
$$

with $\beta < 0$ at high density ("bounce"), $\beta = -1$ at max. density.

($\beta = \cos(2\ell c)$, or second derivative of holonomy modification function in spherical symmetry)

[J Reyes 2009; A Barrau, T Cailleteau, J Grain, J Mielczarek 2011]

Mode equation elliptic when $\beta < 0$.

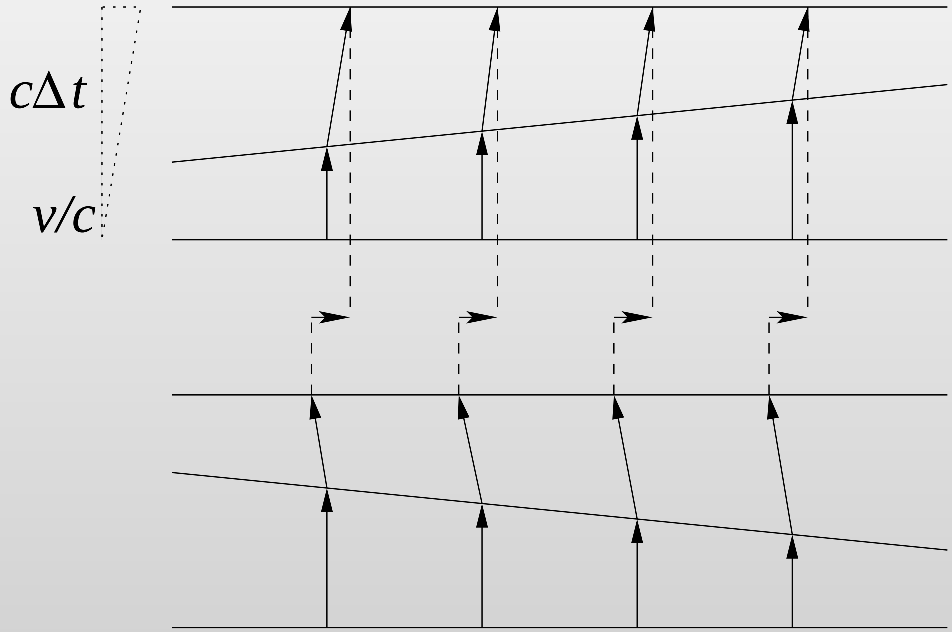
$$
-w'' + \beta\Delta w + (\tilde{a}''/\tilde{a})w = 0
$$

Operators in $2+1$. [A Perez, D Pranzetti 2010]

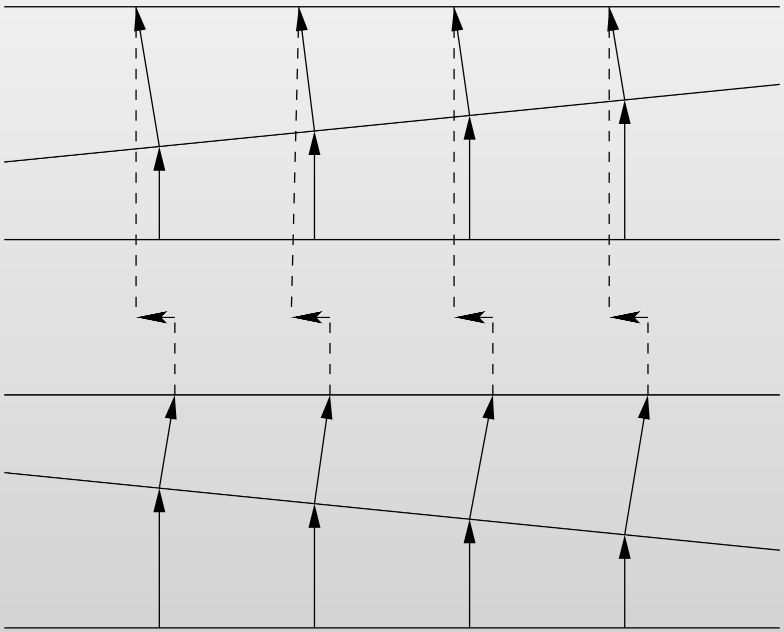
# Signature change

Space-time signature Euclidean.

[with G Paily 2011]




Minkowski geometry



Euclidean geometry

Reminiscent of Hartle–Hawking wave function.

## Status

Main incompleteness:

$\longrightarrow$ higher spatial derivatives (integrated holonomies)

$\longrightarrow$ higher time derivatives (quantum back-reaction).

At high curvature, $\partial c$ as significant as $c^2$. ($R \sim \partial\Gamma + \Gamma^2$)

But:

$\longrightarrow$ Higher time derivatives would affect background evolution as well.

$\longrightarrow$ Higher derivatives cannot cancel algebra deformation: order of derivatives preserved by integrations by parts, and Poisson bracket of moments produces other moments.

$\longrightarrow$ Random cancellation might be possible with strong fine-tuning of numerical values (moments of state).
But no deformation at all for higher-curvature corrections.

## Abhayisms I

$\longrightarrow$ The Wheeler–DeWitt equation can be hyperbolic or elliptic, and it says nothing about the signature of space-time.

Well, . . .

$\longrightarrow$ How can small perturbative inhomogeneity possibly have such a drastic effect as signature change?

Signature change is not a consequence of inhomogeneity; inhomogeneity is just used as a test field.
The equation $-w'' + \beta\Delta w + (\tilde{a}''/\tilde{a})w = 0$ is elliptic for $\beta < 0$ no matter how small $w$ is. Spherically symmetric models.

$\longrightarrow$ The Hojman–Kuchar–Teitelboim analysis, in the deformed case, does not give a consistent [Riemannian] space-time interpretation.

Sure. We are talking about effective quantum space-time.

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# Abhayisms II

$\longrightarrow$ The phase space in the presence of quantum corrections is assumed to be the same as classically.

No. The phase space changes when moment variables are included. Semiclassical expansion possible.
Even when they are not included as a first approximation, loop quantum gravity implies modifications for expectation values.

$\longrightarrow$ Sometimes, effective equations are re-quantized even though they are supposed to describe a quantum theory.

Quantum states in effective equations are sometimes used as a shortcut to apply established methods to fix initial conditions for an inflaton. They can be formulated at the effective level.

# Abhayisms III

$\longrightarrow$ There are consistent versions of inhomogeneity around bounce models that do not give rise to signature change.

In such versions, one uses gauge-fixing, deparameterization, or solves some first-class constraints before quantization. One therefore assumes some classical space-time properties without being able to test whether they give the correct quantum space-time structure. All known loop models that do not assume anything about space-time lead to signature change.

$\longrightarrow$ We always solve classical constraints before quantization when we discuss mini or midi-superspace models.
Why is this suddenly an issue?

For mini or midi-superspace models, we solve classical second-class constraints to remove some degrees of freedom. We do not solve or fix first-class constraints that encode space-time structure.

# Multiverse?

- → Inhomogeneous collapse combined with transition to expansion ("bounce") may lead to causally disconnected regions.

- → Hard to control with non-perturbative quantum gravity, but effective methods help to understand space-time structure.

- → *Big bounce blunder:* Modifications that limit the density have radical implications at Planckian densities.
  4-dimensional Euclidean space instead of space-time.

  No propagation of structure through high density.
  Everything is causally disconnected. Ultimate multiverse.

- → Instead, non-singular beginning of Lorentzian phase.
  Natural place to pose initial conditions for inflaton state.