Geometry Transition in Covariant Loop Quantum Gravity
Black to White

"Planck star tunneling time: An astrophysically relevant observable from background-free quantum gravity", PRD 2016, C.Rovelli, S.Speziale, I.Vilensky and MC

"Gravitational tunnelling in spinfoams", F. d'Ambrosio and MC

" Characteristic Planck star timescales from spinfoams", F. d'Ambrosio, C. Rovelli and MC

"Geometry Transition in Covariant Loop Quantum gravity" – PhD thesis, MC

Marios Christodoulou

Centre de Physique Théorique, Aix-Marseille Université

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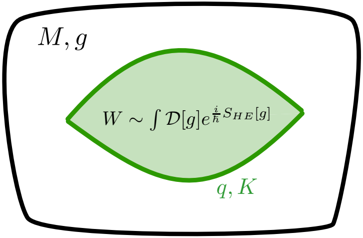
Geometry transition

Treat **birth of white hole** region from a **black hole** region through quantum gravitational effects as a **geometry transition problem**. The mental picture is that of a lens-shaped region enclosing strong, spatio–temporally localized, QG effects. The process is described by a spinfoam transition amplitude, which displays an emergent behaviour as a sum over geometries.



The phenomenon is **not forbidden** by any known physical principle. **In principle**, quantum theory should describe the relevant physics, even if they turn out not to be relevant experimentally. In particular, it should be possible to **estimate characteristic timescales** with currently available technology.

Summary

- We have not so far seen indications for lifetime shorter then Hawking evaporation time. We do not exclude the possibility, have preliminary indications for a much longer lifetime.
- An explicit simple estimate for transition amplitudes describing what appears to be gravitational tunneling.
- Better understanding of how the relevant physics are encoded in the black to white transition amplitude.
- Simpler form of the metric describing the Haggard-Rovelli ("fireworks") spacetime and a cleaner construction.
- Results will be reported shortly. Many open questions remain. Developments and extensions in progress.

Black holes age: Bouncing black holes?

> *"Of course, this calculation* **ignores** *the back reaction of the particles on the metric, and* **quantum fluctuations of the metric***. These might alter the picture."* *[Hawking, Black Hole Explosions , 1974]*

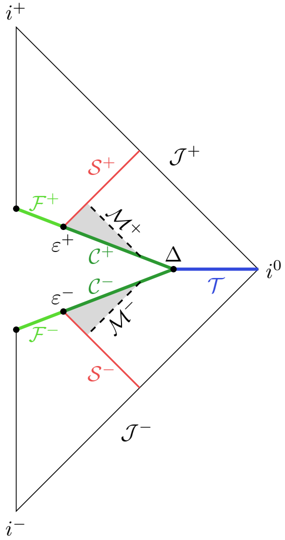
> *"Once the density and curvature enter the Planck scale quantum geometry effects become dominant creating an effective repulsive force which rises very quickly, overwhelms the classical gravitational attraction, and causes a bounce thereby resolving the big bang singularity"* *[Ashtekar, The Big Bang and the Quantum, 2010]*

Hawking evaporation, Page time, Firewall debate.

Dynamical evolution of interior and development of large volumes. *[Rovelli, MC, De Lorenzo, Bengtsson, Jacobsson, Ong . . . . . .]*
$V_S(v) \approx C\, m^2\, v + \mathcal{O}(m)\ \ ,\ v >> m\ \ ,\ C \sim 1$

BH singularity resolution in LQG. *[Modesto, Ashtekar, Bojowald, Singh, Corichi, Gambini, Pullin, Olmedo, Saini, Alesci . . .]* , Black to white transition, an idea with history in other approaches. *[Hajicek, Kiefer, Bojowald, Singh, Goshwami, Maarteens, Husain, Winkler, Barcelo, Carballo, Garay . . .]*

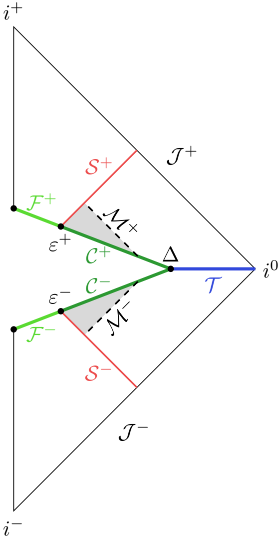
Haggard - Rovelli spacetime: exploding black holes



- Models transition of trapped region formed by collapse to anti-trapped region from which matter is released.
- Spherical Symmetry (Irrotational).
- Matter dynamics and dissipation neglected: Null shells $\mathcal{S}^{\pm}$.
- EFE's exactly solved.
- Inside the shells Minkowski, outside the shells patches of Kruskal.
- Trapped region in past of anti-trapped region.
- Interior boundary $F^- \bigcup C^- \bigcup F^+ \bigcup C^+$ separates part of system treated as classical and quantum. Spacelike, extends outside trapped surfaces $\mathcal{M}^{\pm}$ and chosen arbitrarily so long as trapped and anti-trapped regions present.

[Haggard, Rovelli, De Lorenzo, Perez, Barrau, Bolliet, Vidotto, Bianchi, Smerlak,...]

Metric à la Vaidya



$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\,\Theta(u - u_{\mathcal{S}^+})\right)\mathrm{d}u^2 - \\ - 2\mathrm{d}u\,\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

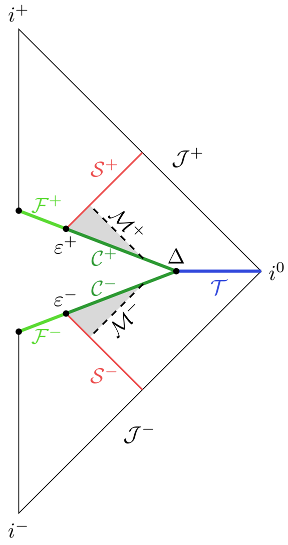
$$r \overset{\mathcal{T}}{=} r$$

$$v - u \overset{\mathcal{T}}{=} 2r^*(r)$$

Fiducial surface $\mathcal{T}$ separates two Kruskal patches.

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\,\Theta(v - v_{\mathcal{S}^-})\right)\mathrm{d}v^2 + \\ + 2\mathrm{d}v\,\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

Length (Mass) scale m and time scale T



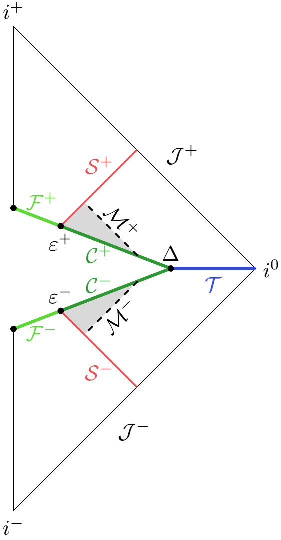
$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\,\Theta(u - T/2)\right)\mathrm{d}u^2 - \\ -2\mathrm{d}u\,\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

$$T = u_{\mathcal{S}^+} - v_{\mathcal{S}^-}$$

$$\begin{aligned} u &\to u - (v_{\mathcal{S}^-} + u_{\mathcal{S}^+})/2 \\ v &\to v - (v_{\mathcal{S}^-} + u_{\mathcal{S}^+})/2 \end{aligned}$$

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\,\Theta(v + T/2)\right)\mathrm{d}v^2 + \\ +2\mathrm{d}v\,\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

Bounce time parameter T



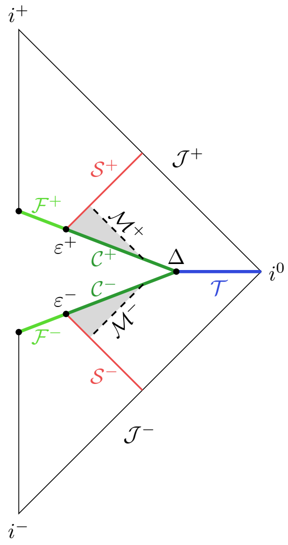
A parameter determining a time scale for the spacetime. It appears naturally in the proper time invariant integral along the orbits Υ of the Killing field, corresponding to stationary observers.

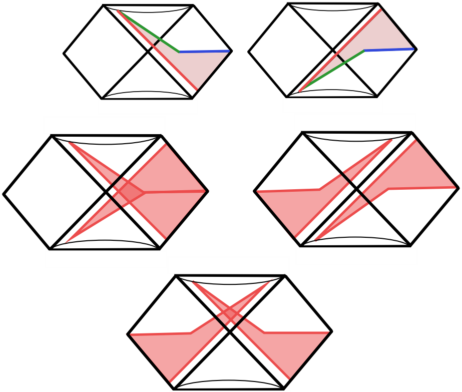
$$T = u_{\mathcal{S}^+} - v_{\mathcal{S}^-}$$

$$\tau_R = \sqrt{f(R)}(u_{\mathcal{S}^+} - v_{\mathcal{S}^-} + 2r^*(R))$$

$$T = \frac{\tau_\Upsilon}{\sqrt{f(A_\Upsilon)}} - r^*(A_\Upsilon)$$

Relation to “crossed fingers”

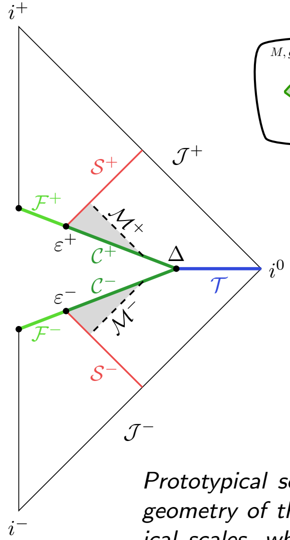


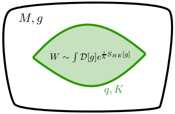


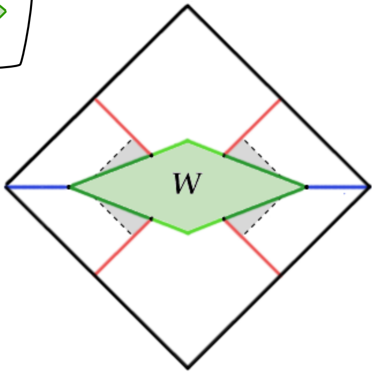
r_δ radius at which null shells cross in “crossed fingers” mapping (middle left figure above).

$$\begin{aligned} r_\delta &= 2m(1+\delta) \\ \delta &= W\left(e^{-\frac{T}{4m}}/e\right) \end{aligned}$$

A minimalistic setup for geometry transition







Prototypical setup for studying geometry transition. The geometry of the spacetime depends on two classical physical scales, which become encoded in the boundary state $|\Psi\rangle$*. In turn, quantum theory correlates the two scales in a probabilistic manner through a transition amplitude* $\langle W|\Psi\rangle$*.*

Covariant LQG in a nutshell: EPRL partition function

General relativity as constrained BF-theory

$$S_{BF}[B,\omega] = \int B \wedge \star F(\omega)$$

$$S_H[B,\lambda,\omega] = \int \left(B + \frac{1}{\gamma} \star B\right) \wedge \star F + $$
$$+\lambda\, C_S[B]$$

$$C_S[B]^{\mu\nu\rho\sigma} \equiv \epsilon_{KLAB} B^{KL}_{\mu\nu} \wedge B^{AB}_{\rho\sigma} -$$
$$-\frac{1}{4!} B^{\alpha\beta}_{CD} B^{\gamma\delta}_{IJ} \epsilon_{\alpha\beta\gamma\delta} \epsilon^{CDIJ} \epsilon_{\mu\nu\rho\sigma}$$

$$B = e \wedge e + \frac{1}{\gamma} \star e \wedge e$$

BF theory topological, can be quantized exactly for compact gauge groups.

$$W^{BF} = \int \mathcal{D}[\omega]\, \delta\left(F[\omega]\right)$$

[Barret,Crane,Freidel,Krasnov,Speziale, Livine, Perreira,Engle,Rovelli, Baratin,Oriti,Han,Haggard,Riello. . .]

$$W_{\mathcal{C}} \sim \int \mu(g_e)\, \delta(\prod_{e\in f} g_e)$$

$$B_f \sim \int_{f^\star} B\ , \quad g_e \sim \mathcal{P} e^{\int_e \omega}$$

Imposition of $C_S[B]$ at the quantum level: expand only in "simple" unitary irreps of $SL(2,\mathbb{C})$.

$$W_{\mathcal{C}} = \sum_{j_f} \mu(j_f) \times$$
$$\underbrace{\times \int_{SL(2,\mathbb{C})} \mu(g_{ve}) \prod_f A_f(g_{ve}, j_f)}_{\text{"partial amplitude"} \sim \exp \frac{i}{\hbar} S_R^{\mathcal{C}\star}}$$

$$A_f(g_{ve}, j_f) = \int_{SU(2)} \mu(h_{vf})\, \delta(h_f) \times$$
$$\times \operatorname{Tr}^{j_f} \left[\prod_{v\in f} Y_\gamma^\dagger g_{e'v} g_{ve} Y_\gamma h_{vf} \right]$$

Boundary state Ψ_Γ

Wavepackets of geometry: coherent spin–network states. Peaked on intrinsic and extrinsic discrete geometry of spacelike tetrahedral triangulation. Here, gauge variant version. Labelled by (dimensionless) data: **boost angles** ζ, **areas** ω per link and 3D normals $\vec{k}$ per half–link. **Semiclassicality parameter** $t \propto \hbar^k \, , k > 0$ (small).

$$\Psi_{\Gamma,t_\ell}(h_\ell;\omega_\ell,\zeta_\ell,\vec{k}_{\ell\mathbf{n}}) = \sum_{\{j_\ell\}} \prod_\ell d_{j_\ell} e^{-(j_\ell-\omega_\ell)^2 t + i\zeta_\ell j_\ell} \psi_\Gamma(h_\ell, j_\ell; k_{\ell\mathbf{n}})$$

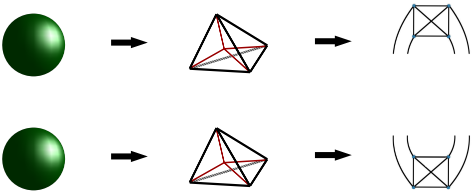
Superpositions of intrinsic coherent states

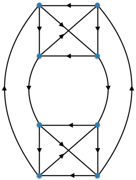
$$\psi_\Gamma(h_\ell, j_\ell; \vec{k}_{\ell\mathbf{n}}) = \bigotimes_\ell \langle j_\ell \vec{k}_{s(\ell)} | D^{j_\ell(h_\ell)} | j_\ell \vec{k}_{t(\ell)} \rangle$$

Up to normalization, correspond to a large-j limit of Thiemann's heat kernel overcomplete basis of coherent states for $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$, parametrized in terms of twisted geometry data. The latter label points in phase space of discrete general relativity corresponding to $\mathcal{H}_\Gamma$ and are easily related to holonomy–flux data.

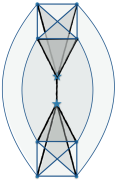
[Thiemann, Winkler, Livine, Speziale, Freidel, Bianchi, Perini, Magliaro,...]

Discretization - Truncation - Skeletonization



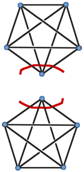


Oriented boundary graph Γ



Spinfoam 2-complex $\mathcal{C}$

Spinfoam skeleton



Dual to two 4-simplices with one tetrahedron in common

A well–defined expression for $W_{\mathcal{C}}(m,T)$

Can perform $SL(2,\mathbb{C})$ integrations explicitly using Cartan decomposition

$$g = u\, e^{r\frac{\sigma_3}{2}}\, v^{-1} \quad , \quad \mathrm{d}g = \frac{\sinh^2 r}{4\pi}\, \mathrm{d}r\, \mathrm{d}u\, \mathrm{d}v$$

Explicit, analytic, finite. Numerics and asymptotics under development *[Speziale, Vilensky, D'Ambrosio, Dona, Sarno, Martin–Dussaud, MC . . .]*

$$W_{\mathcal{C}}(m,T) = \sum_{\{j_\ell\}} w(m,T,j_\ell) \sum_{\{J_\mathrm{n}\},\{K_\mathrm{n}\},\{l_\ell\}} \left(\bigotimes_{\mathrm{n}} N^{J_\mathrm{n}}_{\{j_\mathrm{n}\}}(\{\vec{n}_\mathrm{n}\},\{\alpha_\mathrm{n}\})\, f^{J_\mathrm{n},K_\mathrm{n}}_{\{j_\mathrm{n}\}\{l_\mathrm{n}\}} \right) \left(\bigotimes_{\mathrm{n}} i^{K_\mathrm{n},\{l_\mathrm{n}\}} \right)_\Gamma$$

$$w(m,T,j_\ell) = c(m) \prod_\ell d_{j_\ell}\, e^{-t(j_\ell - \omega(m,T)_\ell)^2}\, e^{i\gamma(m,T)\zeta_\ell j_\ell}$$

$$N^{J_\mathrm{n}}_{\{j_\mathrm{n}\}} = \left(\overleftarrow{\bigotimes_{\ell\in\mathrm{n}}} D^{j_\ell}_{m_\ell j_\ell}(\{\vec{k}_\mathrm{n}\},\{\alpha_\mathrm{n}\}) \right)\, i^{J_\mathrm{n},\{j_\mathrm{n}\}}_{\{\overrightarrow{m}_\mathrm{n}\}}$$

$$f^{K_\mathrm{n},J_\mathrm{n}}_{\{j_\mathrm{n}\}\{l_\mathrm{n}\}} \equiv d_{J_\mathrm{n}}\, i^{J_\mathrm{n},\{j_\mathrm{n}\}}_{\{\overrightarrow{p}_\mathrm{n}\}} \left(\int dr_\mathrm{n}\, \frac{\sinh^2 r_\mathrm{n}}{4\pi} \overrightarrow{\bigotimes_{\ell\in\mathrm{n}}} d_{j_\ell l_\ell p_\ell}(r_\mathrm{n}) \right)\, i^{K_\mathrm{n},\{l_\mathrm{n}\}}_{\{\overleftarrow{p}_\mathrm{n}\}}\, d_{K_\mathrm{n}}$$

The mass m and bounce time parameter T encoded only in the weights.
Use the semiclassical limit of the EPRL to estimate the partial amplitude.
How to perform the spin sum?

Emergence of WMH sum–over–geometries in covariant LQG

$$W_{\mathcal{C}} = \sum_{\{j_f\}} \mu(j_f)\; I(j_f)$$

$$\begin{aligned}
W^{WMH} &\sim \int \mathcal{D}[g] \quad e^{\frac{i}{\hbar} S_{HE}[g]} \\
W^{NR}_{\mathcal{C}^\star} &\sim \int \mu(\ell_s) \quad e^{\frac{i}{\hbar} S_R^{\mathcal{C}^\star}[\ell_s]} \\
W^{EPRL}_{\mathcal{C}} &\sim \int \mu(a_f) \;\; e^{\frac{i}{\hbar} S_R^{\mathcal{C}^\star}[a_f]}
\end{aligned}$$

Subtleties: Multiple semiclassical critical points for given areas (cosine feature). When considering Regge-like boundary data, critical points reconstruct Lorentzian 4D, Euclidean 4D, and 3D (degenerate) simplicial geometries.

Performing the spin sum

We restrict to amplitudes defined on fixed 2-complexes $\mathcal{C}$ without interior faces, dual to simplicial topological triangulations. We consider coherent spin–network boundary states built on the boundary graph $\Gamma = \partial\mathcal{C}$. Each face f is labelled by its link ℓ. Work in Han-Krajewski rep., combine with results by Han and Zhang.

$$W_{\mathcal{C}}(\omega_\ell, \zeta_\ell, k_{\ell\mathtt{n}}, t) = \sum_{\{j_\ell\}\in\Omega(\{j_\ell\},t,K)} \mu(j) \left(\prod_\ell e^{-t_\ell(j_\ell-\omega_\ell)^2} e^{ij_\ell\gamma\zeta_\ell}\right) \times$$
$$\times \underbrace{\int_{\Omega(g,z)} \mu(g)\, \mu(z) \prod_\ell e^{j_\ell\; F_\ell(g,z;k_{\ell\mathtt{n}})}}_{\text{"partial amplitude"} \sim \exp \frac{i}{\hbar} S_R^{\mathcal{C}\star}}$$

Idea of calculation: Assume $\omega, \vec{k}$ Regge-like. Bring amplitude to form

$$W_{\mathcal{C}}(\lambda, \delta_\ell, \zeta_\ell, t_\ell) = f(\omega_\ell, \zeta_\ell) \int_{\Omega(g,z)} \mu(g)\, \mu(z)\, e^{\lambda\, \Sigma(g,z;\,\delta_\ell, \vec{k}_{\ell\mathtt{n}})}\, \mathcal{U}(g,z;t,\zeta_\ell)$$

and use fixed–spin asymptotics: $\mathrm{Re}\Sigma = \delta_g\Sigma = \delta_z\Sigma = 0$. **Critical point of partial amplitude determined only by area data and 3D normals, giving dihedral angles $\phi(\omega, \vec{k})$ that do not in general agree with boost data ζ.**

Sum over spin fluctuations

Split spins into **fluctuations** s_ℓ and area data ω_ℓ. In turn, split ω_ℓ into a large (dimensionless) parameter λ, to be identified to a macroscopic physical area scale, and the **spin data** δ_ℓ. $K \sim 1$ acts as regulator.

$$j_\ell = \omega_\ell + s_\ell = \lambda\, \delta_\ell + s_\ell$$
$$s_\ell \in \{-\frac{K}{\sqrt{2t}}, \frac{K}{\sqrt{2t}}\}$$

Subtle to exchange integrations over g and z and summation over s_ℓ. The fluctuations must not be such that the dimension of the $SU(2)$ intertwiner space vanishes. Sufficient condition, **semiclassicality condition** $\omega\sqrt{t} \ll 1$, always satisfied for coherent spin–network states

$$\begin{aligned} \Omega_\Gamma \quad \equiv \quad & \{\{s_\ell\} : \forall e, \{\max(|\lambda_0(\delta_i - \delta_k) + (s_i - s_k)|), \\ & \min(\lambda_0(\delta_i + \delta_k) + (s_i + s_k))\} \neq \emptyset, (i,j) \ \in e\} \end{aligned}$$

$$\omega\sqrt{t} \ll 1 \Rightarrow \Omega(\{s_\ell\}, t, K) \subset \Omega_\Gamma$$

Estimate for decaying amplitude

Then, at a critical point

$$F_\ell(g_c, z_c; \vec{k}_{\ell\mathtt{n}}) \to -i\,\phi_\ell(s_c(v); \delta_\ell, \vec{k}_{\ell\mathtt{n}})$$

$$\mathcal{U}(g_c, z_c; t, \zeta_\ell) = \sum_{\{s_\ell\}\in\Omega(\{s_\ell\},t,K)} \prod_\ell e^{-s_\ell^2 t_\ell} e^{i s_\ell \left(\gamma\zeta_\ell - \beta\phi_\ell(s_c(v);\delta_\ell,\vec{k}_{\ell\mathtt{n}})\right)}$$

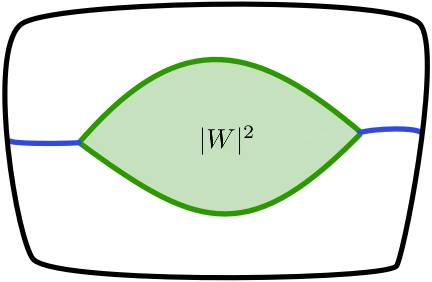
The sum can be done exactly. Morally, we get gaussians in the boost data and the dihedral angles (periodicity in ζ neglected here):

$$W_{\mathcal{C}}(\lambda, \delta_\ell, \zeta_\ell, t) \sim \Bigg[\sum_{s(v)} (\lambda)^N \mu(\delta_\ell) \times$$
$$\times \prod_\ell \exp\left(-\frac{1}{4t} \left(\gamma\zeta_\ell - \beta\phi_\ell(s(v); \delta_\ell, \vec{k}_{\ell\mathtt{n}}) + \Pi_\ell \right)^2 + i\lambda\delta_\ell(\gamma\zeta - \beta\phi(\delta_\ell; \vec{k})) \right) \Bigg]$$
$$\times (1 + O(1/\lambda))$$

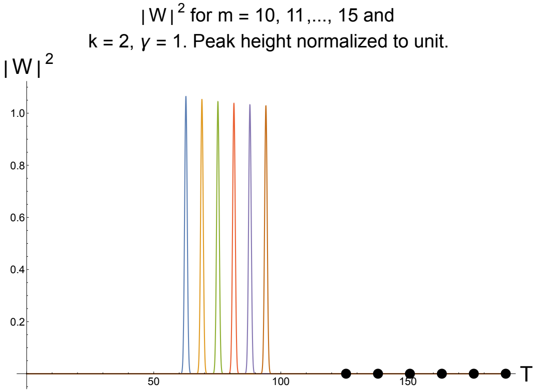
Gravitational tunneling

Solving an initial value problem for Einstein's equations with Cauchy data the intrinsic and extrinsic geometry of the hypersurface formed by the blue hypersurfaces and the upper (lower) boundary surface, and evolving towards the direction in which the foliation time increases (decreases), gives the upper (lower)–half of the spacetime.

$$|W(\omega, \zeta, \vec{k}, t)|^2 \sim \prod_f e^{-(\gamma\zeta - \phi(\omega;\vec{k})^2/4t} \ , \ t \propto \hbar^k \ , \ k > 0$$



Transition amplitude for fixed mass m

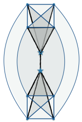


$$\omega_\Delta = 2\left(\frac{m}{\sqrt{2\hbar\gamma}}(1+e^{-T/2m})\right)^2$$

$$\omega^\pm = 2\left(\frac{m}{\sqrt{12\hbar\gamma}}(1+e^{-T/2m})\right)^2$$

$$\zeta_\Delta = \frac{T}{2m}$$

$$\zeta_\pm = \mp\frac{32}{9}\sqrt{6}$$

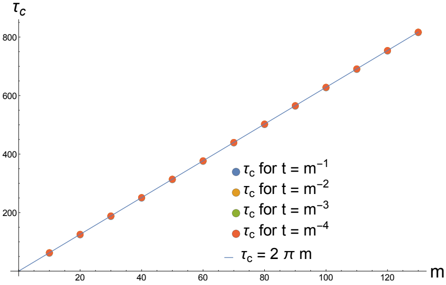


Amplitude is periodic in boost data: $W(\zeta + \frac{4\pi}{\gamma}) = W(\zeta)$. In twisted geometry parametrization, extrinsic curvature encoded through ζ in $SU(2)$ group element, interpreted as AB-holonomy. Truncation, estimate reliable when $\zeta < \frac{4\pi}{\gamma}$. Pronounced peak within each period.

Crossing time

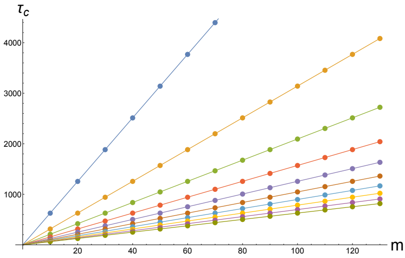
$$\tau_c = \frac{2\pi m}{\gamma}$$

τ_c for m = 10, 20,..., 130 with t = m^{-k},
k = 1,2,3,4 and γ = 1



Plot of τ_c for γ = 0.1, 0.2,...,1

Interpolation: $\frac{2\pi m}{\gamma}$



Checks: Boundary area and normals determine degenerate 3D simplicial geometry. Modified area data to account for arbitrariness in discretization and allow intrinsic curvature. Thus, critical point of partial amplitude Lorentzian or Euclidean 4D. Cosine feature.

Crossing time on general grounds

$$A(m,T) = m^2 \tilde{A}(x) \quad , \quad \zeta(x) \quad , \quad x \equiv T/m$$

$$|W(m,T/m,t)|^2 \sim m^N \mu(T/m) \prod_\ell e^{-(\gamma\zeta_\ell(T/m)-\beta\phi_\ell(T/m)+\Pi)^2/4t}$$

$$\tau_c(m) = m\frac{\int \mathrm{d}x \; x\, F(x) \prod_\ell e^{-(\gamma\zeta_\ell(x)-\gamma\phi_\ell(x)+\Pi_\ell)^2/4t}}{\int \mathrm{d}x \; F(x) \prod_\ell e^{-(\gamma\zeta_\ell(x)-\gamma\phi_\ell(x)^2+\Pi_\ell)^2/4t}}$$

$$\tau_c(m) \approx m\, x_c(\gamma)\,(1+\mathcal{O}(t)\,) \qquad \text{or} \qquad \tau_c(m) \approx m\; \mathcal{O}(e^{-1/t})$$

Integration is over one period, monotonicity of boost angles in x exploited. An example of an **estimate from covariant LQG that is not strongly dependent on a specific choice of hypersurface, discretization and 2-complex.** Cosine feature and different kind of geometrical critical points do not alter conclusion.

Lifetime

- The linear scaling in the mass for τ_c appears to be corroborated by two other groups *[Hajicek,Kiefer,Garay,Barcelo,Carballo]*. Does not make sense as a lifetime: absence of $\hbar$ and far too short.
- The definition we initially gave for the lifetime corresponds to a different timescale of the phenomenon: its characteristic duration **when it takes place**.
- The lifetime is expected to be roughly the inverse of the **probability** of the phenomenon to take place. The former should go to infinity and the latter to zero as $\hbar \to 0$. Transition amplitude has an overall exponential suppression factor depending on t.

$$|W|^2 \sim e^{-1/t} \quad , \quad t = \hbar^{k/2}/m^n \quad , \quad 4 > k > 0$$

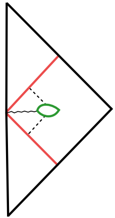
$$p \sim e^{-m^2/\hbar} \quad , \quad \tau_L \sim e^{m^2/\hbar} \quad , \quad \mathrm{d}P(T)/\mathrm{d}T \stackrel{?}{=} P(T)/\tau_L$$

Regarding experimental relevance, $\tau_L \sim e^{m^2/\hbar}$ would be disappointing.
Perspective: as a matter of principle, does LQG predict that black end their life by exploding as white holes?

Perspectives and directions

Plenty remain to be done. . .

- Relation to results in canonical framework. *[Singh, Corichi, Olmedo, Saini]*
- Code black and white hole phase duration in boundary state and take into account limitations from WH instabilites. *[De Lorenzo, Perez]*
- Include interior faces. Use symmetry reduced spinfoam model? Cuboids? *[Bahr, Steinhouse, Kloser, Rabuffo]*
- Analogy with 1D QM propagator in tunneling phenomena. Complexify spinfoam variables? *[Han, Haggard]*
- Concentrate on region close to trapped and anti-trapped surface, encode their presence in boundary state. Allow singularity surface in spacetime? *[Synge, Peeters, Schweigert, van Holten]*



That's all