

# Symplectic Analysis of Null Raychaudhuri

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ILQGS

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*Based on 2309.03932, 2402.xxxx and WIP  
with Laurent Freidel and Rob Leigh*

# Geometry, Dynamics, and Symplectic Structure induced by Gravity on Null Hypersurfaces

Motivations

Geometry & Dynamics

Symplectica

Boost Charge

Perturbation

Atomic

Conclusions

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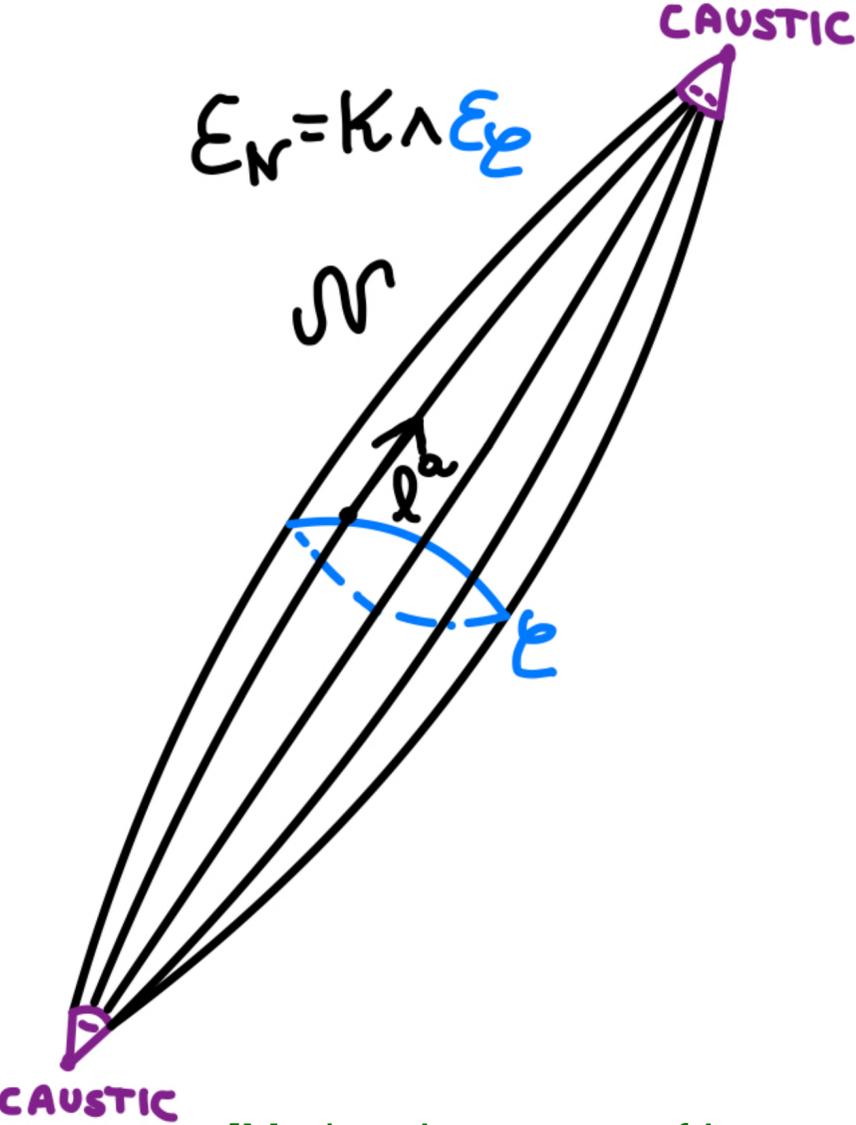
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Non-Perturbative Quantum Geometry: Spin-0 Atomic Quantization

# Geometry of Null Hypersurfaces

$\mathcal{N}$  3d Null Manifold

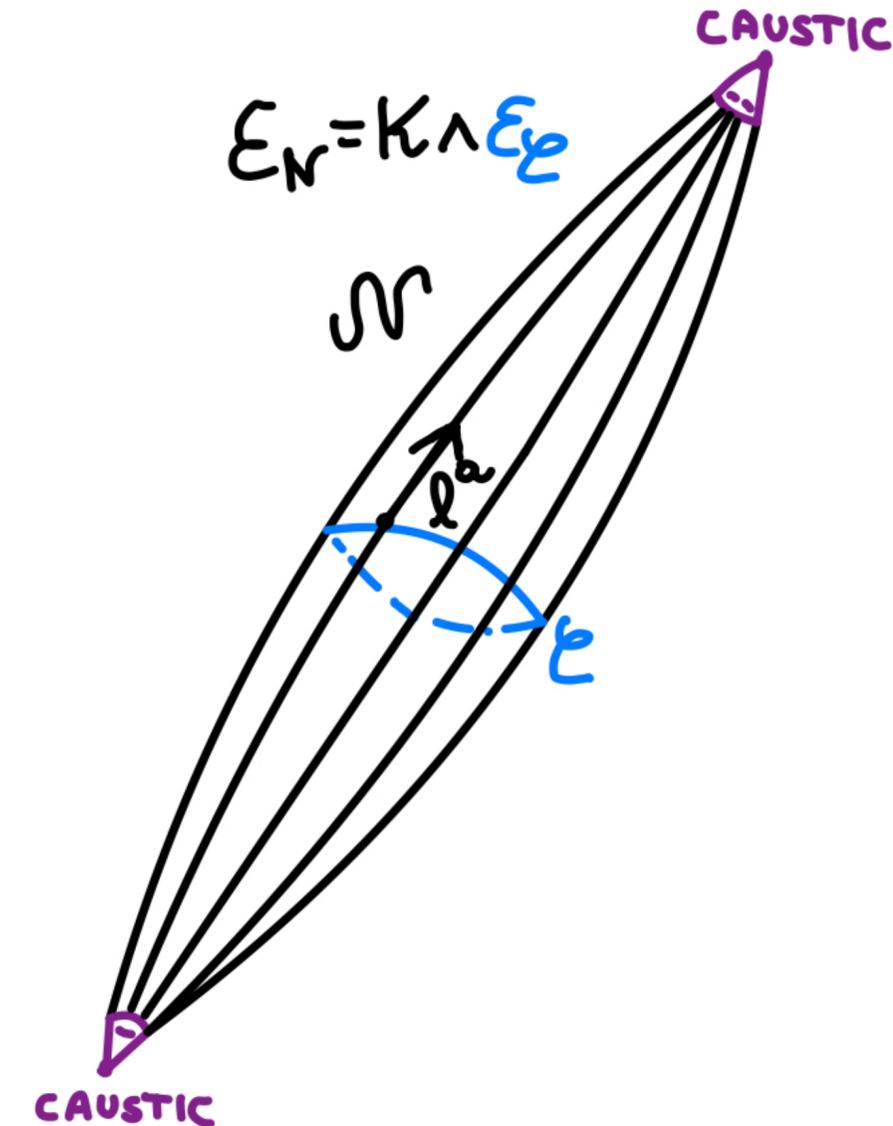


[Modern Language of Levy-Leblond '64, Ashtekar '78 -'24, Henneaux '81, Dautcourt '97, Duval-Gibbons-Horvarthy '14, and many many others]

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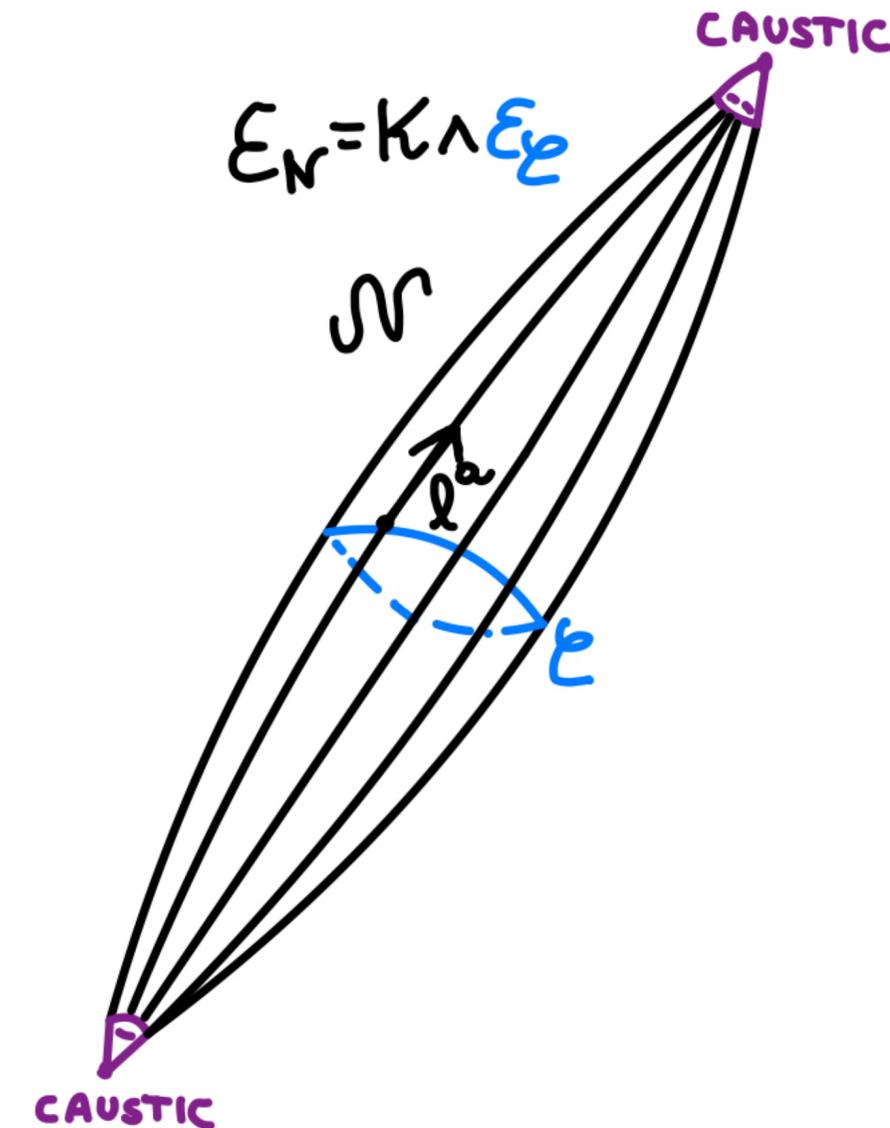
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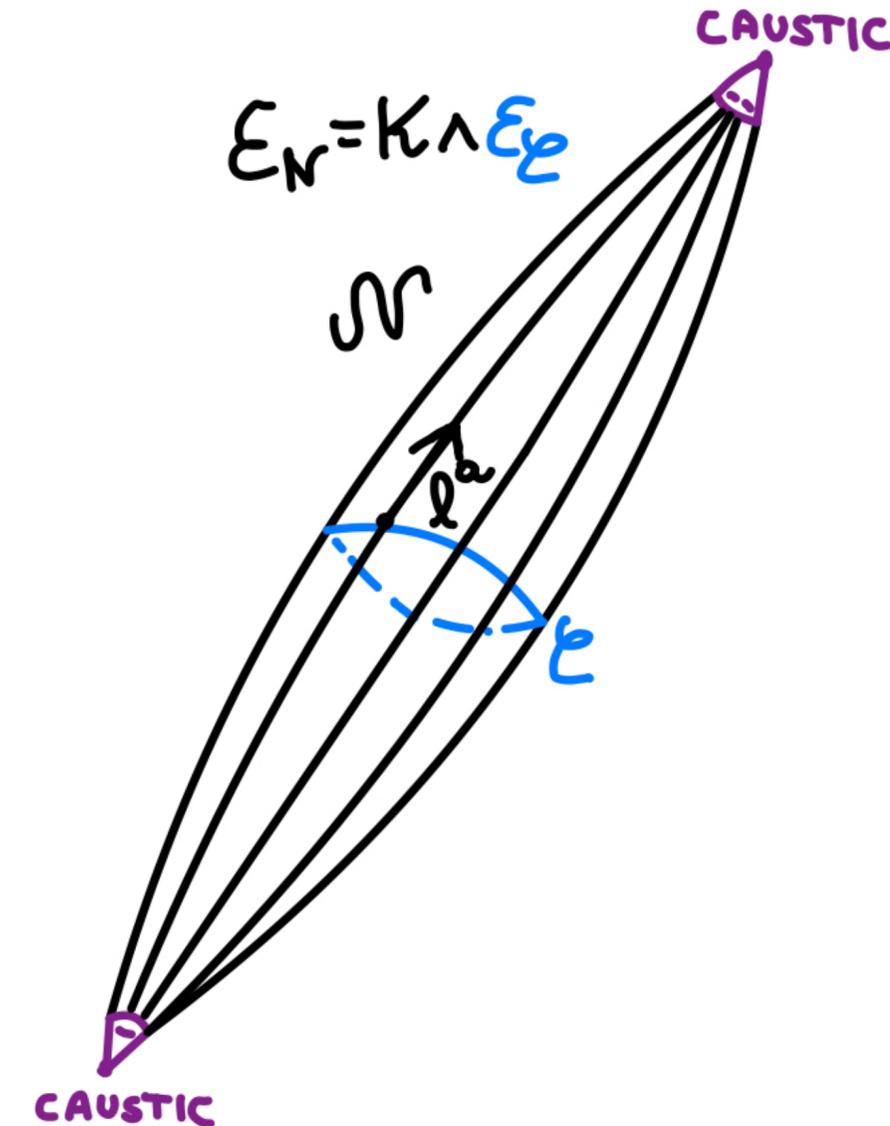
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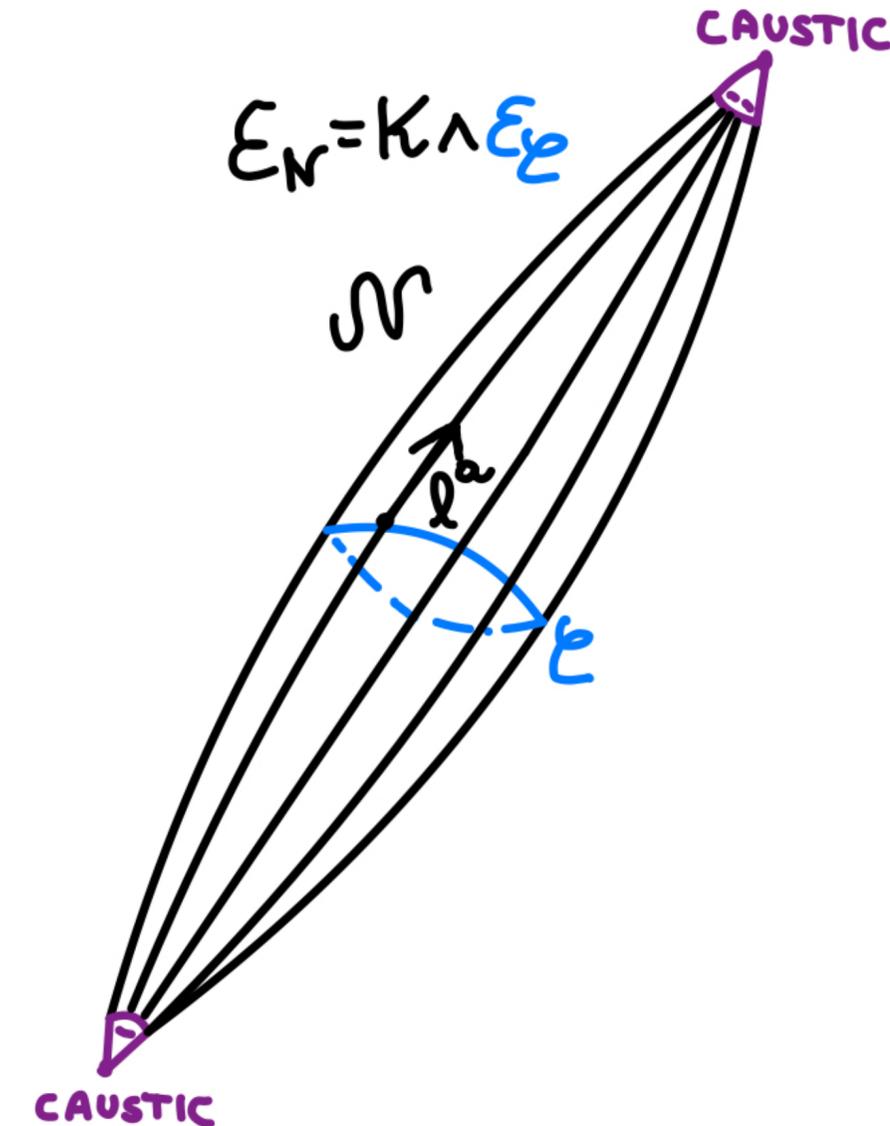
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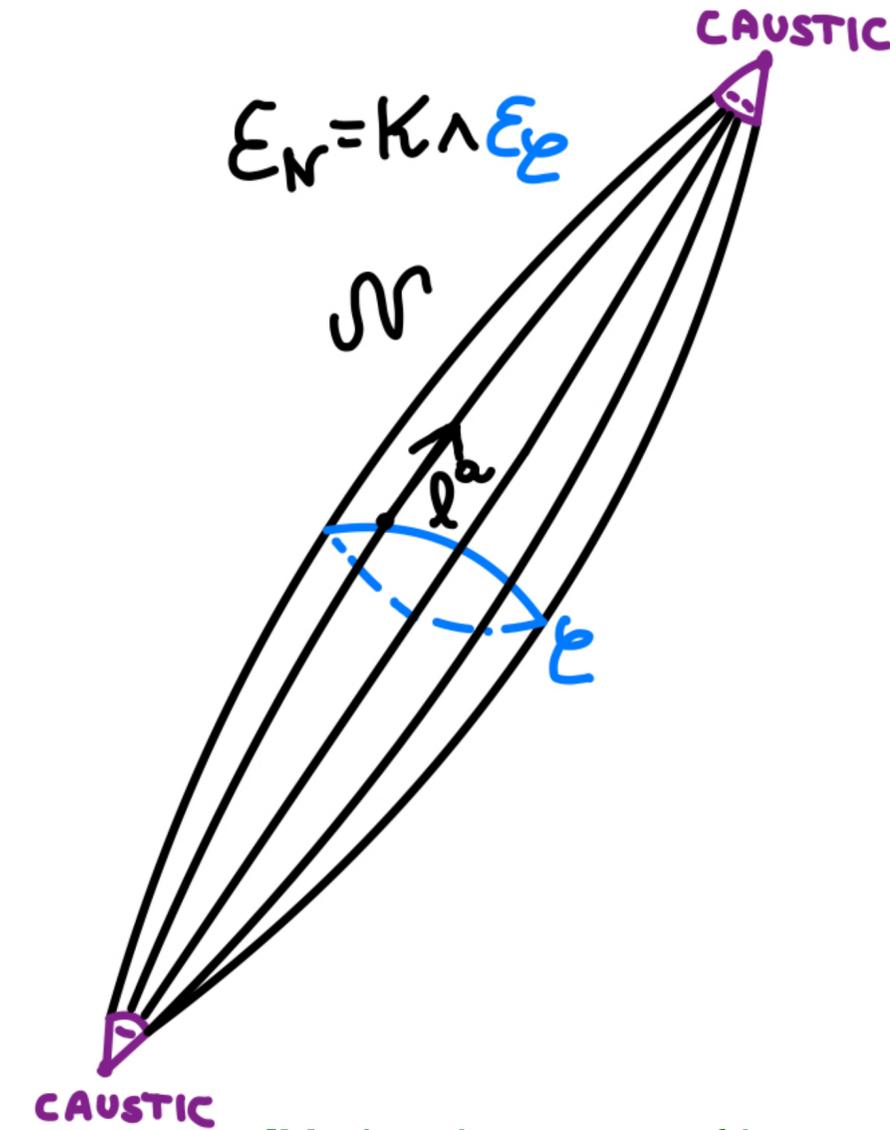
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Carrollian Connection  $D_a \varepsilon_{\mathcal{N}} = -\omega_a \varepsilon_{\mathcal{N}}$  With  $\omega_a = \kappa k_a + \pi_a$



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$$\text{Projected to } \ell^a : (\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a^b \sigma_b^a - R_{\ell\ell} \text{ Null Raychaudhuri Equation}$$

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$$\mu = \kappa + \frac{\theta}{2} \quad \text{Surface Tension}$$

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[Raychaudhuri '55, Sachs '61, Landau '75]



## Null Raychaudhuri

$$(\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a{}^b\sigma_b{}^a - 8\pi GT_{\ell\ell}^{\text{mat}}$$

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## Null Raychaudhuri

[Damour '79, Thorne-Price-Macdonald '86, Penna '17,  
Donnay-Marteau, LC-Leigh-Marteau-Petropoulos '19,  
Freidel-Jai-akson '22, LC-Freidel-Leigh '23]

## Intrinsic Null Conservation Law With Matter Source

$$(\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a{}^b\sigma_b{}^a - 8\pi GT_{\ell\ell}^{\text{mat}} \quad \Leftrightarrow \quad \ell^a D_b T_a{}^b = T_{\ell\ell}^{\text{mat}}$$

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Dynamical data: Spin-0  $\{\Omega, \mu\}$  Spin-1  $\{\pi_a, \ell^a\}$  Spin-2  $\{\sigma^{ab}, \bar{q}_{ab}\}$

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Dressing  $\delta'_f \alpha = (\delta_f + \delta_{\lambda_f})\alpha = 0 \Leftrightarrow \lambda_f = \ell(f)$

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Time Reparameterization Charge:

$$Q'_f = \frac{1}{8\pi G} \int_{\mathcal{N}} \varepsilon_{\mathcal{N}}^{(0)} f C + \frac{1}{8\pi G} \int_{\mathcal{C}} \varepsilon_{\mathcal{C}}^{(0)} (\Omega \partial_v f - f \partial_v \Omega)$$

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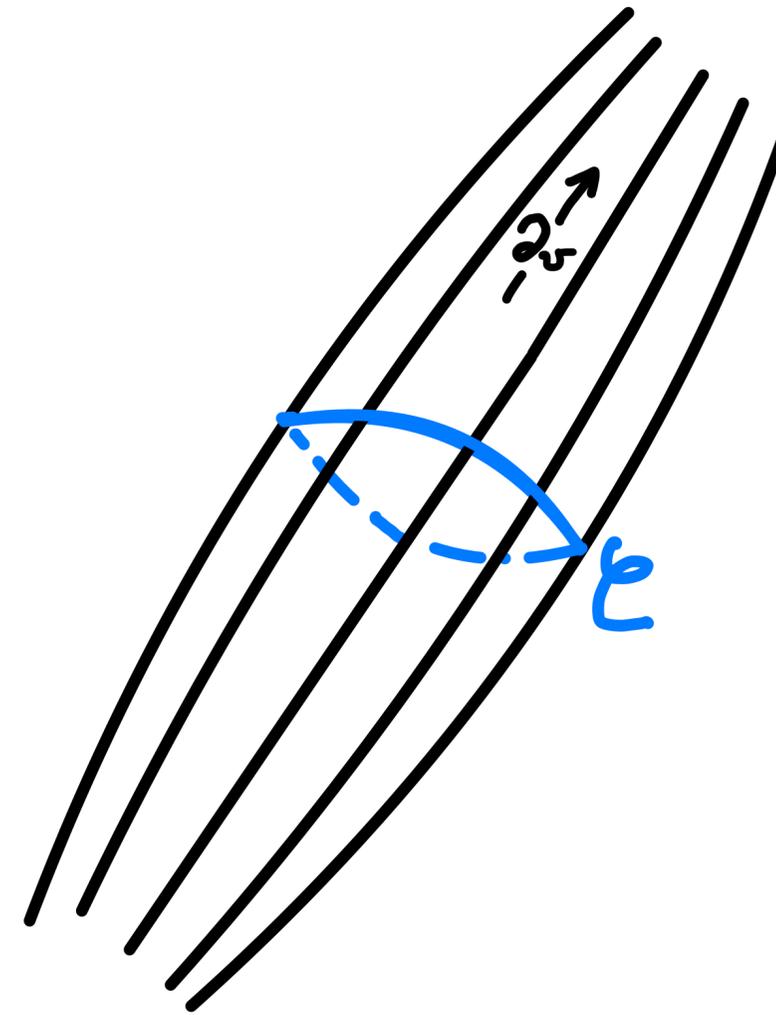
Define the Propagator:  $\bar{\mathcal{P}}_{12} = \frac{1}{\sqrt{\Omega_1 \Omega_2}} e^{-\int_{v_1}^{v_2} (\zeta \partial_v \bar{\zeta} - \bar{\zeta} \partial_v \zeta) dv} \theta(v_1 - v_2) \delta^{(2)}(z_1 - z_2)$

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$$\{\bar{\zeta}_1, \zeta_2\} = 4\pi G \beta_1 \beta_2 \bar{\mathcal{P}}_{12}$$

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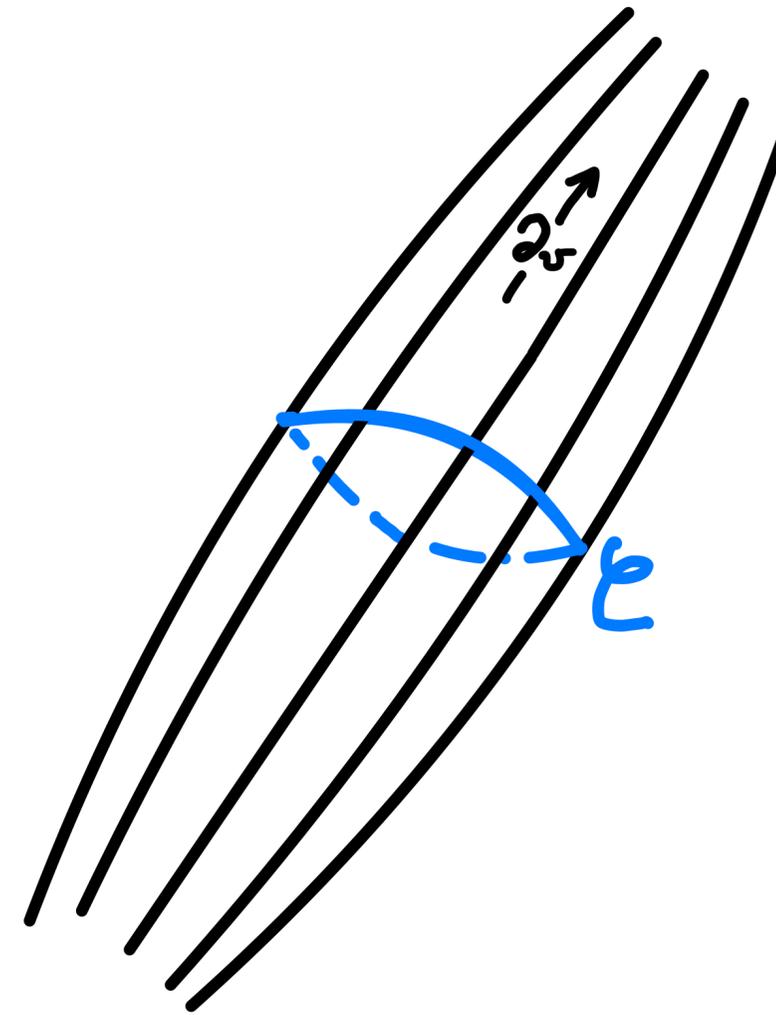
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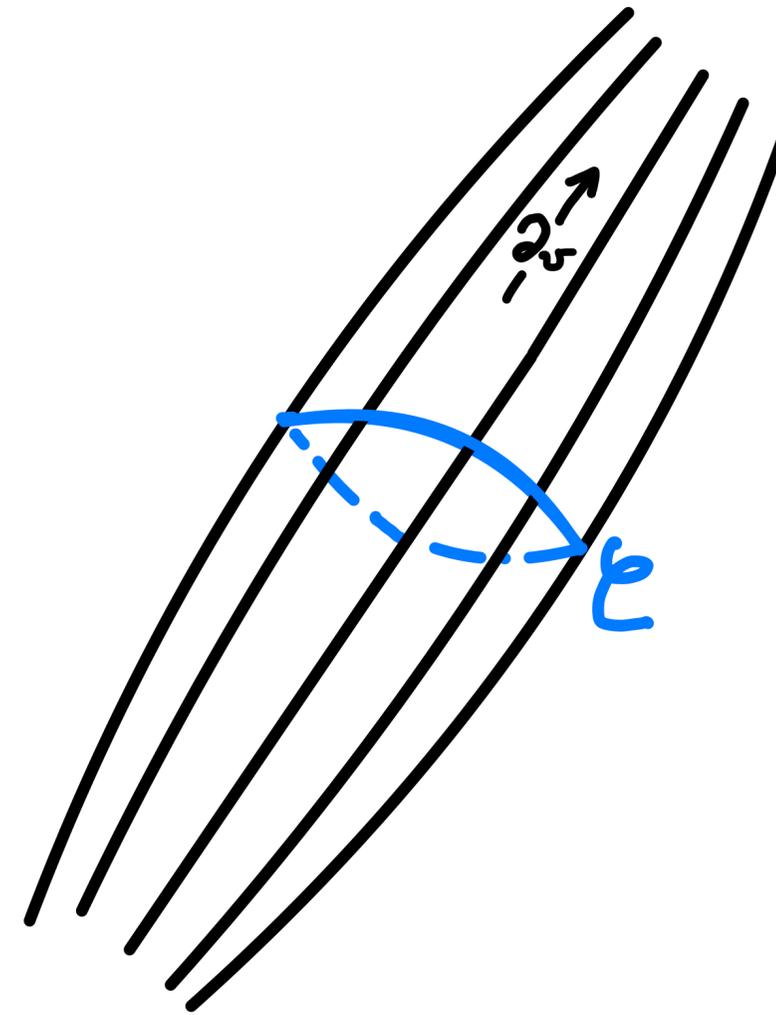
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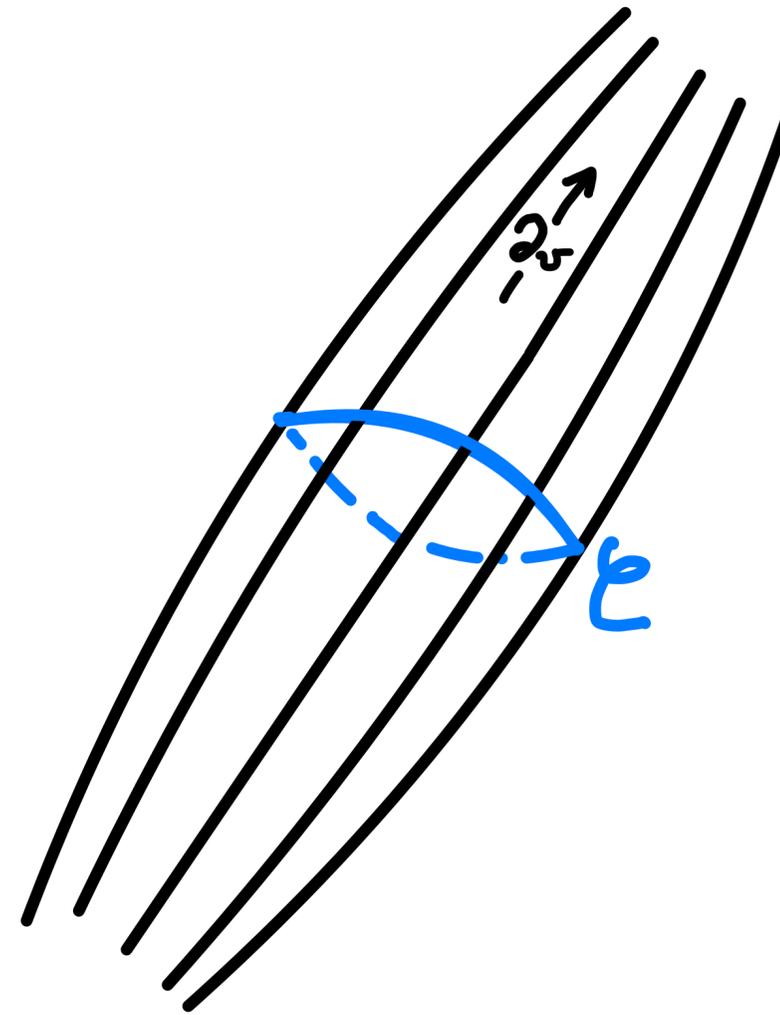
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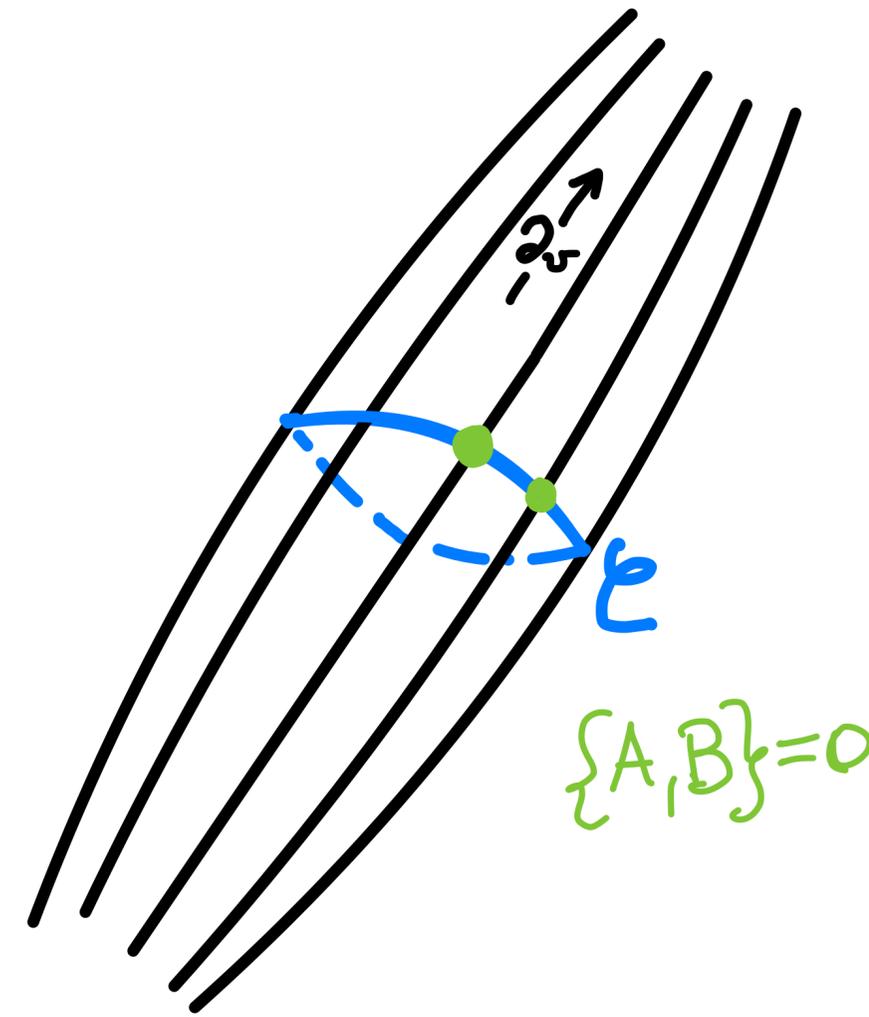
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Ultralocal Structure on the Cut



# Kinematical Poisson Brackets

[Sachs '62, Gambini-Restuccia '78, Penrose '80, Torre '86, Goldberg-Robinson-Soteriou '96, Ashtekar '00, Lewandowski '04, Reisenberger '07 '12 '17 '18, Wieland '17 '19 '21, LC-Freidel-Leigh '23]

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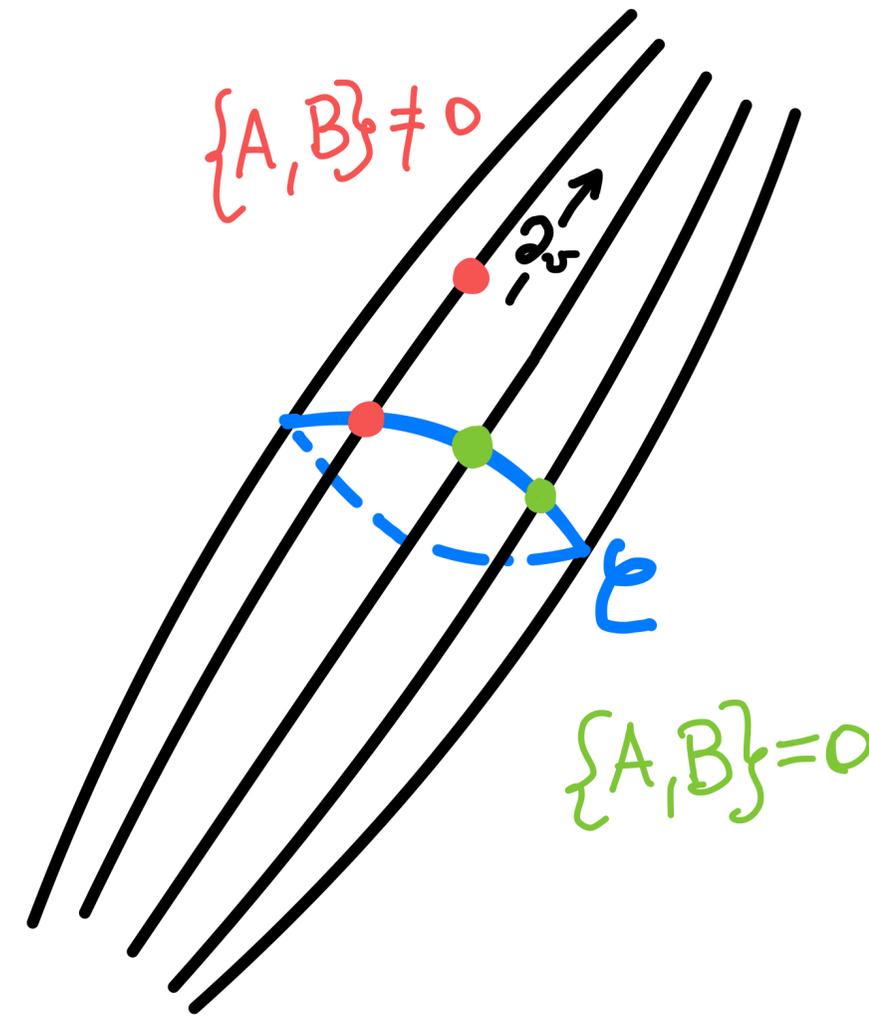
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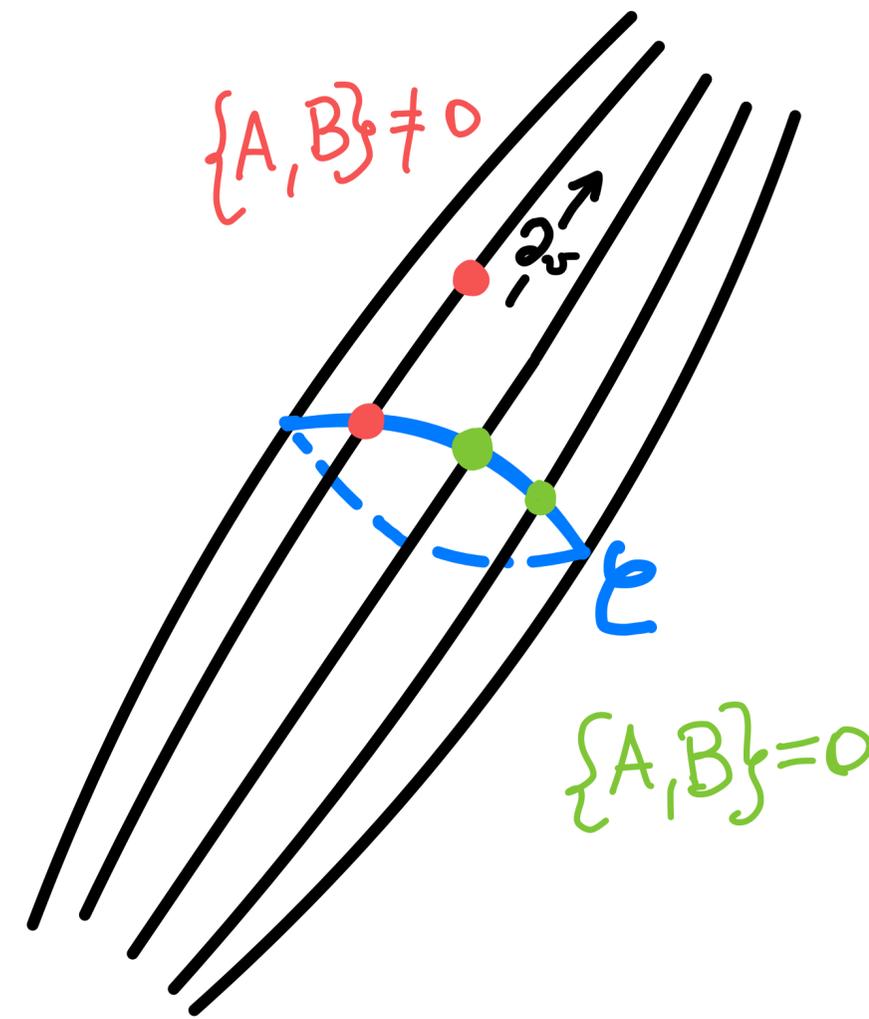
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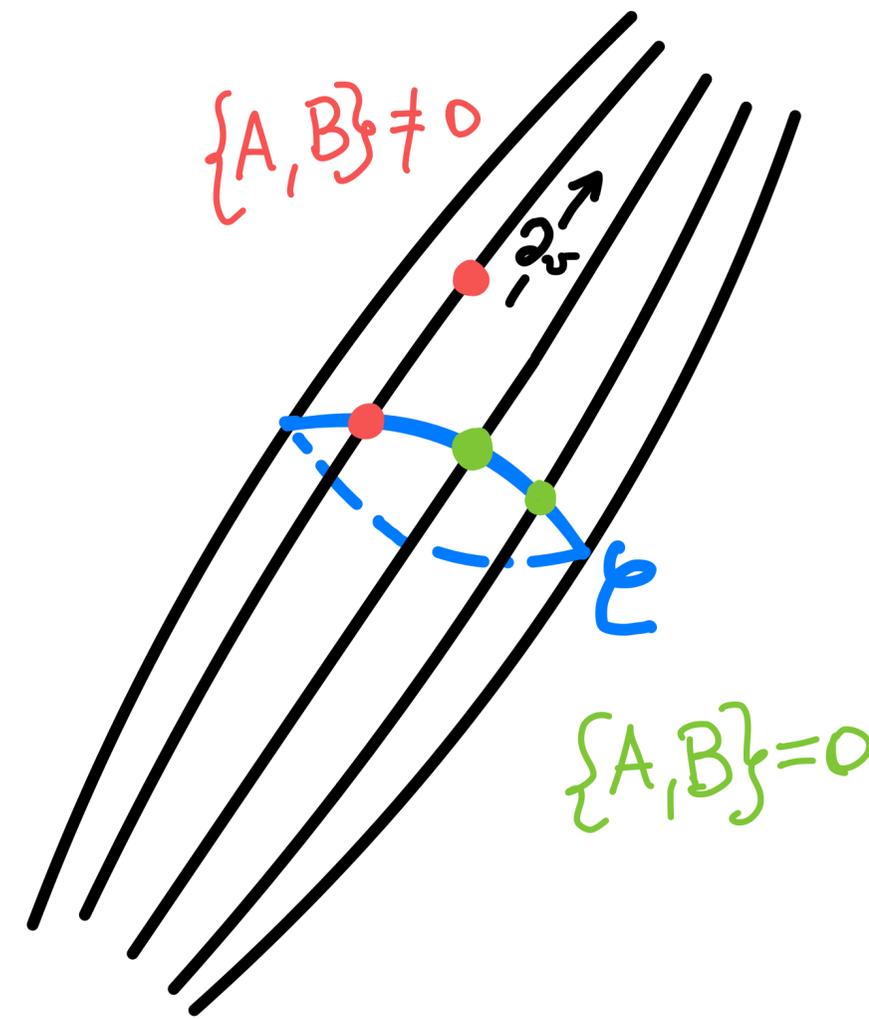
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Ultralocality of Carrollian Physics

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Non-Expanding Horizons  $\tilde{\mu} = \tilde{\kappa} = 0$ , Dressing Time=Affine Time

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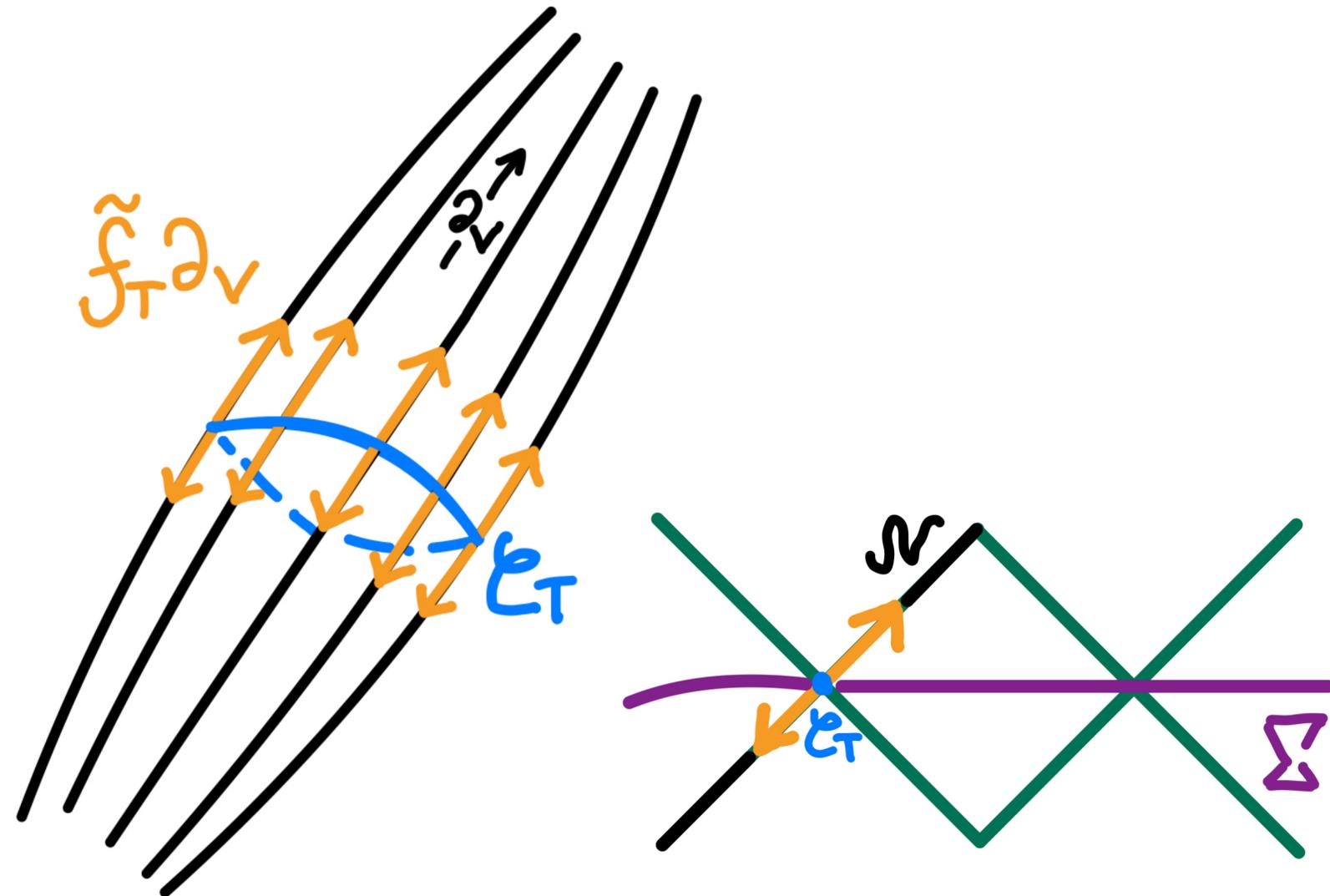
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Spin-0 Sector: The Constraint is the Symplectic Pair of the Dressing Time

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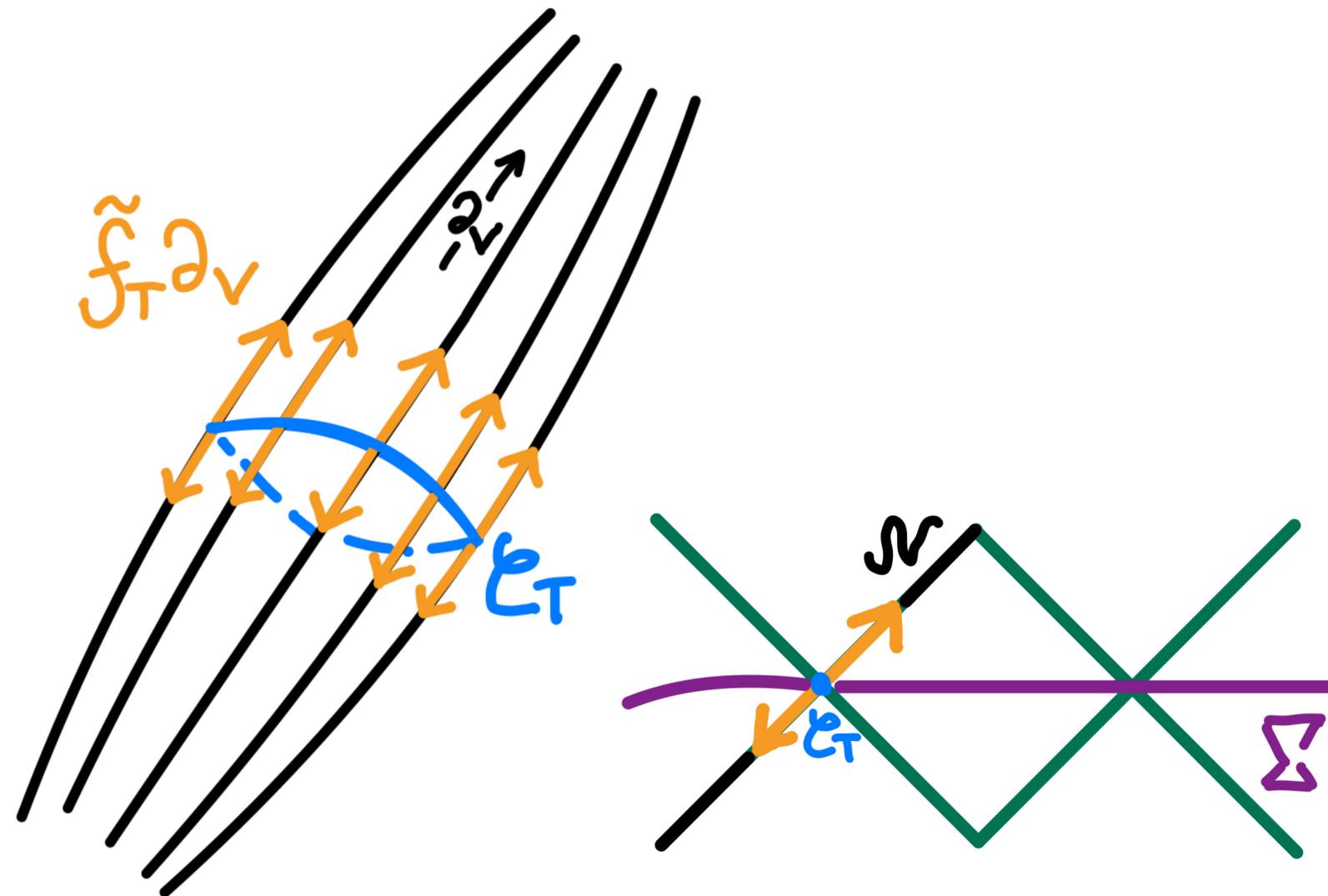


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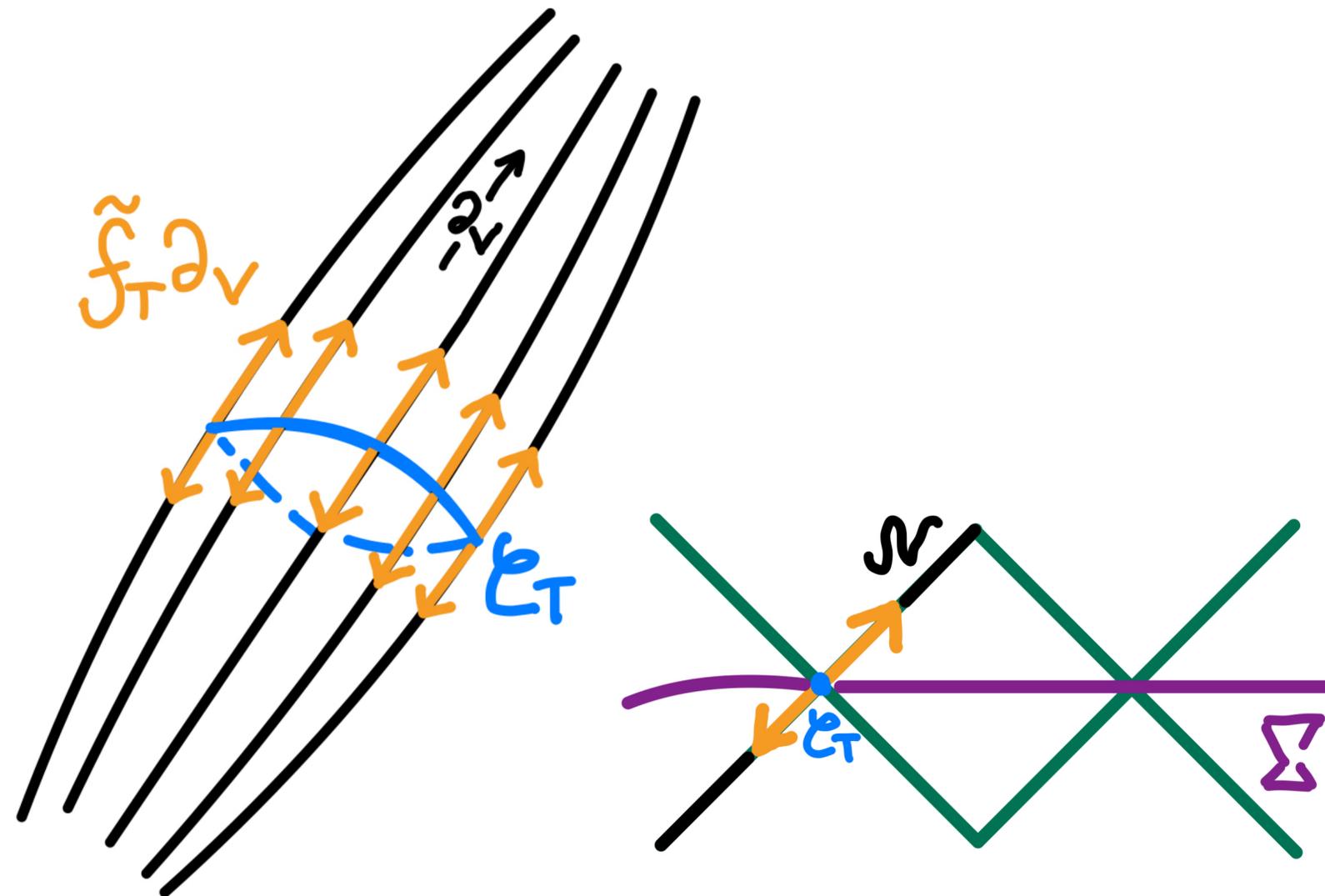
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At the Cut  $\mathcal{C}_T$

$$Q'_T(\mathcal{C}_T) \hat{=} \frac{1}{8\pi G} \int_{\mathcal{C}} \tilde{\varepsilon}_{\mathcal{C}}^{(0)} \tilde{\Omega} \simeq A_{\mathcal{C}} > 0$$



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$$\tilde{S}(V) - \tilde{S}(T) = \Delta \tilde{S} = \frac{1}{8\pi G} \left( \Delta \tilde{\Omega} - (V - T) \tilde{\theta}(V) \right)$$

[Jacobson-Parentani '03, Bianchi '12, Rignon-Bret '23, LC-Freidel-Leigh '23]

[Wald-Zhang, Visser-Yan To Appear]

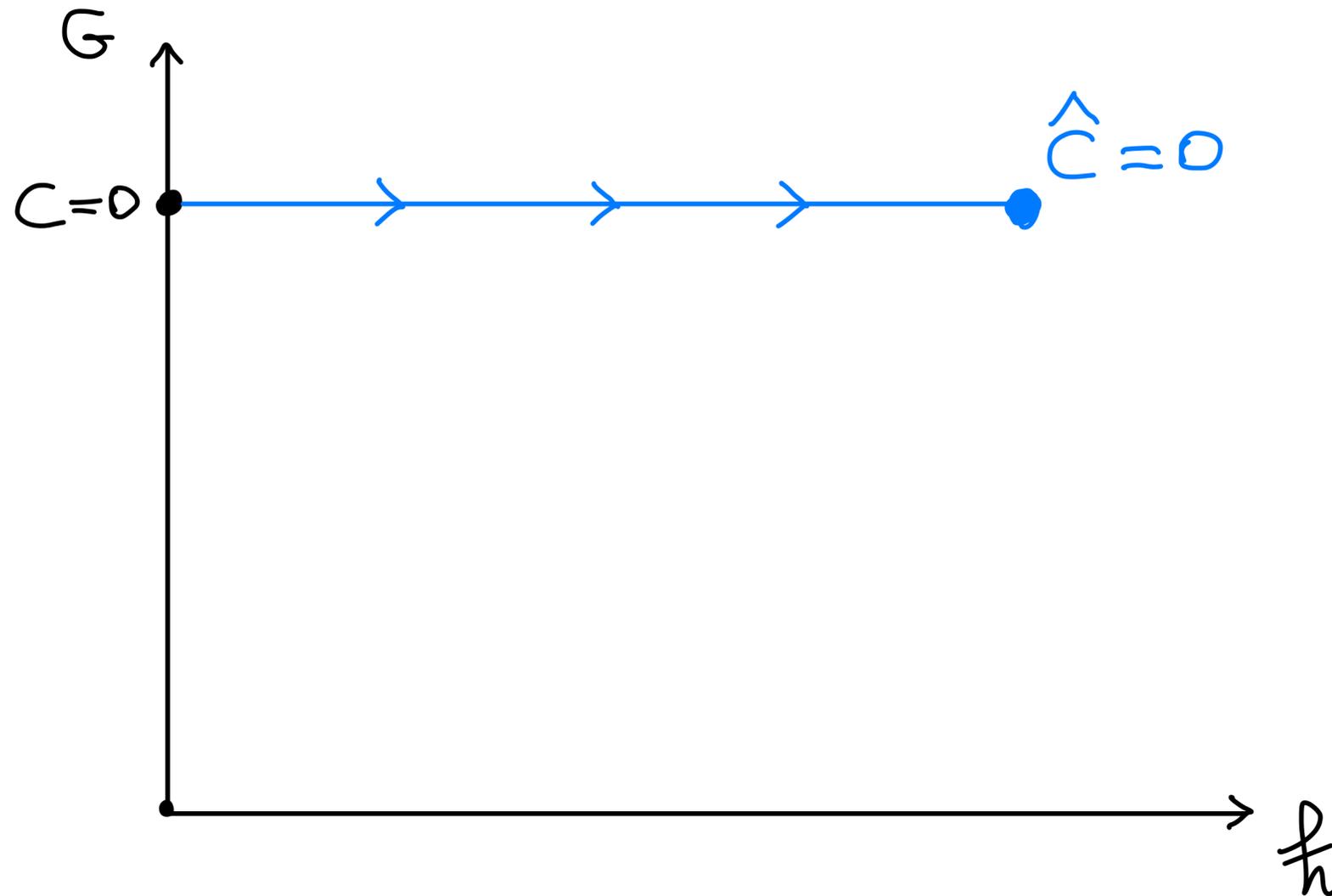
# Perturbative Quantum Geometry

$$C = \partial_v^2 \Omega - \mu \partial_v \Omega + \Omega (\sigma^2 + 8\pi G T_{vv}^{\text{mat}})$$



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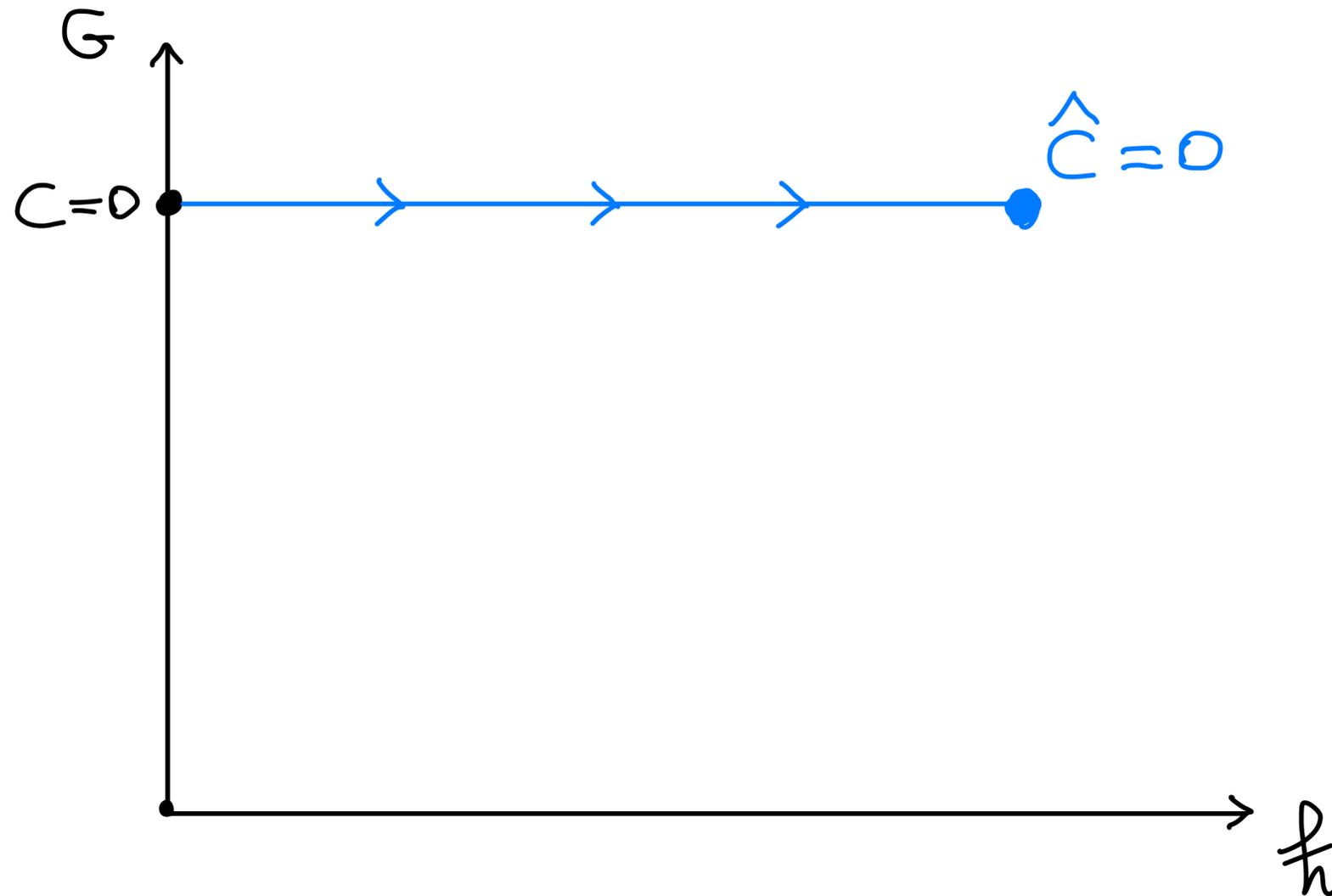
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Quantum Gravity Operatorial Statement

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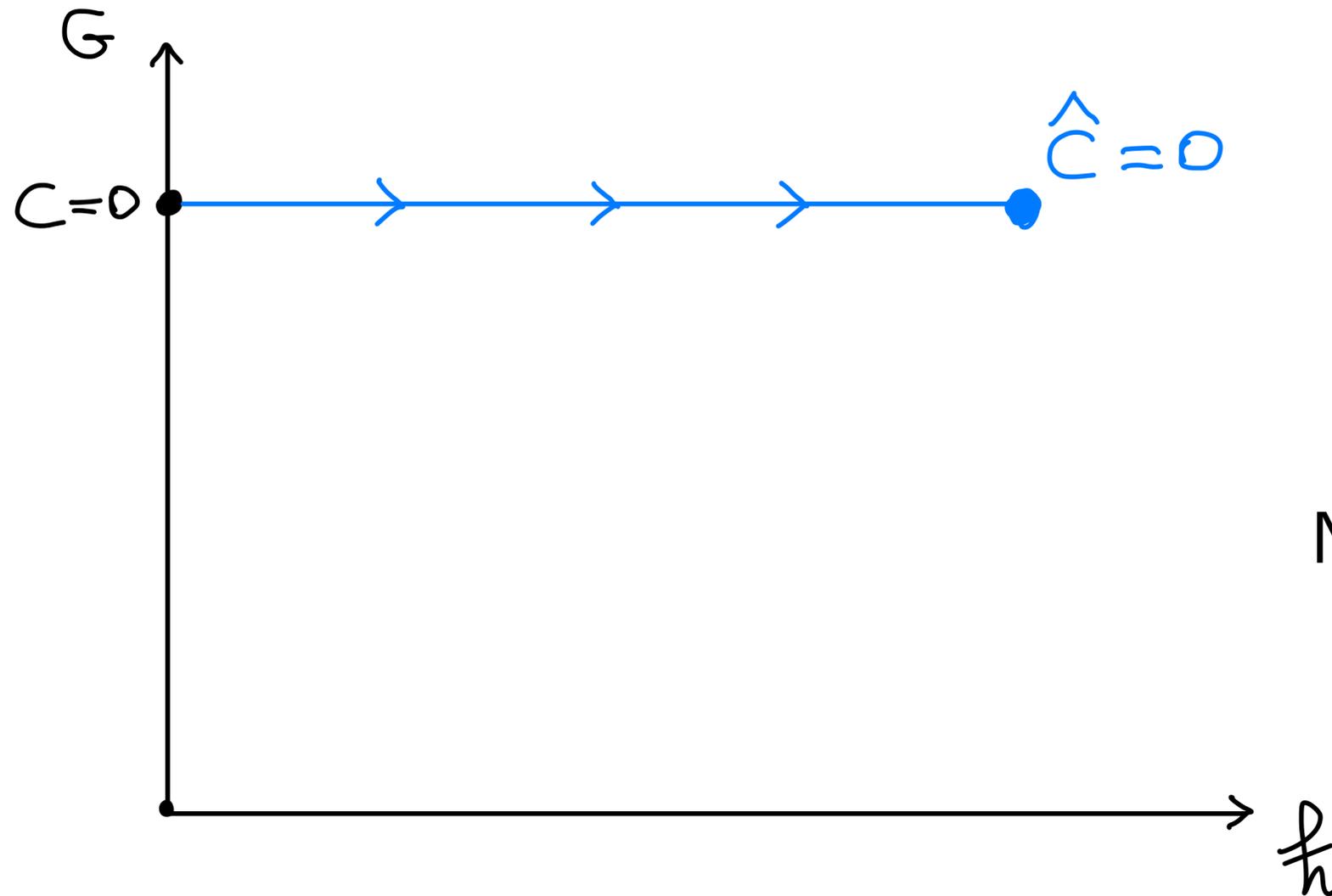


Quantum Gravity Operatorial Statement

The Sectors Mix: Challenging Task

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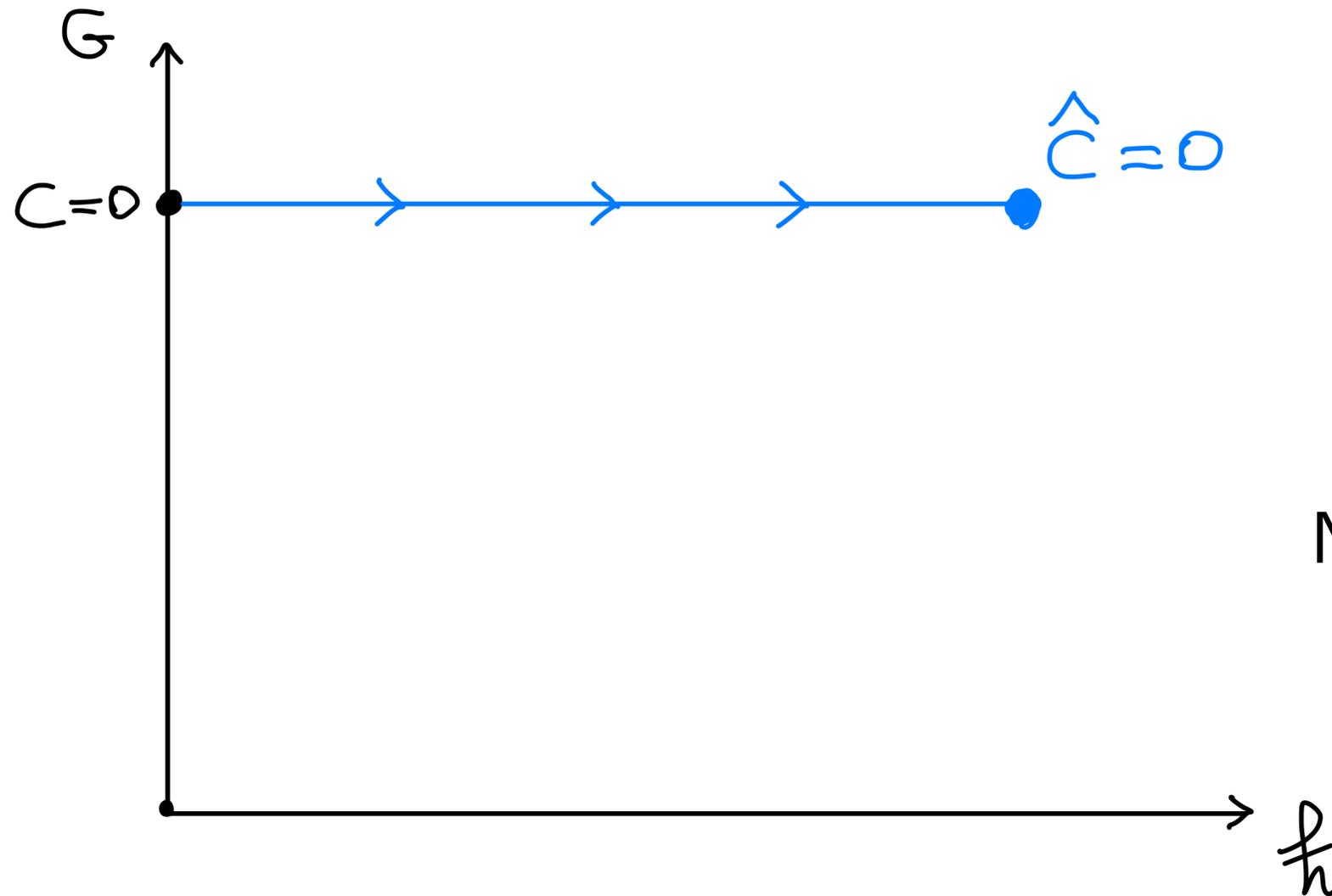
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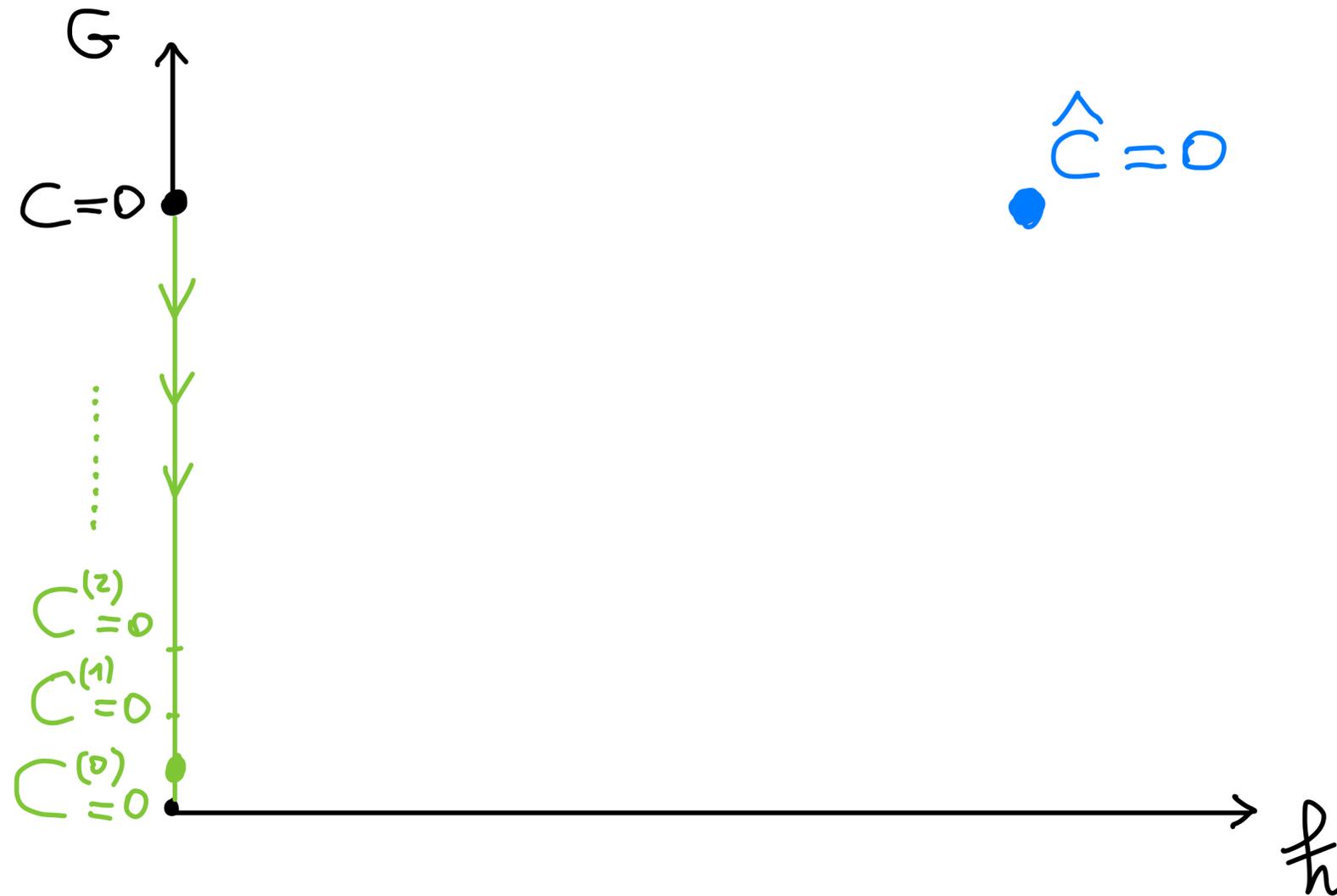
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The fate of Symmetries?

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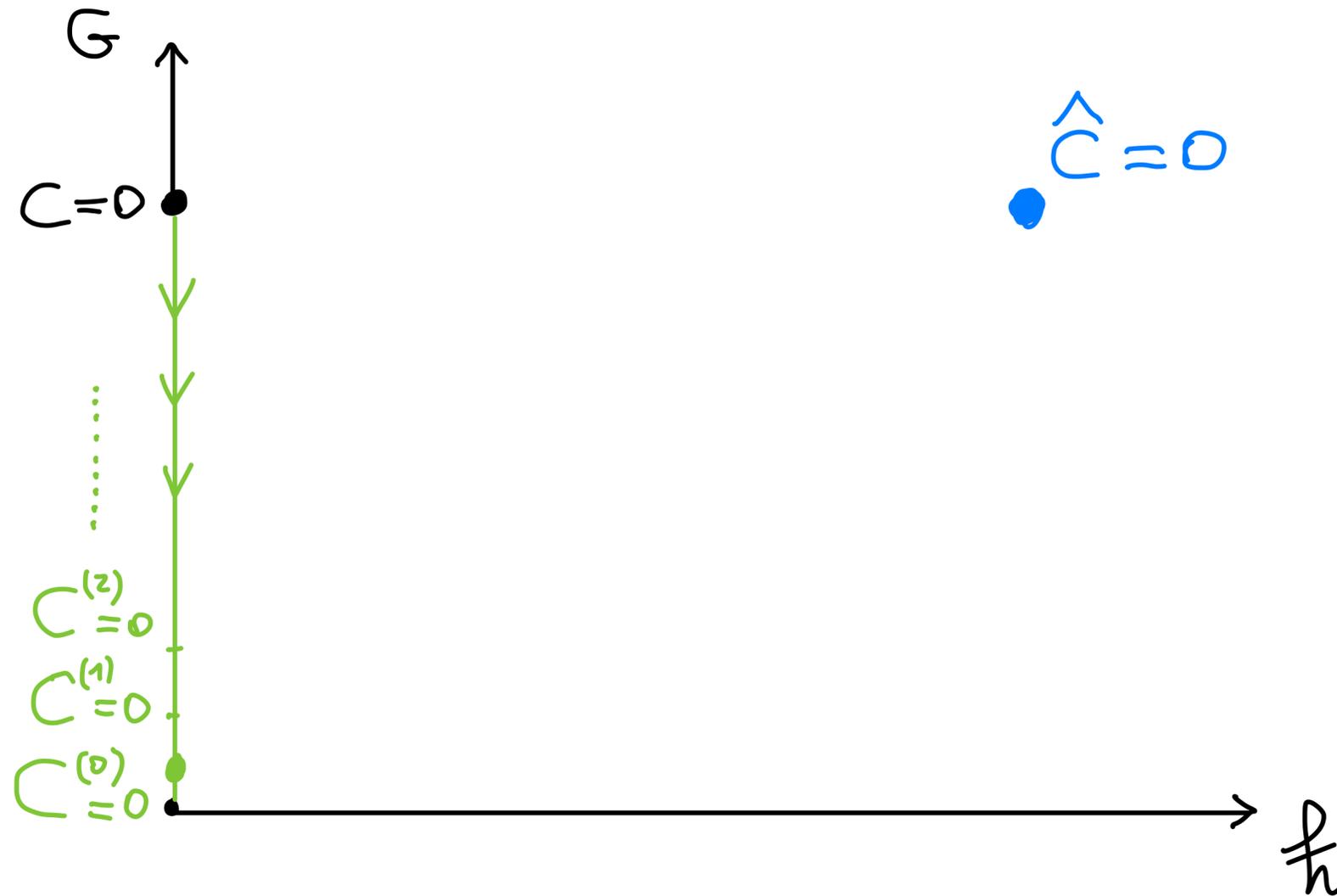
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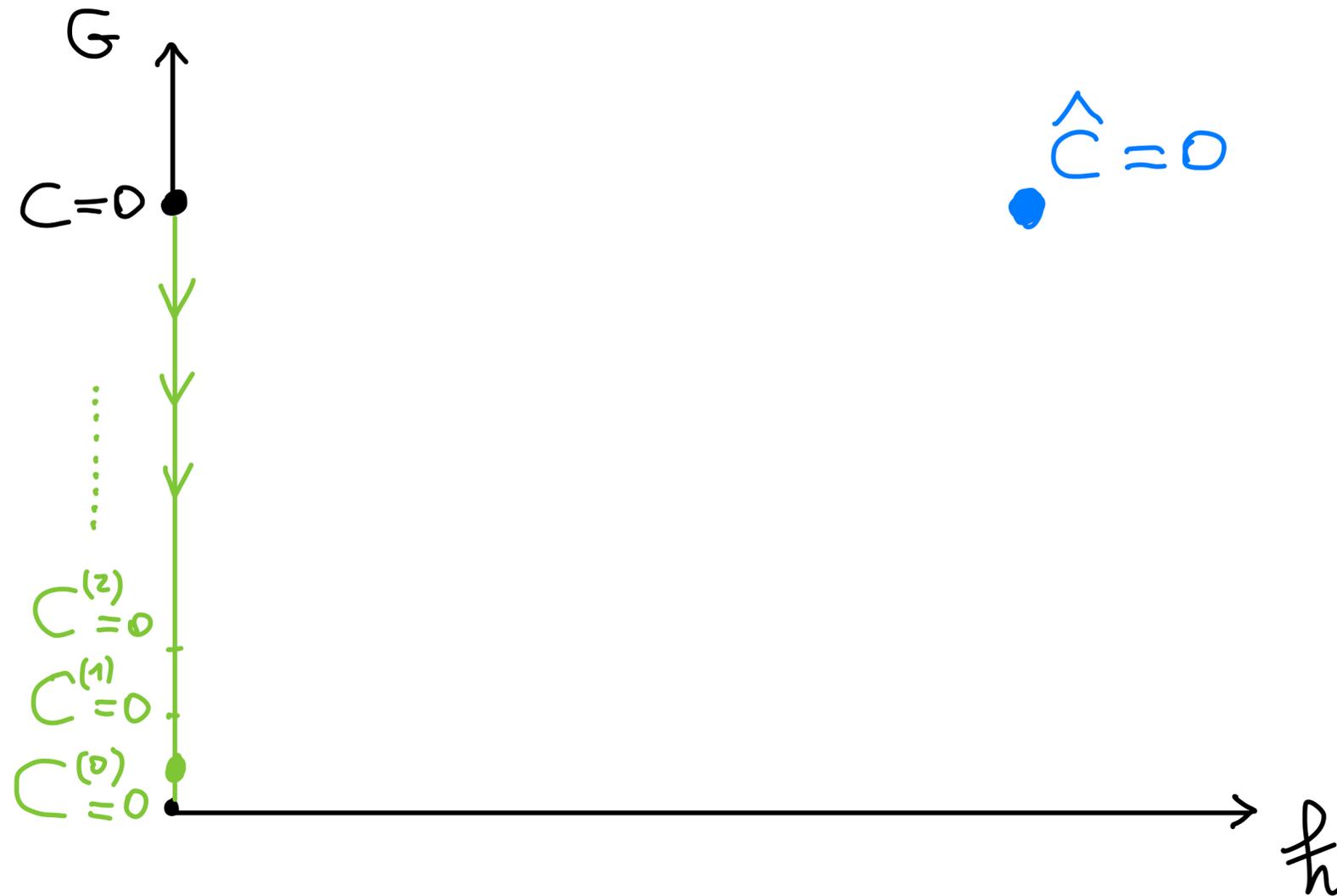


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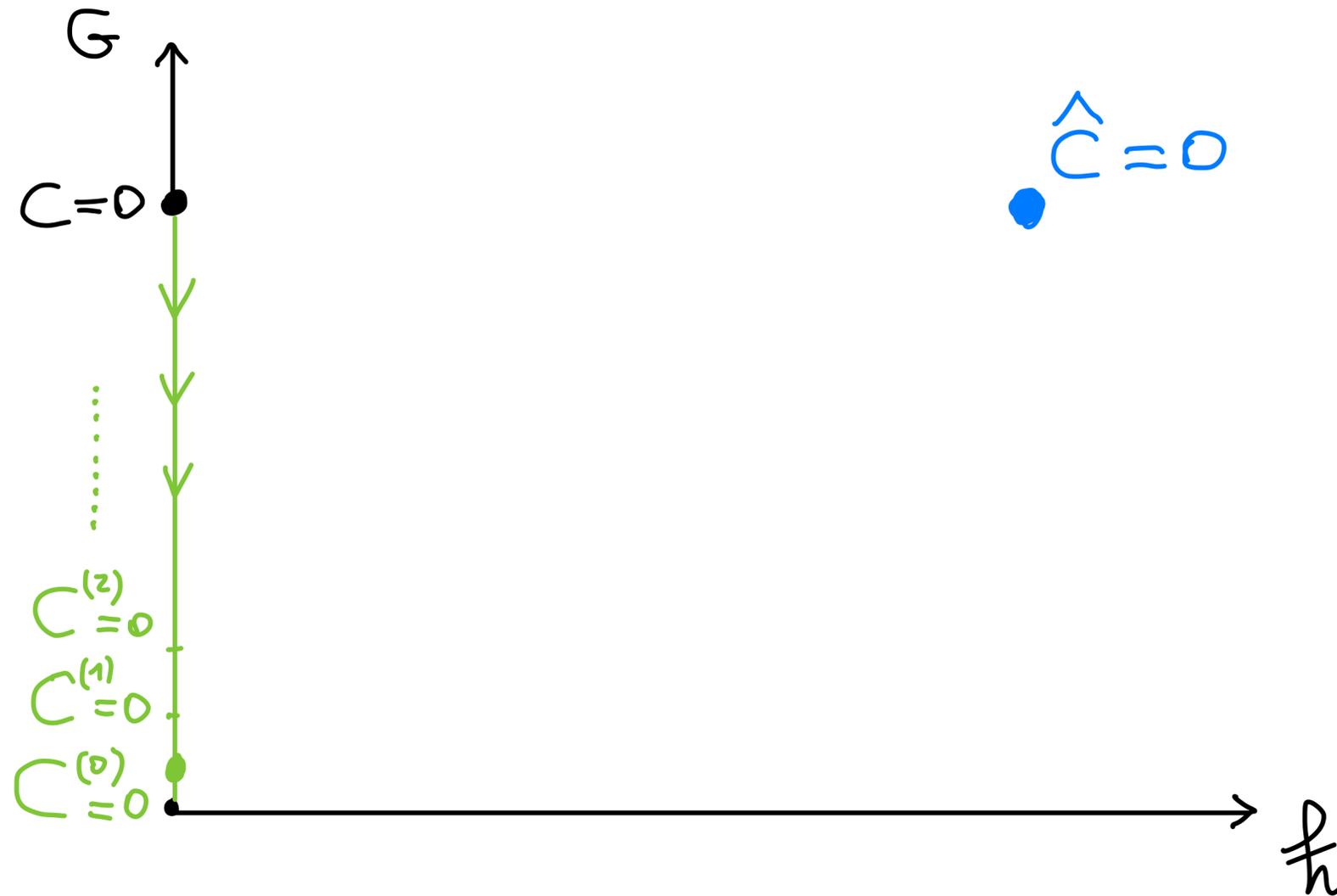
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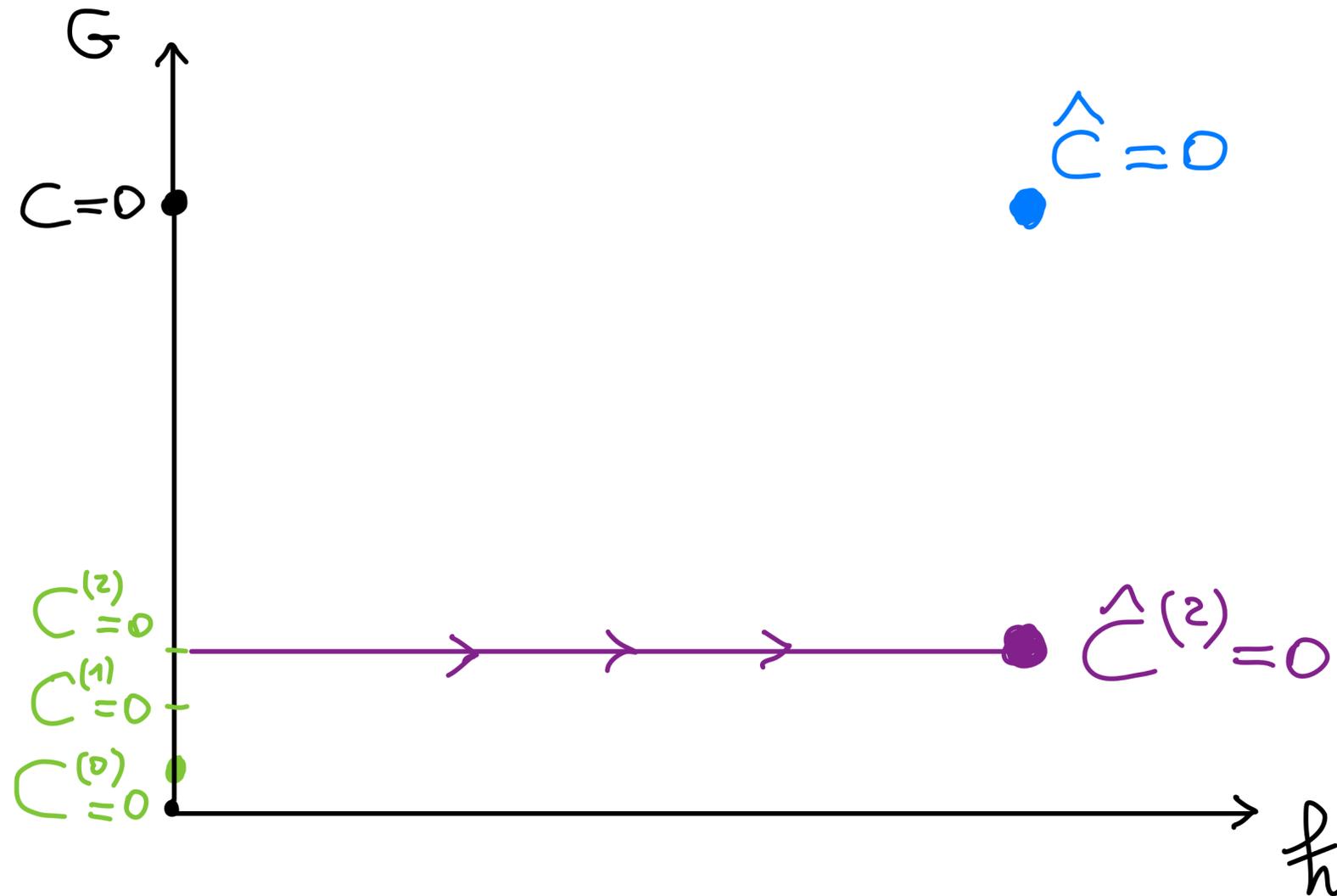
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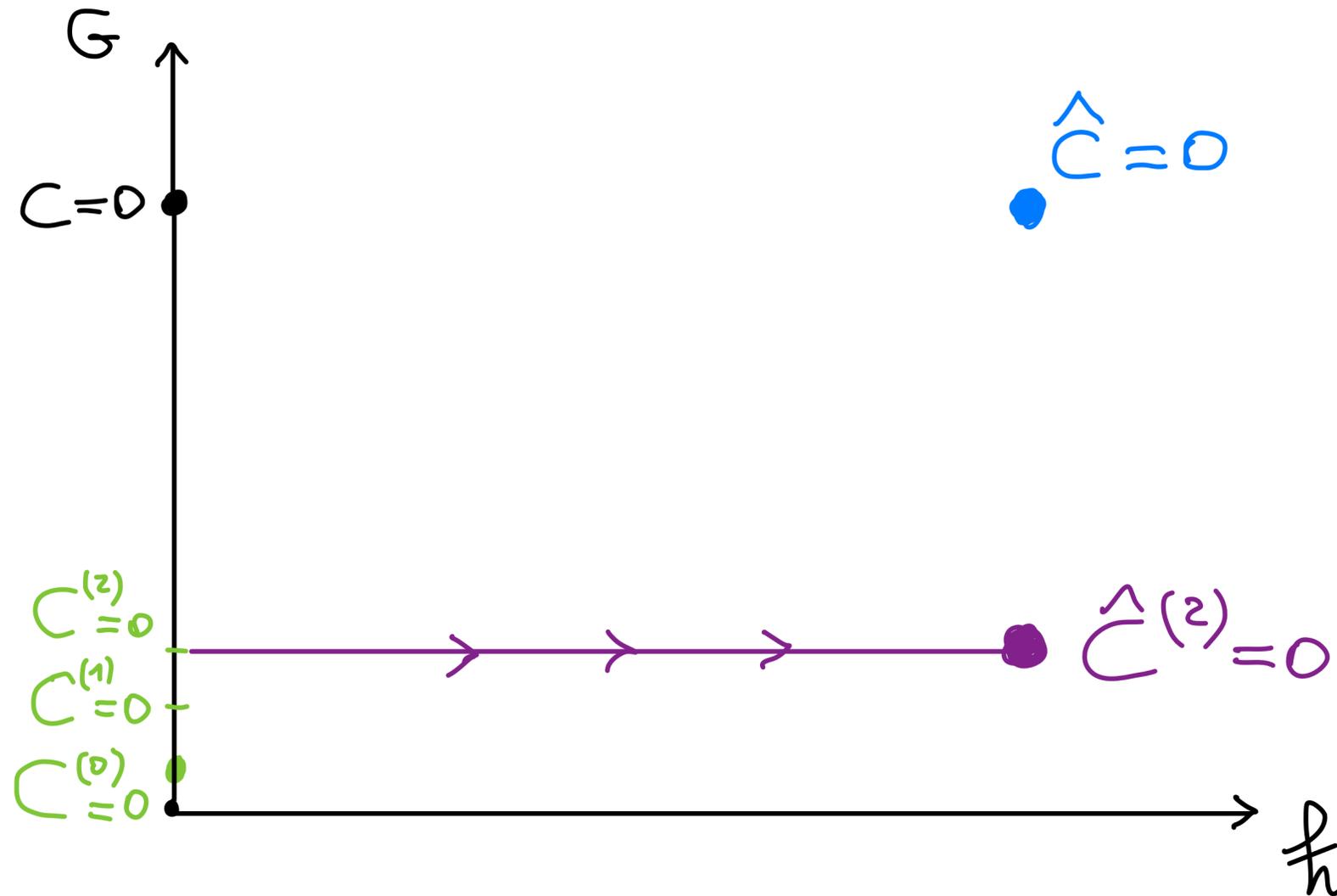
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Symplectic Quantization

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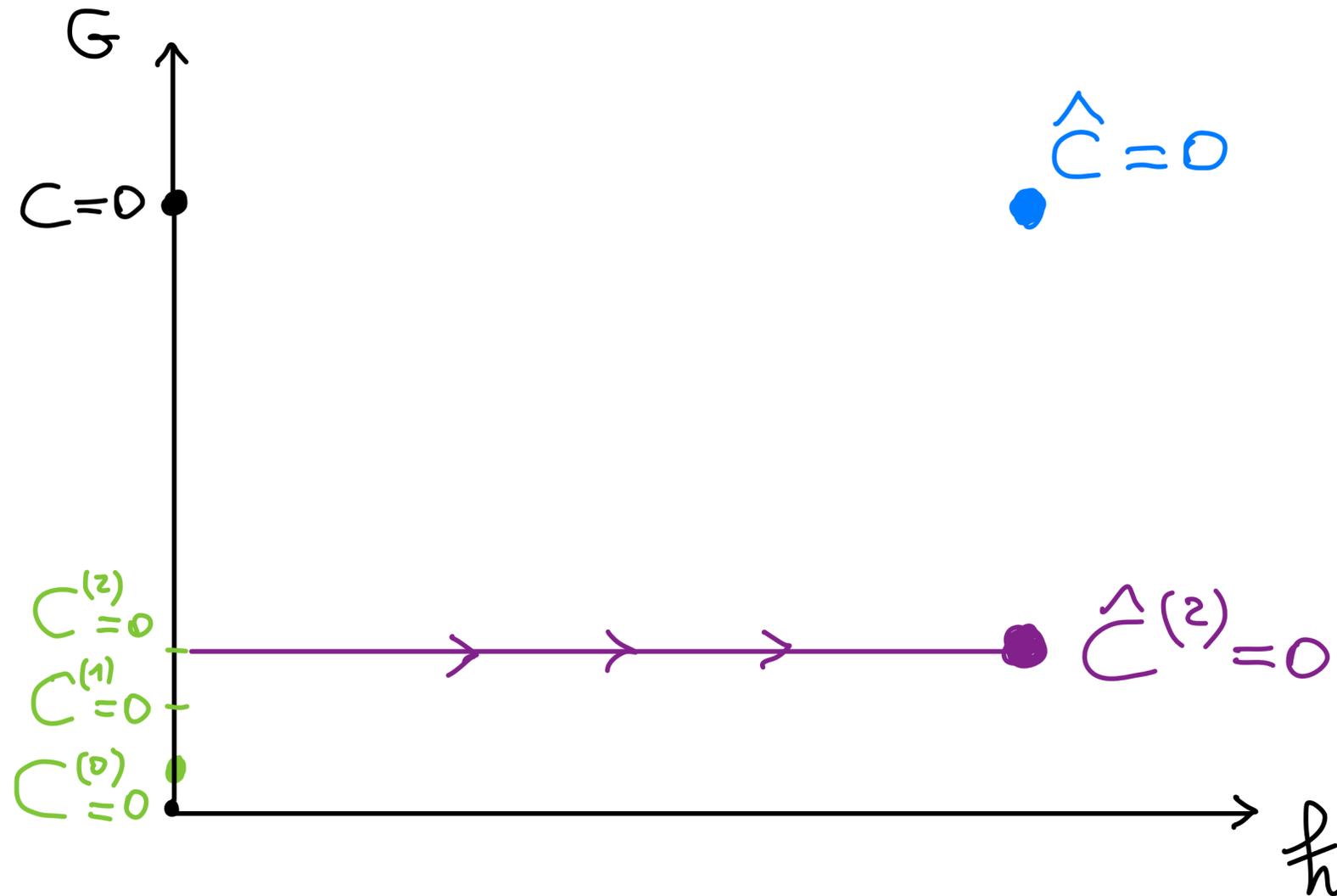


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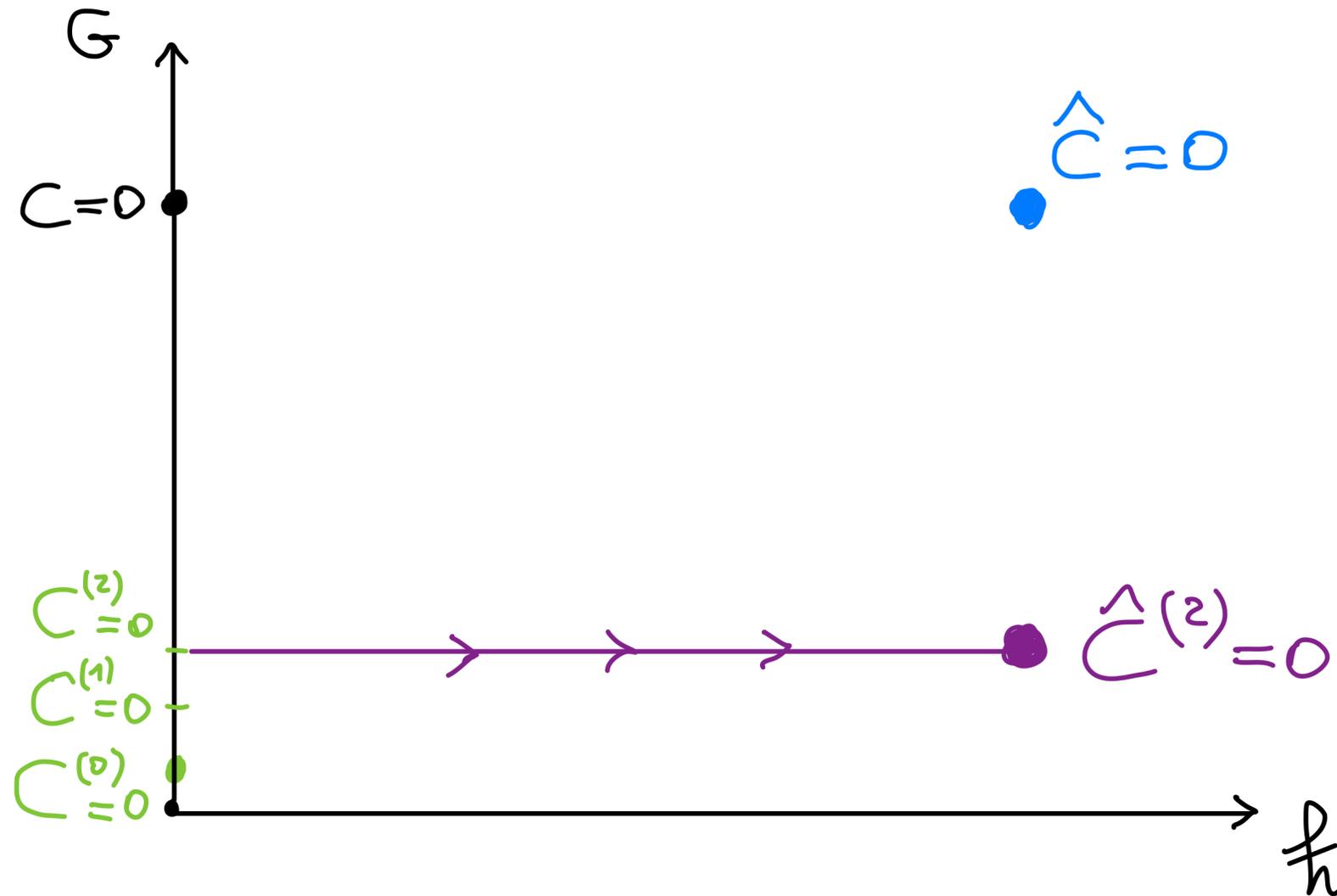
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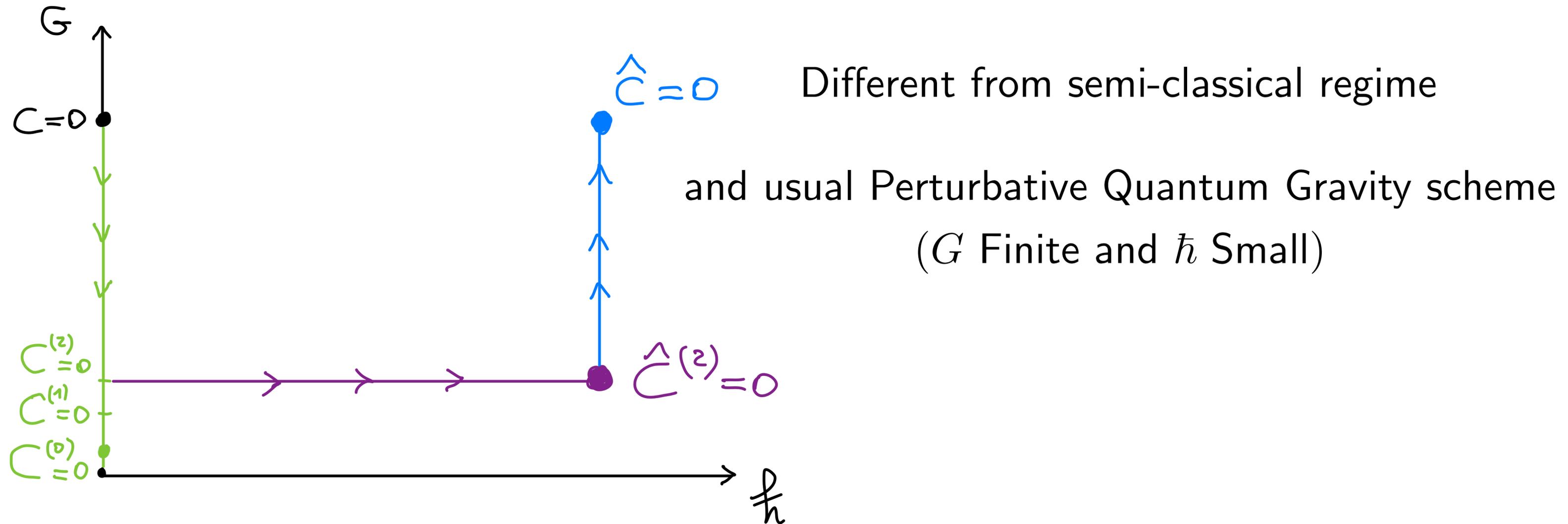
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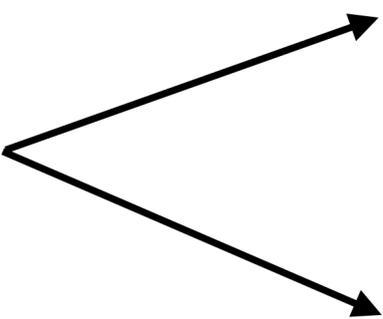
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$$\text{Gravitons } \Leftrightarrow \text{Beltrami Differentials } \zeta = \epsilon X + O(\epsilon^2) \quad \Rightarrow \quad \sigma^2 = 2\epsilon^2 \partial_v X \partial_v \bar{X}$$

$$C = C^{(0)} + \epsilon C^{(1)} + \epsilon^2 C^{(2)} + O(\epsilon^3)$$

$$C^{(0)} = \partial_v^2 \Omega^0 - \bar{\mu} \partial_v \Omega^0 = 0$$


Stationary Background  $\partial_v \Omega^0 = 0$ ,  $\bar{\mu}$  free

Generic Background  $\partial_v \Omega^0 \neq 0$

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Spin-0 Geometry on equal footing as the other sectors: perturbative quantum geometry

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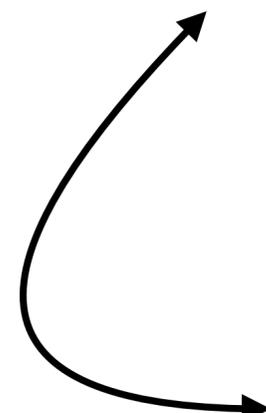
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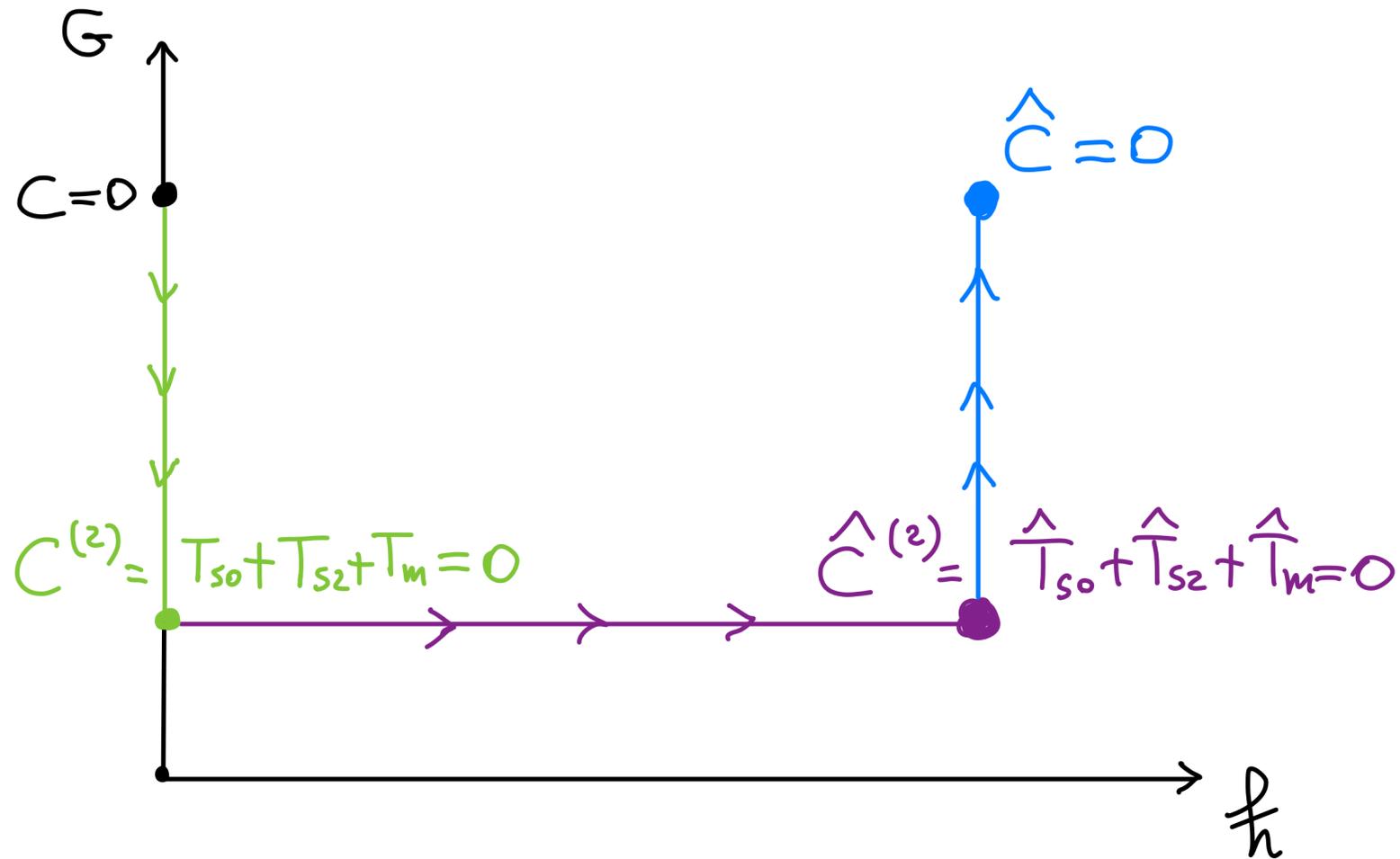
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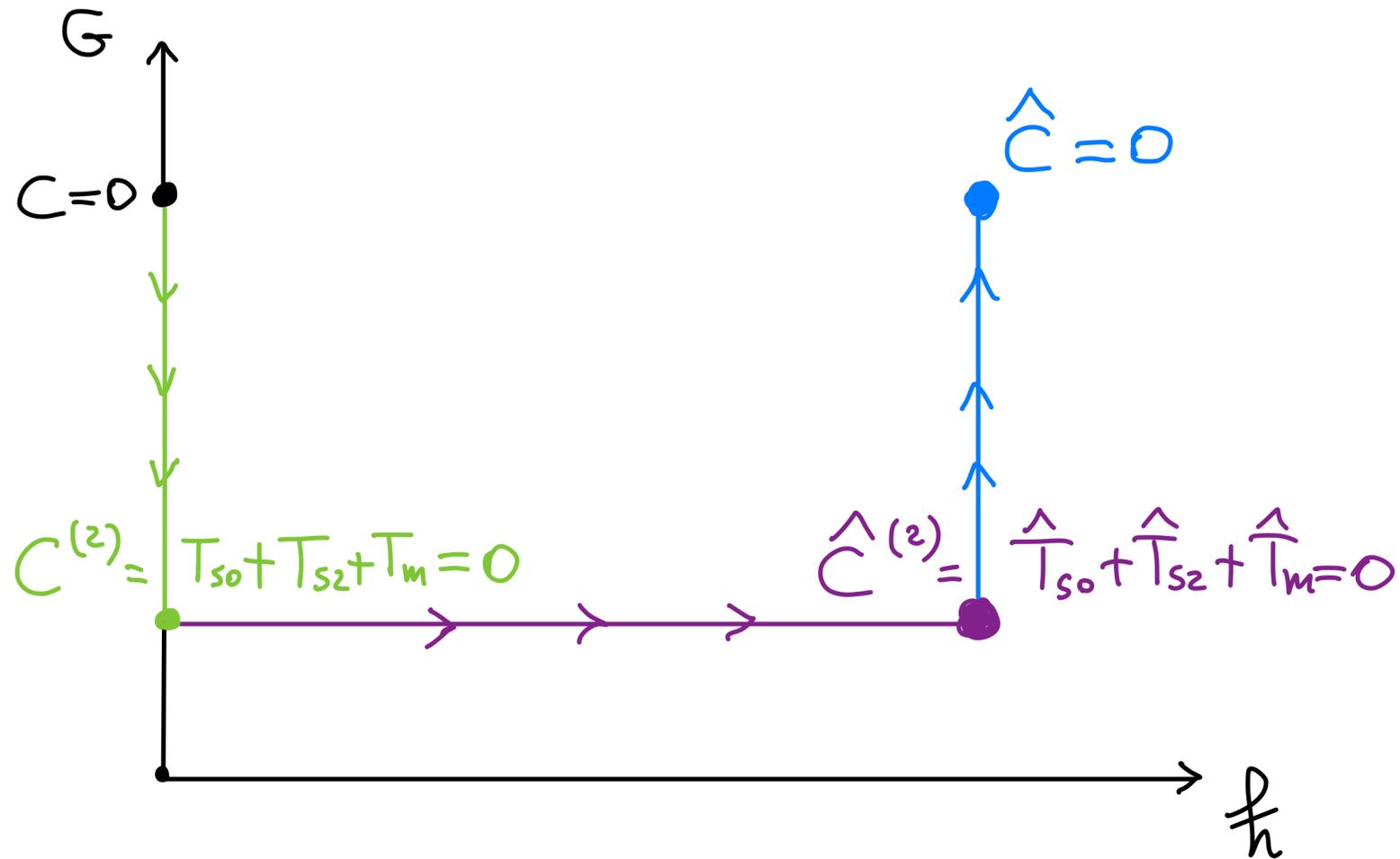
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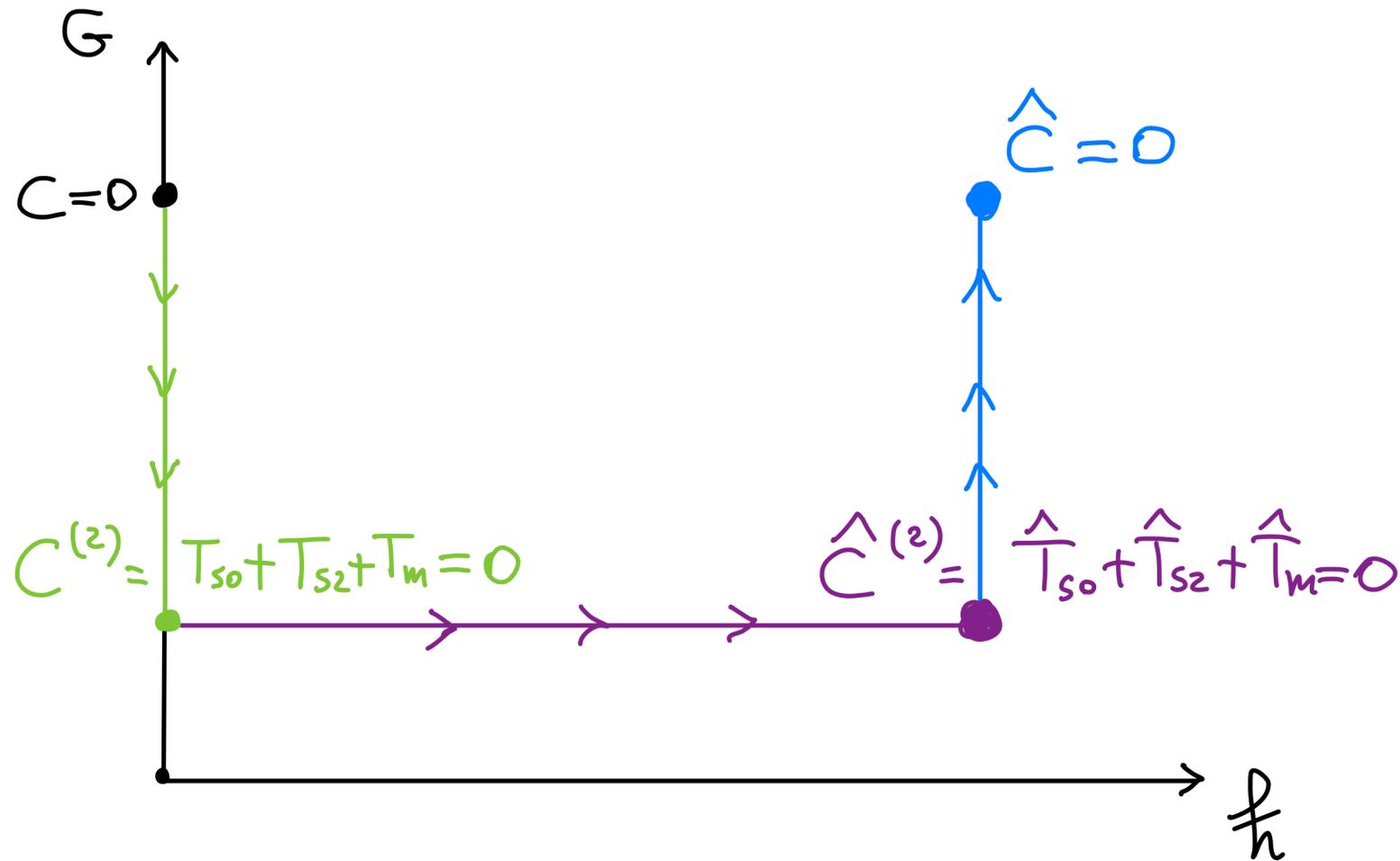


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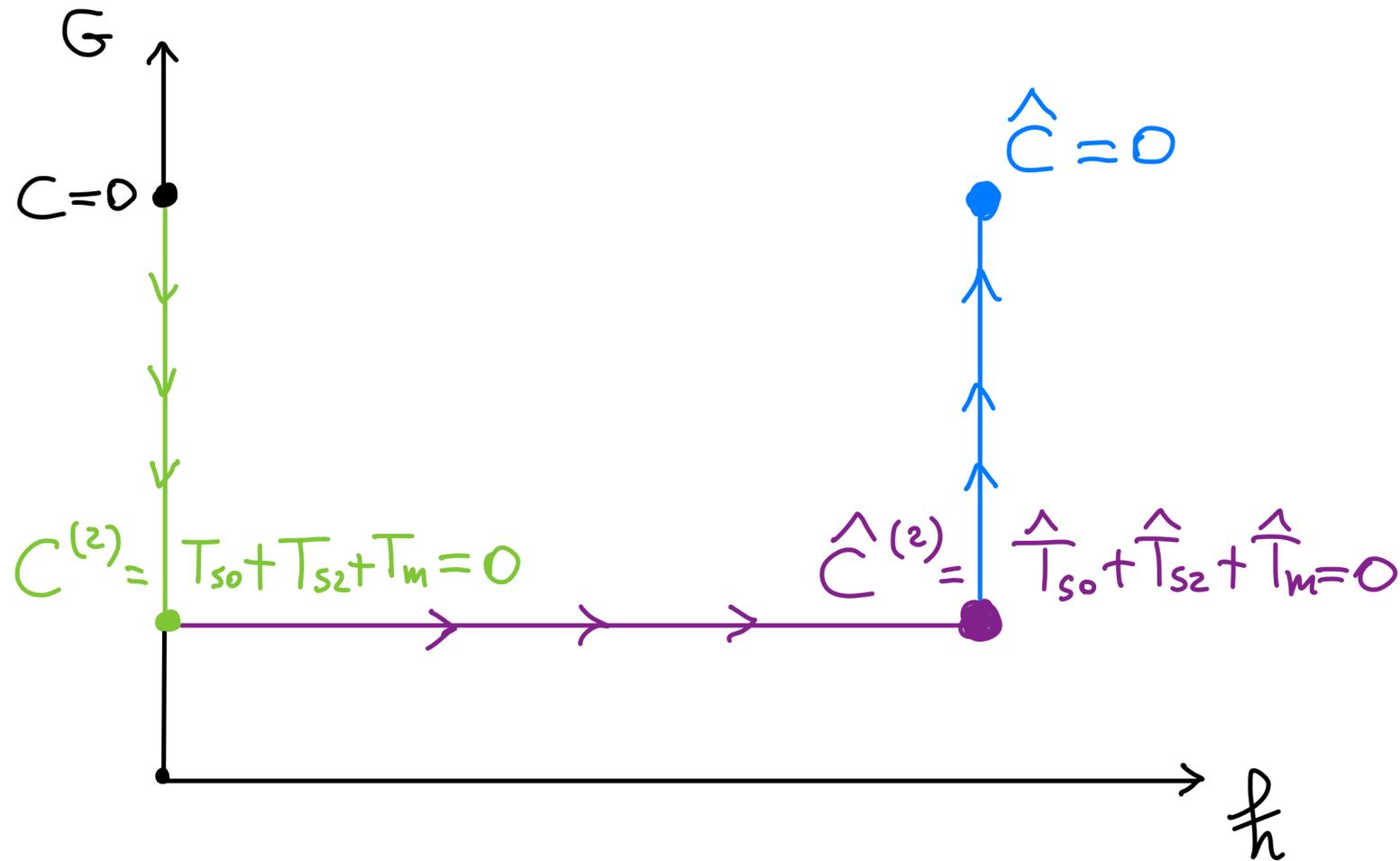
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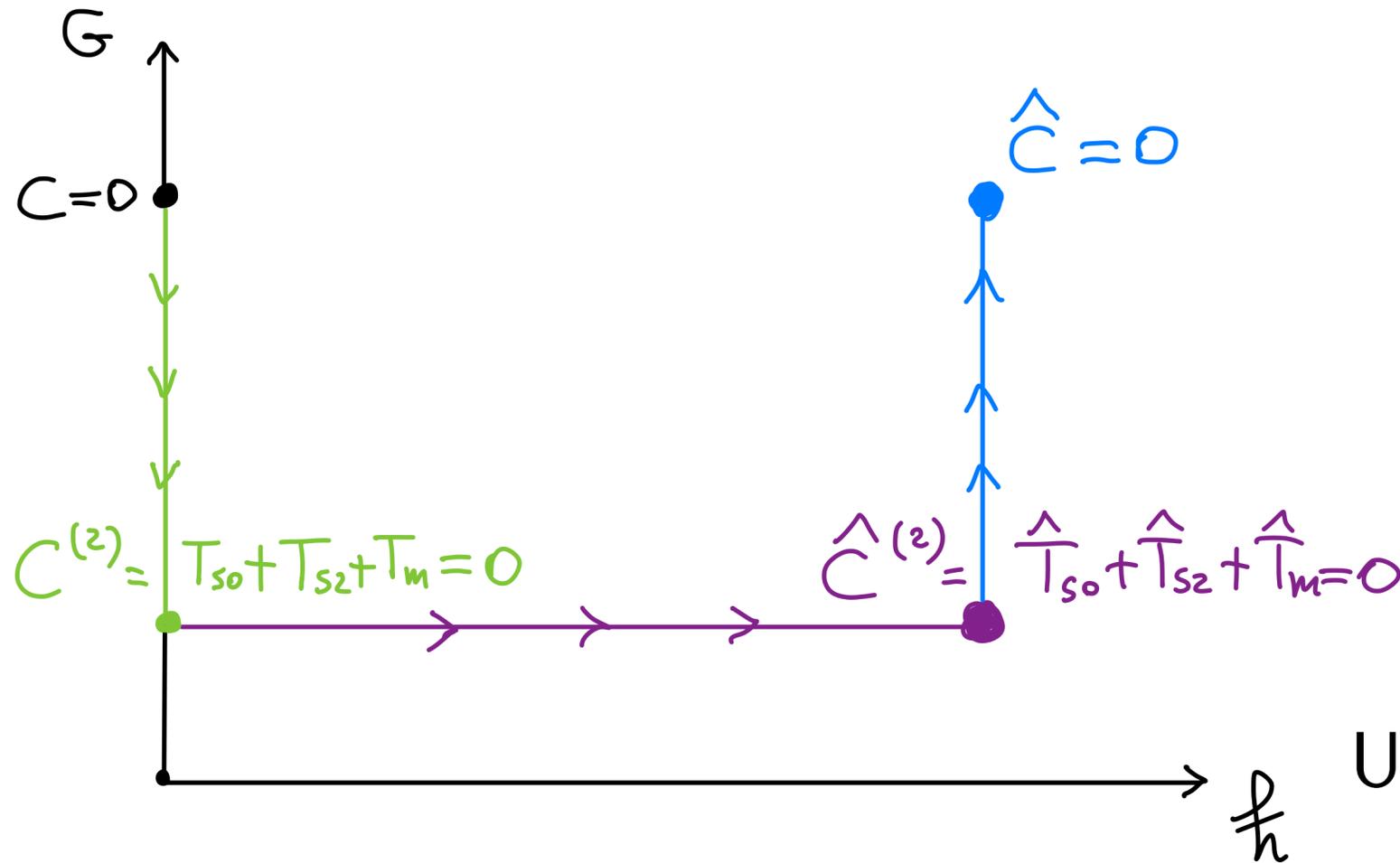


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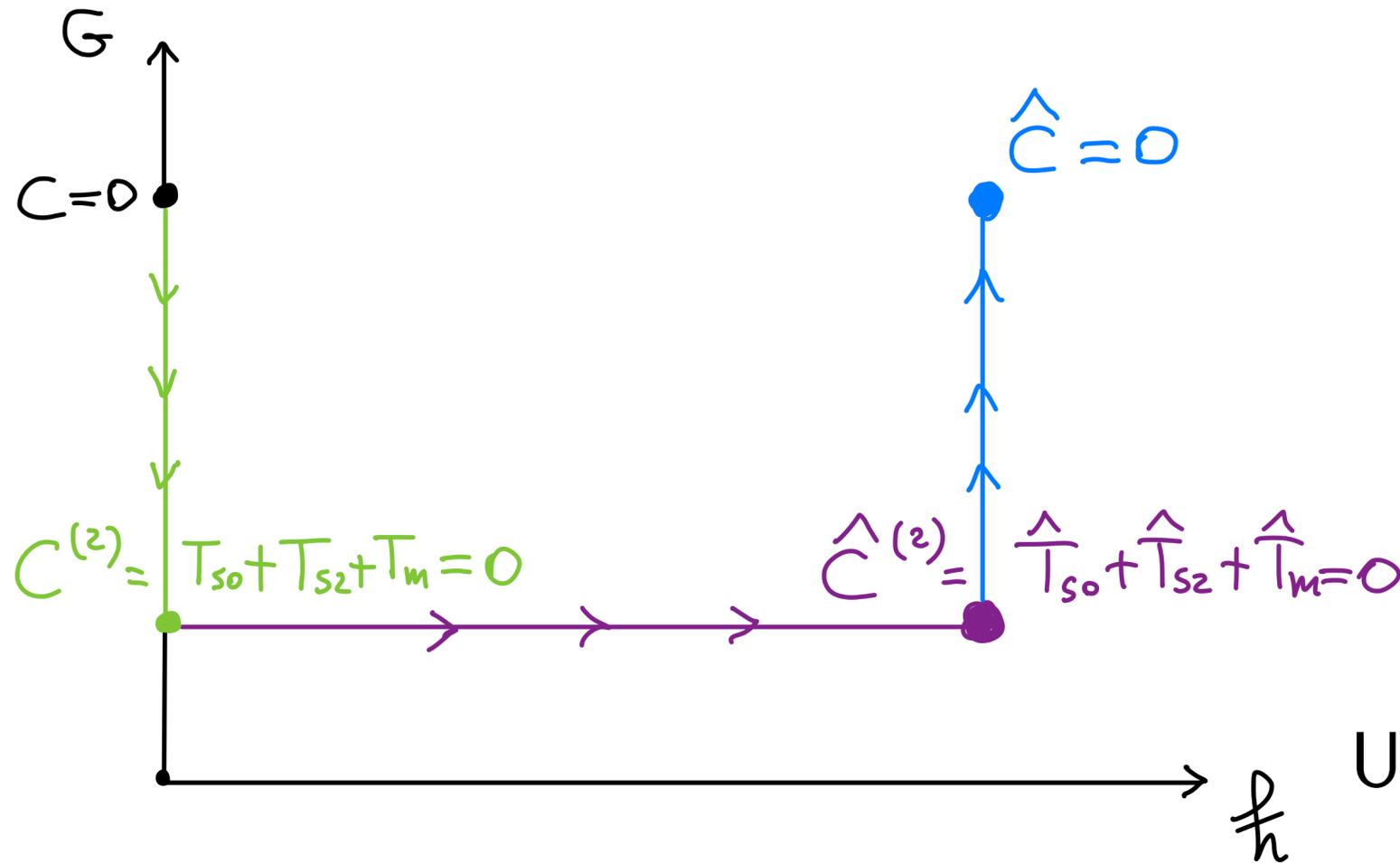
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[In the same vein of: Rovelli-Smolin '94, Ashtekar-Lewandowski '96, Rovelli '96, and many many other]

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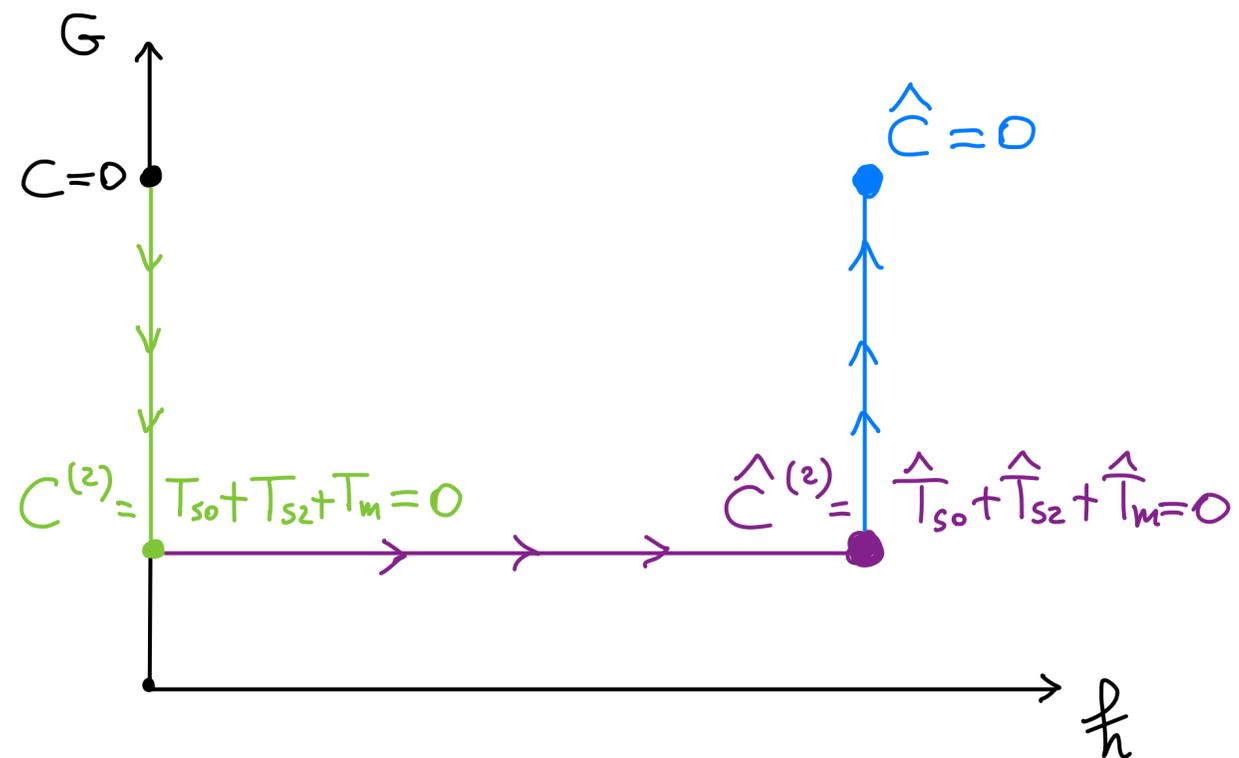
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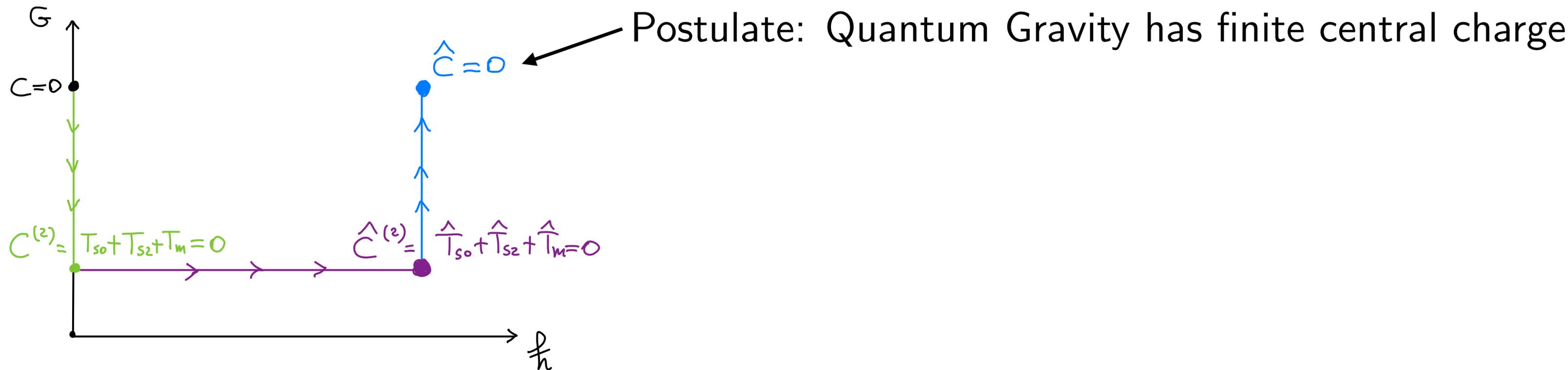


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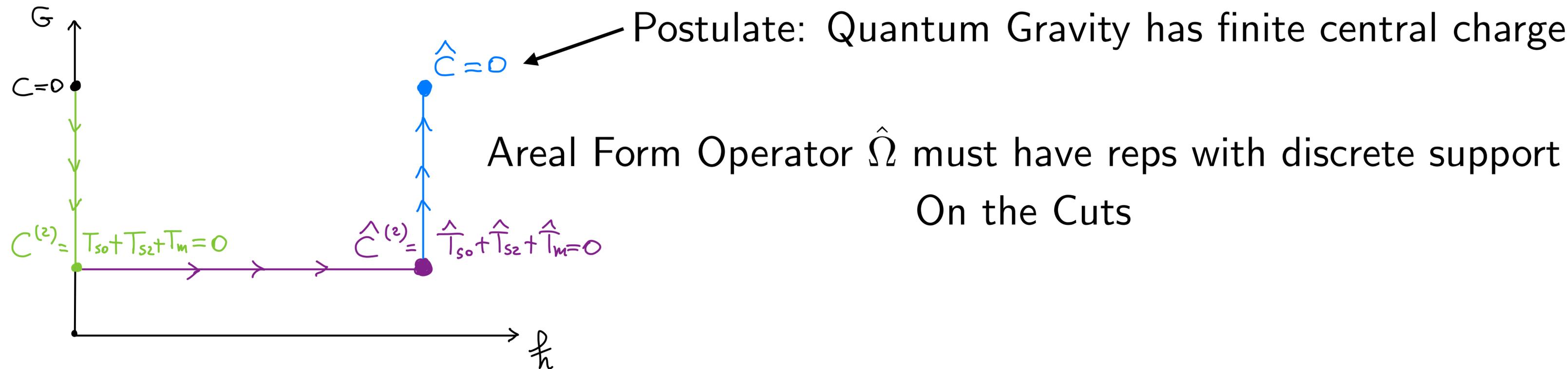


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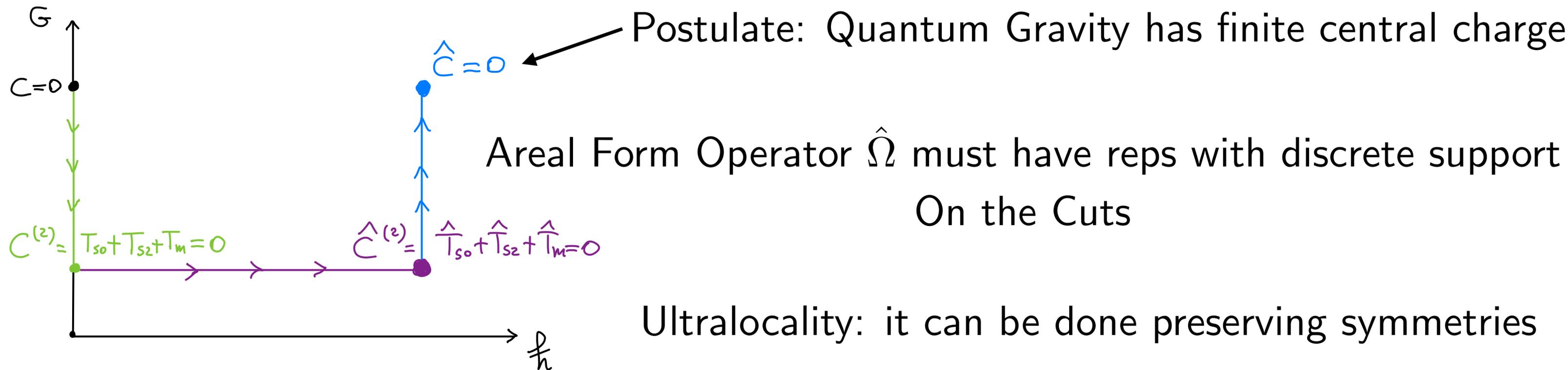


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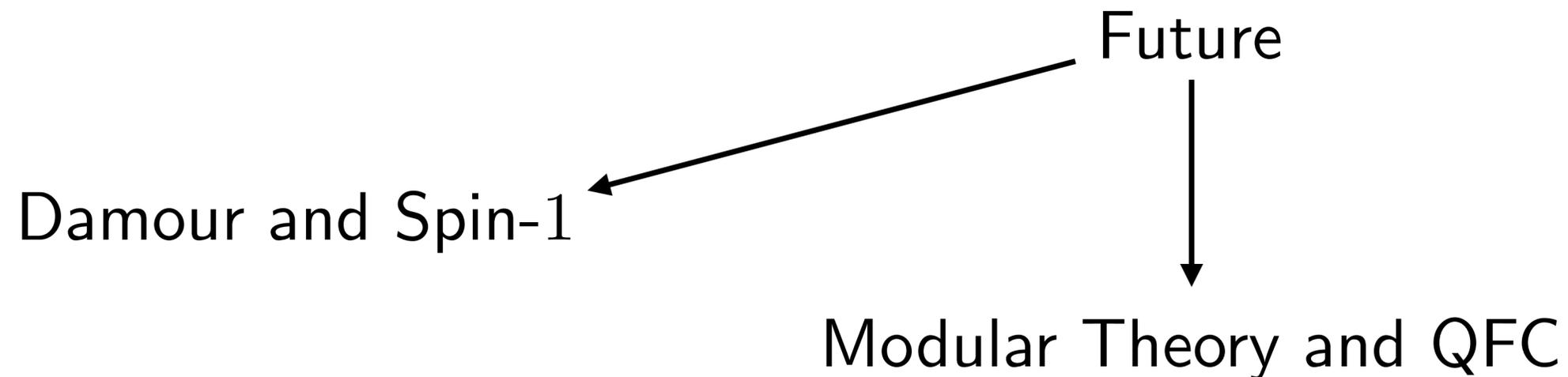
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