Quantum Reduced Loop Gravity I

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International Loop Quantum Gravity Seminars.

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Plan of the talk

_Inhomogeneous extension Bianchi I model

_Reduced quantization

_Introduction to Quantum-reduced Loop Gravity

Inhomogeneous extension Bianchi I model

Proposal

Motivation: can we preserve “more” of the full Loop Quantum Gravity structure in Quantum Cosmology??

We want to define a weaker reduction of gravity phase space which captures the relevant cosmological degrees of freedom such that

_a residual diffeomorphisms invariance is retained and the scalar constraint can be regularized.

Inhomogeneous extension Bianchi I model

Diagonal Bianchi line element

Only part which depends on x

$$ds^2 = N^2(t)dt^2 - e^{2\alpha(t)}(e^{2\beta(t)})_{ij}\,\omega^i \otimes \omega^j$$

Universe volume

diagonal and with vanishing trace

Fiducial 1-forms

Two independent components: Anisotropies

$$d\omega^i = C^i_{jk}\omega^j \wedge \omega^k.$$

Constant depending on the kind of Bianchi model

One considers only type A $\quad C^i_{ij} = 0$

Most relevant cases: I,II, IX...

Reduced phase-space

Momenta: $E_i^a = p^i(t)\omega\omega_i^a,$ $p^i = e^{2\alpha}e^{-\beta_{ii}}$

Not summed

Connections: $A_a^i = c_i(t)\omega_a^i,$ $c_i = \left(\frac{\gamma}{N}(\dot{\alpha} + \dot{\beta}_{ii}) + \alpha_i\right)e^{\alpha}e^{\beta_{ii}}$

It depends on the kind of Bianchi model adopted

Poisson brackets:

$$\{p^i(t), c_j(t)\}_{PP} = \frac{8\pi G}{V_0}\gamma\delta^i_j$$

Fiducial volume (it can be avoided by rescaling variables)

Holonomies:

edge length

$$h_{e_i} = e^{i\mu_i c_i \tau_i}$$

edge along ω_i

Bianchi I

The simplest case is Bianchi I model

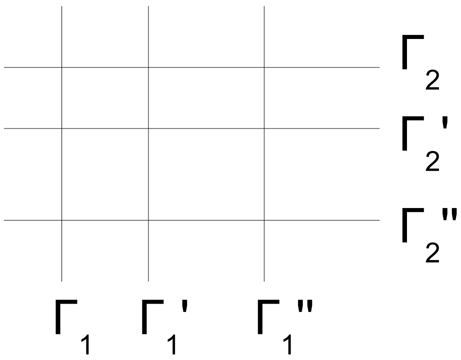
$$C^i_{jk} = 0 \qquad\qquad \omega^i = \delta^i_a dx^a$$

$$ds^2 = N^2(t)dt^2 - a_1^2(t)(dx^1)^2 - a_2^2(t)(dx^2)^2 - a_3^2(t)(dx^3)^2$$

Three scale factors along Cartesian coordinates $\qquad x^i = \delta^i_a x^a$

Let us consider the integral curve Γ_i of the dual vector field $\omega_i = \delta_i^a \partial_a$

$$\Gamma_i = \begin{cases} x^i = x^i(s) \\ x^j = const. \end{cases}$$



Phase-space variables

$$E^a_i = p^i(t)\delta^a_i \qquad\qquad A^i_a = c_i(t)\delta^i_a$$

If we retain a dependence on spatial coordinates in the reduced variables of a Bianchi I model....

$$E_i^a = p^i(t,x)\delta_i^a \qquad\qquad A_a^i = c_i(t,x)\delta_a^i$$

1) <u>Re-parametrized Bianchi I model:</u>

$$ds^2 = N^2(t,x)dt^2 - a_1^2(t,x^1)(dx^1)^2 - a_2^2(t,x^2)(dx^2)^2 - a_3^2(t,x^3)(dx^3)^2$$

the three scale factors are functions of the associated Cartesian coordinate

2)<u>Kasner epoch:</u> it describes the behavior of the generic cosmological solution during each Kasner epoch.

$$ds^2 = N^2(t,x)dt^2 - a_1^2(t,x)(dx^1)^2 - a_2^2(t,x)(dx^2)^2 - a_3^2(t,x)(dx^3)^2$$

Spatial gradients negligible with respect to time derivatives.

Reduced phase-space

Momenta:

$$E_i^a = p^i(t,x)\delta_i^a$$

Connections:

$$A_a^i = c_i(t,x)\delta_a^i$$

Poisson brackets:

$$\{p^i(t,x), c_j(t,y)\} = 8\pi G\gamma\delta_j^i\delta^3(x-y)$$

(Given a metric tensor, all triads related by a rotation are equally admissible. ⇒ A unique choice implies a gauge-fixing of the rotation group)

$$\widetilde{E}_i^a = R^k{}_i E_k^a \qquad R^k{}_i = \delta_i^k$$

Gauge fixing condition:

$$\chi_i = \sum_{l,k} \epsilon_{il}{}^k E_k^a \delta_a^l = 0$$

Restriction of admissible diffeomorphisms to preserve the expression of connections and momenta. ⇒ **Reduced diffeomorphisms**

Kinematical constraints

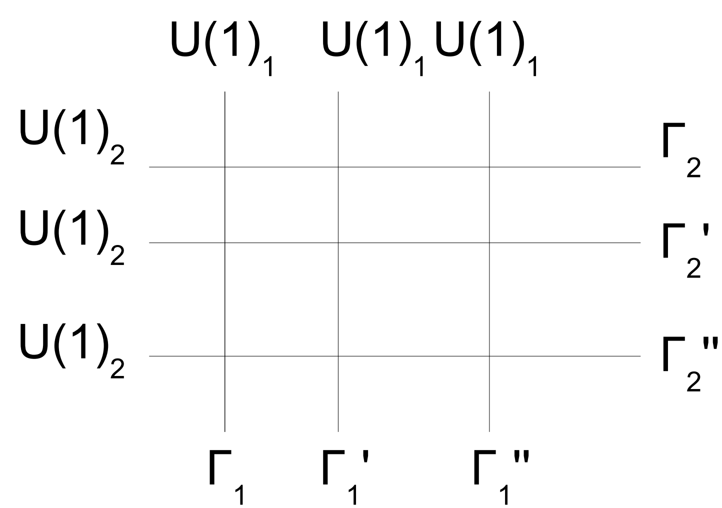
U(1)$_i$ Gauss constraints: $G_i = \partial_i p^i = 0$ not summed

Along each Γ_i

$$G_i = \partial_i p^i = 0$$ spatial index

<u>generates U(1) gauge transformations.</u>

c_i and p^i are the connection and the momentum of a U(1) gauge theory on each Γ_i .



By varying i one gets three independent U(1) gauge groups.

Reduced diffeomorphims: $D_i = \sum_j [p^j \partial_i c_j - \partial_i (p^j c_j)]$

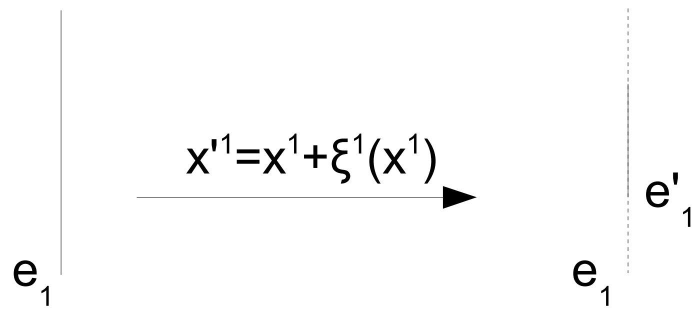
on each Γ_k

$$x'^i = x^i + \xi^i$$

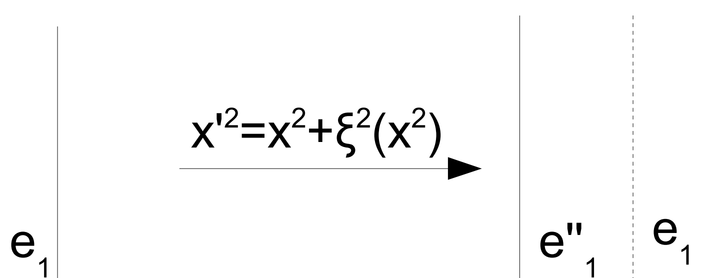
Infinitesimal parameter

$$\xi^i = \xi^i(x^i)$$

Given an edge e_1 along $\omega_1 = \delta_1^{\ a} \partial_a$ a reduced diffeomorphisms acts as



A generic diffeomorphisms in the 1-dimensional space generated by ω_i



A rigid translation along the directions generated by ω_j for $j \neq i$

A reduced diffeo maps an edge e_i into another edge e'_i which is still parallel to the vector field ω_i

Reduced Quantization

Reduced quantization

Let us quantize the algebra of holonomies along reduced graphs and fluxes along dual surfaces:

edges e_i

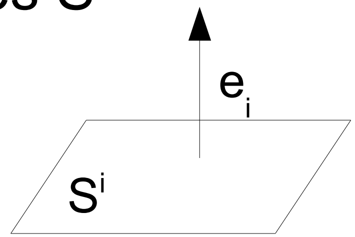
$U(1)_i$ holonomies along e_i

$$h_{e_i} = P\left(e^{i\int_{e_i} c_i dx^i}\right)$$

$U(1)_i$ group element

Fluxes across dual surfaces S^i

$$p^i(S^i) = \int_{S^i} p^i n_i du dv$$



Kinematical Hilbert space:

$U(1)_i$ Haar measure

$$H = \oplus_\Gamma H_\Gamma \qquad H_\Gamma = \otimes_i \otimes_{\{e_i \subset \Gamma\}} L^2(U(1)_i, d\mu^i)$$

graph structure!

A generic functional over a graph is given by

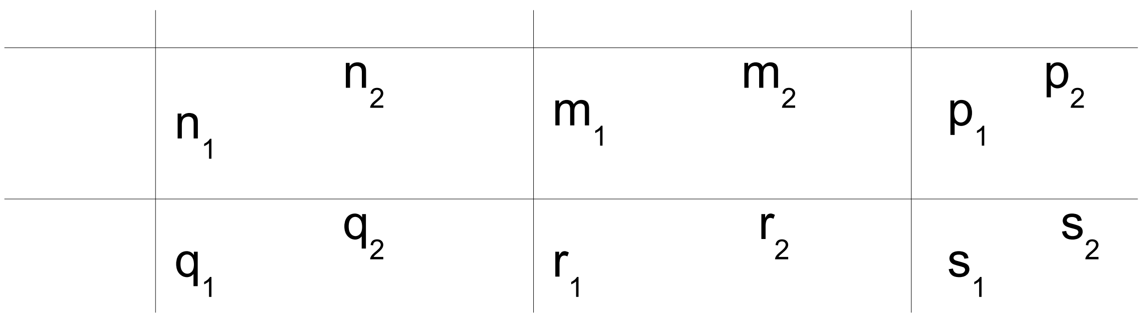
$$\psi_\Gamma = \otimes_i \otimes_{\{e_i \subset \Gamma\}} \psi_{e_i}$$

functions of U(1)$_i$ group element

$$\psi_{e_i} = \sum_{n_i} e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

U(1)$_i$ Irreps

Basis: U(1)$_i$ networks



Momenta act as invariant vector fields of the U(1)$_i$ groups

$$p^i(S^i)\psi_{e_i} = 8\pi\gamma l_P^2 \sum_{n_i} n_i e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

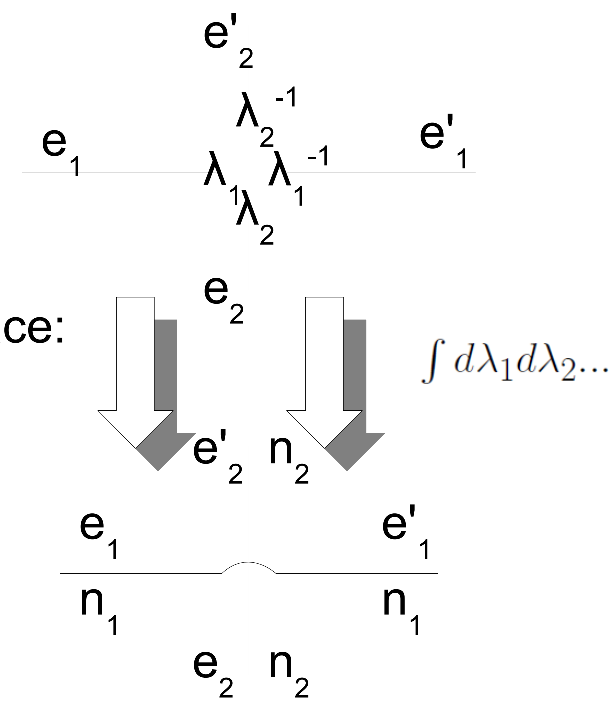
Kinematical constraints:

1) Relic Gauss constraint $G_i = \partial_i p^i = 0$

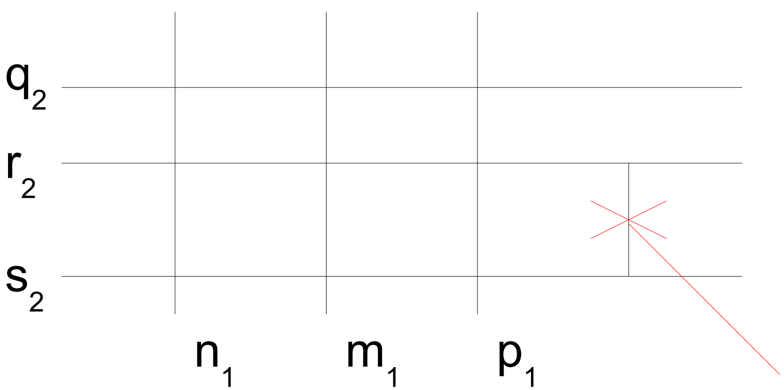
they generate $U(1)_i$ gauge transformations.

$$h_{e_i} \rightarrow \lambda_i(x_0) h_{e_i} \lambda_i^{-1}(x_1)$$

Projection on the $U(1)_i$ gauge-invariant Hilbert space:



$U(1)_i$ quantum numbers conserved along ω_i

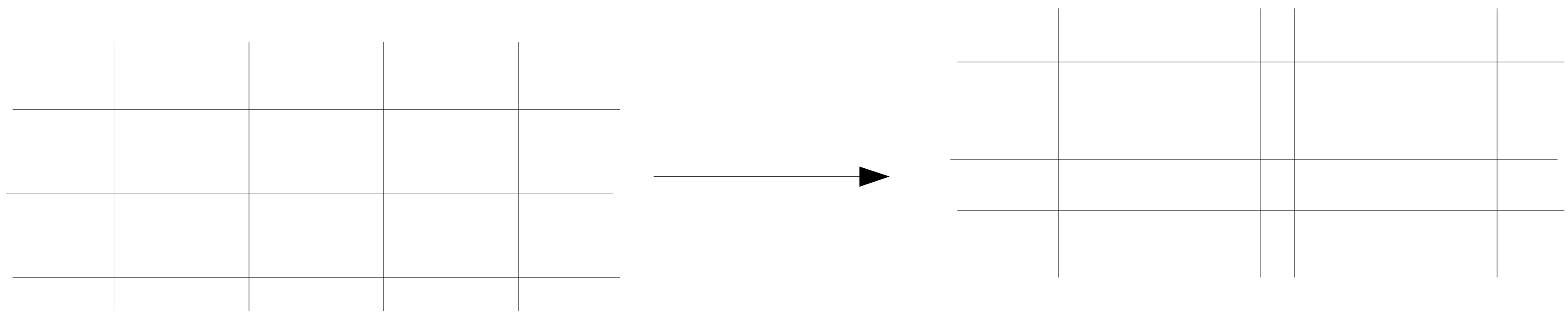


Lattice structure

Not allowed!

2) Reduced diffeomorphisms:

Action of reduced diffeomorphisms:



Invariant states via a sum over reduced s-Knots.

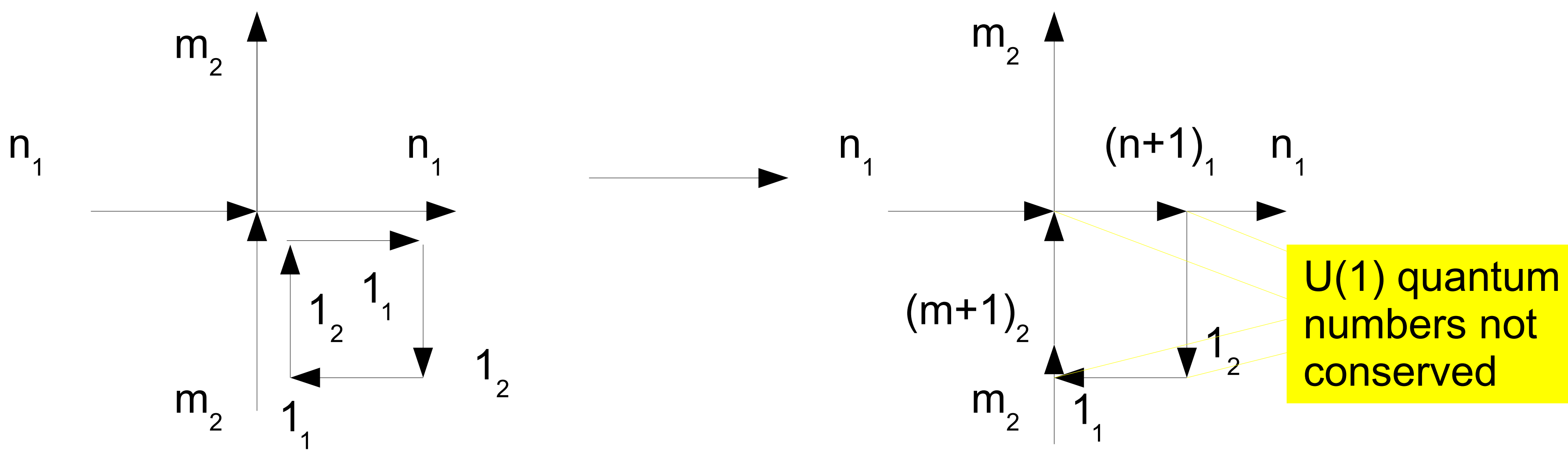
$$\psi^*_s(.) = \sum_{\Gamma \in s} \psi^*_\Gamma(.)$$

s: equivalence class of graphs under reduced diffeomorphisms

Can we implement the dynamics (Thiemann prescription) ??? NO

We cannot attach the holonomies needed to regularize the superHamiltonian

The attachment of a U(1) group element spoils U(1) gauge invariance



The drawback is the absence of a real 3-dimensional vertex structure.

We need a nontrivial interplay between $U(1)_i$ quantum numbers

Truncation of the full theory

Introduction to Quantum-reduced Loop Gravity

Proposal

$$
\begin{tikzcd}
\{A^i_a(t,x), E^b_j(t,y)\} = 8\pi G\gamma\delta^b_a\delta^i_j\delta^3(x-y) \arrow[r, "\text{reduction}", "\substack{\text{reduced}\\ \text{phase-space}}"'] \arrow[d, "\text{quantization}"'] & \{p^i(t,x), c_j(t,y)\} = 8\pi G\gamma\delta^i_j\delta^3(x-y) \arrow[d, "\text{quantization}"'] \\
H_\Gamma = \otimes_{\{e\subset\Gamma\}} L^2(SU(2), d\mu) \arrow[r, "???", "???"'] & H_\Gamma = \otimes_i \otimes_{\{e_i\subset\Gamma\}} L^2(U(1)_i, d\mu^i)
\end{tikzcd}
$$

Can we infer the KINEMATICS of the reduced model from the full theory??

generic graphs, SU(2) group elements, invariant intertwiners, background independence..

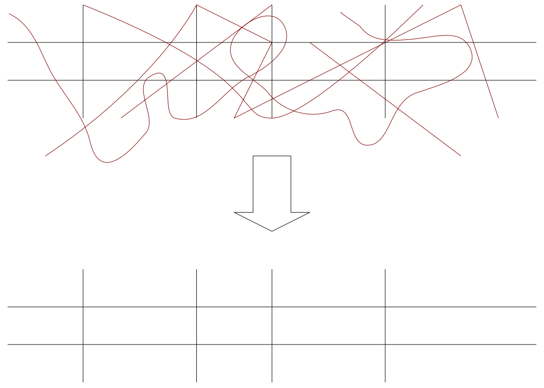
Metodology: Truncation of LQG Hilbert space in order to get

1) the same lattice structure as in reduced quantization → Projection to graphs with edges e_i → Reduced diffeomorphisms

2) U(1) group elements → Projection from SU(2) group to U(1) subgroups → Non trivial vertex structure from SU(2)-invariant Hilbert space!

Emanuele's talk.......

1) Projection to reduced graphs (with edges e_i)



$$Ph_e = \begin{cases} h_e & e = e_i \\ 0 & otherwise \end{cases}$$

projector

Action of diffeomorphisms

$$U_\varphi h_e = h_{\varphi(e)}$$

$$^{red}U_\varphi = P U_\varphi P$$

Diffeo in reduced space

$$^{red}U_\varphi h_{e_i} = P U_\varphi P h_{e_i} = P U_\varphi h_{e_i} = P h_{\varphi(e_i)} = U_{red_\varphi} h_{e_i} \qquad ^{red}U_\varphi = U_{red_\varphi}$$

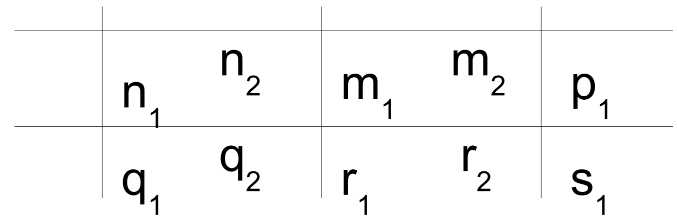
The truncation of admissible edges restricts the class of admissible diffeomorphisms to reduced ones.

Invariant states as in reduced quantization by summing over reduced s-knots.

(Partial) Conclusions:

reduced quantization inhomogeneous Bianchi I model:

_ Hilbert space: square integrable functions of $U(1)_i$ group elements attached to edges e_i.



_ kinematics: OK! $U(1)_i$ gauge invariance via invariant intertwiners

reduced diffeo-invariance via reduced s-knots

_ dynamics (Thiemann-like prescription): NO!!

Quantum reduced Loop Gravity:

_ truncation to reduced graphs: only reduced diffeomorphisms are implemented.

_ what has to be done? Reduction from SU(2) to $U(1)_i$ elements.

Quantum Reduced Loop Gravity II

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ILQGS

12th March 2013

Plan of the Talk

- Reduced Kinematical Hilbert Space: Cosmological LQG
- Constraints
- Hamiltonian
- News

Cosmological LQG

GOAL:

Implement on the SU(2) Kinematical Hilbert space of LQG the classical reduction:

$$\boxed{A_a^i = c_i(t,x)\omega_a^i} \qquad \boxed{E_i^a = p^i(t,x)\omega\omega_i^a}$$

$$\boxed{\{p^i(x,t), c_j(y,t)\} = 8\pi G\gamma\delta_j^i\delta^3(x-y)}$$

First truncation: we <u>restrict the holonomies to curves along edges</u> e_i parallel to fiducial ω_i^a vectors

The SU(2) classical holonomies associated to the reduced variables are

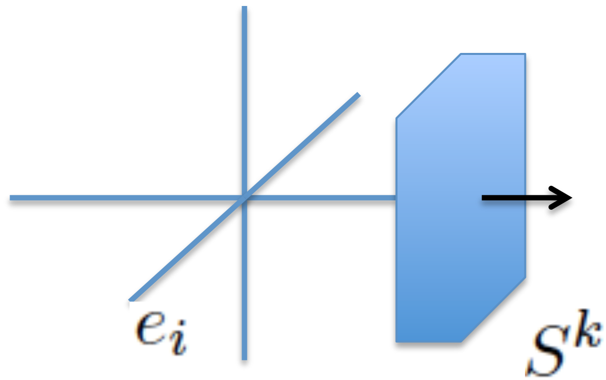
$$^R h_{e_i}^j = P(e^{i\int_{e_i} c^i\omega_a^i dx^a(s)\tau_i})$$

$$^R h_{e_i}^j = \exp\left(i\alpha^i\tau_i\right)$$

NO sum over i

Holonomy belong to the U(1) subgroup generated by τ_i

Consider fluxes across surfaces $x^a(u,v)$ with normal vectors parallel to the fiducial ones



The classical reduction implies

$$E_i(S^k) = \int E_i^a \frac{1}{\omega} \omega_a^k du dv = \delta_i^k \int p_i \frac{1}{\omega} du dv$$

For consistency only the diagonal part of the matrix $E_i(S^j)$ is non vanishing

$$\chi_i = \sum_{l,k} \epsilon_{il}{}^{k} E_k(S^l) = 0$$

Second class with the Gauss constraint

How to implement the reduction on the holonomies and consistently impose $\chi_i=0$?

Strategy: Mimic the spinfoam procedure

Impose the second class constraint weakly to find a "Physical Hilbert space"

Engle, Pereira, Rovelli,Livine '07- '08

Imposing a Master constraint strongly on the SU(2) holonomies:

$$\boxed{\chi^2 = \sum_i \chi_i \chi_i = \sum_{i,m,k,l} [\delta^{im} \delta_{kl} E_i(S^k) E_m(S^l) - E_i(S^k) E_k(S^i)]}$$

$$\boxed{\chi^2 h^j_{e_i} = (8\pi\gamma l_P^2)^2 (\tau^2 - \tau_i \tau_i) h^j_{e_i}} = 0$$

Different i for each direction

To solve it is convenient to introduce SU(2) coherent states

SU(2) coherent states

$$|j,\vec{u}>= D^j(\vec{u})|j,j>=\sum_m |j,m> D^j(\vec{u})_{mj}$$

The Master constraint condition acting at the endpoint
(the conjugate condition at the starting point):

$$\chi^2 D^j(g)|j,\vec{u}>= D^j(g)(\tau^2-(\vec{e_i}\cdot\vec{\tau})^2)|j,\vec{u}>= D^j(g)(j(j+1)-(\vec{e_i}\cdot\vec{\tau})^2)|j,\vec{u}>$$

Using the property $\vec{v}\cdot\vec{\tau}|j,\vec{v}>= j|j,\vec{v}>$

If $\vec{e_i}=\vec{u}$ in the large j limit up to $\mathrm{L_p}$ corrections the basis element will satisfy:

$$\chi^2 D^j(g)|j,\vec{u}>=0$$

Reduced basis Elements

$$\boxed{\langle j,\vec{e_i}|D^j(g)|j,\vec{e_i}\rangle}$$

There is a natural way of embedding U(1) cylindrical functions in SU(2) ones:

Projected spinnetworks (Alexandrov, Livine '02)
with the Dupuis-Livine map (Dupuis Livine '10)

$$f : U(1) \to SU(2)$$

$$\tilde{\psi}(g) = \int_{U(1)} dh\, K(g,h)\psi(h), \qquad g \in SU(2)$$

$$K(g,h) = \sum_n \int_{U(1)} dk\, \chi^{j(n)}(gk)\chi^n(kh)$$

SU(2) trace

U(1) trace

These SU(2) functions have the remarkable property that they are completely determined by their restriction to their U(1) subgroup

$$\tilde{\psi}(g)|_{U(1)} = \psi$$

If we consider projected functions defined over the edge e_i choosing the subgroup

$U(1)_i$ as the one generated by τ_i

$$\tilde{\psi}(g)_{e_i} = \sum_{n_i} {}^iD^{j(n_i)}_{m=n_i\, r=n_i}(g)\psi^{n_i}_{e_i}$$

$< j, \vec{e}_i | D^j(g) j, \vec{e}_i >$

U(1) quantum number

The Master constraint equation selects the degree of the map: $\boxed{|n_i| = j(n)}$

The strong quadratic condition implies the linear one weakly (restriction to symmetric matrix) !

$$< \tilde{\psi}'_i | E_k(S^l) | \tilde{\psi}_i >= 8\pi\gamma l_P^2 \sum_{j,j'} \psi^{j'}_{e_i} \int dg\, {}^iD^{j'}_{j'j'}(g)\tau_k {}^iD^j_{jj}(g)\psi^j_{e_i} = 0, \qquad k \neq i$$

The quantum states associated with an edge e_i are entirely determined by their projection into the subspace with maximum magnetic numbers along the internal direction i

$$\psi_{e_i} = \tilde{\psi}(g)_{e_i}|_{U(1)_i} = \sum_j e^{i\theta^i j} \psi^j_{e_i} = \sum_j {}_i< j,j|^R h^j_{e_i} |j,j>_i \psi^j_{e_i}$$

The action of fluxes $E_l(S^k)$ on the reduced space is nonvanishing only for $l = k = i$

$$\boxed{E_i(S^i)\tilde{\psi}_{e_i} = 8\pi\gamma l_P^2 \sum_j j D^j_{jj} \psi^j_{e_i}}$$

This is how we find in the SU(2) quantum theory the classical reduction

$\boxed{A^i_a = c_i(t,x)\omega^i_a}$ $\boxed{E^a_i = p^i(t,x)\omega\omega^a_i}$ $\boxed{\{p^i(x,t), c_j(y,t)\} = 8\pi G\gamma\delta^i_j\delta^3(x-y)}$

Analogy with Spinfoam Quantization:

SL(2,C) basis elements

$$\langle g|p,k,j,m,j',m'\rangle = D^{p,k}_{jm,j'm'}(g)$$

$$\tilde{\psi}(g) = \sum_{jmn} d_j\, \psi_{jmn}\, D^{p(j),j}_{jm,jn}(g)$$

SU(2) basis elements

$$\langle g|j,m,r\rangle = D^j_{mr}(g)$$

$$\tilde{\psi}(g)_{e_i} = \sum_{n_i} {}^i D^{j(n_i)}_{m=n_i\; r=n_i}(g)\psi^{n_i}_{e_i}$$

Linear simplicity constraint

$$\vec{K} + \gamma \vec{L} = 0$$

$$\tau^k h^j_{e_i} = 0 \quad \forall k \neq i$$

Quadratic part of the constraint imposed strongly:

$$(2\gamma C_1 - (\gamma^2 - 1)C_2)|\tilde{\psi}\rangle = 0$$

$$(\tau^2 - \tau_i\tau_i)h^j_{e_i} = 0$$

Select j(n)=n

$$p = \gamma k$$

Only one condition (SU(2)
has only one parameter label for the irrep.)

Weakly satisfied in the large limit

$$\langle\tilde{\psi}|\vec{K} + \gamma\vec{L}|\tilde{\psi}'\rangle = 0$$

Weakly satisfied in the large limit

$$< \tilde{\psi}_i'|E_k(S^l)|\tilde{\psi}_i > = 0$$

Select $k = j$

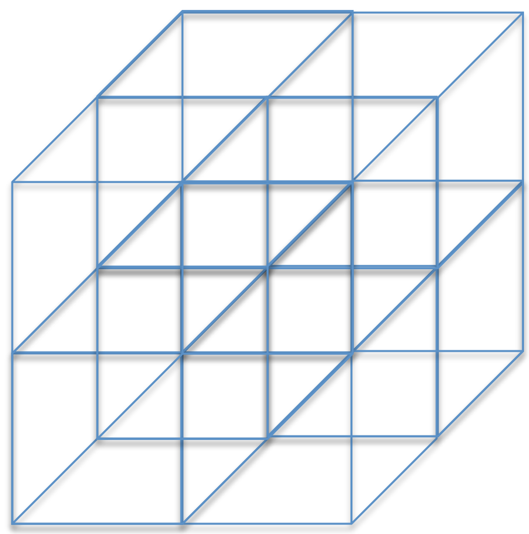
$$g|_K = D^{\gamma j,j}_{jm,jn}(g) = \int_{SU(2)} dh\; K(g,h)\; D^j_{mn}(h)$$

$$g|_K = {}^i D^{|j|}_{jj}(g)$$

If we define a Projector P_χ on Physical reduced states:

The projector P_χ acting on ψ_Γ SU(2) cylindrical functions defined on general Graphs Γ :

- Restrict the Graphs to be part of a cubical lattice



- Select the states belonging to the SU(2) subspace where our constraint conditions hold weakly:

$$\tilde{\psi}(g)_{e_i} = \sum_{n_i} {}^{i}D^{j(n_i)}_{m=n_i\ r=n_i}(g)\psi^{n_i}_{e_i}$$

What is the fate of the GR constraints ?

Gauss Constraint

$$\hat{G}_i(A, E) \quad \Rightarrow \quad P_\chi^\dagger \hat{G}_i P_\chi$$

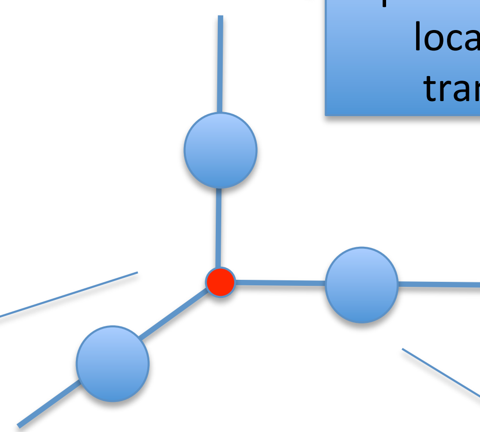
The Gauss constraint of the full theory is implemented by group averaging

$$P_{\mathcal{G}} = \int dg\, U_{\mathcal{G}}(g) \qquad U_{\mathcal{G}}(g) D^j_{mn}(h_e) = D^j_{mn}(g_{s(e)} h_e g_{t(e)})$$

Operator that generates local SU(2) gauge transformations

Spinnetwork states:

$$< h|\Gamma, \{j_e\}, \{x_v\} >= \prod_{v\in\Gamma} \prod_{e\in\Gamma} x_v \cdot D^{j_e}(h_e)_{mn}$$



SU(2) intertwiner

SU(2) holonomy

Implementing $\boxed{P_\chi^\dagger \hat{G}_i P_\chi}$

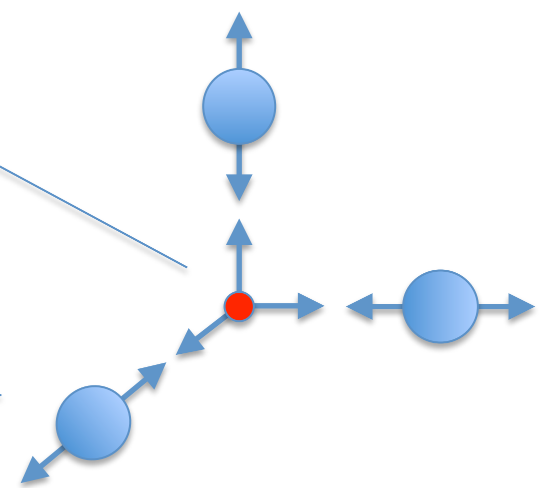
The reduced states will be of the form :

$$< h|\Gamma, j_e, x_v\rangle_R = \prod_{v\in\Gamma}\prod_{e\in\Gamma} < \mathbf{j_i}, \mathbf{x}|\mathbf{j_i}, \vec{\mathbf{u}}_i > \cdot {}^{i}D^{j_{e_i}}(h_{e_i})_{j_i j_i}$$

Projection on the intertwiner base of the Livine Speziale Intertwiner: Livine, Speziale ‘07

$$|\mathbf{j_i}, \vec{\mathbf{u}}_i >= |j_1, \cdots, j_i, \vec{u_1}, \cdots, \vec{u_i} >= \int dg \prod_i |j_i, \vec{u_i} >$$

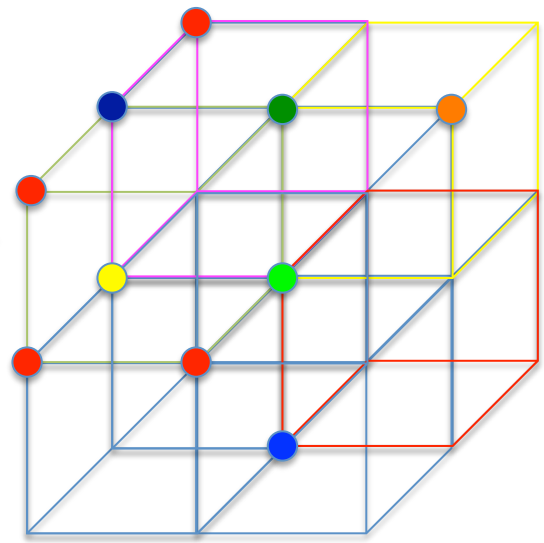
SU(2) intertwiner
projected on coherent states:
Reduced intertwiner



SU(2) holonomy
Projected on coherent states
Reduced holonomy

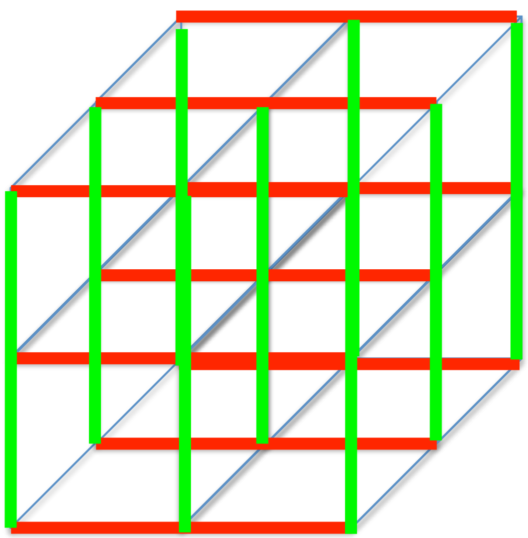
The Inhomogenous sector

Different
Reduced SU(2)
intertwiners:
inhomogeneities

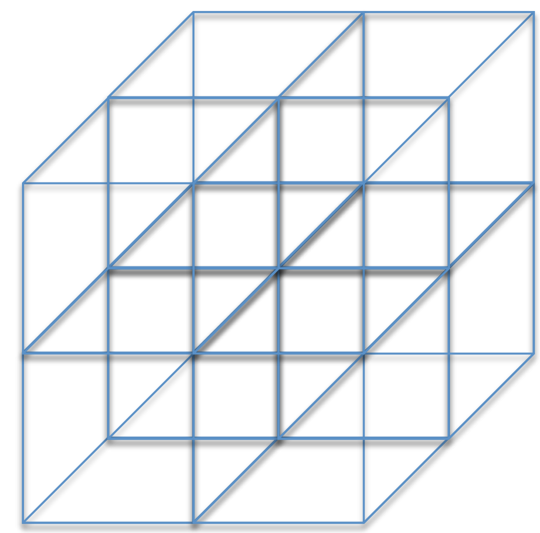


Different
Spin labels:
Anisotropies

Homogeneous and anisotropic sector



Homogeneous and Isotropic sector



Diff Constraint

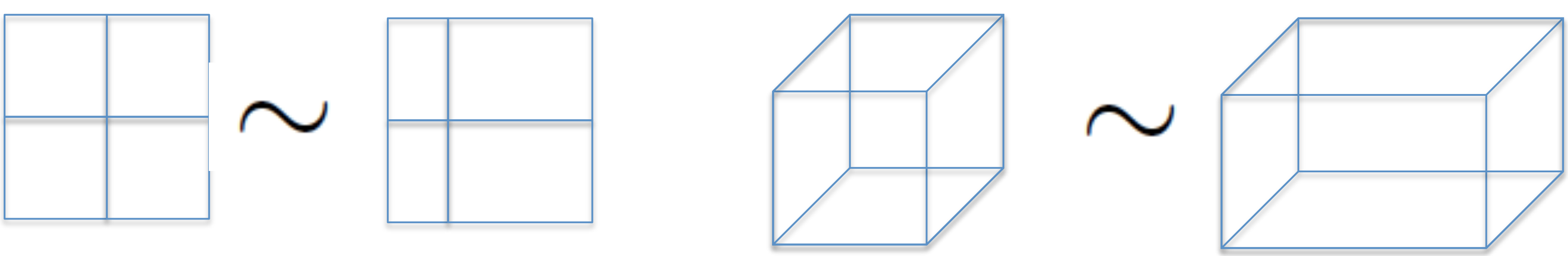
$$\hat{V}_a(A,E) \Rightarrow P^{\dagger}_{\chi}\hat{V}_a P_{\chi}$$

Full Theory: s-knot state

Ashtekar, Lewandowski, Marolf, Mourao, Thiemann

On the reduced space: Reduced s-knot states

Equivalence class of graphs that preserve the cellular structure:



Hamiltonian Constraint

$$\boxed{\hat{H}(A,E)} \quad \Rightarrow \quad \boxed{P_\chi^\dagger \hat{H} P_\chi} \quad ?$$

The regularized Euclidean constraint in the full theory reads:

T. Thiemann '96-'98

$$H^m_{\square}\,[N] := \frac{N(\mathfrak{n})}{N_m^2}\,\epsilon^{ijk}\,\mathrm{Tr}\Big[h^{(m)}_{\alpha_{ij}} h^{(m)}_{s_k} \{h^{(m)-1}_{s_k}, V\}\Big]$$

We regularize à la Thiemann, but using only elements of the reduced space:

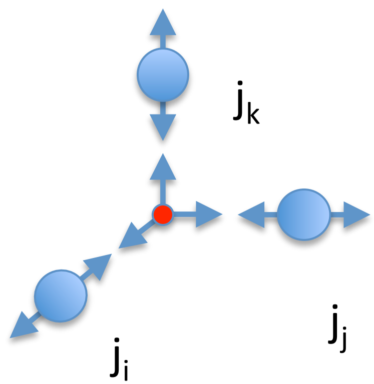
$$^{R}H^m_{\square}\,[N] := \frac{N(\mathfrak{n})}{N_m^2}\,\epsilon^{ijk}\,\mathrm{Tr}\Big[{}^{R}h^{(m)}_{\alpha_{ij}}\, {}^{R}h^{(m)}_{s_k} \{{}^{R}h^{(m)-1}_{s_k}, V\}\Big]$$

Action of the operator

on a tri-valent node:

Rovelli, Gaul '00
Alesci, Thiemann, Zipfel '11

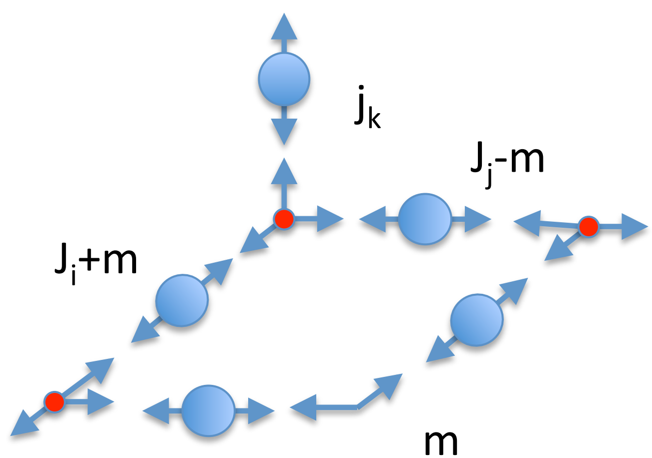
$^{R}H^{m}_{\square}\,[N]$



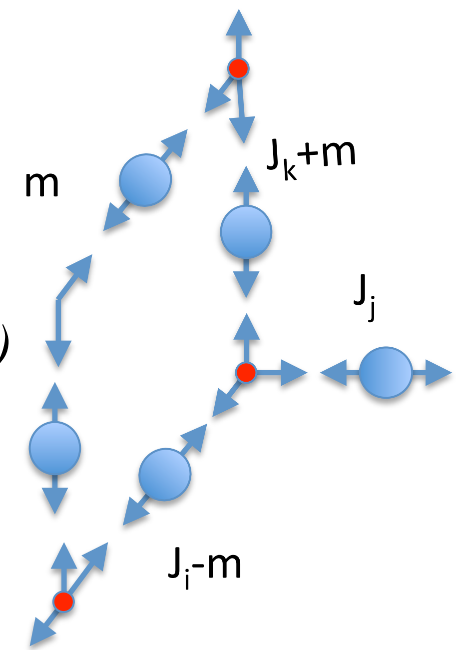
$=$

Computed with recoupling theory adapted to the reduced case

$A_{ij}(j_i, j_j, j_k, m)$



$+$ $A_{ki}(j_i, j_j, j_k, m)$



+ Permutations

$$\boxed{A_{ij}(j_i, j_j j_k, m)} =$$

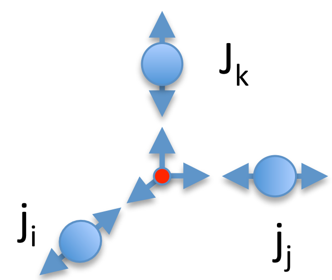
$$\sqrt{j_i j_j j_k + 1}\left[\begin{Bmatrix} j_i+m & j_j & j_k+m \\ j_k & m & j_i \end{Bmatrix}\begin{Bmatrix} j_i+m & j_j-m & j_k \\ m & j_k+m & j_j \end{Bmatrix} - \begin{Bmatrix} j_j+m & j_i & j_k+m \\ j_k & m & j_j \end{Bmatrix}\begin{Bmatrix} j_i+m & j_i-m & j_k \\ j_k+m & m & j_i \end{Bmatrix}\right]$$

Diagonal volume

Remarkably this expression for m=1 and large values simplify to

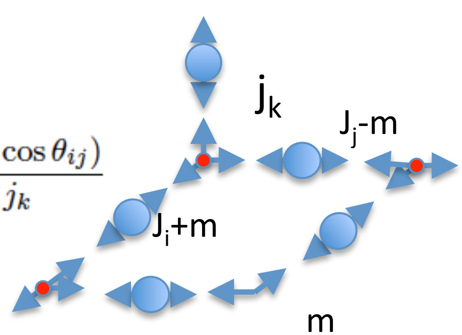
$$\boxed{A_{ij}(j_i, j_j j_k, 1) = \frac{j_i + j_j}{j_k}\,\frac{(1+\cos\theta_{ij})}{j_k}}$$

$${}^{R}H^{m}_{\square}\ [N]$$

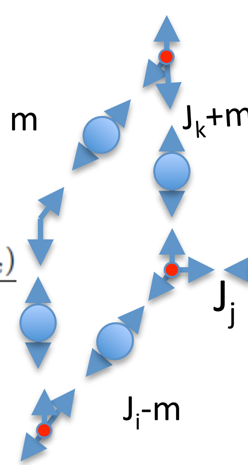


=

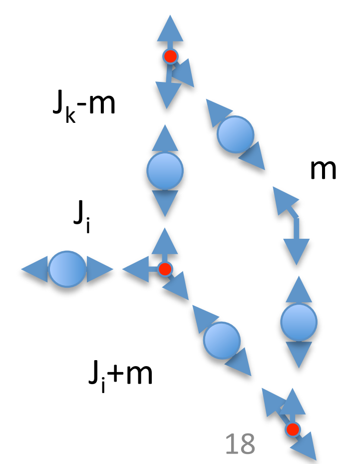
$$\frac{j_i + j_j}{j_k}\,\frac{(1+\cos\theta_{ij})}{j_k}$$



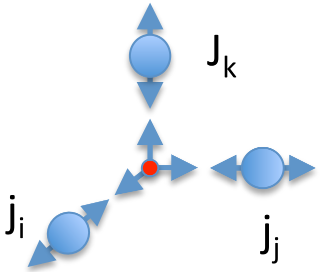
$$+\ \frac{j_i + j_k}{j_j}\,\frac{(1+\cos\theta_{ik})}{j_j}$$



$$+\ \frac{j_j + j_k}{j_i}\,\frac{(1+\cos\theta_{jk})}{j_i}$$

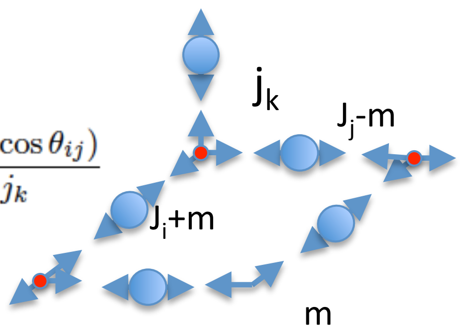


$${}^{R}H^{m}_{\square}\,[N]$$

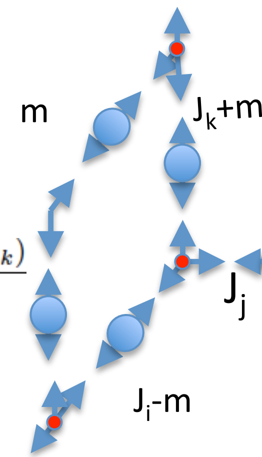


$$=$$

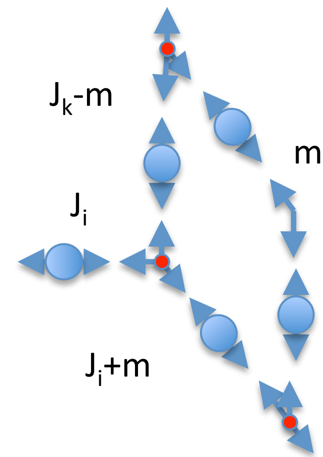
$$\frac{j_i + j_j}{j_k}\,\frac{(1+\cos\theta_{ij})}{j_k}$$



$$+\ \frac{j_i + j_k}{j_j}\,\frac{(1+\cos\theta_{ik})}{j_j}$$



$$+\ \frac{j_j + j_k}{j_i}\,\frac{(1+\cos\theta_{jk})}{j_i}$$



Large j limit "seems":

$$\boxed{\frac{c_1 c_2}{p_3} + \frac{c_1 c_3}{p_2} + \frac{c_2 c_3}{p_1} = 0}$$

News

Semiclassical limit

$$\Psi_{\Gamma,H_l}(h_l) = \int \prod_n dg_n \prod_l K_{\alpha_l}(h_l,\, g_{s(l)}\, H_l\, g_{t(l)}^{-1})$$

Heat Kernel coherent states

$$H_l = \; h_l \; \exp(i\frac{\alpha_l E_l}{8\pi G\hbar\gamma})$$

SL(2,C) element coding classical data

Hall, Thiemann, Winkler, Sahlmann, Bahr

$$\Psi_{H_l}(h_l) = \sum_{j_l, i_n} \psi_{H_l}(j_l, i_n)\Psi_{j_l, i_n}(h_l)$$

intertwiner base

Large distance asymptotic behaviour Bianchi Magliaro Perini

$$\Psi_{H_l}(h_l) \simeq \sum_{j_l, i_n} \prod_l e^{-\frac{(j_l - j_l^0)^2}{2\sigma_l^2}} e^{-i\xi_l j_l} \left(\prod_n \Phi_{i_n}\right) \Psi_{j_l, i_n}(h_l)$$

Codes the intrinsic geometry

Codes the extrinsic curvature

Livine-Speziale Intertwiners

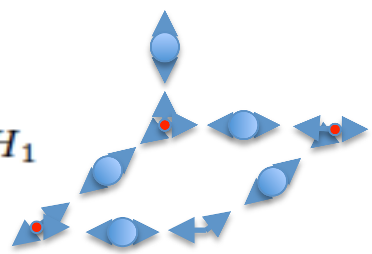
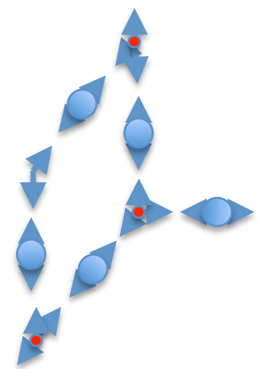
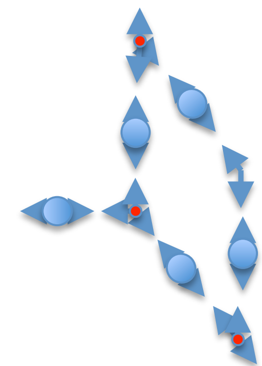
$$j_0 = \frac{|E|}{8\pi G\hbar\gamma}$$

$$\xi \sim K = c$$

Project in our reduced space the coherent states

$$P_\chi|\Psi_{H_l}\rangle = |\Psi_{H_l}\rangle_R$$

Single cell:

$|\Psi_{H,\square}\rangle_R$ = ${}^R\Psi_{H_1}$  + ${}^R\Psi_{H_2}$  + ${}^R\Psi_{H_3}$ 

Expectation value of the Hamiltonian on coherent states for a single cell:

$$\boxed{{}_R\langle\Psi_{H,\square}|{}^R\hat{H}^m_{\square}|\Psi_{H,\square}\rangle_R} =$$

$$\boxed{\sqrt{\frac{p^1p^2}{p^3}}c_1c_2 + \sqrt{\frac{p^2p^3}{p^1}}c_2c_3 + \sqrt{\frac{p^3p^1}{p^2}}c_3c_1}$$

Classical Bianchi I Hamiltonian

Perspectives

This analysis opens the way to

- Study the Physical solutions on the Dual Diff invariant Space and eventually construct a Physical Scalar Product

- Add matter as a clock: Big Bounce ? QFT on quantum spacetime ?

- Link to LQC ? *Ashtekar, Agullo, Barrau, Bojowald, Campiglia, Corichi, Giesel, Hofmann, Grain, Henderson, Kaminski, Lewandowski, Mena Marugan, Nelson, Pawlowski, Pullin, Singh, Sloan, Taveras, Thiemann, Winkler, Wilson-Ewing*

- Spinfoam Cosmology? *Bianchi, Krajewski, Rovelli, Vidotto*

- Something Different ?

(In the homogeneous anisotropic case the scale factors are not independent)

- Arena for the canonical theory:
  AQG, Master constraint, deparametrized theories.. Computable!