Rainbows from Quantum Gravity

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Assanioussi, AD, Lewandowski 2014 [arXiv:1412.6000]

Outline

Introduction

Idea:

Construct QFT on quantum spacetime and study how modes of the field probe the QG

What do we expect to find?

classical spacetime ⇒ "eigenstate of geometry" $g_{\mu\nu}^{\text{class}}$

⇒ every mode lives on $g_{\mu\nu}^{\text{class}}$

quantum spacetime ⇒ semiclassical state Ψ_o, superposition of "metric-eigenvalues" peaked on $g_{\mu\nu}^{\text{class}}$

⇒ maybe different modes $\vec{k}$ of the test field live on different components

Outline

Spacetime manifold: $M = \mathbb{R} \times \Sigma$. For simplicity in treating quantum fields, Σ is topologically a 3-torus.

The theory:

$$S[g,\phi] = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{1}$$

Canonical analysis:

- geometry, $g_{\mu\nu} \;\longrightarrow\; (q_{ab}(x); \pi^{ab}(x))$
- K-G matter field, $\phi \;\longrightarrow\; (\phi(x); \pi_\phi(x))$

$\Rightarrow$ Every point γ in phase space Γ is uniquely defined by coordinates

$$\gamma = \left(q_{ab}(x), \phi(x); \pi^{ab}(x), \pi_\phi(x)\right) \tag{2}$$

Not all of Γ is physical: 4 constraints per each point $x \in \Sigma$,

$$C(x), \qquad C_a(x) \tag{3}$$

Coordinates on Γ can be splitted into *homogeneous* and *inhomogeneous*:

(4)
$$q_{ab} = q_{ab}^{(0)} + \delta q_{ab}, \qquad \phi = \phi^{(0)} + \delta\phi$$
$$\pi^{ab} = \pi^{ab}_{(0)} + \delta\pi^{ab}, \qquad \pi_\phi = \pi_\phi^{(0)} + \delta\pi_\phi$$

where in particular

(5)
$$a^2 := \int_\Sigma d^3x\ \delta^{ab} q_{ab}(x) \quad \text{defines} \quad q_{ab}^{(0)} = a^2\delta_{ab}$$

Note that $q_{ab}^{(0)}$ is of the Robertson-Walker type, but in general *does not* satisfy 0th order Einstein equation (i.e., Friedmann equation).

Physical phase space: solve the constraints $C = 0$ and $C_a = 0$. To linear order in the inhomogeneities, one can show [AD, Lewandowski, Puchta 2013] that the only physical degrees of freedom are:

- homogeneous geometry $\rightarrow a$
- tensor modes of geometry (graviton-to-be) $\rightarrow \delta q^{+}_{\vec{k}}, \delta q^{\times}_{\vec{k}}$
- scalar modes of matter (scalar field) $\rightarrow \delta\phi_{\vec{k}}$

where we already performed spatial Fourier transform, and $\vec{k} \in \mathcal{L} = (2\pi\mathbb{Z})^3 - \{\vec{0}\}$.

Homogeneous part of Hamiltonian constraint, $\int d^3x C(x) = 0$, is solved for momentum $\pi^{(0)}_{\phi}$ of $\phi^{(0)}$. Hence, $\phi^{(0)}$ is *physical time*, and we obtain a *physical Hamiltonian*:

$$\frac{d}{d\phi^{(0)}} F = \{F, h_{\text{phys}}\} \tag{6}$$

where

$$h_{\text{phys}} = H_{\text{hom}} - \sum_{\vec{k}} \frac{H_{\text{hom}}^{-1}}{2} \left[\delta\pi^2_{\vec{k}} + \left(a^4 k^2 + a^6 m^2\right) \delta\phi^2_{\vec{k}}\right] + \text{Hamiltonian for } \delta q^{i}_{\vec{k}} \tag{7}$$

and $H_{\text{hom}} = \sqrt{\kappa/6}\; a\pi_a$.

Outline

Focus on the scalar part:

$$\mathcal{H} = \mathcal{H}_{\text{hom}} \otimes \mathcal{H}_\phi \tag{8}$$

and quantum dynamics driven by Hamiltonian

$$\hat{H} = \hat{H}_{\text{hom}} \otimes \hat{I} - \frac{1}{2}\sum_k \left(\hat{H}_{\text{hom}}^{-1} \otimes \delta\hat{\pi}_k^2 + \hat{\Omega}(k,m) \otimes \delta\hat{\phi}_k^2 \right) \tag{9}$$

where

$$\hat{\Omega}(k,m) := k^2 \frac{\hat{H}_{\text{hom}}^{-1}\hat{a}^4 + \hat{a}^4\hat{H}_{\text{hom}}^{-1}}{2} + m^2 \frac{\hat{H}_{\text{hom}}^{-1}\hat{a}^6 + \hat{a}^6\hat{H}_{\text{hom}}^{-1}}{2} \tag{10}$$

$\hat{H}$ acts on a state $|\Psi(t,a,\phi)\rangle \in \mathcal{H}$ via Schroedinger equation:

$$-i\frac{d}{dt}|\Psi\rangle = \hat{H}|\Psi\rangle \tag{11}$$

Test field approximation (0th order Born-Oppenheimer) $\Rightarrow$ Geometry and matter are disentangled:

$$|\Psi(t,a,\phi)\rangle = |\Psi_o(t,a)\rangle \otimes |\varphi(t,\phi)\rangle \tag{12}$$

where

$$-i\frac{d}{dt}|\Psi_o\rangle = \hat{H}_{\text{hom}}|\Psi_o\rangle \tag{13}$$

Plugging this in the Schroedinger equation, and projecting on $\langle\Psi_o|$, gives

$$i\frac{d}{dt}|\varphi\rangle = \frac{1}{2}\sum_k \left[\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle\delta\hat{\pi}_k^2 + \langle\Psi_o|\hat{\Omega}(k,m)|\Psi_o\rangle\delta\hat{\phi}_k^2\right]|\varphi\rangle \tag{14}$$

Not surprising: a collection of harmonic oscillators. But the parameters of this h.o. are given in terms of expectation values of geometric operators on quantum state of geometry, Ψ_o.

Outline

QFT on quantum spacetime sandwitched on $|\Psi_o\rangle \in \mathcal{H}_{\text{hom}}$:

$$i\frac{d}{dt}|\varphi\rangle = \frac{1}{2}\sum_k \left[\langle\Psi_o|\hat{H}^{-1}_{\text{hom}}|\Psi_o\rangle\delta\hat{\pi}^2_k + \langle\Psi_o|\hat{\Omega}(k,m)|\Psi_o\rangle\delta\hat{\phi}^2_k\right]|\varphi\rangle \tag{15}$$

QFT on classical Robertson-Walker spacetime

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -\bar{N}^2 dt^2 + \bar{a}^2\left(dx^2+dy^2+dz^2\right) \tag{16}$$

$\Rightarrow$

$$i\frac{d}{dt}|\varphi\rangle = \frac{1}{2}\sum_k \left[\frac{\bar{N}}{\bar{a}^3}\delta\hat{\pi}^2_k + \frac{\bar{N}}{\bar{a}^3}\left(\bar{a}^4k^2+\bar{a}^6m^2\right)\delta\hat{\phi}^2_k\right]|\varphi\rangle \tag{17}$$

The comparison gives

$$\begin{cases} \bar{N}/\bar{a}^3 = \langle\hat{H}^{-1}_{\text{hom}}\rangle \\ \bar{N}\left(\bar{a}^4k^2+\bar{a}^6m^2\right)/\bar{a}^3 = \langle\hat{\Omega}(k,m)\rangle \end{cases}$$

$\Rightarrow$ Only one real and positive solution:

$$\bar{N} = \langle\hat{H}^{-1}_{\text{hom}}\rangle\bar{a}^3, \qquad \bar{a} = \bar{a}(k/m) \tag{18}$$

Striking conclusion:

fundam. quantum gravity+matter $\Longleftrightarrow$ QFT on effective, k-dependent spacetime

The effective scale factor:

$$(19)\quad \bar{a}(k/m)^2 = \begin{cases} u_+ + u_- - \dfrac{k^2}{3m^2} & \text{if } k < k_o \\ \dfrac{2k^2}{3m^2}\cos\left[\dfrac{1}{3}\arccos\left(-1 + \dfrac{27m^6}{2k^6}\delta\right)\right] - \dfrac{k^2}{3m^2} & \text{if } k \geq k_o \end{cases}$$

where

$$(20)\qquad u_\pm := \sqrt[3]{\frac{\delta}{2} - \frac{k^6}{27m^6} \pm \sqrt{\frac{\delta^2}{4} - \frac{k^6}{27m^6}\delta}}, \qquad \delta = \frac{\langle\hat{\Omega}(k,m)\rangle}{m^2\langle\hat{H}_{\mathrm{hom}}^{-1}\rangle}$$

remark: if we started with massless field, $m = 0$, the solution turns out to be k-independent and given by [Ashtekar, Kaminski, Lewandowski 2009]

$$(21)\qquad \bar{a}^2_{\mathrm{m}=0} = \sqrt{\frac{\langle\hat{H}_{\mathrm{hom}}^{-1}\hat{a}^4 + \hat{a}^4\hat{H}_{\mathrm{hom}}^{-1}\rangle}{2\langle\hat{H}_{\mathrm{hom}}^{-1}\rangle}}$$

This is consistent with the "high energy" limit $k \gg m$ of the massive solution (19).

In the "low energy" limit $k \ll m$, we have

$$\bar{a}\,(k/m)^2 \approx \bar{a}_o^2 \left[1 + \frac{\beta}{3}\left(\frac{k/\bar{a}_o}{m}\right)^2\right] \tag{22}$$

where

$$\bar{a}_o^2 = \frac{1}{\sqrt[3]{2\langle \hat{H}_{\text{hom}}^{-1}\rangle}} \langle \hat{H}_{\text{hom}}^{-1}\hat{a}^6 + \hat{a}^6\hat{H}_{\text{hom}}^{-1}\rangle^{\frac{1}{3}}, \qquad \beta := \frac{1}{\sqrt[3]{2\langle \hat{H}_{\text{hom}}^{-1}\rangle}} \frac{\langle \hat{H}_{\text{hom}}^{-1}\hat{a}^4 + \hat{a}^4\hat{H}_{\text{hom}}^{-1}\rangle}{\langle \hat{H}_{\text{hom}}^{-1}\hat{a}^6 + \hat{a}^6\hat{H}_{\text{hom}}^{-1}\rangle^{\frac{2}{3}}} - 1$$

Interpretation

- Scale factor $\bar{a}_o$ defines the low-energy, k-independent metric

  $$\bar{g}^o_{\mu\nu}\,dx^\mu dx^\nu = -\bar{N}_o^2 dt^2 + \bar{a}_o^2(dx^2 + dy^2 + dz^2) \tag{23}$$

  We can think of it as the semiclassical metric seen by an observer performing macroscopic measurements.
- Parameter β encodes the quantum nature of spacetime: if product of expectation values = expectation value of products, then $\beta = 0$.

Outline

A cosmological observer (4-velocity u^μ, metric $\bar{g}^o_{\mu\nu}$) measures a particle of wave-vector k_μ passing through the lab:

- Energy: $E := u^\mu k_\mu = k_0/\bar{N}_o$
- Momentum: $p^2 := (\bar{g}_o^{\mu\nu} + u^\mu u^\nu)k_\mu k_\nu = k^2/\bar{a}_o^2$

But the particle satisfies the mass-shell relation wrt metric $\bar{g}_{\mu\nu}(k/m)$:

(24) $$-m^2 = \bar{g}^{\mu\nu} k_\mu k_\nu = -\frac{k_0^2}{\bar{N}^2} + \frac{k^2}{\bar{a}^2} = -f^2 E^2 + g^2 p^2$$

where

(25) $$f := \frac{\bar{N}_o}{\bar{N}}, \qquad g := \frac{\bar{a}_o}{\bar{a}}$$

are the so-called *rainbow functions* [Magueijo, Smolin 2004].

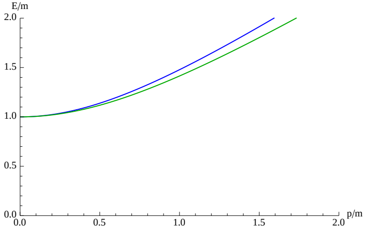
$\Rightarrow$ Modified dispersion relation:

(26) $$E^2 = \frac{1}{f^2}\left(g^2 p^2 + m^2\right) = m^2 + (1+\beta)p^2 + O(p^4)$$

$$E \approx \sqrt{m^2 + (1+\beta)p^2} \tag{27}$$

The standard dispersion relation is recovered two independent limits:

- semiclassical matter (i.e., modes with $p \ll m$): in this case $E \approx m$ (the most famous formula of physics!)
- semiclassical gravity (i.e., $|\Psi_o\rangle$ such that $\beta \ll 1$): in this case, $E \approx \sqrt{m^2 + p^2}$



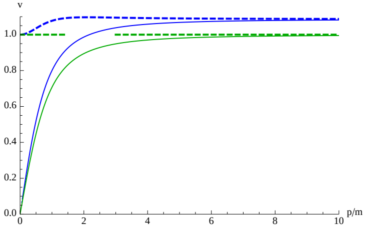
Green = semiclassical spacetime ($\beta \approx 0$), Blue = quantum spacetime ($\beta \approx 0.2$)

remark: No particular role is played by $E_{\text{Planck}} \approx 10^{19}\,GeV$. Indeed, if $\beta \approx 1$ (i.e., $|\Psi_o\rangle$ is a very non-classical state), then modifications are present for $p \approx m$, which for a proton would be around 1 GeV. We do not see Lorentz-violations in accelerators because $|\Psi_o\rangle$ is extremely classical today!

(28) $$v = \frac{dE}{dp} = \frac{1+\beta}{\sqrt{m^2 + (1+\beta)p^2}}p$$

remark: For massless particles, $m \ll p$, we do not get 1 but rather $\sqrt{1+\beta}$. Hence, we have a modified velocity of light. Just a shift by β! Where is the big deal? Well...

1) β is a function of expectation values of geometric operators on $|\Psi_o\rangle$, and as such it depends on time.
2) This simple form for $v_{m=0}$ is due to the approximation we considered. For the exact $\bar{a}^2$ of equation (19) above, numerics give



Green = semiclassical spacetime ($\beta \approx 0$); Blue = quantum spacetime ($\beta \approx 0.2$); Dashed = light

Outline

Main result

quantum gravity+matter $\Longleftrightarrow$ QFT on effective, k-dependent spacetime $\bar{g}_{\mu\nu}$

This is true in any theory of quantum gravity based on a Hamiltonian formulation, if the following approximations hold:

- linearized inhomogeneities around homogeneous isotropic background (e.g., LQC)
- test-field approximation: $|\Psi(t, a, \phi)\rangle = |\Psi_o(t, a)\rangle \otimes |\varphi(t, \phi)\rangle$

What is the effect of this result?
k-dependence of $\bar{g}_{\mu\nu}$ implies a modified dispersion relation controlled by the scale

$$\beta = \frac{1}{\sqrt[3]{2\langle \hat{H}_{\text{hom}}^{-1} \rangle}} \frac{\langle \hat{H}_{\text{hom}}^{-1} \hat{a}^4 + \hat{a}^4 \hat{H}_{\text{hom}}^{-1} \rangle}{\langle \hat{H}_{\text{hom}}^{-1} \hat{a}^6 + \hat{a}^6 \hat{H}_{\text{hom}}^{-1} \rangle^{\frac{2}{3}}} - 1 \tag{29}$$

Only one parameter, in spite of the microscopic structure of quantum spacetime $|\Psi_o\rangle$!
$\Rightarrow$ compare with crystals' refractive properties: described uniquely by refractive index n

Can we test this result?
Today the geometry is classical, so $\beta \ll 1$: no Lorentz-violation today :(

$\Rightarrow$ However, in the primordial Universe the geometry is expected to be "very quantum", in which case $\beta \approx 1$, and hence the Lorentz-violation is present even for $p \approx m$:o thank you