

Imprints of black hole area quantization in gravitational waves

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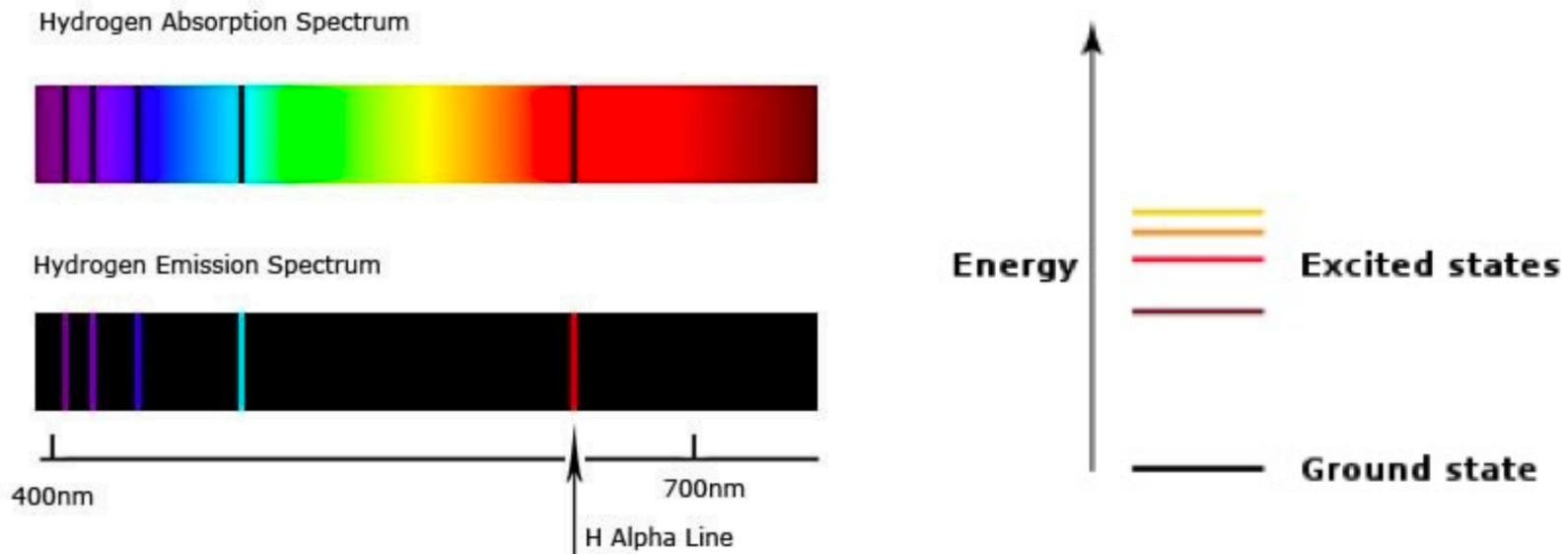
Based on: Agullo, Cardoso, d.R., Maggiore, Pullin, PRL 126 4 041302 (2021); IJMPD Vol. 30, 2142013 (2021)

Outline

- Motivation.
- Expectations from Bekenstein quantization. Absorption transitions and (no) overlapping
- Imprints of area quantization: GW echoes after ringdown
- Imprints of area quantization: absence of tidal heating during inspiral
- Expectations from LQG. Open problems and possible avenues

Spectroscopy

- Instrumental in revealing the **quantum structure of atoms** and molecules in the early XX century

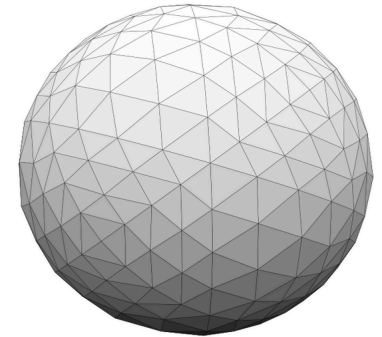


- Quantization of atomic energy spectrum** changes qualitatively the way **matter and electromagnetic radiation interact**, making **absorption/emission non-trivial**.

Non-trivial BH absorption spectrum?

Quantum BHs behave as atoms [Bekenstein 1974]

- Area and angular momentum constitute a complete set of observables to specify physical states of astrophysical BHs.
- Quantum mechanics is expected to discretize these parameters

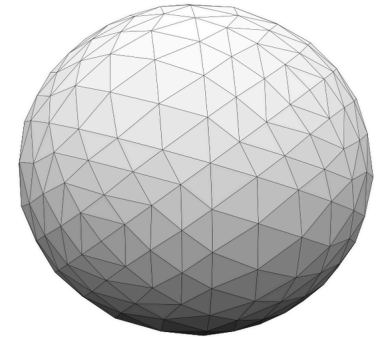


(A, J)

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(A, J)

BH mass spectrum $M(A, J)$ **is not continuous** \longrightarrow could change drastically the way BHs interact with gravitational radiation [Foit-Kleban 2016, Cardoso-Foit-Kleban 2019]

Could the gravitational radiation play, for the quantum aspects of BHs, a similar role as electromagnetic radiation did for atoms one century ago?

Are there chances to be observable?

Why should we expect that a discretization at the **Planck scale** of an **astrophysical** BH horizon can affect significantly the GWs that we observe in our detectors?

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Given any quantum theory of gravity, the **transition between two energy levels** for macroscopic BHs should be governed by the **1st Law of BH mechanics*** (classical limit):

$$w_{\text{abs}} \equiv \frac{c^2 \Delta M}{\hbar} = \frac{\kappa c^2}{8\pi G} \frac{\Delta A}{\hbar} + \Omega_H \frac{\Delta J}{\hbar}$$

*quasi-local formulation developed by Ashtekar, Beetle, Lewandowski for isolated horizons

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If discretization occurs at the Planck scale, for solar-mass BHs the **absorption frequencies lie in LIGO-Virgo frequency band** → can be tested with GW observations!

*quasi-local formulation developed by Ashtekar, Beetle, Lewandowski for isolated horizons

Bekenstein-Mukhanov quantization

Bekenstein-Mukhanov model

- BH area behaves as an **adiabatic invariant**: arguments on semiclassical quantization lead to a **discrete area spectrum** [Bekenstein74, Mukhanov86, Bekenstein-Mukhanov95]

$$A \sim n\Delta A = n (\alpha \ell_p^2) \quad n \in \mathbb{N}$$

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- Upon quantization of the angular momentum in the standard way, $J = \hbar j$ we get a discrete energy spectrum:

$$j \in \mathbb{N}/2 \\ 0 \leq j < \frac{\alpha}{8\pi} n$$

$$M = \sqrt{\frac{A}{16\pi} + \frac{4\pi J^2}{A}} \longrightarrow M_{n,j} = \sqrt{\hbar} \sqrt{\frac{\alpha n}{16\pi} + \frac{4\pi j^2}{\alpha n}} \quad \text{Highly irregular}$$

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- Heuristic, but reasonable: take the model as **working hypothesis and explore its consequences**.

Can we constrain the value of alpha from GW observations?

The quantum-mechanical problem of absorption

- Interested in **radiative transitions of the BH** that are **induced by the interaction with an incident GW**. **Phenomenological approach**: apply familiar results of time-dependent perturbation theory

$$H(t) = H_{BH} + H_{int}(t)$$

$$H_{BH} |M_{n,j}\rangle = M_{n,j} |M_{n,j}\rangle$$

$$H_{int}(t) \sim V e^{-i\omega t} + V^\dagger e^{i\omega t} \quad (\text{GW mode})$$

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- Due to the **interaction**, state of quantum BH becomes a linear combination of stationary states: (linear regime)

$$|M\rangle = \sum_{n,j} \lambda_{n,j} |M_{n,j}\rangle$$

“Probability of observing the BH in state (n,j)” \longleftrightarrow Microscopic theory of gravity

What are the relevant BH absorption transitions?

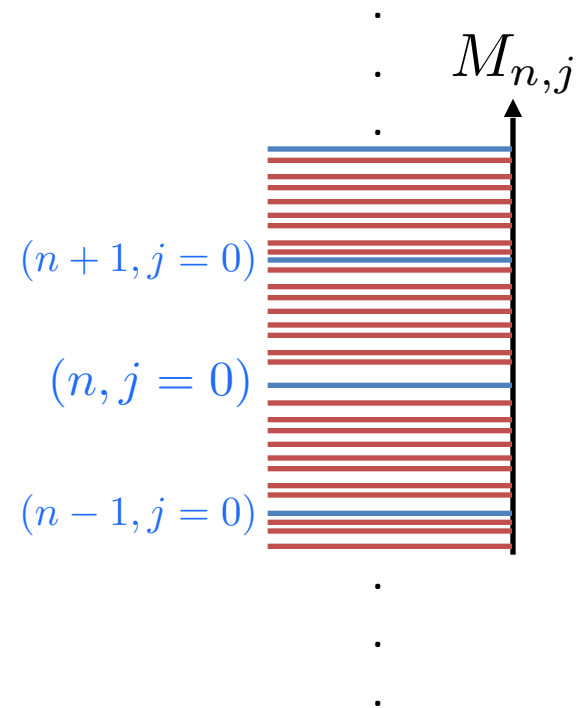
- **Fermi Golden rule**: absorption probability distribution is peaked around

$$M_f = M_i + \hbar\omega$$

- Angular momentum conservation imposes **selection rules**:

$$J_f = J_i + m$$

Typical dominant GW-mode in astrophysics: $(\ell = 2, m = 2)$



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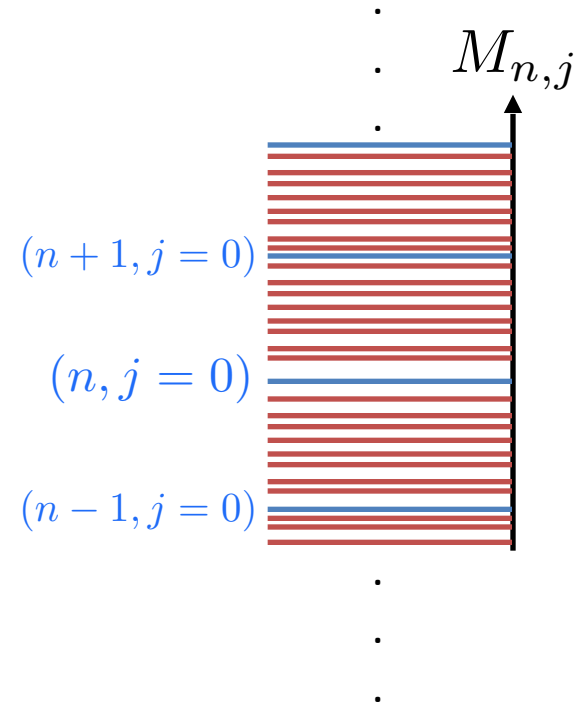
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- The BH is unable to **absorb** the incident dominant GW mode $(\omega, 2, 2)$ unless

$$\hbar\omega \sim \hbar\omega_{\Delta n} \equiv M_{n+\Delta n, j+2} - M_{n, j} = \frac{\hbar \kappa_{n, j}}{8\pi} \alpha \Delta n + 2\hbar\Omega_{n, j} + O(n^{-1})$$

Do consecutive accessible levels overlap?

- BH energy states must **spontaneously decay** due to Hawking radiation \longrightarrow **non-zero linewidth**

$$\Gamma_{n,j} = \frac{\hbar}{\tau_{n,j}} \quad \tau_{n,j} = \text{Hawking decay rate [Page, 1976]}$$

Linewidth increases with BH rotation (due to super-radiance). Energy levels may overlap and an **effective continuous spectrum may emerge** [Coates, Volkel, Kokkotas 2019]

$$\tau \equiv -\frac{\hbar \langle \omega \rangle}{\dot{M}} \quad \begin{aligned} \dot{M} &= -\sum_{\ell m} \int_0^\infty d\omega \hbar \omega \langle N_{\ell m}(\omega) \rangle \\ \langle \omega \rangle &= \frac{\sum_{\ell, m} \int_0^\infty d\omega \omega \langle N_{\ell m}(\omega) \rangle}{\sum_{\ell, m} \int_0^\infty d\omega \langle N_{\ell m}(\omega) \rangle} \end{aligned}$$

- Massless fields dominate the Hawking emission (neutrinos, photons, gravitons)

Do consecutive accessible levels overlap?

- Astrophysical BHs are described by (M,a). **The question of overlapping is independent of mass:**

$$\left. \begin{aligned} \Gamma_{n,j} &\sim \frac{1}{M_{n,j}} \\ \omega_{\Delta n} &\sim \frac{1}{M_{n,j}} \end{aligned} \right\} \quad \frac{\Gamma_{n,j}}{\hbar(\omega_{\Delta n} - \omega_{\Delta n-1})} = R(a)$$

There isn't overlapping if $R(a) < 1$

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- In the B-M model there is a 1-parameter quantization ambiguity. We find **overlap occurs for** $\alpha < \alpha_{\text{crit}}(a)$

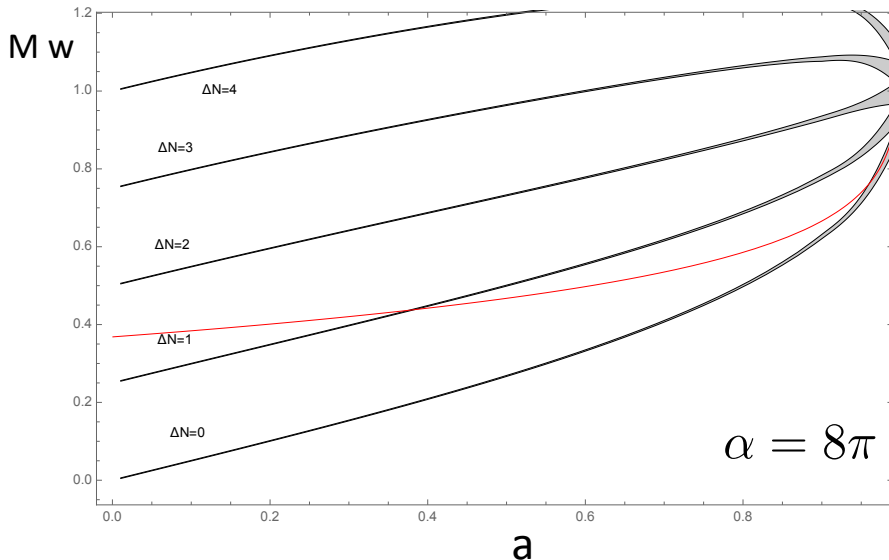
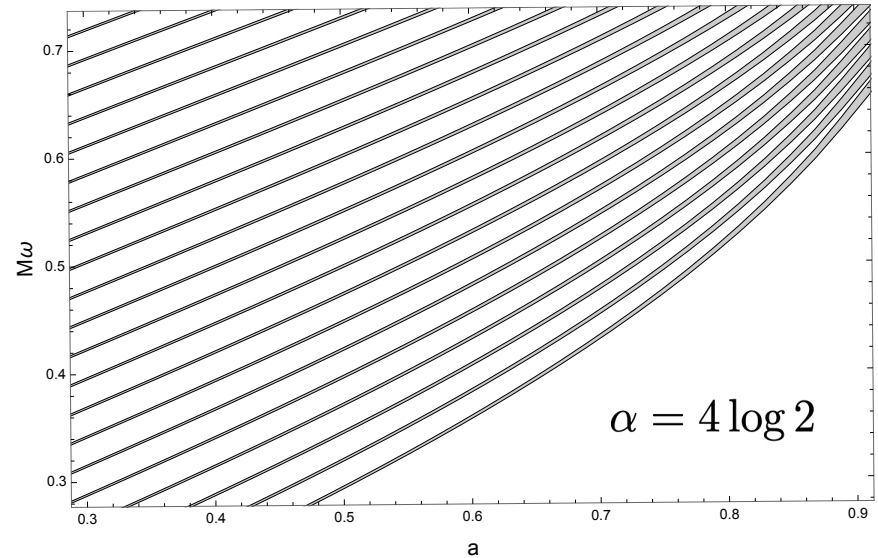
$$\alpha_{\text{crit}}(a) \approx 0.0842 + 0.2605 a^2 + 0.0320 e^{5.34 a^3}$$

Example $a = 0.7 \longrightarrow \alpha_{\text{crit}}(0.7) = 0.42 \ll 4 \log 2$ (smallest value in literature)

Do consecutive accessible levels overlap?

Absorption frequencies for different values of alpha, including linewidths

$$\hbar\omega_{\Delta n} = \frac{\kappa\hbar}{8\pi}\alpha\Delta n + 2\hbar\Omega_H$$

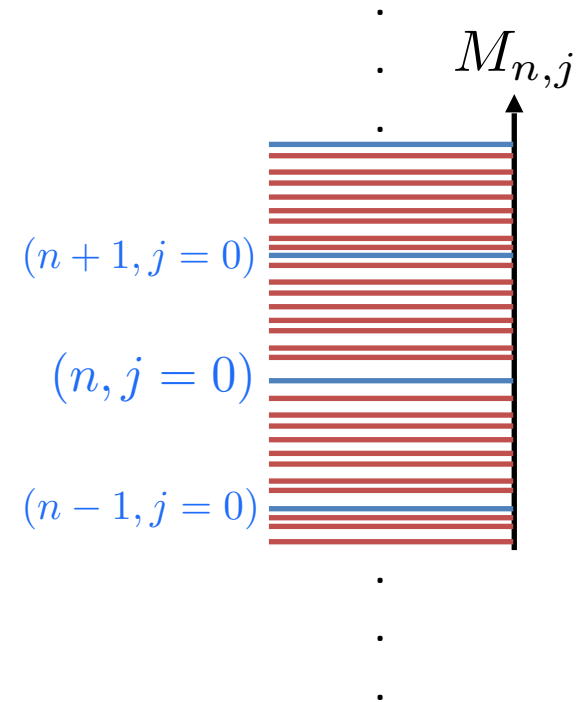


Conclusion: for reasonable values of alpha and BH spin there is no overlap of spectral lines

Absorption vs Emission spectra

- BH absorption spectrum is discrete because of the restriction $\Delta j = \pm 2$
Fixed by the incoming GW mode.

- During Hawking evaporation any mode (l,m) can be emitted, all energy levels in the Bekenstein spectrum are accessible.

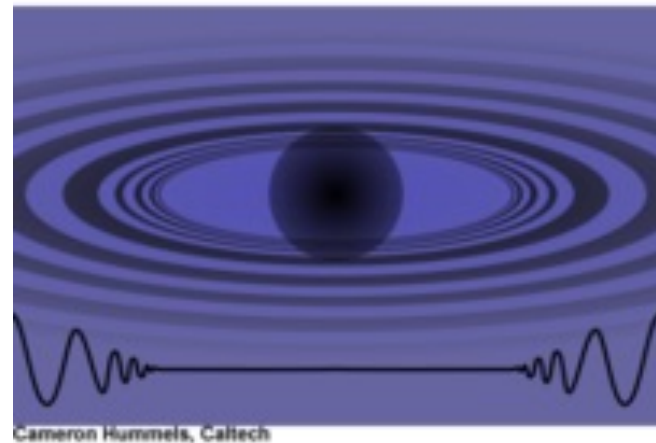
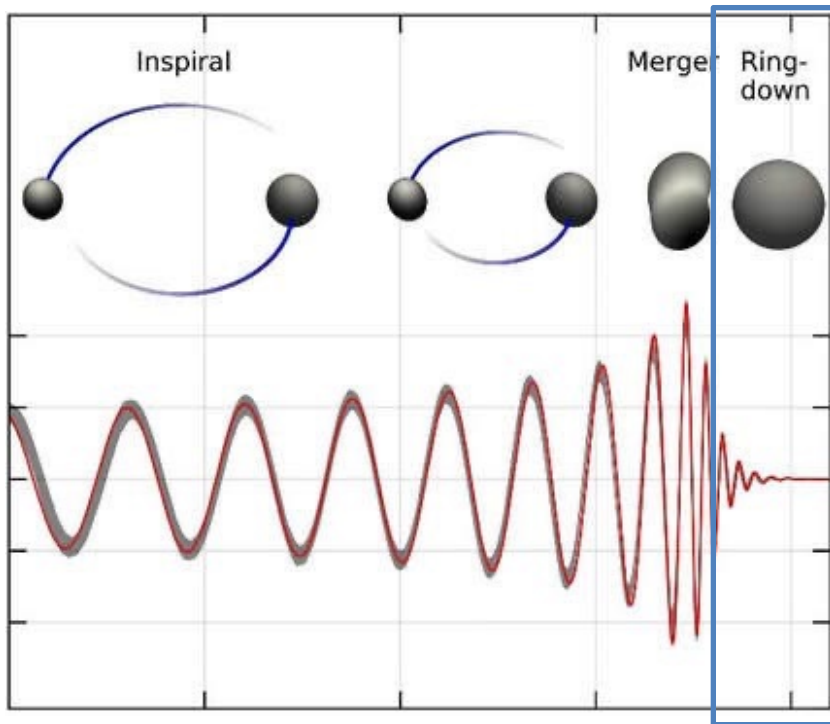


- The full spectrum is complex and highly irregular. The existence of many more accessible levels together with the linewidth partially recovers the continuous Hawking emission spectrum.

Imprints of B-M area quantization on GWs

Imprints of BH area quantization: GW echoes

- The **ringdown** signal from a binary BH merger is well described by **BH perturbation theory**



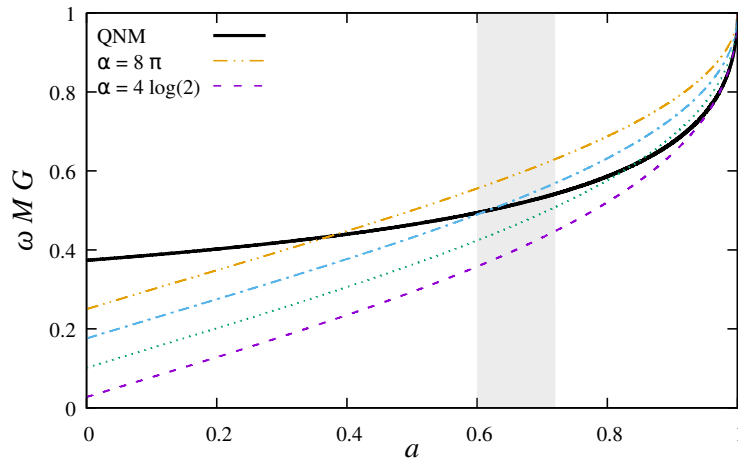
(quasi) normal modes of vibration
of the final Kerr geometry

$$\omega_{nlm} = \text{Re } \omega_{nlm} + i \text{Im } \omega_{nlm}$$

- Numerical simulations show that the collision of two BHs **excite efficiently** the QNMs of the final BH

Imprints of BH area quantization: GW echoes

Foit-Kleban (2016) proposed to think of the ringdown waves as being affected by the Bekenstein area quantization, so that they should oscillate with the same characteristic frequencies.



Detecting two ringdowns of different BH spins compatible with classical GR would rule out the uniform area quantization proposal.

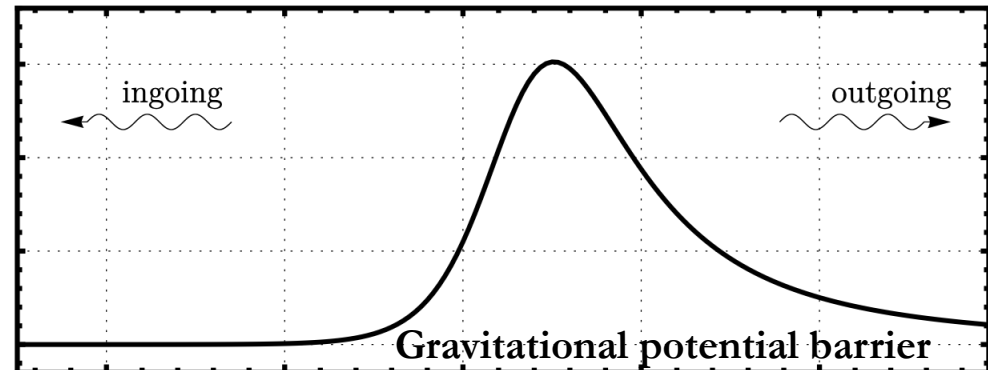
This is not justified: **ringdown waves are associated to the light-ring local geometry, not related to the horizon at all.**

Cardoso-Foit-Kleban (2019), instead, studied the scattering of Gaussian packets by a Schw BH horizon, introducing an ad hoc gravitational potential that “filters” some specific frequencies and disperses the rest.

Imprints of BH area quantization: GW echoes

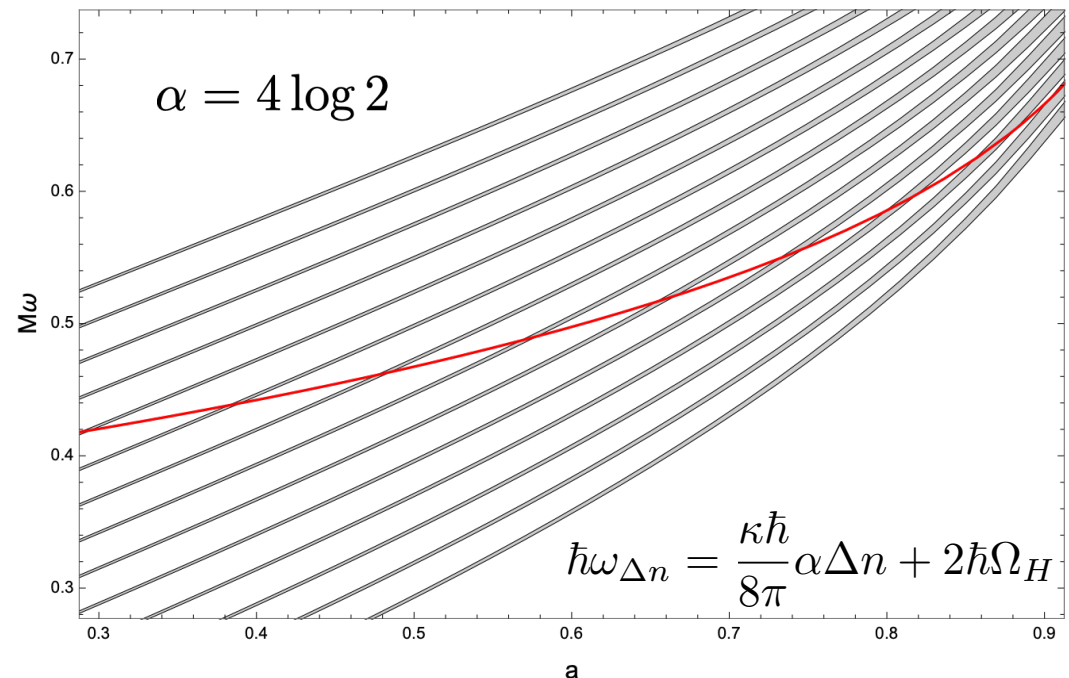
BH QNM are excited around the light-ring

A significant fraction of GWs is directed towards the horizon, and absorbed according to GR.



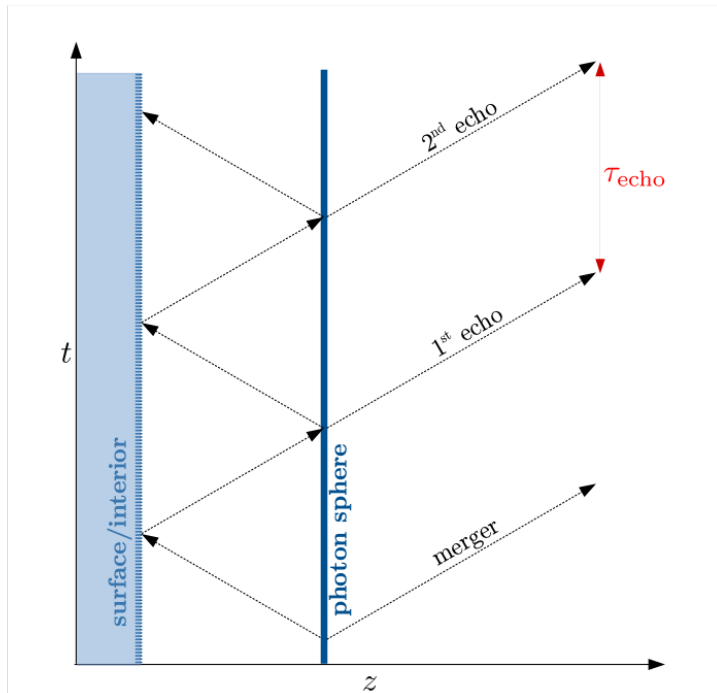
With area quantization, **absorption occurs** iff QNM oscillation frequency matches a **characteristic absorption frequency**

$$\text{Re } \omega_{022} \in \left[\omega_{\Delta n} - \frac{\Gamma}{2\hbar}, \omega_{\Delta n} + \frac{\Gamma}{2\hbar} \right]$$

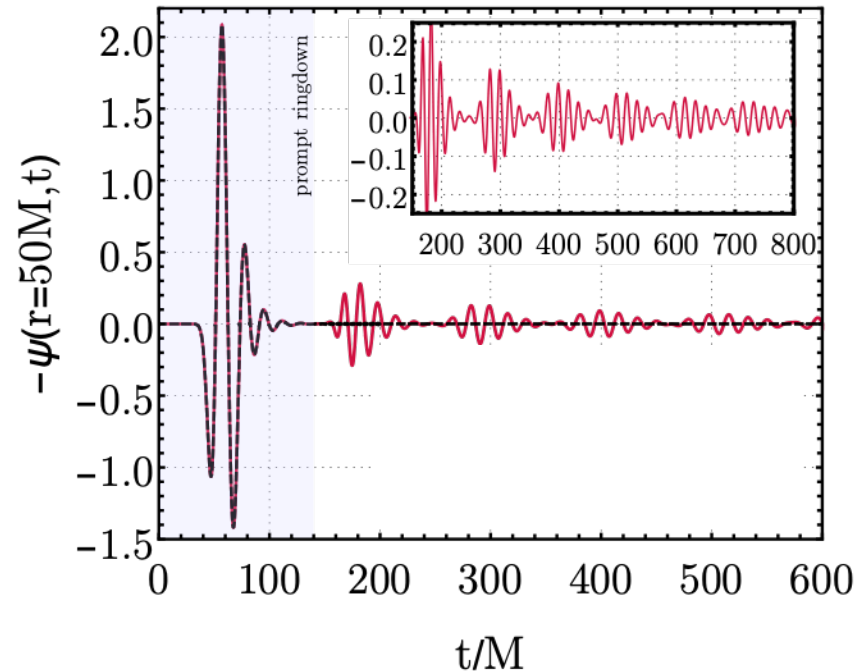


Imprints of BH area quantization: GW echoes

Otherwise, an external observer should see a GW signal followed by **echoes** [Cardoso, Franzin, Pani 2016]

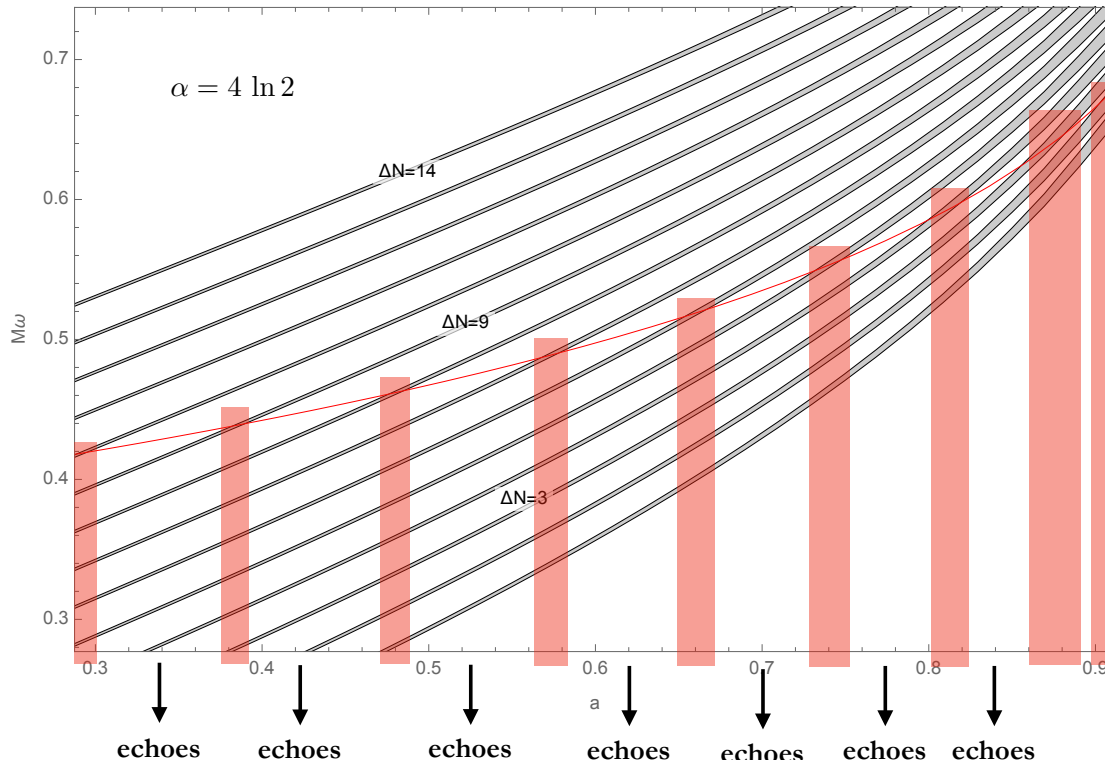


Credit: Cardoso and Pani, Liv. Rev. Rel.



Imprints of BH area quantization: GW echoes

- **Echoes** are expected for **specific values of BH spin**: **offers a way to measure α**

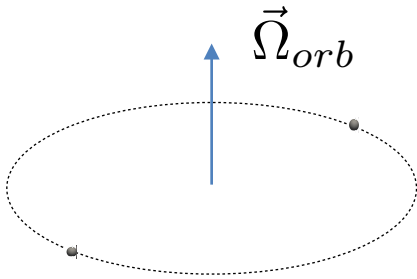


- Bayesian analysis on LIGO/Virgo data [Nielsen et al 2018] rule out echoes as large as 0.2 the signal peak. Constraints will improve with future-generation detectors.

Imprints of BH area quantization: tidal heating

- The **inspiral** signal from a binary BH merger is well described by the **Post-Newtonian formalism**

Point-particle orbital motion in flat space (leading order contrib.):

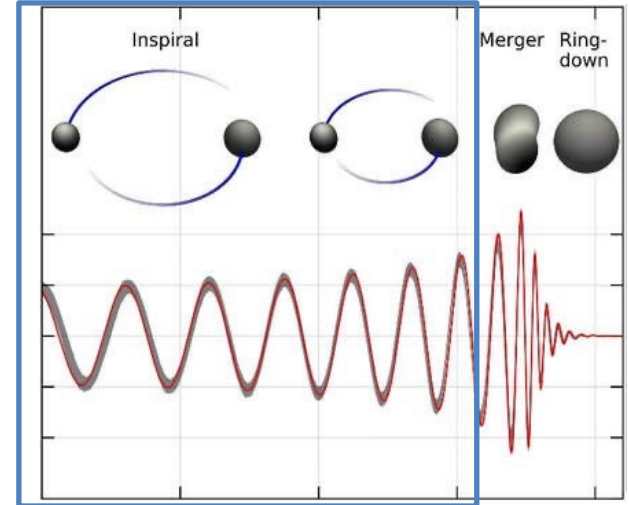


Rate of inspiral is determined by energy conservation:

$$\tilde{h}(\omega) = \mathcal{A}(\omega)e^{i\psi_{PP}(\omega)}$$

$$\omega \sim 2\Omega_{orb}$$

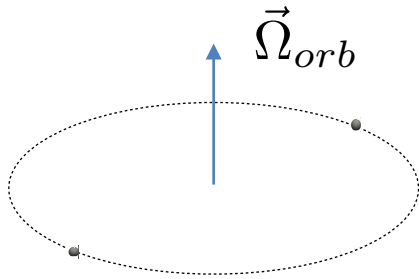
$$\dot{E}_{orb} = -\dot{E}_{GWs} < 0$$



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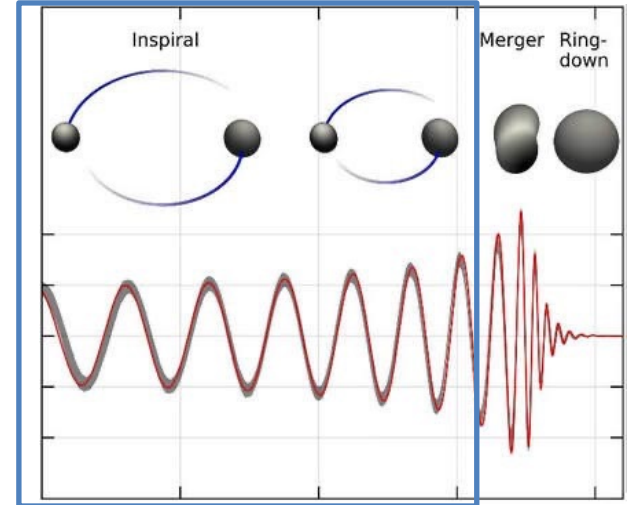


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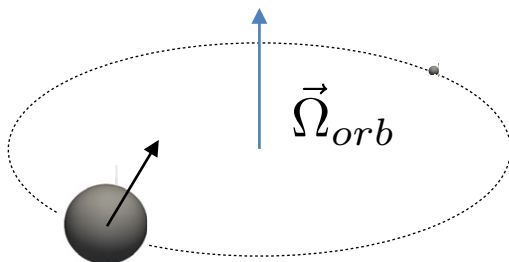
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$$\omega \sim 2\Omega_{\text{orb}}$$

$$\dot{E}_{\text{orb}} = -\dot{E}_{\text{GWs}} < 0$$



Finite size effects, spin-orbit interaction, etc introduce corrections to the GW phase waveform, encoded in higher PN orders:

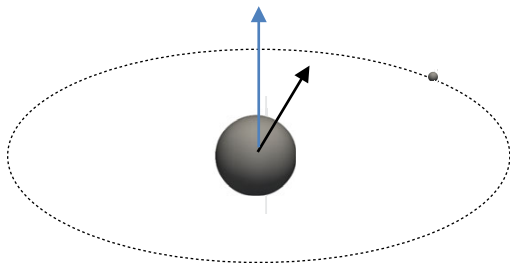


$$\tilde{h}(\omega) = \mathcal{A}(\omega)e^{i(\psi_{\text{PP}}(\omega) + \psi_{\text{1PN}}(\omega) + \dots)}$$

Imprints of BH area quantization: tidal heating

The BH is immersed in a sea of gravitational perturbations that alters the gravitational field of the background Kerr metric (tidal deformations)

The **absorption of energy and angular momentum by the horizon** from these GWs **backreacts on the binary's evolution**, resulting in a correction to the GW phase:



$$\dot{E}_{\text{orb}} = -\dot{E}_{\text{GWs}} - \dot{E}_{\text{BH}}$$

$$\tilde{h}(\omega) = \mathcal{A}(\omega)e^{i(\psi_{\text{PP}}(\omega) + \psi_{\text{TH}}(\omega) + \dots)}$$

Tidal heating: the work done by the tidal forces is used partially to extract the BH rotational energy, and is partially dissipated into area increase (“heating”). $\frac{\kappa}{8\pi}\delta A = T\delta S = \delta Q$

Imprints of BH area quantization: tidal heating

- For quasicircular, extreme mass-ratio inspirals, such tidal effects can significantly increase the duration of the signal

$$\psi_{\text{TH}} = \psi_{\text{PP}} \left(F(a_i, q) v^5 \log v + G(q) v^8 [1 - 3 \log v] \right) \quad (2.5\text{PN} \times \text{Log } v \text{ correction})$$

- The existence of a **threshold** in the BH energy spectrum makes **absorption of low frequency GWs highly suppressed during most of the inspiral**: different binary evolution with respect to GR prediction.

$$\omega_{\Delta n} \geq 2\Omega_H \qquad \omega \sim 2\Omega_{\text{orb}}(r)$$

- Therefore the lack of absorption will leave its imprint in the phase of GWs emitted: $\psi_{TH} \approx 0$

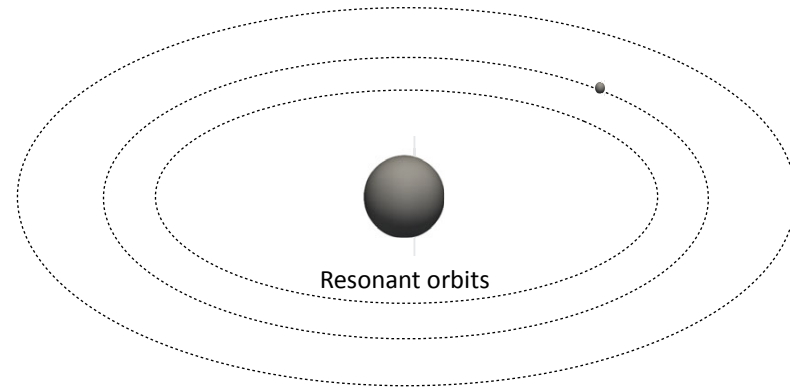
Imprints of BH area quantization: tidal heating

- Phenomenological approach:

$$\tilde{h}(\omega) = \mathcal{A}(\omega) e^{i(\psi_{\text{PP}}(\omega) + \gamma(\omega)\psi_{\text{TH}}(\omega) + \dots)}$$

- As inspiral shrinks, GW frequency increases, and a set of absorption “resonances” [Cardoso, dR, Kimura, 2019] could emerge.

$$\gamma(\omega) = \begin{cases} 1 & \text{if } \omega = 2\Omega_H + \frac{\kappa\alpha}{8\pi^2}\Delta n \geq 2\Omega_H \\ \ll 1 & \text{otherwise} \end{cases}$$



- Some **analysis** [Maselli et al, 2018] show that LISA and ET will reach the desired **sensitivity** to discriminate among different values of γ

What are the expectations from LQG?

Properties of area spectrum in LQG

- Standard area operator in LQG: **spectrum is not equally spaced**

$$A^{\text{LQG}}(j_1, \dots, j_n) = 8\pi\gamma\ell_P^2 \sum_{i=1}^n \sqrt{j_i(j_i + 1)}$$

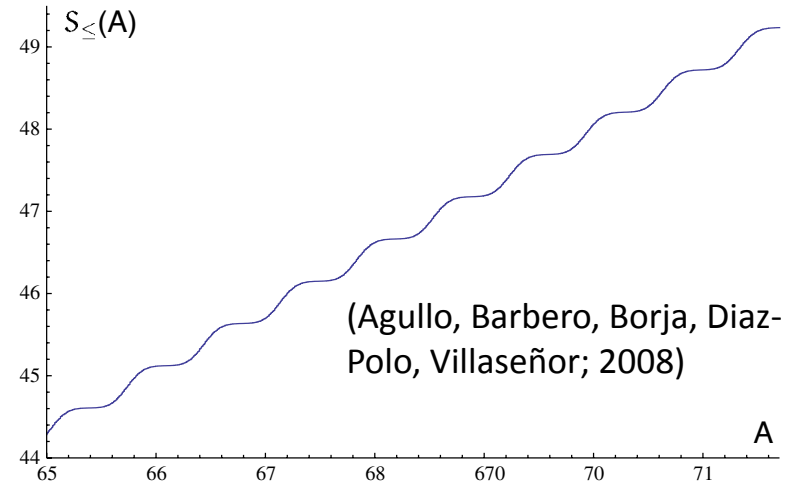
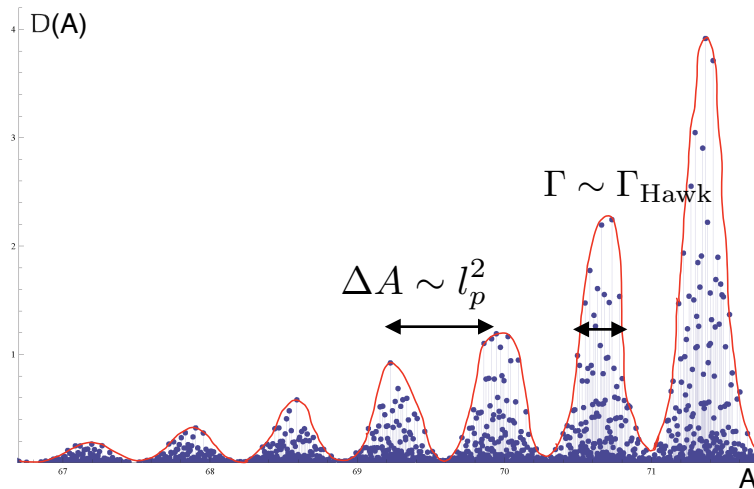
- Spacing between neighbouring eigenvalues can be estimated as [Barreira, Carfora, Rovelli; 1996]:

$$\Delta A \sim e^{-\sqrt{A}/l_p} \longrightarrow \text{energy spectrum is quasi-continuous}$$

- An effectively **continuous absorption spectrum would be recovered for macroscopic BHs**.
Echoes would not be expected, a priori.

Properties of area spectrum in LQG

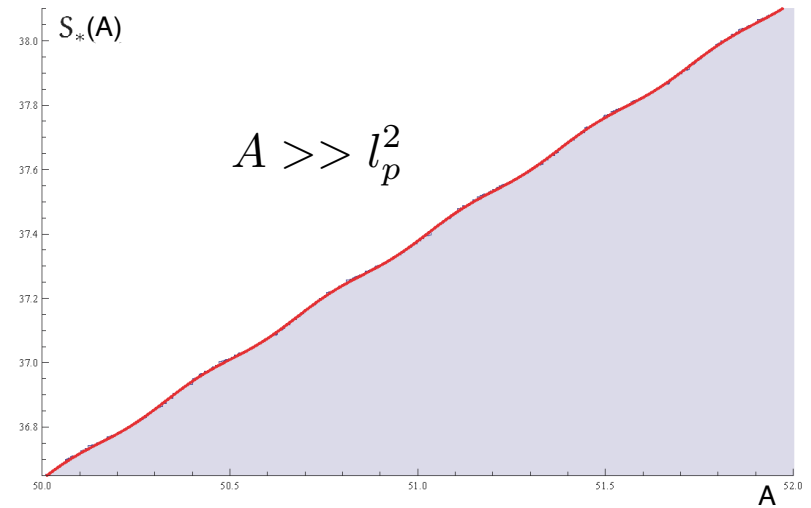
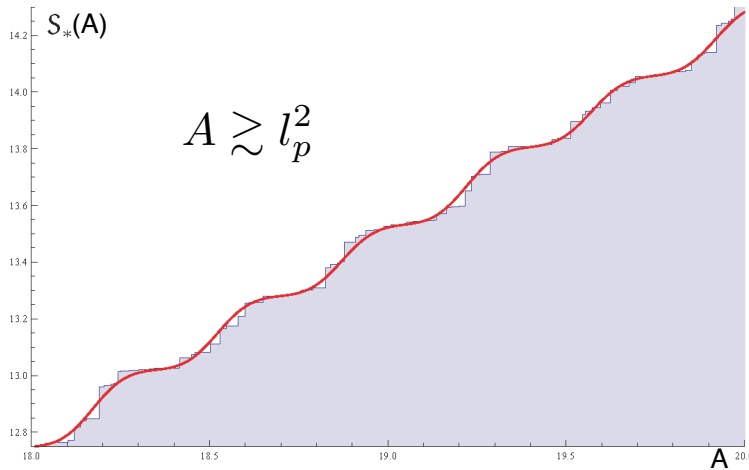
- For microscopic area values and no rotation, **BH degeneracy spectrum shows a periodic band structure**



- BH entropy can only increase in periodic steps: it constrains absorption of GWs
- Effectively reproduces the B-M uniform area quantization**, with linewidths compatible with Hawking evaporation. Echoes would be expected for microscopic BHs.

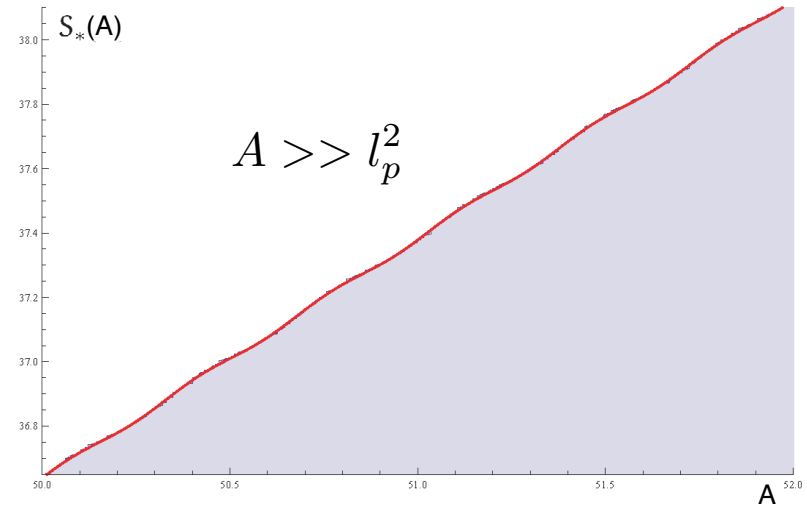
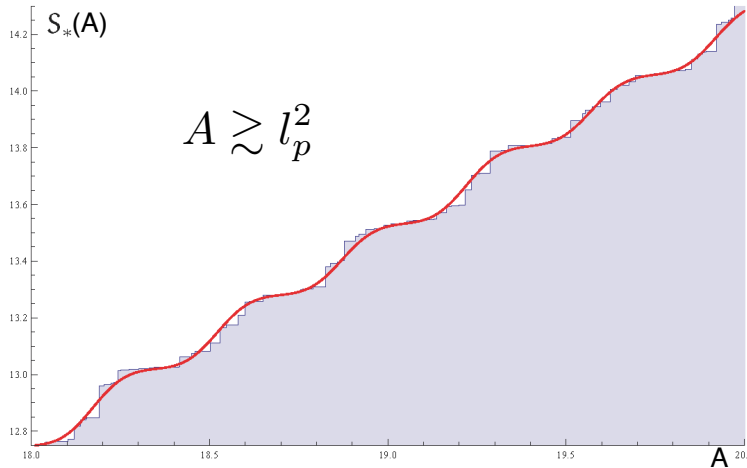
Properties of area spectrum in LQG

- Periodic band structure **tends to disappear as BH area becomes macroscopic** (Barbero, Villaseñor; 2011)



Properties of area spectrum in LQG

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- This conclusion is obtained counting only the number of microstates compatible with the macrostate (M, J=0). **Rotation is completely ignored.**

Does the periodic structure emerge for macroscopic BHs if the full degeneracy spectrum is calculated?

$$S(A, J) = ?$$

(Frodden et al; 2012)

Properties of area spectrum in LQG

- Flux-area operator: alternative choice within LQG

(Barbero, Lewandowski, Villaseñor; 2011)

$$A^{\text{flux}}(m_1, \dots, m_n) = 8\pi\gamma\ell_P^2 \sum_{i=1}^n |m_i|$$

Area spectrum is equally spaced: $\Delta A = 4\pi\gamma\ell_P^2$

- Similar to Bekenstein-Mukhanov quantization. Echoes are expected even for macroscopic BHs with this proposal. GW observations could be used to measure or bound the Barbero-Immirzi parameter

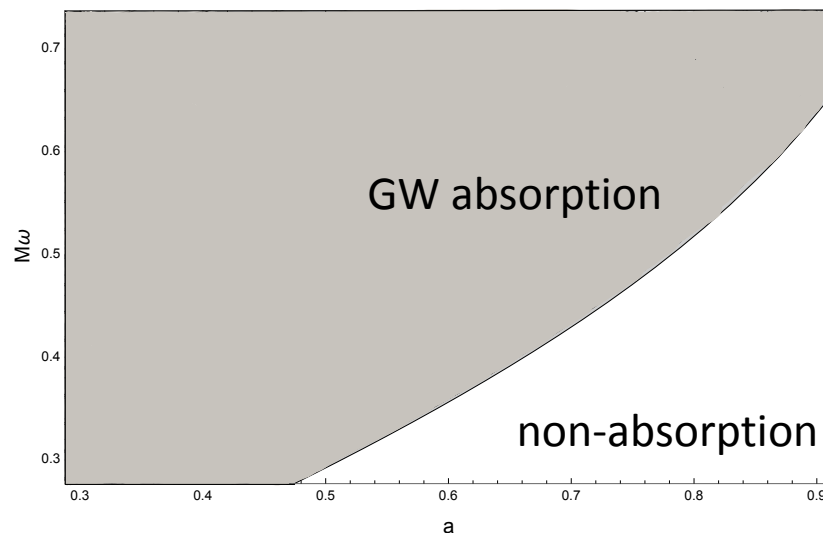
Properties of area spectrum in LQG

- What about tidal heating?

$$w_{\text{abs}} \equiv \frac{c^2 \Delta M}{\hbar} = \frac{\kappa c^2}{8\pi G} \frac{\Delta A}{\hbar} + \Omega_H \frac{\Delta J}{\hbar} \sim \frac{M_\odot}{M} \left[\frac{\Delta A}{\ell_p^2} + f(J/M^2) \frac{\Delta J}{\hbar} \right] \text{ kHz}$$

- If angular momentum is quantized in the usual way, we still have a threshold even if area spectrum is effectively continuous.

$$\omega_{\text{abs}} \gtrsim f(a) \frac{M_\odot}{M} \text{ kHz}$$

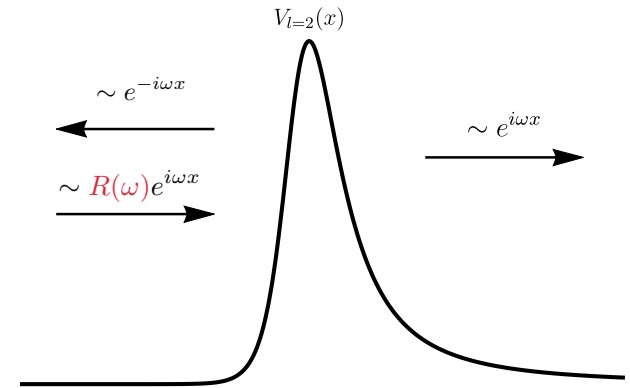


- LQG could still affect the GW signal through tidal heating.

Possible avenues for LQG

- **Effective boundary conditions:** study how a quantum horizon interacts with GWs to calculate the absorption probability distribution accurately

Regge-Wheeler eqn:
$$[\partial_t^2 - \partial_{r^*}^2 + V_l(r)] \psi(r^*, t) = 0$$



$$H(t) = H_{BH} + H_{int}(t) \longrightarrow R(\omega, \ell, m) \sim \langle M + \hbar\omega, J + m\hbar | H_{int} | M, J \rangle \otimes |\omega, \ell, m\rangle$$

- H_{int} is given by dipole approximation in the electromagnetic analogue $H_{int}(t) = -\vec{\mu} \cdot \vec{E}(t)$

- Can LQG do this calculation (quadrupolar moment approximation)?

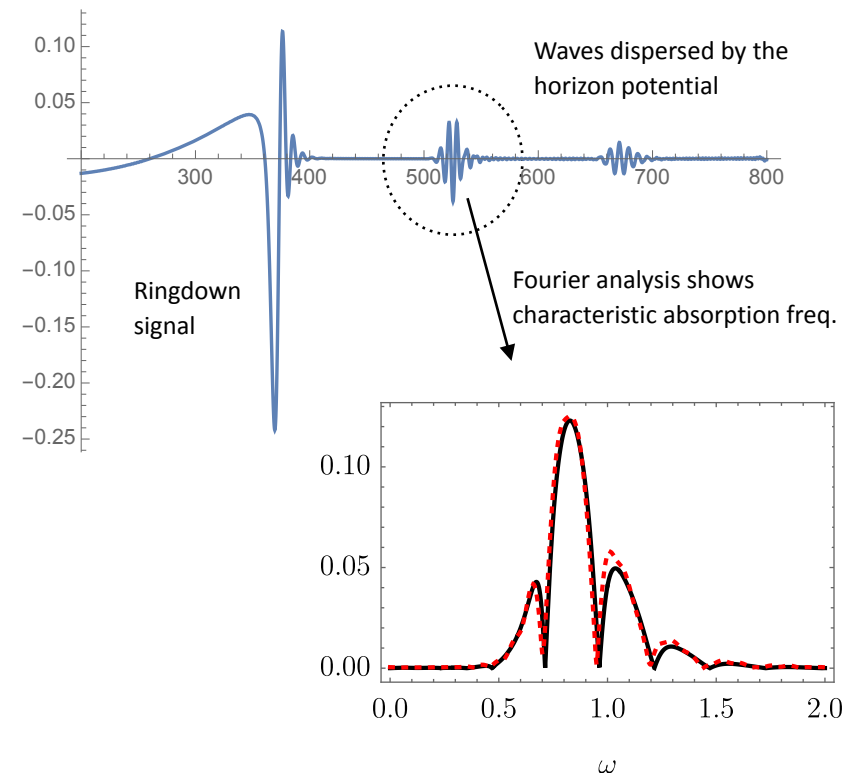
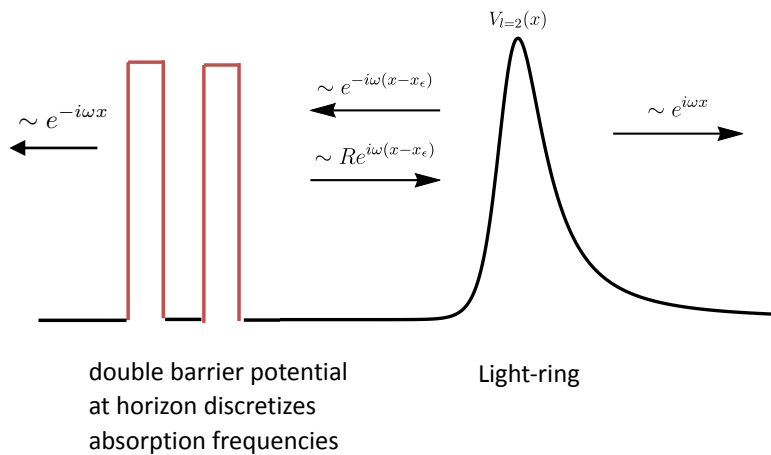
$$H_{int}(t) \sim Q_{ab} E^{ab} ?$$

Possible avenues for LQG

- **Effective geometry approach** (analogous to LQC): same boundary conditions, but area quantization is encoded in corrections to the gravitational potential at the horizon

$$\left[\partial_t^2 - \partial_{r^*}^2 + V_l^{\text{eff}}(r) \right] \psi(r^*, t) = 0 \quad (\text{Effective Regge-Wheeler eqn})$$

Example: Cardoso, Foit, Kleban (2019)



- Can LQG predict an effective potential at the horizon?

Conclusions

The study of the **interaction between BHs and GWs** could be helpful in revealing quantum aspects of BHs

If we assume:

- BHs admit a **quantum description** in terms of a Hamiltonian operator H in a **suitable Hilbert space**
- Existence of orthonormal basis of eigenstates $|M\rangle$, with eigenvalues M determined by **uniform, Planck scale area quantization**

Then there is basis to expect imprints of BH area quantization in GW physics.

Conclusions

- **GW echoes and suppressed tidal heating** could be signs of area quantization, from which the fundamental quantum of black hole area could be measured.
- **The possibility of detecting these imprints strongly depends on having accurate waveforms.** Accurate calculations require details of the microscopic BH degrees of freedom and the way they interact with the radiation field. LQG could provide these details.

THANK YOU