Black hole evaporation in loop quantum gravity

Based on:
A. Barrau, T. Cailleteau, X. Cao, J.D.P, J. Grain

Jacobo Diaz-Polo
Louisiana State University
Plan of the Talk

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• Black hole entropy computation
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  • Exact solutions in the Plank regime
  • Analytical exact solutions and large area limit
• Exact degeneracy of eigenstates and Hawking radiation
• Emission lines in the Planck regime
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  • Statistical analysis
• Low energy emission in the Planck regime
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Introduction

• Quantum gravity theories lack of observable tests.

• Interest: search for observational signatures of quantum gravity theories.
  • Microscopic black holes produced in high energy particle collisions?
  • Evaporation of primordial black holes?

• Loop quantum gravity provides a very precise description of the area spectrum and the associated degeneracy (entropy) of each eigenvalue.

• The particular – discrete – structure of this spectrum affects the process of black hole evaporation.

• Question: Does the structure of the black hole area spectrum in LQG produce observational signatures in Hawking radiation?

• Could these signatures be used to discriminate LQG from other theories through observation of black hole radiation?

• We will try to argue that the answer is YES.
Isolated Horizons
(Ashtekar, Beetle, Fairhurst)

- Local (intrinsic) geometrical definition of horizon
  - No need to know the whole “history” of space-time.
  - No restrictions on the surrounding space-time.
  - We restrict here to non-rotating case.

- Sketch of the definition:
  - Null hyper-surface with a preferred foliation and topology \( S^2 \times \mathbb{R} \).
  - Null normal with vanishing expansion and certain normalization conditions.
  - Field equations hold.
  - “Reasonably mild” energy conditions.

- Main condition expressed in connection variables:
  - \( F_{ab} \rightarrow \) Curvature of the U(1) connection on the horizon.
  - \( \Sigma^i_{ab} \rightarrow \) Pull-back of the triad to the horizon.
  - \( r_i \rightarrow \) Internal vector fixing de gauge (SU(2) \( \rightarrow \) U(1)).
  - \( a_0 \rightarrow \) Area of the horizon.

\[
F_{ab} = -\frac{2\pi \gamma}{a_0} \Sigma^i_{ab} r_i
\]
Black holes in loop quantum gravity
(Ashtekar, Baez, Corichi, Krasnov)

• Consider a classical space-time containing an isolated horizon:
  • Inner boundary satisfying IH boundary conditions.
  • Phase space splits into a volume and surface parts.

• Quantize separately, then impose “quantum” boundary conditions

\[ \mathcal{H}_{BH} = \mathcal{H}_S \otimes \mathcal{H}_V \]

• Qualitative picture:

  • Bulk described by “open” spin-networks, piercing the horizon.

  • Horizon described by a Chern-Simons theory over a punctured sphere.
Black holes in loop quantum gravity

• Horizon: surface in the bulk
  • Pierced by spin-network at a finite number $n$ of points.
  • Horizon area characterized by a list $(j_1, \ldots, j_n)$ of half integer ‘spins’.
  • Punctures carry an additional ‘magnetic’ number $m$ satisfying $m_i \in \{-j_i, -j_i+1, \ldots, j_i\}$

• Horizon: intrinsic geometry
  • Flat connections (Chern-Simons) except at the punctures (topological defects).
  • Basis for geometrical states: holonomies of paths around each individual puncture, characterized by integer (mod $k$) numbers $(a_1, \ldots, a_n)$.
  • Each puncture concentrates distributional curvature (conical singularities) given by discrete angle deficits $(2\pi a_i/k)$.
  • Analogous to Gauss-Bonnet theorem: $a_1 + \ldots + a_n = 0 \mod k$

• Isolated horizon boundary conditions:
  • Promoted to the quantum level as an operator equation.
    \[(1 \otimes \hat{F})\Psi = \left(-\frac{2\pi \gamma}{a_0} \sum \hat{r} \otimes 1\right)\Psi\]
  • Boils down to a simple relation between horizon and bulk labels: $2m_i = -a_i \mod k$
The Entropy computation
(Domagała, Lewandowski) (Meissner)

• Classically, boundary states fully determined by bulk states.
• Distributional quantum states: Independent d.o.f. at the horizon.
  • Black hole entropy associated to these horizon geometric d.o.f.
• For a horizon area $A_0$, the logarithm of the number of horizon states with areas in a small interval $[A_0 - \delta A, A_0 + \delta A]$ gives the entropy of the BH.

• Problem: Area is a property related to the bulk states.
  • A horizon state $(a_1, \ldots, a_n)$ only sets a lower bound for its area: $j_i = |m_i|$
  • Find a way to translate area constraint in terms into horizon states
    • If the ‘lower bound’ is smaller than $A_0$ and the interval ‘large enough’, one can always find the ‘right area’.
    • Area condition translated into the inequality.
• $N(A)$ is the number of all the finite, arbitrarily long, sequences $(m_1, \ldots, m_n)$ of non-zero half-integers, satisfying:

\[ 8\pi \gamma \ell_P^2 \sum_i \sqrt{|m_i|(|m_i| + 1)} \leq A_0 \quad \sum_i m_i = 0 \]

• The entropy of the black hole is: $S(A) = \log N(A)$

• Solving this, one obtains:
  (Large area limit)
  \[ S(A) = \frac{\gamma c}{\gamma 4\ell_P^2} A - \frac{1}{2} \log \frac{A}{\ell_P^2} + O(A^0) \]
The Planck regime

- Exact calculation on a computer for microscopic black holes.
  - Brute force algorithm.
  - Large area regime cannot be reached.

- Even in the Planck regime, linear behavior of entropy confirmed.
  - Numerical fit compatible with predicted Barbero-Immirzi parameter.
  - Also the logarithmic correction is observed with the ‘right’ coefficient.

- But also.... new quantum effects, resulting from the discrete structure:
  - Band structure in the degeneracy spectrum.
  - Effective discretization of entropy as a function of area.
Analytic techniques

Do these quantum effects extend to the large area regime?

Analytic techniques: allow to compute entropy in the large area limit while keeping the information about the discrete features of the spectrum.

Techniques related with number theory
- Pell’s equation, partition problem, diophantine equations.
- Availability of many standard techniques for solving those problems.

Generating functions
- Complete control on the whole combinatorial problem.
- Provide analytic expressions.
- Starting point for asymptotic computations.

Analytic combinatorics
- Asymptotic limit of the distributions generated by the generating functions.
- Study of their statistical properties in the large area limit.
- Band structure disappears at higher areas.
- However, not possible to rule out a ‘revival’ of the effect in the asymptotic limit.

Exact degeneracy

- Previous computation relies on the area interval.
- However, in order to study radiation we are interested in the exact degeneracy of each single area eigenvalue.
- We cannot use the ‘raw’ inequality anymore.
  - If the ‘lower bound’ is smaller than $A_0$, we still need to make sure that a horizon configuration exists, with $j_i \geq |m_i|$, reproducing the exact value $A_0$.
  - Compute, case by case, the horizon states compatible with each area eigenvalue.
- Result:
Hawking radiation

• Isolated horizons are not dynamical states.

• However, we can study the spectroscopy resulting from considering transitions between the different stationary states.

• Following the philosophy of Fermi’s Golden Rule, the probability of transition can be considered as a function of the final state degeneracy.

• One could expect two sources of quantum signatures on the observed Hawking radiation spectrum:
  • The particular structure of the black hole degeneracy spectrum
  • Perhaps more importantly, the particular discrete structure of the area spectrum.

• We are going to test this fact by means of Monte-Carlo simulations.

• Look at three different signatures:
  • Emission lines in the Planck regime.
  • Low energy emission in the Planck regime.
  • Peaks in the higher mass regime.
Emission lines in the Planck regime

• Discrete structure of the area spectrum in LQG could produce some emission lines in the spectrum, particularly during the last stages of evaporation.

• These lines would be, in principle, expected to be different (or non-existing) in the case of other black hole evaporation models.

• Obtain the spectrum for three different cases:
  • Loop Quantum Gravity.
  • Semi-classical Hawking model.
  • Model based on Bekenstein-Mukhanov conjecture.

• Use a Monte-Carlo simulation to obtain the emission spectrum.

• Perform statistical tests to check if they can be observationally discriminated.
Monte-Carlo simulation

• Start from a black hole of $A = 200 \, A_{Pl}$.

• Modulate the probability of transition with a factor: $P_{1 \rightarrow 2} = N(\lambda, M)e^{-\Delta S}$
  • $\Delta S$ is the entropy difference ($P_{1 \rightarrow 2}$ proportional to the degeneracy of the final state).
  • $N$ is the gray-body factor (computed numerically).

• Then, let the black hole randomly decay all the way till the minimal area state, recording the energy of each individual transition.

• Repeat the process for 1 million black holes.

• Plot the spectrum:

```
lqg
Entries  1000000
Mean    0.171
RMS    0.06511

Energy (E_{Pl})
0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
```

```
Hawking
Entries  1000000
Mean   0.1712
RMS    0.06446

Energy (E_{Pl})
0.15 0.2 0.25 0.3 0.35 0.4 0.45
0
200
400
600
800
1000
1200
1400
```

```
Mukhanov
Entries  1000000
Mean   0.1712
RMS    0.06545

Energy (E_{Pl})
0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0
1000
2000
3000
4000
5000
```

$P_{1 \rightarrow 2} = N(\lambda, M)e^{-\Delta S}$
Monte-Carlo simulation

- Compare with the result of the same simulation for the semiclassical model:
  - Maximally conservative
  - Same gray-body factors
  - Same minimal value of area

- And for the model based on the Bekenstein-Mukhanov conjecture:
Monte-Carlo simulation

- Spectroscopic plot:
Statistics analysis

- Can these two spectra (LQG/Hawking) be differentiated by observation?
- Run a Kolmogorov-Smirnov test
  - Measure the difference of the cumulative functions of both distributions
- Plot the number of black holes needed to discriminate both models as a function of the relative error and the confidence level
- Either with a sufficiently small relative error in the measurement, or a sufficiently large number of observed black holes, both models can clearly be distinguished!
Statistical analysis

• Comparing LQG with Bekenstein-Mukhanov spectrum:
  • Both spectra have lines, but in different positions.
  • Much easier to discriminate!
Low energy emission, Planck regime

• Last stages of evaporation in the semi-classical approach have to be modulated by a cut-off, ensuring that the black hole does not emit more energy than it has left.

• This effectively makes the energy of the emitted particles tend to zero as the area goes to zero.

• This low-energy emission can be differentiated from the low energy particles occurring in the initial lower-temperature states.
  • For energies lower than $8 \times 10^{15}$ GeV the final state emission occurs $100\mu$s after the initial emission.
  • Characteristic decreasing energy of the emitted quanta.

• This low energy particles, however, do not occur in the final stages of evaporation in LQG, where the discreteness of the area spectrum only allows for some very precise (high energy) transitions.

• This fact can be observed on our Monte-Carlo simulations
Peaks in the higher-mass regime

• Does the band structure in the spectrum also produce a characteristic signal?

• To observe it, we need to look at the ‘instantaneous’ spectrum.
  • Periodicity in area is constant, but quadratic relation between area and mass (energy) washes out the periodic signals in a dynamical, mass-changing process.
  • We need to observe a ‘sample’ of quasi-static black holes of the same mass.

• Candidates: Primordial black holes.

• Continuous mass distribution ⇒ Again messing up the observation.

• However, some models of PBH production ⇒ unique initial mass.

• Somewhat tricky, but:
  • If primordial black holes originated from a phase transition, all with the same mass,
  • If that initial mass is higher than $M_C \approx 10^{15}$ g, so they are still out there,
  • If there are enough of them so we can observe a detectable signal,
  • If there is a ‘revival’ of the band structure on the degeneracy spectrum for large areas,
• Then, a signal, originating from this band structure, could be observed.
• Following these hypotheses, we run another Monte-Carlo simulation.
• The resulting instantaneous spectrum would be:
Conclusions

• LQG description of black holes is somewhat effective, and expected to be best suited to the large area regime

• There are also different models being currently developed (SU(2), rotation, Grand Canonical ensemble)

• However, most of the features studied here do not depend on the particular black hole model or the details of the entropy computation, but only on the particular features of the area spectrum in LQG - a much better established prediction in the Plank regime.

• We might be far from being able to experimentally detect Hawking radiation from microscopic black holes (who knows? maybe not so far...)

• Nevertheless, it is still highly non-trivial to obtain observable predictions that could be tested by such kind of hypothetical measurements

• Now we know that, if microscopic black holes are to be found at LHC, they can tell us if loop quantum gravity is on the right track!