A new vacuum for loop quantum gravity

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ILQGS, Feb 2014

BD, Sebastian Steinhaus,
Time evolution as refining, coarse graining and entangling
arXiv: 1311.7565[gr-qc]

BD, Marc Geiller,
A new vacuum for loop quantum gravity
arXiv: 1401.6441[gr-qc]
and to appear

Overview

Motivation.

How to construct continuum physical theory with refining time evolution.

As an exercise construct BF vacuum: dualize the Ashtekar-Lewandowski construction.

Need to dualize everything!

BF refinement and BF cylindrical consistent observables.

Holonomies and integrated fluxes

BF measure and cylindrically consistent inner product.

Compactification /discretization of excitations via inductive limit construction.

Remarks on diffeomorphism symmetry and (full) dynamics.

Simplicity constraints again

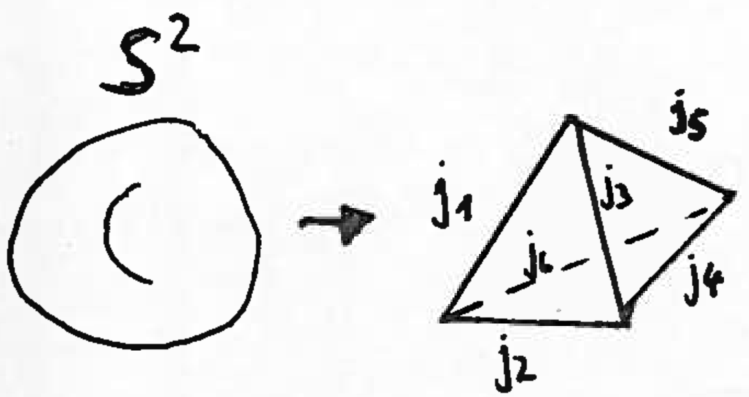
Conclusion and outlook

How to construct (continuum) physical vacuum.

[Bahr: cylindrical consistent path integral measure]
[BD,12: dynamical cylindrical consistency]
[BD, Steinhaus 13: Refining by time evolution]

What is vacuum?

coarse representation of vacuum



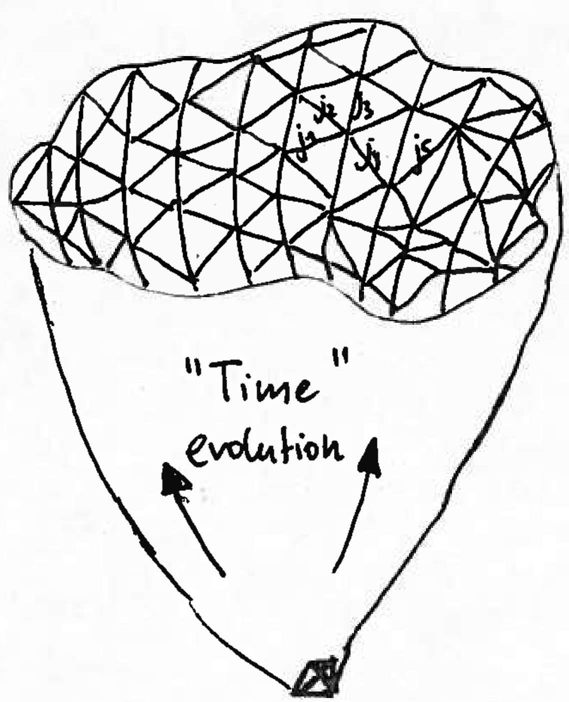
$$\rightarrow \psi(j) \sim e^{iS(j)}$$

(Hartle-Hawking) wave function
associated to closed boundary
of space time,
given by amplitude for basic space time
building block

[Oeckl: generalized boundary proposal]
[Conrady, Rovelli et al 03]
...

[Hoehn: discrete generalized boundary proposal 14]

finer representation of vacuum



$$\rightarrow \psi(j_{bdry}) \sim e^{iS(j_{bdry})}$$

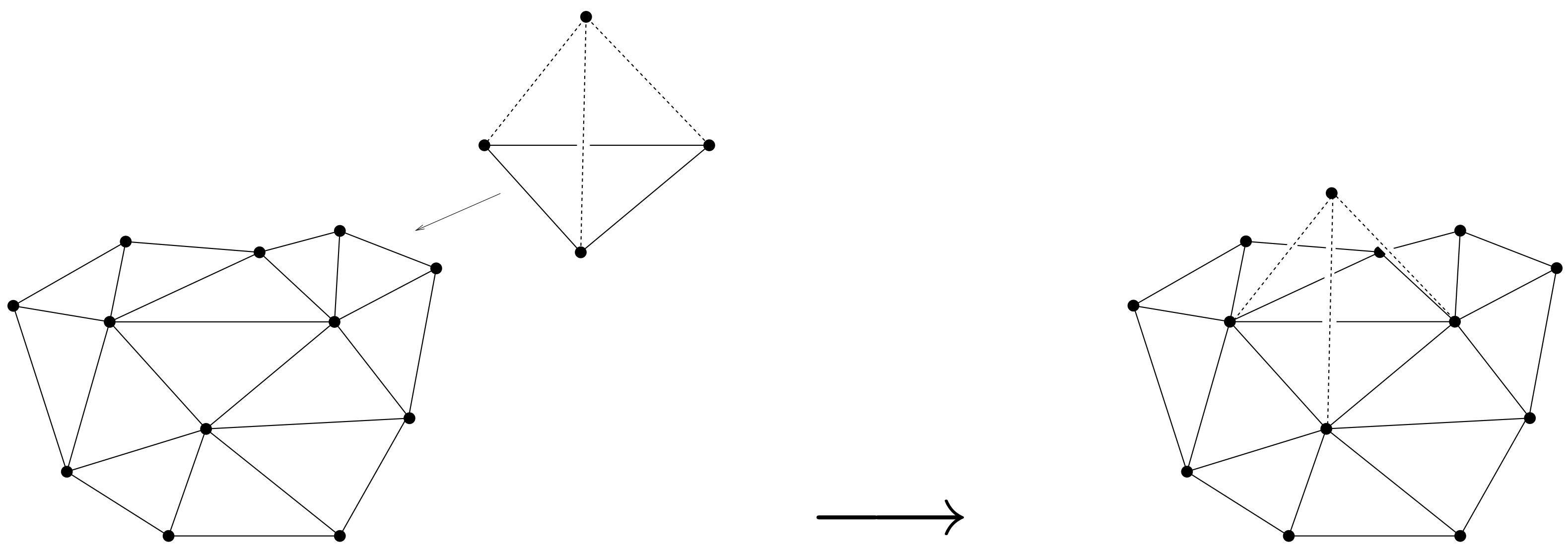
Refined vacuum wave function
by evolving i.e. with refining
Pachner moves.

[BD, Steinhaus 13: Refining by time evolution]

Refining: adding degrees of freedom in (interpolating) vacuum state

Same refining can be applied to a general state:

[BD, Hoehn, Steinhaus, ...]

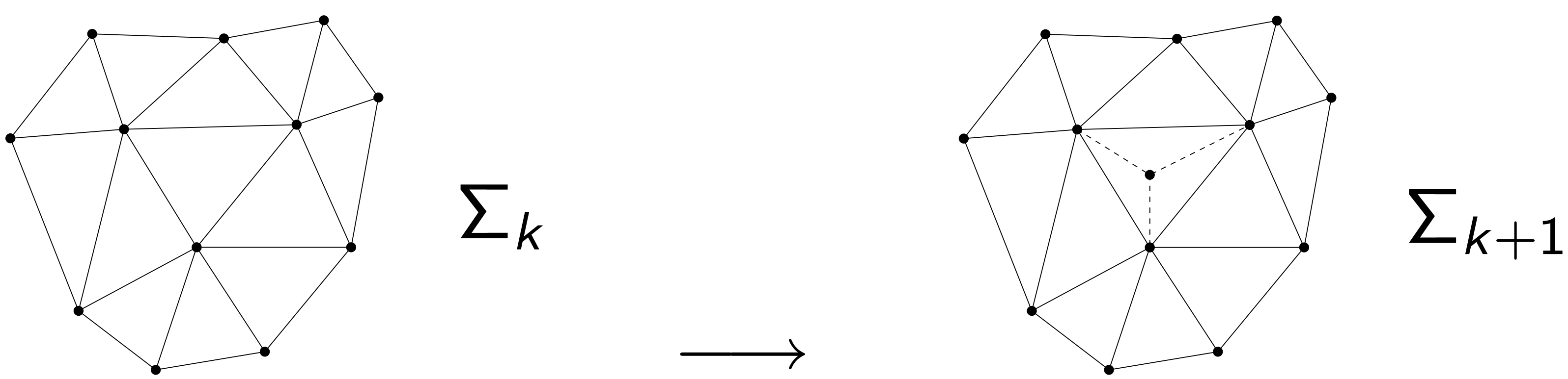


general state

refined state

fits nicely the heuristics of tensor network renormalization

[BD 12]



[BD, Steinhaus 13: Refining by time evolution]

Lesson: think about refining with Pachner moves.

Can we use this to construct continuum limit

in the same way standard LQG is based on

(dual) graph refinements?

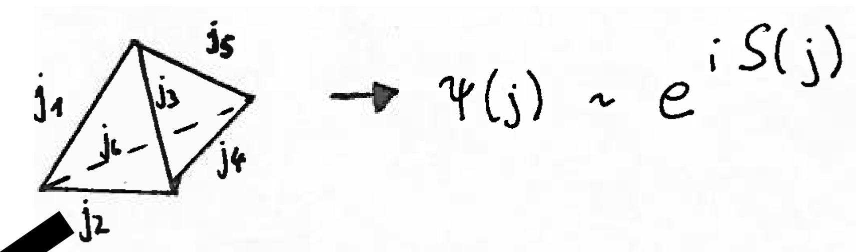
As we will see this makes sense for topological theories and allows the construction of the continuum theory via inductive/projective limit used in LQG.

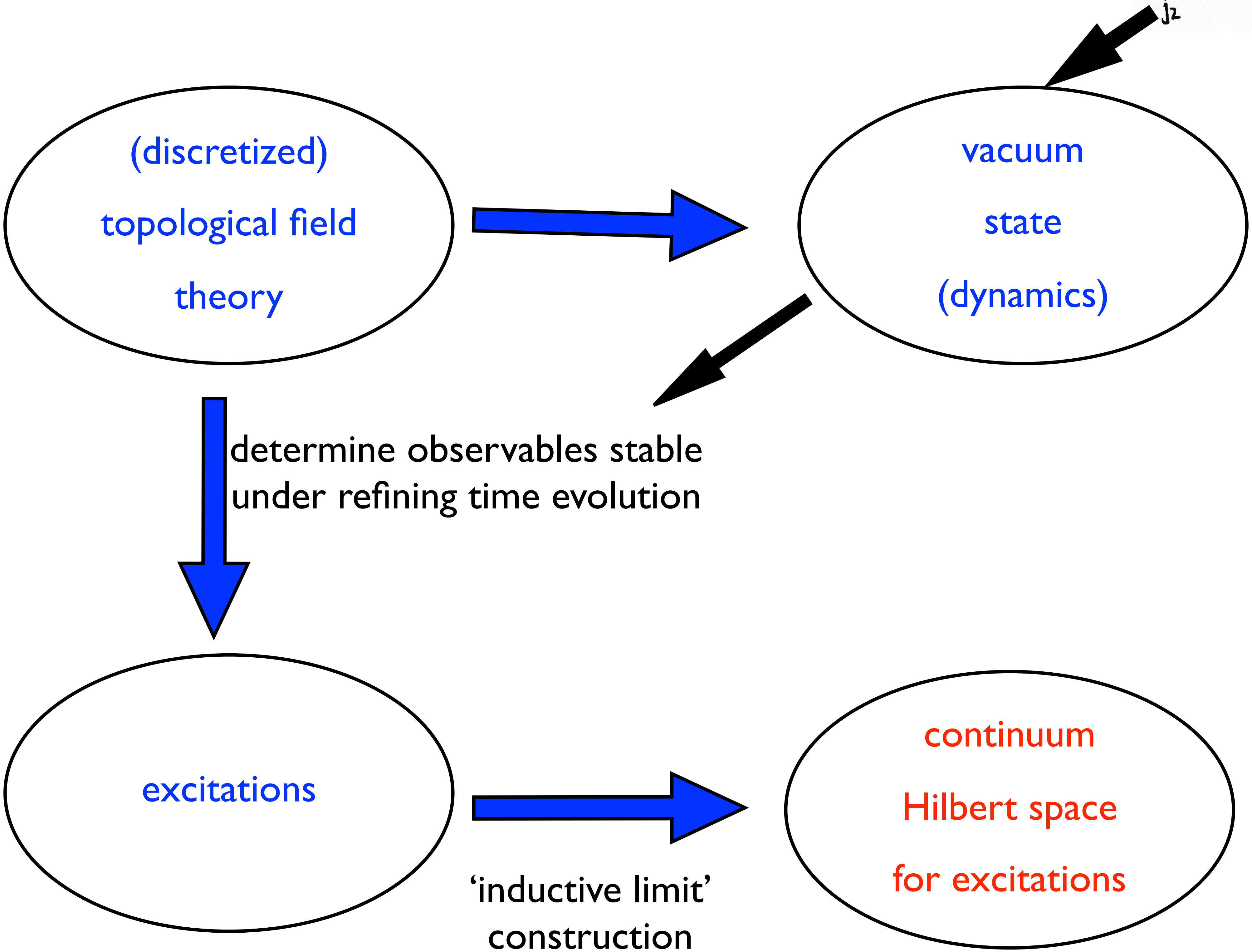
This limit does not only describe the topological theory but also excitations!

In fact we obtain a representation (of gauge invariant projection) of the (modified) holonomy flux algebra.

Thus the space of `refining Dirac observables' = cylindrically consistent observables is (unexpectedly) large.

Excitations and vacuum (general)





Cylindrically consistent amplitude maps

For non-topological theories:

Refining operations (Pachner moves) do not necessarily commute.

[Comparable to the requirement to satisfy Dirac constraint algebra.]

However can hope that after some refining-coarse graining discretization independence is restored.

This allows the construction of a family of cylindrically consistent amplitudes

associated to a family of discretizations (not necessarily simplices) of boundary manifolds.

[BD 12, ILQGS talk 2012] [Read BD, Steinhaus 13!]

Cylindrical consistency allows an interpretation of these amplitudes as acting on continuum boundary Hilbert space (only labelled by topology!).

Defines the continuum (physical) theory starting from spin foam amplitudes.

From AL to BF: Dualize everything!

Main lesson: dualize everything!

(BF refining needs triangulation)

[Gambini, Griego, Pullin 97,

Bobienski, Lewandowski, Mroczek 01: A two-surface quantization of Lorentzian gravity]

[Bianchi 09]

LQG as theory of curvature defects.

[Freidel, Geiller, Ziprick 11]

LQG continuum phase space with BF gauge fixing.

[Baratin, Oriti & Baratin, BD, Oriti, Tambornini 10]

Non-commutative flux representation of LQG.

[BD, Guedes, Oriti 12]

LQG in terms of E-bar instead of A-bar?

Problem: Usual refining is not consistent with E-bar!

[LOST-F 05/04]

Uniqueness theorem for AL vacuum.

Can there be another vacuum?

Loop quantum gravity vacua

geometric variables: $\{A\,,\,E\} = \delta$

connection

flux: spatial geometry

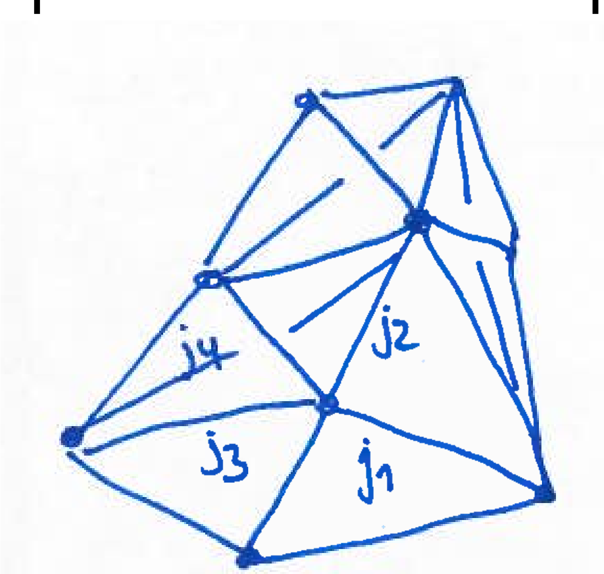
condense

Ashtekar - Lewandowski vacuum (90's)

$$\psi_{vac}(A) \equiv 1 \quad , \qquad E \equiv 0$$

peaked on degenerate (spatial) geometry
maximal uncertainty in connection

excitations:
spin network states supported on graphs



(representation) labels for edges

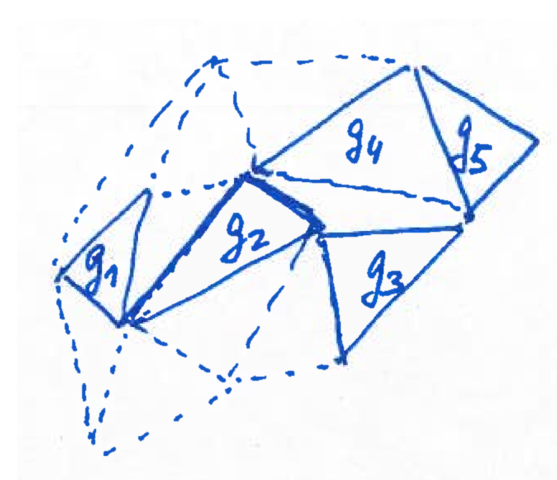
[Koslowski: vacuum with shifted E]

BF (topological) theory vacuum

$$\psi_{vac}(E_{Gauss}) \equiv 1 \quad , \qquad F(A) \equiv 0$$

peaked on flat connections
maximal uncertainty in spatial geometry

excitations:
flux states supported on (d-1)D-surfaces



(group) labels for faces

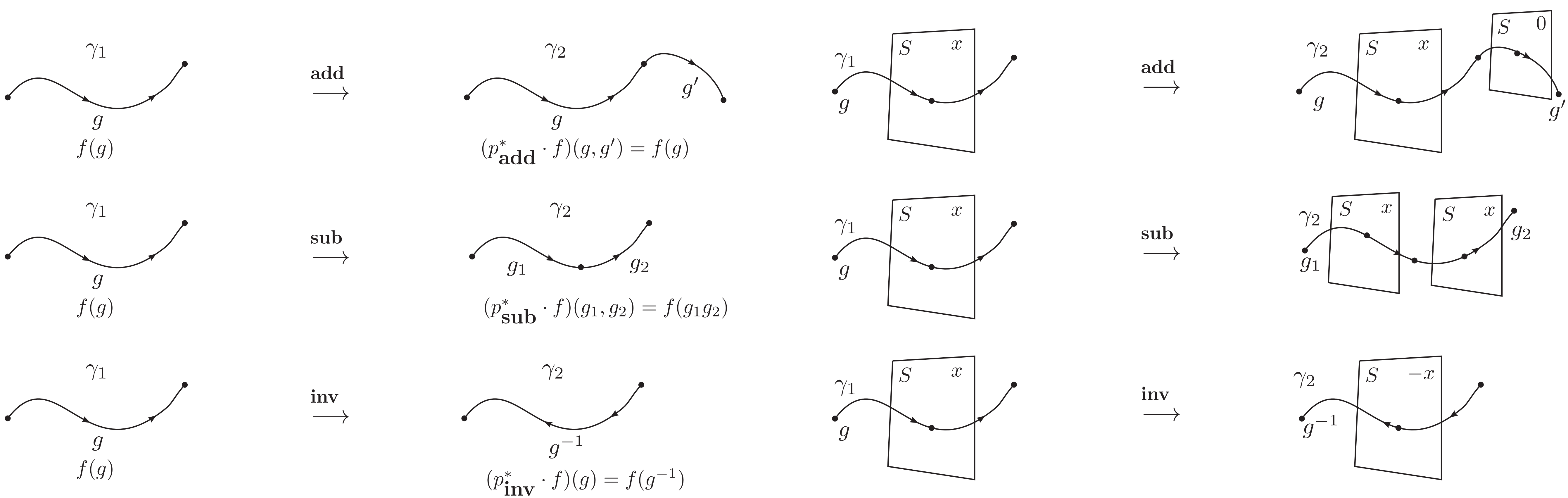
shift connection to homogeneous curvature?

Loop quantum gravity with AL vacuum

[...Ashtekar, Isham, Lewandowski 93]

- based on dual graphs: carry excitations (spin networks = functional of holonomies labelled by spins)
- refining operations on this graph: matches composition of holonomies
- cylindrically consistent holonomy and flux observables: commute with this refining
- allows the construction of a cylindrically consistent measure

and inner product and the definition of the continuum Hilbert space via a so-called inductive limit



Refinement operation on holonomies and fluxes. [Thiemann 00, QSD7] [BD, Guedes, Oriti 12]

BF refinement and BF cylindrically consistent observables

[BD, Geiller 14]

Set-up for BF vacuum

More (regular) structure than just dual graph!

[Bonzom, Smerlak 12: needed for BF quantization]

[Gurau: colored triangulation]

- simplicial version of LQG, see also [Thiemann 00, QSD7]
- instead of embedding dual graph, embed triangulation (vertices)
- refining given by operation on triangulation (and not on dual graph)
- classical phase space: projective limit of phase spaces as in [Thiemann 00, QSD7]
- however change AL embedding to BF embedding

Set-up

- manifold with auxiliary metric
- set of embedded triangulations
    - embedded vertices: carry coordinate labels
    - edges: geodesics with respect to auxiliary metric (replaces piecewise linear)
    - triangles, tetrahedra: given by minimal surfaces
- dual complex (for instance barycentric, however details do not matter)

  with a root node (fixing a reference frame)

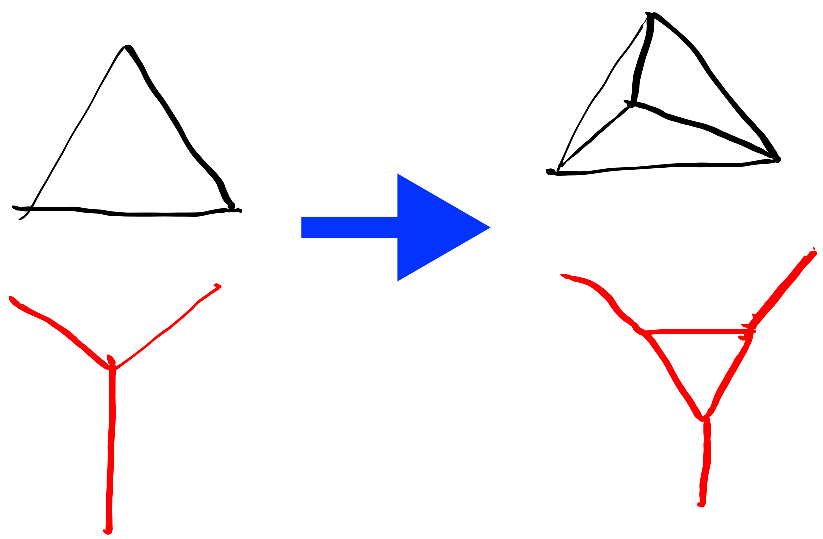
- refining operations given by refining Alexander moves

  (alternative: set of refining Pachner moves)
- equips the set of triangulations with a partial (directed)* order

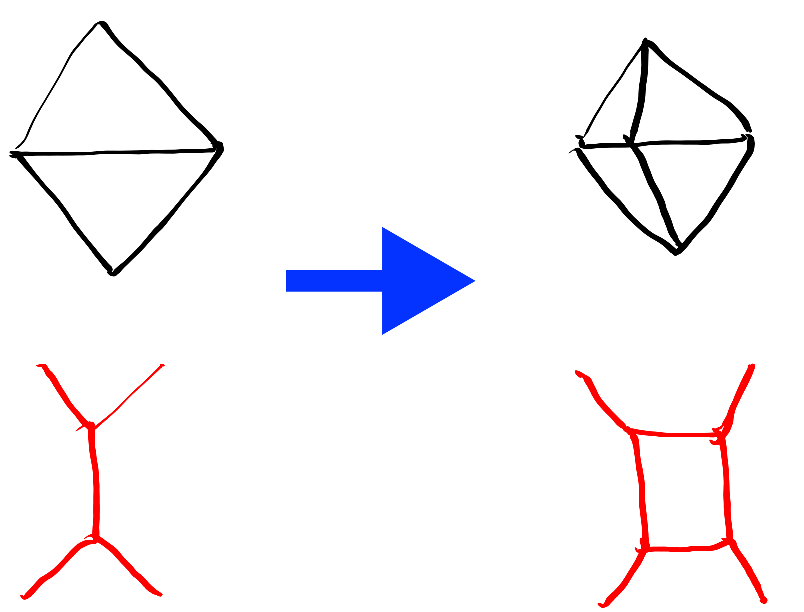
Alexander moves

subdividing (sub) simplices

In d=2: subdividing a triangle



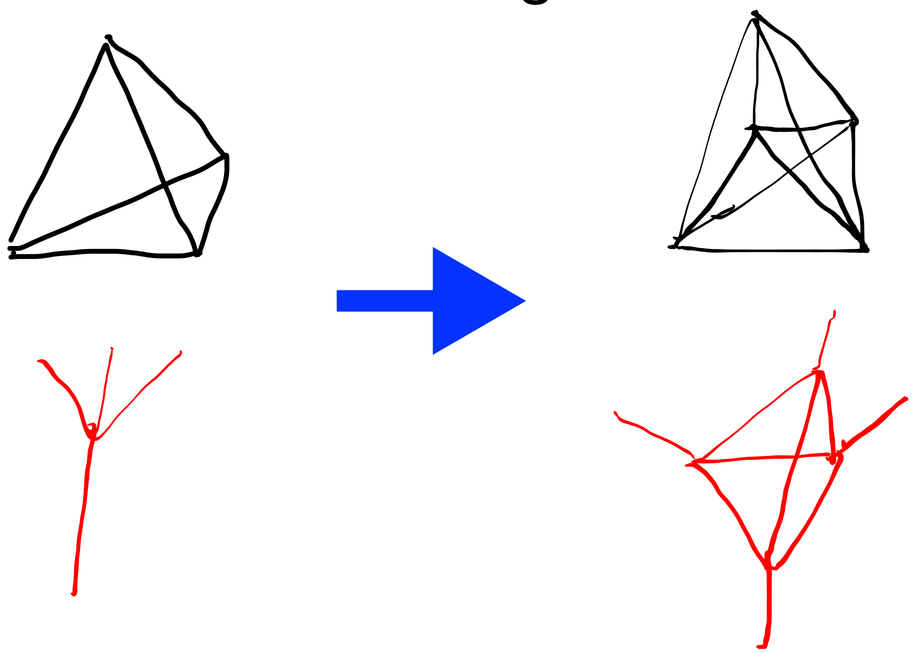
subdividing an edge



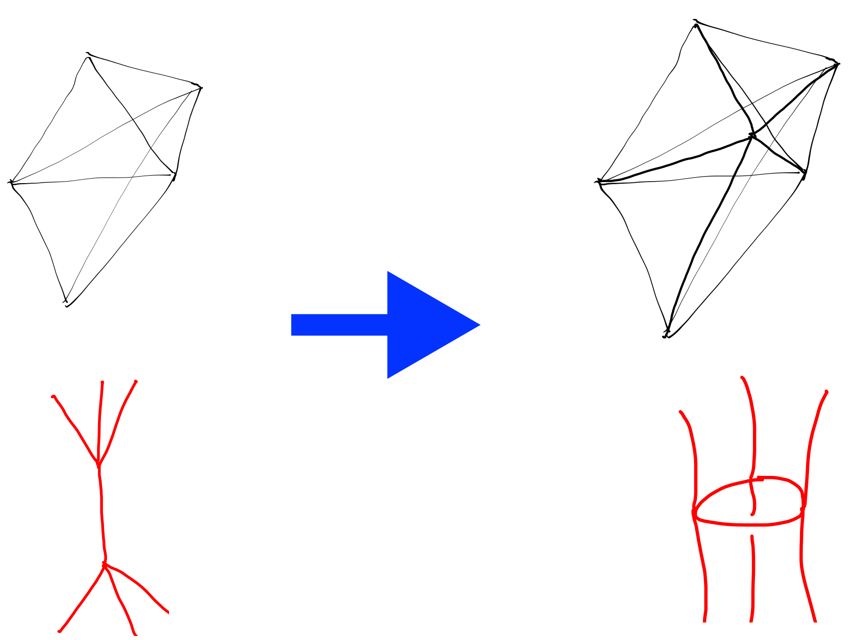
Alexander moves

subdividing (sub) simplices

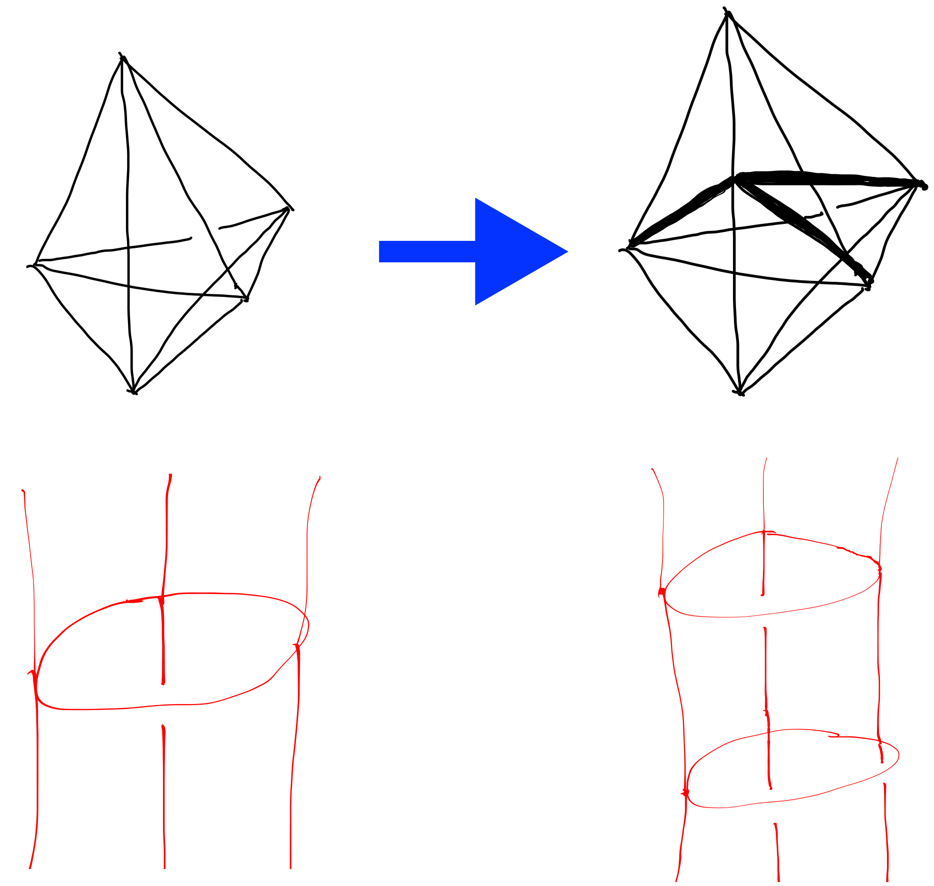
In d=3: subdividing a tetrahedron:



subdividing a triangle:



subdividing an edge:



(there are infinitely many of those moves)

Phase spaces

- for now fix triangulation and a root node
- specify a set of point separating functions
- gauge invariance: we consider phase space functions invariant under gauge transformations at all nodes except at the root

-closed holonomies with source at root

$$h_\gamma$$

-integrated (simplicial) fluxes transported to the root

(can understand these as vector fields acting on functions of holonomies)

d=2: $\mathbf{X}_\pi$ d=3: $\mathbf{X}_\sigma$ (+transport to root (tree), + tree on σ)

-Poisson brackets deducible from basic (standard) Poisson brackets:

$$\{X^k_e, g_e\} \quad = \quad g_e T^k \qquad\qquad \{X^k_e, X^l_e\} \quad = \quad f^{klm} X^m_e$$

[Thiemann 00, QSD7]

Integrated simplicial fluxes

[Husain 91]

[Thiemann 00, QSD7]

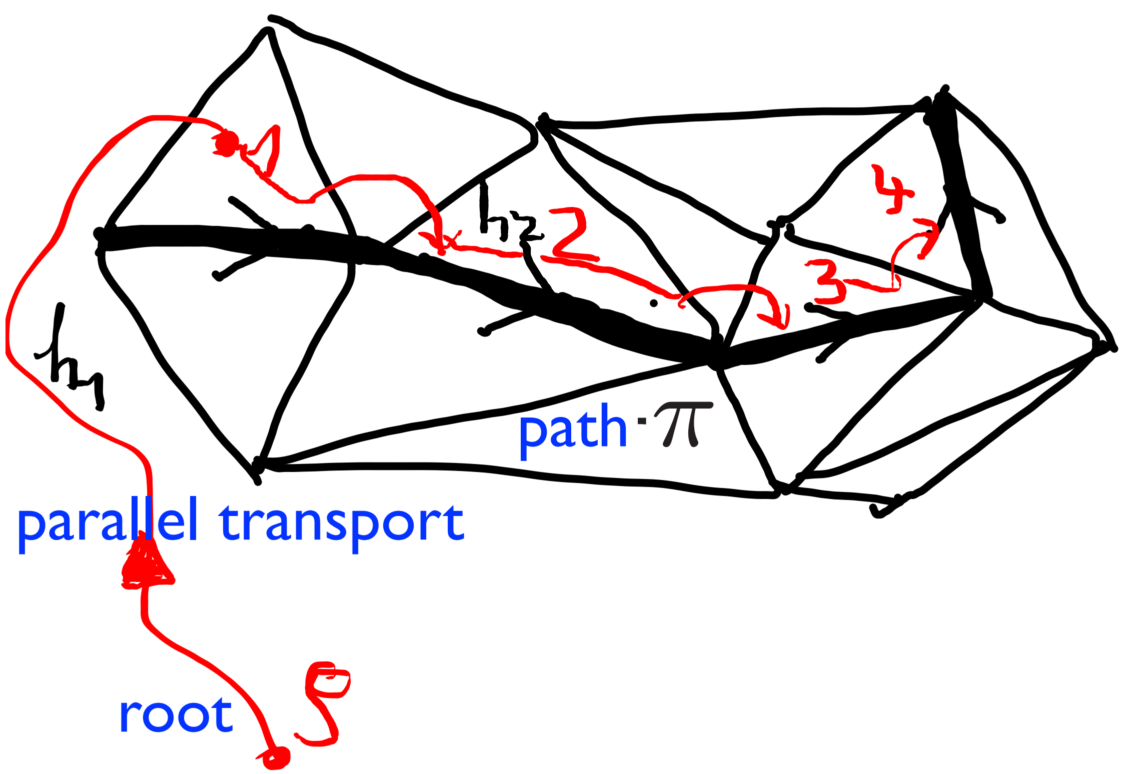
[Freidel, Louapre 04]

In d=2 (with an almost canonical choice of parallel transport):

for one edge:

$$X_e = \int_{e^*} h_{e^*(t),e(0)} E_a\big(e^*(t)\big)(\dot{e}^*)^a(t) h^{-1}_{e^*(t),e(0)}\, \mathbf{d}t,$$

for a path in the triangulation:



$$X_\pi = h_1^{-1} X_1 h_1 + h_2^{-1} X_2 h_2 + h_3^{-1} X_3 h_3 + h_4^{-1} X_4 h_4$$

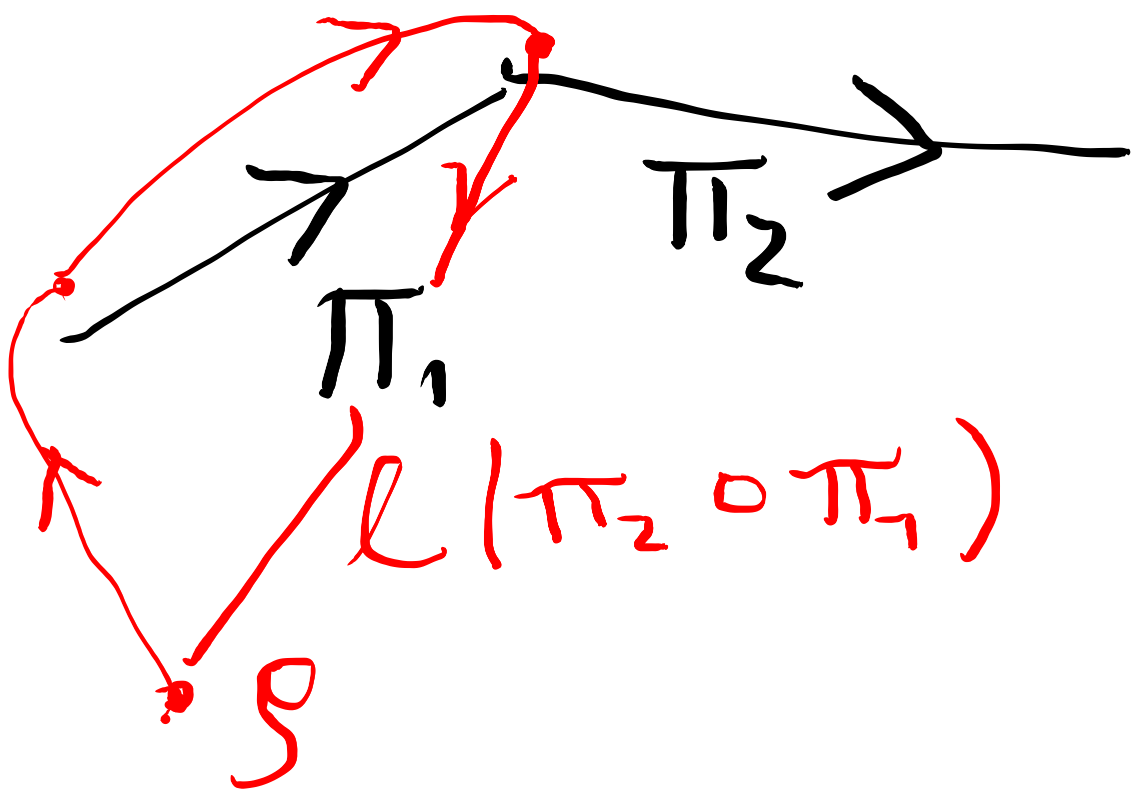
Interpretation: vector from source vertex to target vertex of path.

In (2+1) gravity closed paths give Dirac observables - but here we allow open paths!

Composition of integrated simplicial fluxes

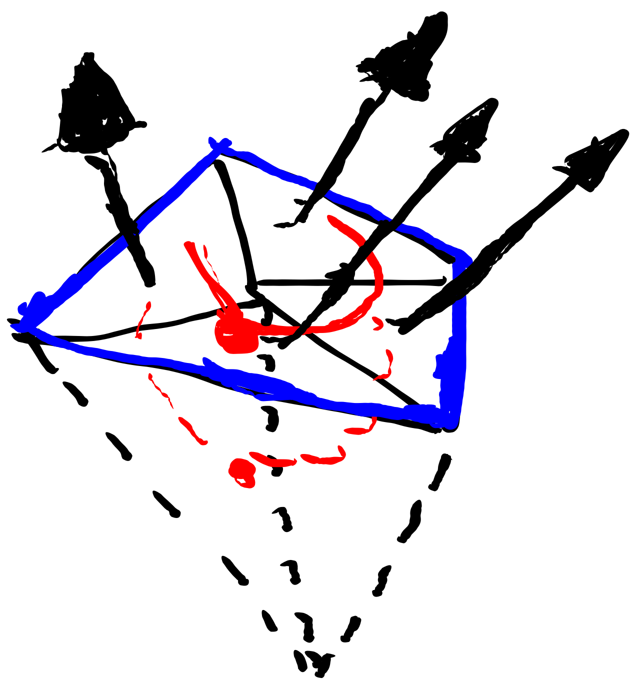
replaces composition of holonomies in AL embedding

$$\mathbf{X}_{\pi_2} \circ \mathbf{X}_{\pi_1} \quad = \quad \ell^{-1}_{\pi_2 \circ \pi_1} \mathbf{X}_{\pi_2} \ell_{\pi_2 \circ \pi_1} \, + \, \mathbf{X}_{\pi_1}$$



Integrated simplicial fluxes

In d=3 (need a surface tree for parallel transport):



black arrows: elementary fluxes

blue: piece of a surface

red: bonsai tree for piece of surface

parallel transport (dashed red) takes place

in tetrahedra `below’ the surface

For composition of integrated fluxes need to specify a `bridge’ edge

(which connects the two surface trees).

Continuum phase space

Can be defined as a projected limit

of phase spaces associated to fixed triangulations.

[Thiemann 00, QSD7: for AL embedding]

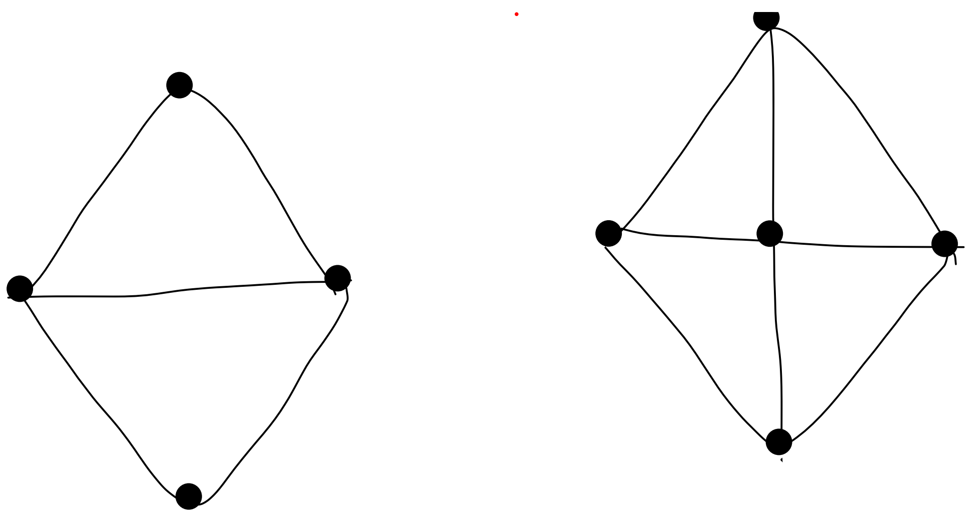
The discussion needed for that is exactly the same as for

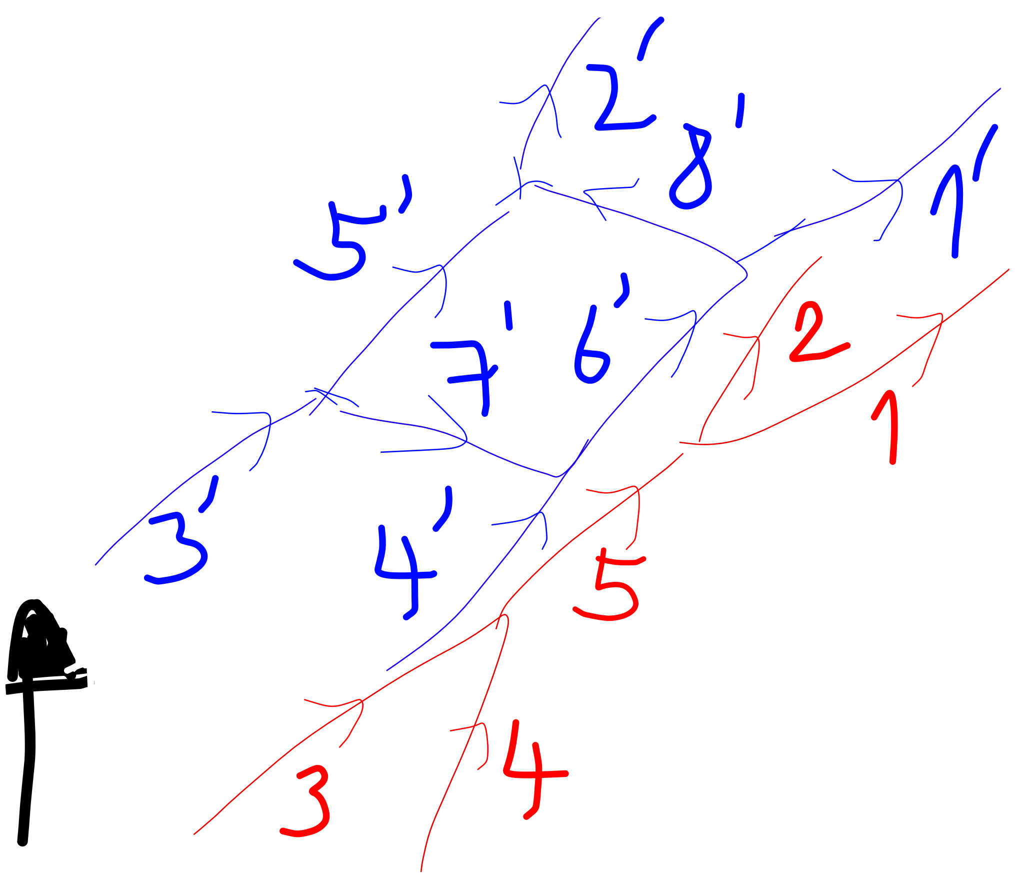
cylindrically consistency of the quantum observables.

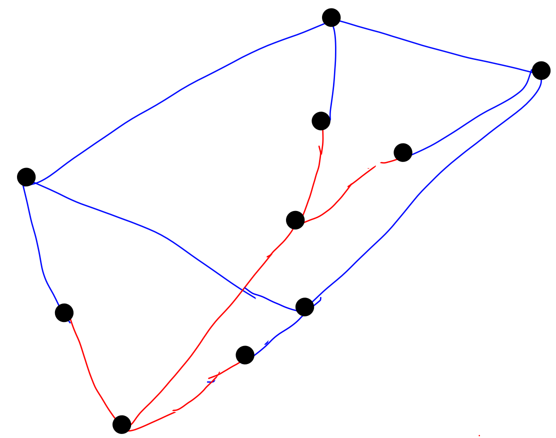
	AL embedding	BF embedding
holonomies	compose	stay constant
fluxes	stay constant	compose

Quantum theory

Refining for (holonomy) wave functions







We glue this dual complex

to the spatial hypersurface:

integrate over red edges.

Impose flat holonomies.

$$\mathbb{E}_{e_5}\psi(g_{1'},\cdots,g_{8'},\cdots)$$

$$= \int \delta(g_2^{-1}g_{2'}g_{8'}g_{1'}^{-1}g_1)\,\delta(g_3g_{3'}^{-1}g_{7'}^{-1}g_{4'}g_4^{-1})\,\delta(g_5g_4g_{4'}^{-1}g_{6'}^{-1}g_{1'}^{-1}g_1)$$

$$\delta(g_{7'}g_{5'}^{-1}g_{8'}g_{6'})\psi(g_1,\cdots,g_5,\cdots)\,\mathbf{d}g_1\cdots\mathbf{d}g_5 \quad .$$

Solving the delta functions and gauge fixing at the \`old' nodes:

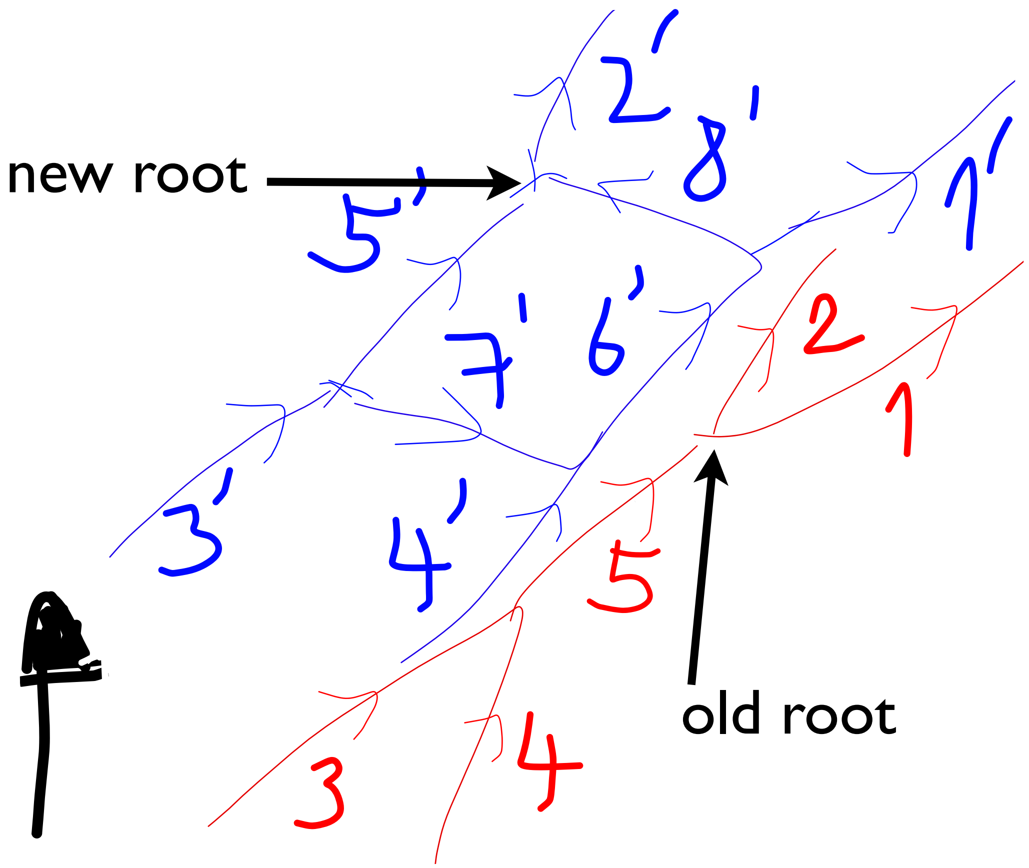
$$\mathbb{E}_{e_5}(\psi)(g_{1'},\cdots,g_{8'},\cdots)$$

$$= \delta(g_{7'}g_{5'}^{-1}g_{8'}g_{6'})\psi(g_1,\cdots,g_5,\cdots)\Big|_{\substack{g_1=g_{1'}g_{8'}^{-1}\\ g_2=g_{2'}\\ g_3=g_{3'}}}\Big|_{\substack{g_4=g_{7'}^{-1}g_{4'}\\ g_5=g_{5'}}}$$

Determines refining map for holonomies.

The root

Gauge fixing determines behaviour of root, in case it coincides with an old node.



Thus not only holonomies going through the region are cylindrically consistent, but also holonomies starting at the root.

$$\mathbb{E}_e\left(f(h_\gamma)\,\psi\right) \quad = \quad f(h_{\mathbb{E}(\gamma)})\mathbb{E}_e(\psi)$$

Moreover gauge action at root commutes with refining:

$$\mathbb{E}_e\left(\mathbf{G}_h^A\psi\right) \quad = \quad \mathbf{G}_h^{\mathbb{E}(A)}\mathbb{E}_e(\psi)$$

$$g_1 = g_{1'}g_{8'}^{-1} \qquad g_4 = g_{7'}^{-1}g_{4'}$$
$$g_2 = g_{2'} \qquad g_5 = g_{5'}$$
$$g_3 = g_{3'}$$

Integrated (exponentiated) fluxes: action

$$\{[h^{-1}T^k X_k h]^i\,,\, g\} \quad = \quad g\, hT^i h^{-1}$$

$$\Big\downarrow$$

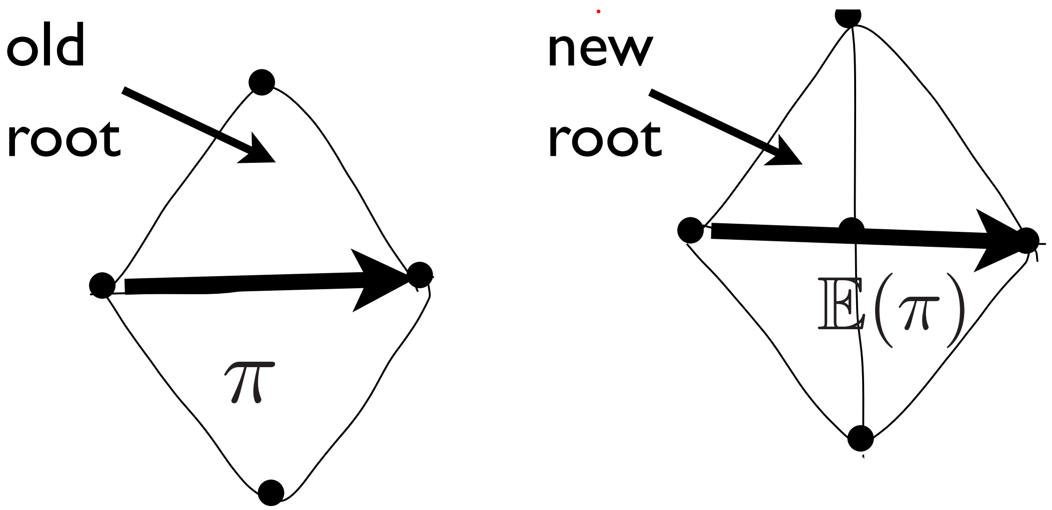
$$\exp(\alpha_i\{[h^{-1}Xh]^i, \cdot\}) \quad = \quad R_{h\,\exp(\alpha_i T^i) h^{-1}}$$

Exponentiated fluxes act by right translations.

Define: $\alpha = \exp(\alpha_i T^i)$

We will need exponentiated fluxes.

Integrated (exponentiated) fluxes: consistency



$$\mathbb{E}_{e_5}\left(\mathbb{X}^{\alpha}_{e_5^{-1}}\right) = \mathbb{X}^{h\alpha h^{-1}}_{e_6^{-1}} \circ \mathbb{X}^{\alpha}_{e_5^{-1}}$$

Basically follows from Gauss constraints and the fact that we add a flat (Gauss-closed) piece of geometry.

Thus a vector pointing from one vertex to the next vertex stays invariant after subdividing (in a flat manner).

$$\psi(\cdots, g_5, \cdots)$$

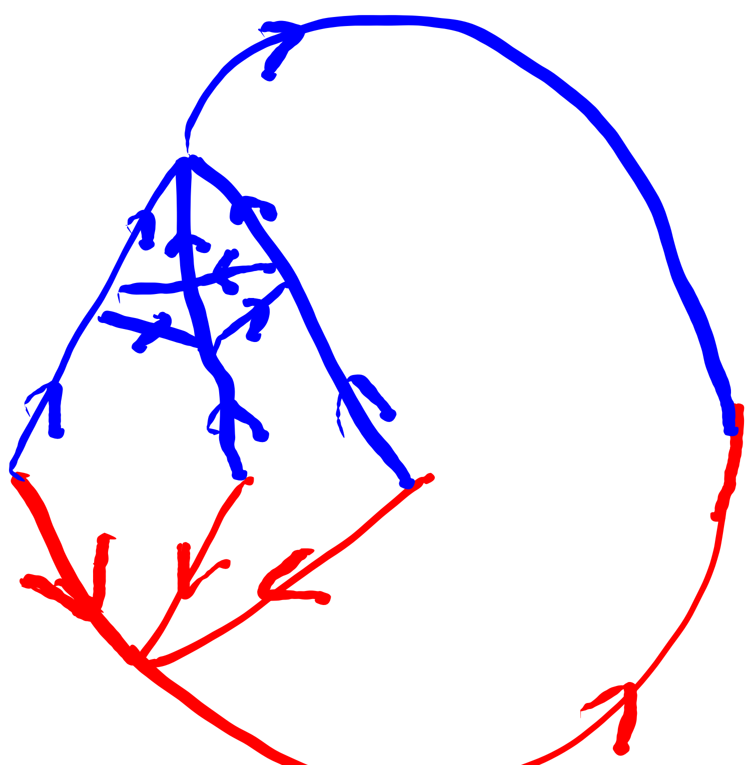
$$\begin{aligned}&\mathbb{E}_{e_5}(\psi)(g_{1'}, \cdots, g_{8'}, \cdots)\\ =\ &\delta(g_{7'} g_{5'}^{-1} g_{8'} g_{6'})\psi(g_1, \cdots, g_5, \cdots)\Big|_{\substack{g_1 = g_{1'} g_{8'}^{-1}\\ g_2 = g_{2'}\\ g_3 = g_{3'}}}\Big|_{\substack{g_4 = g_{7'}^{-1} g_{4'}\\ g_5 = g_{5'}}}\end{aligned}$$

$$\begin{aligned}&\mathbb{X}^{h\alpha h^{-1}}_{e_6^{-1}} \circ \mathbb{X}^{\alpha}_{e_5^{-1}}(\mathbb{E}_{e_5}\psi)\\ =\ &\delta(g_{7'} g_{5'}^{-1} g_{8'} g_{6'})\psi(g_1, \cdots, g_5, \cdots)\Big|_{\substack{g_1 = g_{1'} g_{8'}^{-1}\\ g_2 = g_{2'}\\ g_3 = g_{3'}}}\Big|_{\substack{g_4 = g_{7'}^{-1} g_{4'}\\ g_5 = \underline{\alpha^{-1} g_{5'}}}}\end{aligned}$$

$$\mathbb{X}^{\alpha}_{e_5^{-1}}\psi(\cdots, g_5, \cdots) = \psi(\cdots, \alpha^{-1} g_5, \cdots)$$

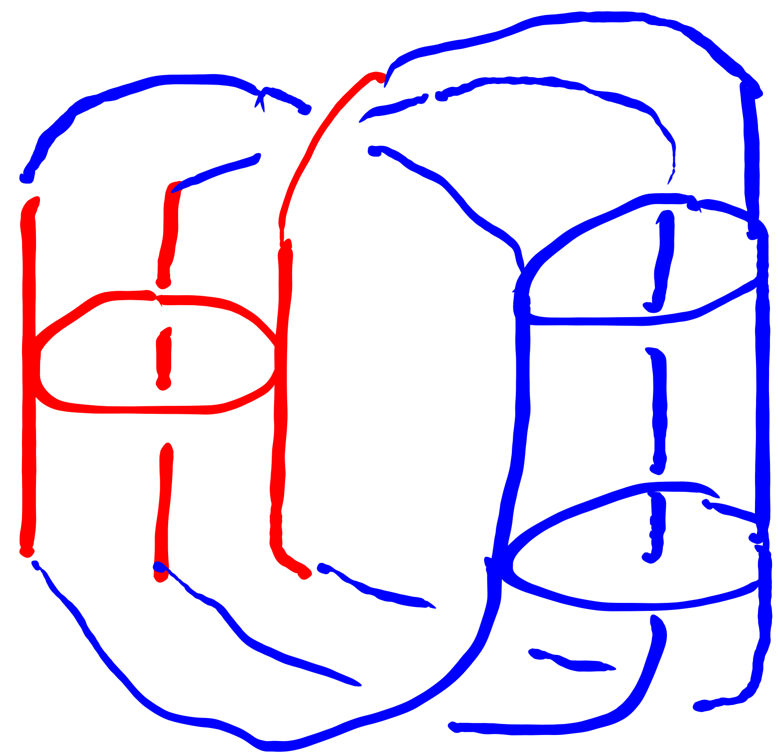
$$\begin{aligned}&\mathbb{E}_{e_5}(\mathbb{X}^{\alpha}_{e_5^{-1}}\psi)(g_{1'}, \cdots, g_{8'}, \cdots)\\ =\ &\delta(g_{7'} g_{5'}^{-1} g_{8'} g_{6'})\psi(g_1, \cdots, \underline{\alpha^{-1} g_5}, \cdots)\Big|_{\substack{g_1 = g_{1'} g_{8'}^{-1}\\ g_2 = g_{2'}\\ g_3 = g_{3'}}}\Big|_{\substack{g_4 = g_{7'}^{-1} g_{4'}\\ g_5 = g_{5'}}}\end{aligned}$$

Same mechanism in (3+1)D



Subdividing a tetrahedron.

(Gluing a 4-simplex)



Subdividing a triangle.

The measure: dualize AL measure

AL-vacuum

$$\eta_{AL}(A) \equiv 1$$

Excitations via

holonomy observables

$$\longrightarrow$$

Spin network basis

$$\psi_{\{j\}}(A) = \psi_{\{j\}}(A) \cdot \eta_{AL}(A)$$

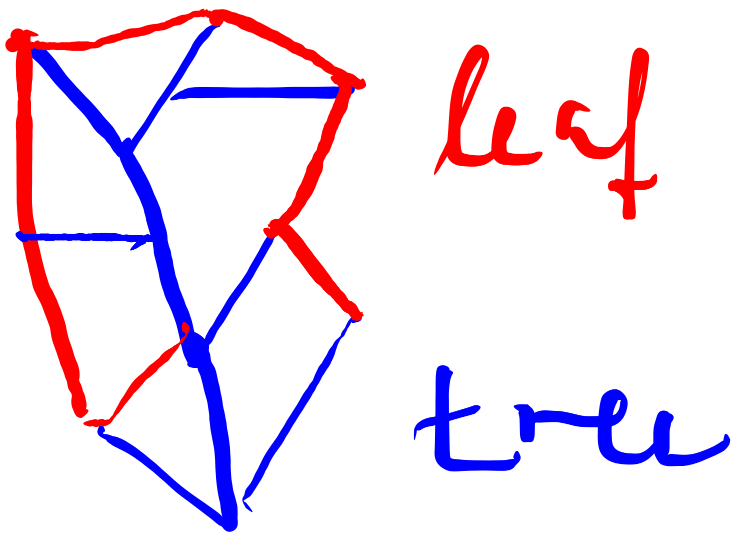
$$\downarrow$$

$$\mu_{\mathrm{AL}}(\psi_{\{j\}}) = \delta_{\varnothing,\{j\}}$$

AL measure

BF vacuum and excitations

First consider fixed triangulation. Gauge fix with maximal tree.



t_ℓ holonomy from root to source of leaf

Leafs are in one-to-one correspondence with fundamental cycles c_ℓ .

BF vacuum (does not depend on tree): constant in (Gauss-) fluxes.

$$\eta_{\mathrm{BF}} = \prod_\ell \delta(\mathcal{C}_\ell) \doteq \prod_\ell \delta(g_\ell)$$

Obtain basis of excitations by action of exponentiated (integrated) fluxes:

$$\chi_{\{\alpha_\ell\}} := R_{\{\mathrm{Ad}_{t_\ell}(\alpha_\ell)\}} \eta_{\mathrm{BF}} \doteq \prod_\ell \delta(g_\ell \alpha_\ell).$$

Excitations are labelled by group elements : α_ℓ

Can group average at root.

Measure

Basis of excitations labelled by group elements. $\chi_{\{\alpha_\ell\}} := R_{\{\mathrm{Ad}_{t_\ell}(\alpha_\ell)\}}\eta_{\mathrm{BF}} \doteq \prod_\ell \delta(g_\ell \alpha_\ell).$

Define measure in the same way as the AL measure:

$$\mu_{AL}(\chi_{\{\alpha_\ell\}}) = \prod_\ell \tilde{\delta}(\alpha_\ell)$$

For Abelian groups:

choice as formal Kronecker Delta leads to

Bohr compactification of the dual group (Z for U(1)).

For Non-Abelian groups:

Bohr compactification of the dual torus group

sufficient? (as in SU(2) quantum group)

Choice as group delta:

Same inner product as with Haar measure:

$$\int \overline{R_{\{\alpha'^{-1}_\ell \alpha_\ell\}} \prod_\ell \delta(g_\ell)} \prod_{\ell'} \delta(g_{\ell'}) \, \mathbf{d}^{|L|} g_\ell \;=\; \prod_\ell \delta(\alpha'^{-1}_\ell \alpha_\ell)$$

Shows independence of choice of tree.

Can now attempt to construct the continuum limit as

an inductive limit of Hilbert spaces in the same way as in standard LQG.

Need to make sure that inner product is cylindrically consistent, i.e. does not depend on the choice of triangulation it is computed on.

For the Bohr compactification this is the case.

Compactification of excitations

If we choose group delta, we need to modify the inner product to make it cylindrically consistent.
With some regulated group delta function:

$$\langle \psi_1, \psi_2 \rangle' = \lim_{\varepsilon \to 0} \frac{\langle \psi_1, \psi_2 \rangle_\varepsilon}{\langle \eta_{\mathrm{BF}}, \eta_{\mathrm{BF}} \rangle_\varepsilon},$$

[Bahr, hopefully to appear very soon]

Heuristically equivalent to a Bohr compactification. Need exponentiated fluxes.

Lesson: Inductive Hilbert space construction puts discrete topology on excitations.

For AL: dual graphs with discrete labels. For BF: (d-2) objects in triangulations with group labels.

[Okolow 13] Constructs (inductive Hilbert space) continuum limit
for non-compact configuration spaces R^N
via a projective limit of density matrices (i.e. functionals).
Results also in a Bohr compactification / almost periodic functions.

Generalize this to (cotangent space of) Lie groups!

Deformations/ Generalizations

Dualising the Koslowski shift (introduction of background triad):

leads to shift of background connection.

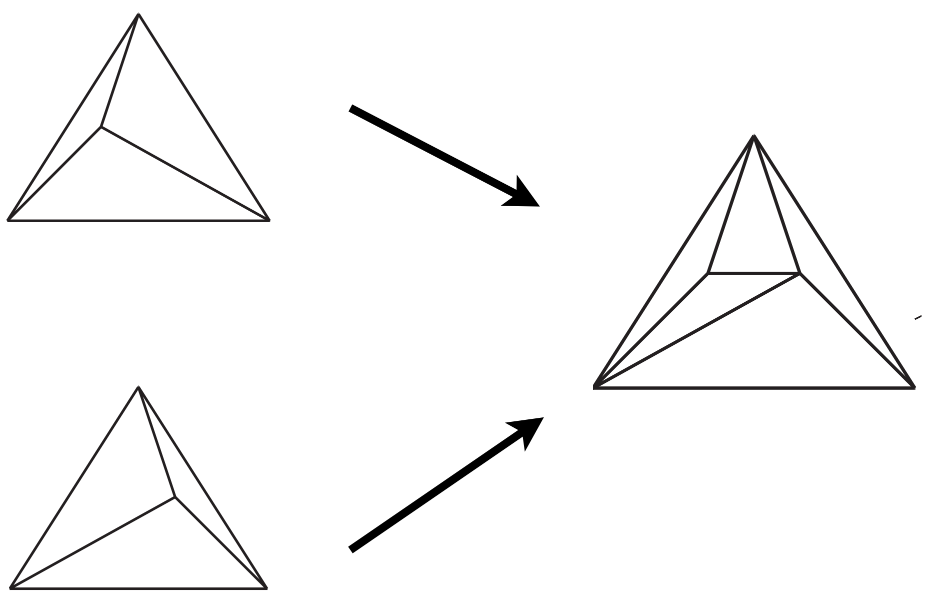
A homogeneous curvature requires flux dependent background connection,

which requires change in (group) measure to keep holonomies as unitary operators.

Thus one expects a deformation of the symmetry group. Derivation of quantum group?

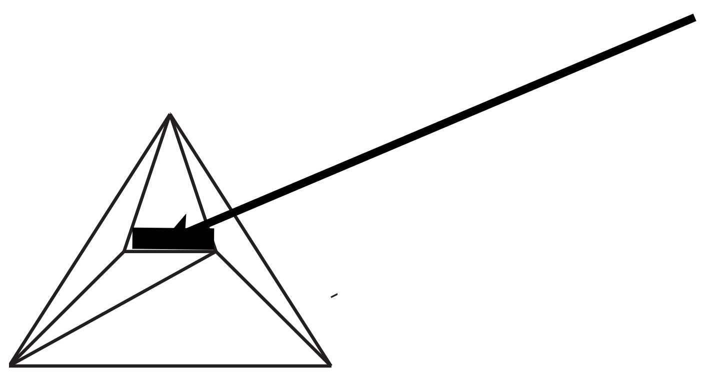
Can we use non-commutative flux representation and compactify this space?

(Spatial) Diffeomorphism symmetry



Common refinement.

Identifies BF vacuum on different triangulations as one (spatial diffeomorphism invariant state).



Finite (spatial) diffeomorphisms: $R_{\mathcal{C}_e} \sim EF$

(Right shift with holonomy around vertex.)

Needs exponentiated flux. (Diffeos and exponentiated fluxes not weakly continuous.)

Might explain `finite action interpretation' of Hamiltonian.

In (3+1)d: complications due to simplicity constraints, but doable. [Zapata 96, BD & Ryan 08]

On Hamiltonian dynamics and simplicity constraints

Good news!

Some Hamiltonian constraints are already there!

Dual regularization mechanism to Thiemann-Hamiltonian.

- non-graph changing (interpretation as tent move)
- for `flat or homogeneous sector' (stacked spheres) and in (2+1)D free of discretization anomalies

[classical: BD & Ryan 08, Bonzom & Dittrich 13 quantum: Barrett , Crane 96, Bonzom 11, Bonzom, Freidel 11,]

- `graph' changing: Pachner or Alexander moves: spin foam dynamics
- dynamics can be understood as first refining and then imposing dynamics [Example: BD, Steinhaus 13]
- could impose dynamics by gluing spin foam amplitudes [Alesci, Rovelli 10]

On Hamiltonian dynamics and simplicity constraints

Coming back to "spin foam amplitudes give the physical vacuum"

(Why are we not already with a very physical vacuum?

This BF vacuum has constant distribution in twisted geoemtries.) [Speziale, Freidel 10]

Can we get (Regge) physical vacuum with (almost) constant distribution in Regge like geometries, and (some) suppression of non-Regge geometries?

This however is a non-local problem (Area constraints are non-local).

Tautological claim :

Imposition of simplicity constraints making everyone happy

equivalent to

Continuum limit, i.e. with construction of physical vacuum.

We therefore need coarse graining and refining ...

... coming in the next ILQGS talk.

Conclusions and outlook

We have an understanding of how to construct the physical vacuum as a continuum object, starting from spin foam amplitudes.

The construction of the BF vacuum /representation is a nice exercise towards this end:

Realization of a condensate state.

Very near to spin foam dynamics.

Might facilitate extraction of low energy physics, cosmology etc.

Many generalizations possible. One is:

Does it allow SL(2,C) Hilbert space, supporting self dual variables?