A holographic description of boundary gravitons

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1511.05441
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1710.04202, 1710.04237, 1803.02759 (CQG Letters)

Seth Asante, BD, Hal Haggard
To appear soon

ILQGS, Nov 2018

Overview

1. Holography and boundary degrees of freedom

2. Holographic description for 3D quantum gravity amplitudes.

3. Can this be generalized to 4D?

4. Outlook

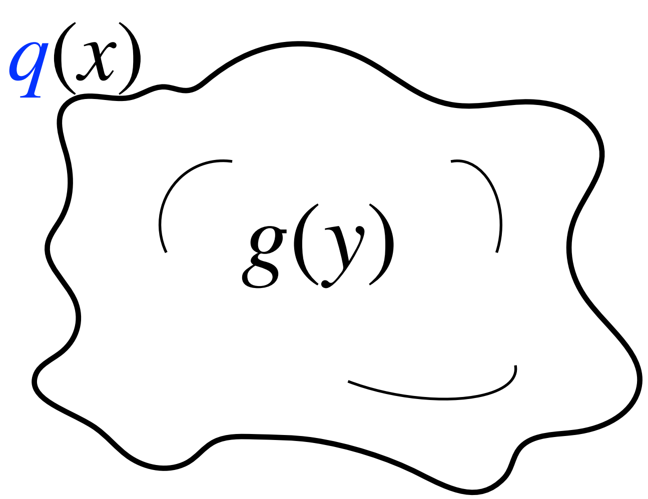
Holography

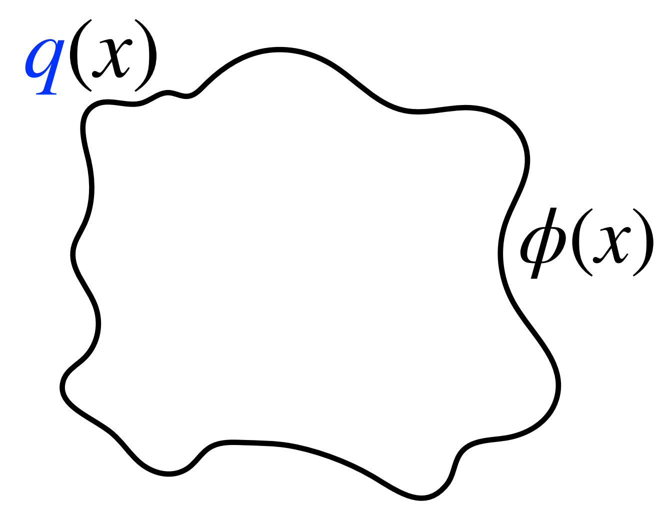
Partition function
for D-dimensional gravity
as functional for boundary geometry

$\simeq$

Partition function
for a (D-1)-dimensional boundary theory
coupled to boundary geometry

$$\int_{g(y(x))=q(x)} \mathscr{D}g(y)\ \ \exp(\mathrm{i}S_{\text{grav}}[g(y)]) \quad \simeq \quad \int \mathscr{D}\phi(x)\ \ \exp(\mathrm{i}S_{\text{dual}}[\phi(x);q(x)])$$





- usually proposed for asymptotic boundaries:
  great restriction on allowed boundary geometries

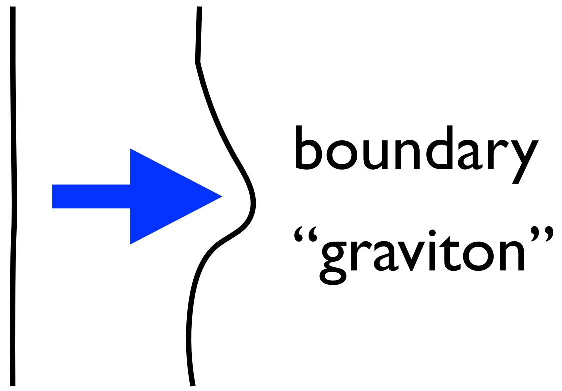
- open problem: bulk reconstruction

Boundary degrees of freedom

Gauge degrees of freedom are converted into "physical" degrees of freedom due to the presence of a boundary.

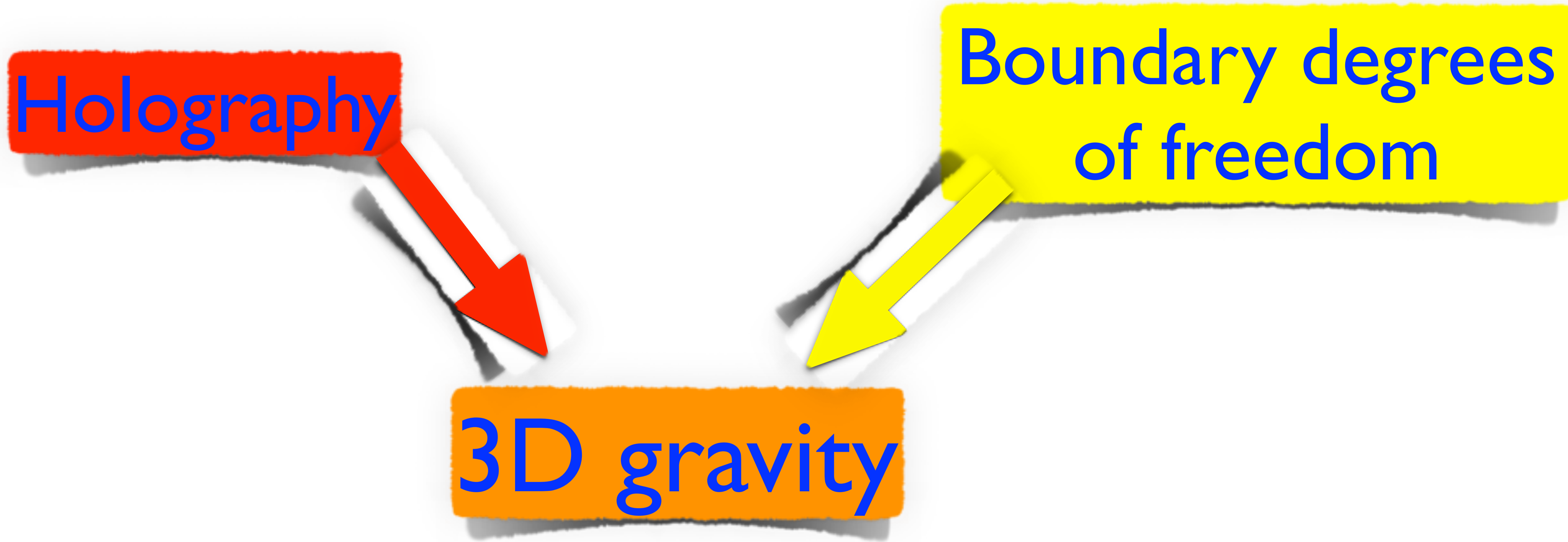
✤ diffeomorphisms deforming the boundary

[Carlip 94: entropy for BTZ black holes]



Why are these boundary degrees of freedom "physical"?

[Barnich, Carlip, Donnely, Freidel, Geiller, Gomes, Perez, Pranzetti, Riello, Strominger, …]

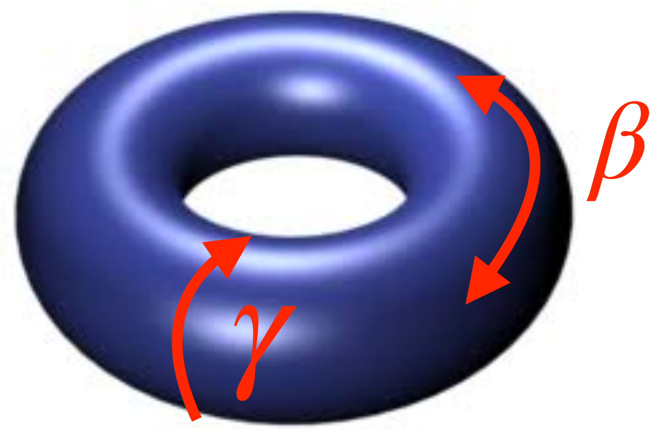


- No propagating bulk degrees of freedom in 3D gravity.
- Only boundary degrees left (and finitely many topological ones).
- These can be used to construct a holographic boundary field theory.

3D gravity: partition function for solid torus

One-loop partition function
(for asymptotically flat boundary)

[Barnich, Gonzalez, Maloney, Oblak 15]



$$\mathscr{A}[\beta,\gamma] = \exp\left(\frac{\beta}{8G}\right) \times \prod_{k\geq 2} \frac{1}{|1-\exp(\mathrm{i}k\gamma)|^2}$$

- Matches a BMS3 character [Oblak 15]

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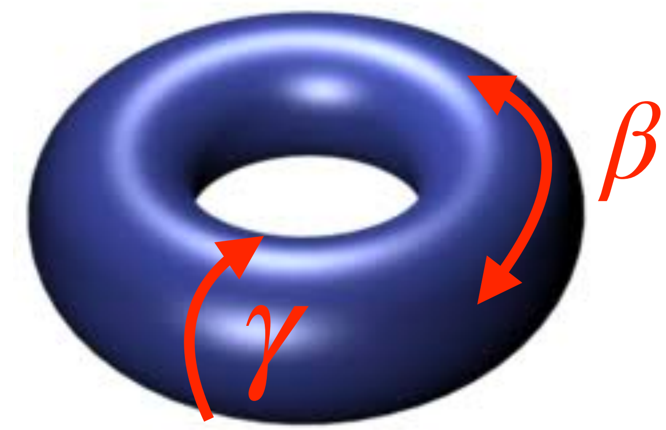


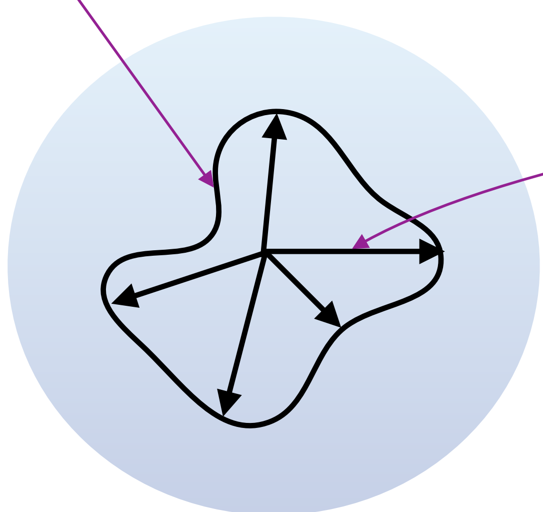
$$\mathscr{A}[\beta,\gamma] = \exp\left(\frac{\beta}{8G}\right) \times \prod_{k\geq 2} \frac{1}{|1-\exp(ik\gamma)|^2}$$

One loop correction
(coming from the determinant of the physical part of Hessian of the action)

We thus have (field) degrees of freedom.

What is the dynamics of these degrees of freedom?

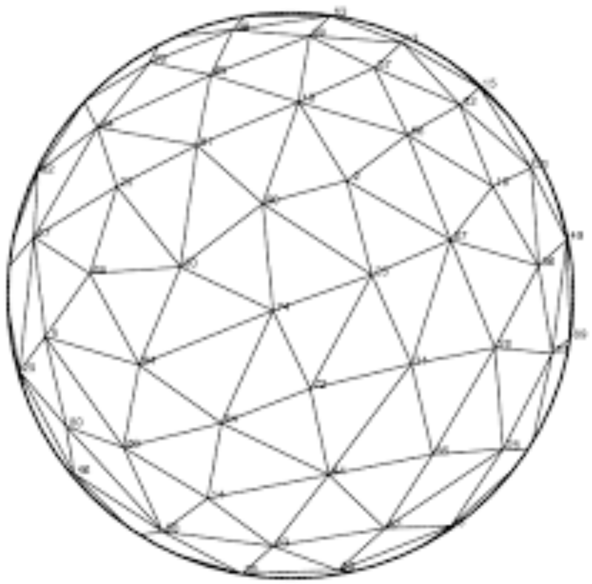
Describing boundary deformations



Define an effective action for the geodesic lengths,
which is induced by dynamics for 3D gravity.

Use Regge calculus to construct boundary theory

- discretization of gravity based on a triangulation
- variables are the lengths of edges: can be identified with geodesic lengths

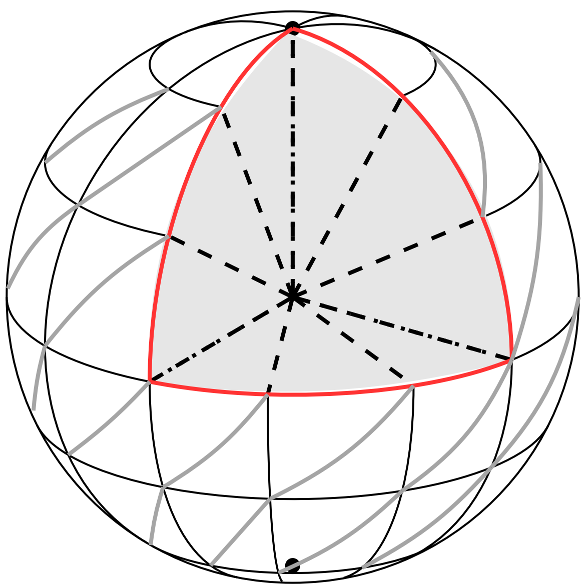


- Triangulate ball-shaped region
- Integrate out all edge length except (possibly) coarse grained lengths from some central point to boundary
- Take continuum limit on boundary.

Congratulations!
You got an (effective) boundary field theory.

Is it local?

In 3D: it is! (For ball shaped regions)



- Action of Regge gravity (evaluated on solution) is invariant under changes of bulk triangulation.
- One loop partition function (that is the path integral measure) is invariant under changes of bulk triangulation. [BD, Steinhaus 11]

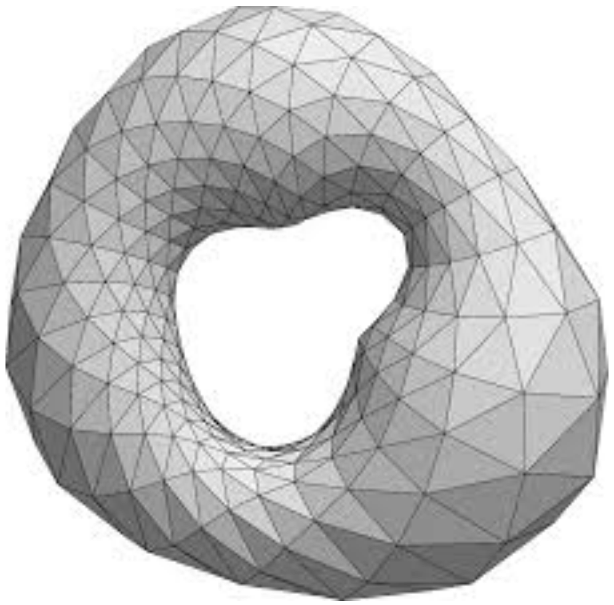
- We can choose for the evaluation of the (one-loop) path integral the coarsest available triangulation.
- This triangulation has only radial edges, whose lengths defines the boundary field.

The effective boundary theory is given by the path integral based on this triangulation, and is therefore local.

'Side results':
✳Works for finite (non-asymptotic) boundaries.
✳Bulk reconstruction for free.

Back to solid torus: Regge calculus

[Bonzom, BD 15]



- need to integrate out some variables
- but still "essentially" a local boundary field theory

One loop partition function:

- reproduces continuum result by Barnich et al
- but holds for finite boundary
- extended to fluctuating boundary metric:
  extract boundary action
- explains puzzling features of the asymptotic one-loop partition function

Back to solid torus: Regge calculus

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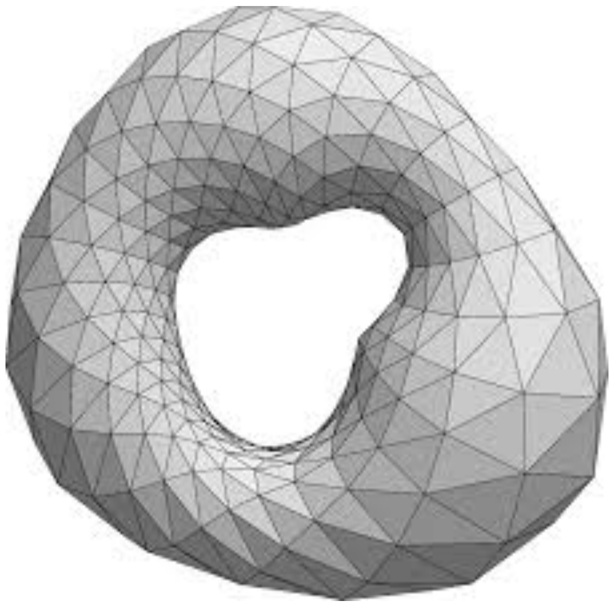

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One loop partition function

$$\mathscr{A}[\beta,\gamma] = \exp\left(\frac{\beta}{8G}\right) \times \prod_{k\geq 2} \frac{1}{|1-\exp(\mathrm{i}k\gamma)|^2}$$

lower modes missing as they describe rigid translations of torus

singular for rational angles γ

Boundary field theory with Liouville coupling

$$S_{\text{bdry}} = \int d^2x \sqrt{h}\, \left(\phi Q^{ab} \nabla_a \nabla_b \phi \,-\, \mathscr{R}\phi\right)$$

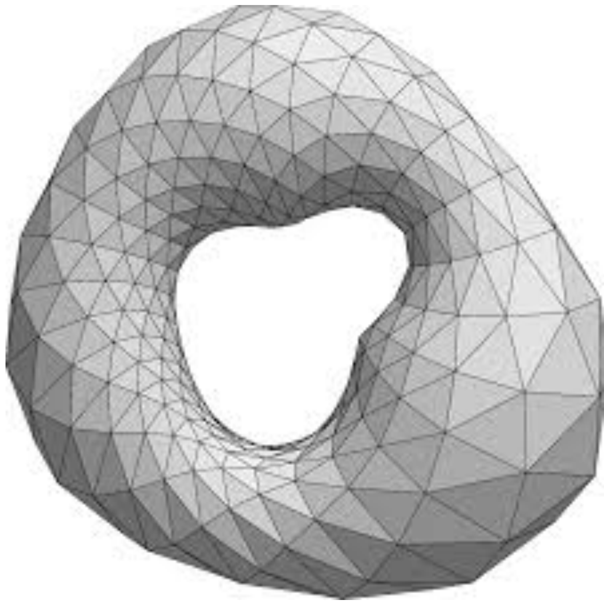
kinetic term is degenerate

$$Q^{ab} = K^{ab} - K h^{ab}$$

Similar actions obtained with completely different and more indirect methods in [Barnich, Gomberoff, Gonzales 13; Carlip 16: in Lorentzian signature]

Fully non-perturbative boundary theory

(for finite boundary)



Use the Ponzano-Regge model which allows straightforward implementation of metric boundary conditions (as opposed to Chern-Simons).

[BD, Goeller, Livine, Riello]
1710.04202, (NPB)
1710.04237, (NPB)
1803.02759 (CQG Letters)

[Riello]
1802.02588 (PRD)

- find different boundary theories depending on choice of boundary quantum geometry and choice of boundary field

⋆ deep quantum: statistical (including integrable) models, in particular six vertex models
⋆ general spin network: RSOS model, boundary field gives radial distance
⋆ semi-classical: non-linear sigma model
  - first evaluation of Hessian determinant for extended triangulation
  - reproduces one-loop result, but it is extended by "Planckian" backgrounds

- singularity structure: only arises in asymptotic or semi-classical limit

3D quantum gravity

Can be understood as a theory describing the embedding of a quantum surface into quantum flat (or homogeneously curved) space.

Boundary degrees of freedom can lead to highly non-trivial boundary theories.

What about 4D?

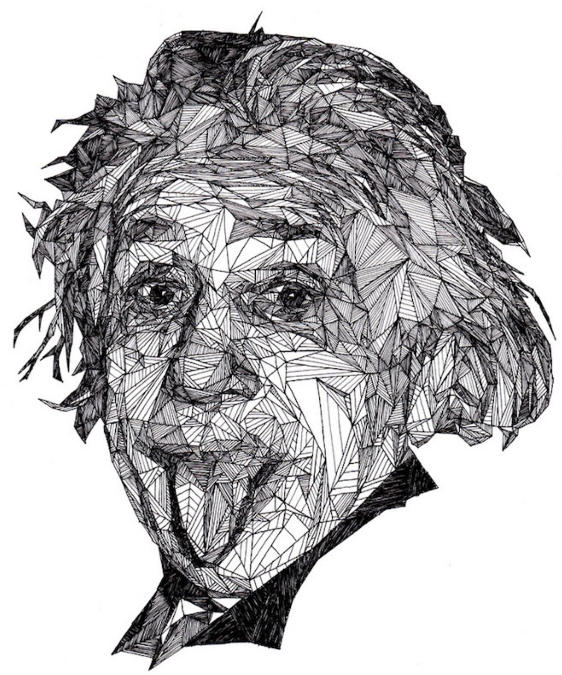
[Asante, BD, Haggard, to appear very soon]

Remark: Can use 4D BF theory to generalize the results for the Ponzano-Regge model.

But we are looking for something closer to gravity.

3D gravity vs 4D gravity

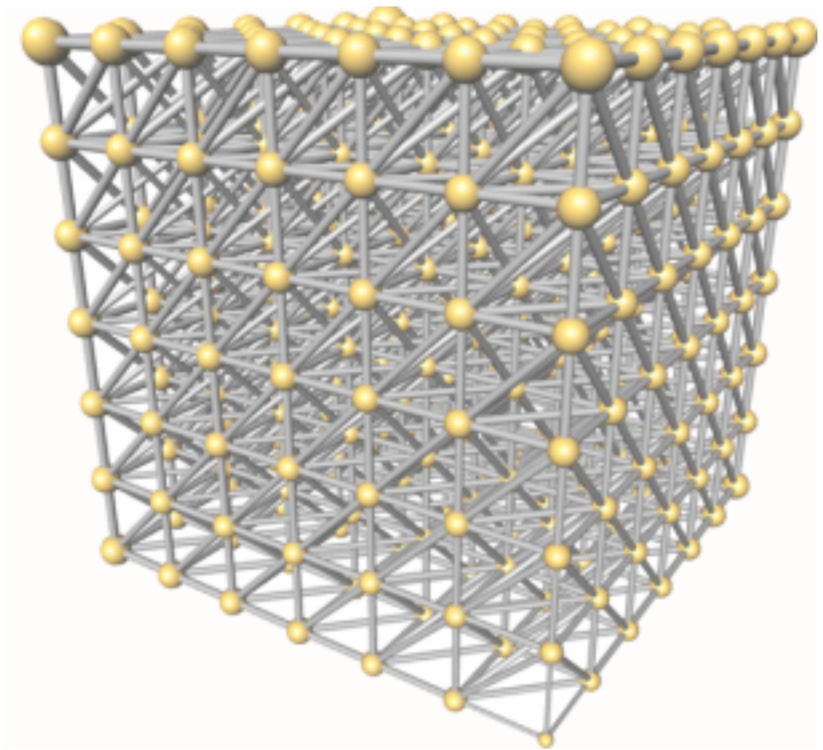
3D gravity



Any (triangulated) 2D surface can be locally embedded into 3D flat space.

There is a flat solution for all boundary data.

4D gravity



A 3D geometry needs to satisfy (4D flatness) conditions to be embeddable in flat 4D space.

But there are 3D internally curved hypersurfaces embeddable into 4D (flat) space.

Flat sector in 4D gravity

3D boundary metric

6 modes

Flat sector

4 modes inducing flat solutions (diffeomorphism)

Boundary diffeos

3 modes induced by diffeos tang. to boundary

Boundary graviton

One mode induced by diffeos, normal to tangential modes

Bulk gravitons

2 modes inducing curved solutions

Can impose the flat sector by considering only boundary metrics inducing flat solutions.

Regge quantum gravity

$$\mathcal{Z}_{\text{Regge}}(l_{\text{bdry}}) =$$

$$\int \prod_{\text{bulk}} d\mu(l)\ \exp(-S_{\text{Regge}}[l])$$

- For flat sector: on-shell action invariant under changes of the bulk triangulation

- Thus as in 3D, we can use the coarsest available bulk triangulation.

- There is no (local) triangulation invariant path integral measure (to one loop).

[BD, Kaminski, Steinhaus 14]

- But singularities in one-loop correction (as appeared in 3D) are bulk triangulation invariant.

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RBF-model of flat space

[Baratin, Freidel 06]

$$\mathcal{Z}_{\text{BFR}}(l_{\text{bdry}}) =$$

$$\int \prod_{\text{bulk}} d\mu_{\text{BFR}}(l) \prod_{\text{bulk}-\text{triang}} \underset{\text{delta functions on deficit angles}}{\delta(\epsilon_t(l))} \;\; \exp(-\underset{\text{reduces to boundary term}}{S_{\text{Regge}}}[l])$$

Highly divergent!

- remove redundant delta functions, so that the result is triangulation invariant.
- diffeo-symmetries as in Regge

Regge quantum gravity

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- Only non-vanishing for flat sector

- On-shell action agrees with Regge on flat sector.

- (One-loop) path integral is triangulation invariant.

Constructing the boundary field theory

- Choose a background space time.
  To have a model we can do computations with:
  4D version of the solid torus — the solid three-torus

- Choose triangulation, such that
  - one can take continuum limit on boundary
  - we have a very coarse bulk triangulation (for now)

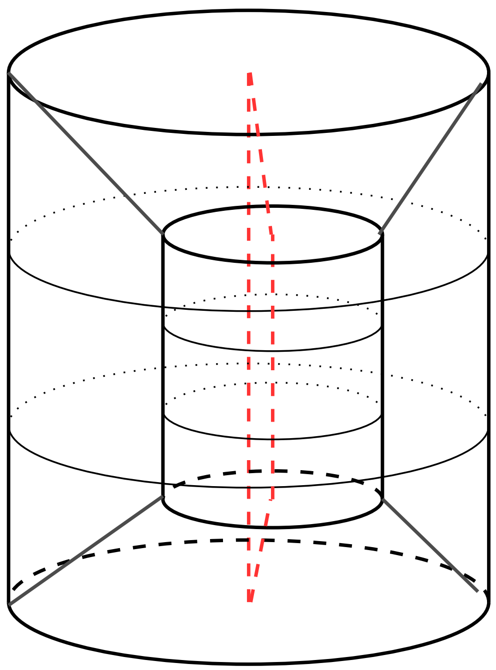
- Construct linearized Regge action for triangulation.

- Integrate out all edge lengths, except radial edges going from the boundary to a central 2d axis.

- Identify flat sector for boundary lengths.

- Restrict effective action to this flat sector.

Background space time: solid three-torus



Flat space: $$ds^2 = dr^2 + r^2 d\theta^2 + dy^2 + dz^2$$

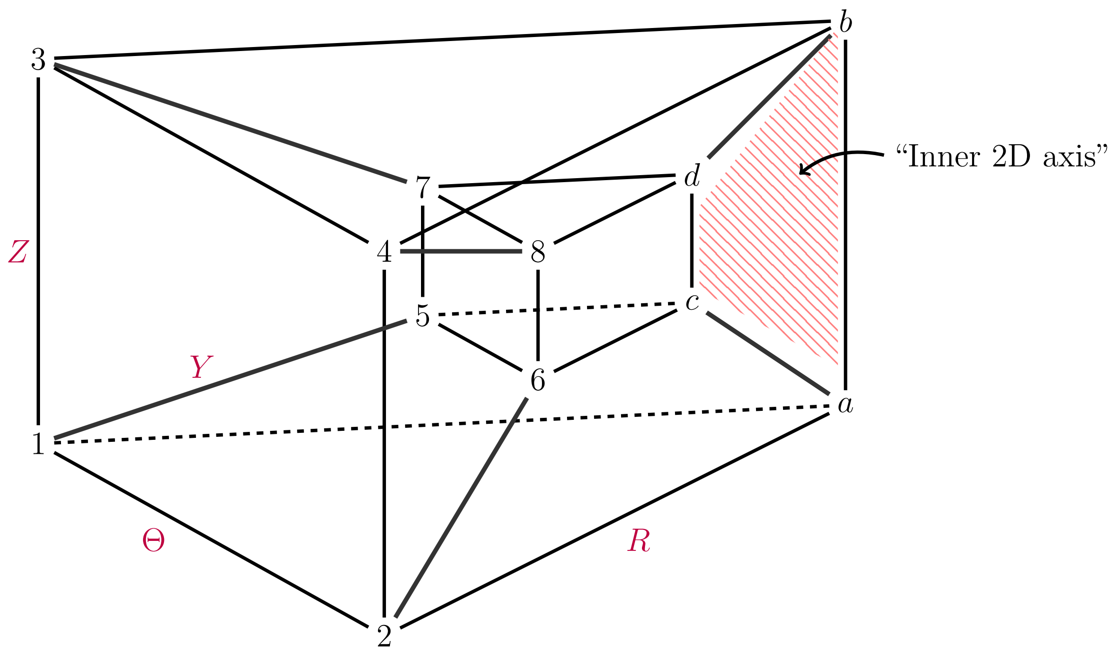
with twists:
$$\begin{aligned}
(r,\theta,y,z) &\sim (r,\theta+2\pi,y,z),\\
(r,\theta,y,z) &\sim (r,\theta+\gamma_y,y+\alpha,z),\\
\text{and}\quad (r,\theta,y,z) &\sim (r,\theta+\gamma_z,y,z+\beta).
\end{aligned}$$

Boundary: $r = R$

Twisted Fourier transform on the boundary:

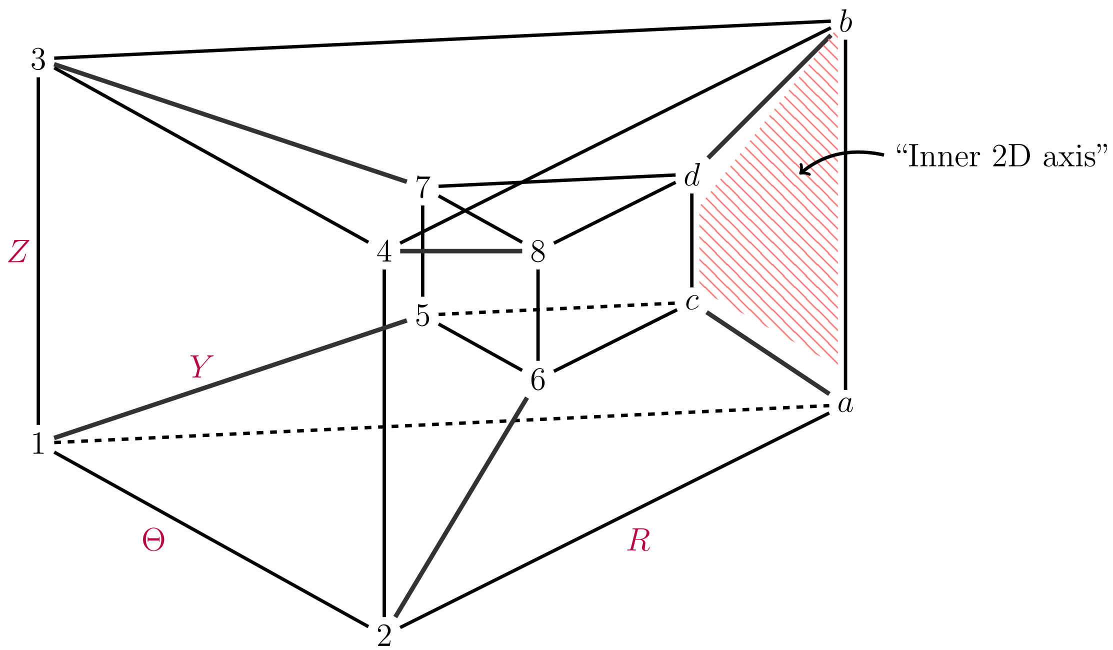
$$f(k_\theta, k'_y, k'_z) \sim \int d\theta dy dz\, f(\theta, y, z)\; e^{-\mathrm{i}\theta k_\theta - \mathrm{i}y\underbrace{(k'_y - \frac{\gamma_y}{\alpha}k_\theta)}_{k_y} - \mathrm{i}z\underbrace{(k'_z - \frac{\gamma_z}{\beta}k_\theta)}_{k_z}}$$

Triangulation



Triangulated Hyper-prisms
-glued to thin solid hyper-cylinders,
-stacked into a thick solid hyper-cylinder,
-whose boundaries are identified to a solid 3-torus.

Triangulation



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Bulk edge length variables		
Edges on hyper-prism	Length of edge(s)	Length fluctuations
$e(1a)$ $e(5c)$ $e(2a)$ $e(6c)$ $e(3b)$ $e(7d)$ $e(4b)$ $e(8d)$	R	ℓ_r
$e(1c)$ $e(3d)$ $e(2c)$ $e(4d)$	$\sqrt{R^2+Y^2}$	ℓ_{ry}
$e(1b)$ $e(5d)$ $e(2b)$ $e(6d)$	$\sqrt{R^2+Z^2}$	ℓ_{rz}
$e(1d)$ $e(2d)$	$\sqrt{R^2+Y^2+Z^2}$	ℓ_{ryz}
$e(ac)$ $e(bd)$	Y	ℓ_φ
$e(ab)$ $e(cd)$	Z	ℓ_ζ
$e(ad)$	$\sqrt{Y^2+Z^2}$	$\ell_{\varphi\zeta}$

Effective action for radial edge length coupled to boundary geometry.

Integrate out.

Boundary edge length variables		
Edges on hyper-prism	Length of edge(s)	Length fluctuations
$e(12)$ $e(56)$ $e(34)$ $e(78)$	Θ	ℓ_θ
$e(15)$ $e(37)$ $e(26)$ $e(48)$	Y	ℓ_y
$e(13)$ $e(57)$ $e(24)$ $e(68)$	Z	ℓ_z
$e(16)$ $e(38)$	$\sqrt{\Theta^2+Y^2}$	$\ell_{\theta y}$
$e(14)$ $e(58)$	$\sqrt{\Theta^2+Z^2}$	$\ell_{\theta z}$
$e(28)$ $e(17)$	$\sqrt{Y^2+Z^2}$	ℓ_{yz}
$e(18)$	$\sqrt{\Theta^2+Y^2+Z^2}$	$\ell_{\theta yz}$

Boundary metric: flat and curved sectors

Boundary metric	Boundary graviton mode	Simple graviton mode	Complicated graviton mode

$$\begin{pmatrix} h_{\theta\theta} \\ h_{yy} \\ h_{zz} \\ h_{\theta y} \\ h_{\theta z} \\ h_{yz} \\ h_{\theta yz} \end{pmatrix} \longrightarrow \begin{pmatrix} (k_y^2+k_z^2)^2 \\ k_\theta^2 k_y^2 \\ k_\theta^2 k_z^2 \\ -\frac{1}{R^2} k_\theta k_y (k_y^2+k_z^2) \\ -\frac{1}{R^2} k_\theta k_z (k_y^2+k_z^2) \\ k_\theta^2 k_y k_z \end{pmatrix} \quad \begin{pmatrix} 0 \\ k_z^2 \\ k_y^2 \\ 0 \\ 0 \\ -k_z k_y \end{pmatrix} \quad \begin{pmatrix} 0 \\ -k_\theta k_y k_z \\ +k_\theta k_y k_z \\ +\frac{1}{2} R^2 k_z^2 (k_y^2+k_z^2) \\ -\frac{1}{2} R^2 k_y^2 (k_y^2+k_z^2) \\ \frac{1}{2} k_\theta (k_y^2-k_z^2) \end{pmatrix}$$

Can be completed to orthonormal basis,
by adding (orthonormalized) boundary diffeos.
Also available for the discretization.
Graviton modes happen to be distinguished dynamically.

Allows to construct projectors onto flat and curved sector.

Result

Second order on-shell action splits into two terms with different R scaling (as in 3D):

$$S_{|\mathrm{sol}} = R\, S_{\mathrm{dom}} \; + \; R^{-1}\, S_{\mathrm{surp}} \; + \; \mathcal{O}(R^{-3})$$

Invariant under boundary diffeos. Quite simple.

Not invariant under boundary diffeos. Quite complicated.

On the flat sector the dominant term of radial effective action is local and given by (second order expansion of):

$$S_{\mathrm{bdry}} = \int d^3x \sqrt{h}\, \left(\phi Q^{ab} \nabla_a \nabla_b \phi \; - \; \mathscr{R}\phi\right)$$

$$Q^{ab} = K^{ab} - K h^{ab}$$

Exactly as in 3D!

Boundary gravitons described by a scalar theory (with degenerate kinetic term)

coupled to boundary Ricci-scalar.

On the one-loop correction

We also have a degenerate kinetic term: $$Q^{ab}\nabla_a\nabla_b \sim (k_y^2 + k_z^2)$$

$$= (k'_y - \frac{\gamma_y}{\alpha}k_\theta)^2 + (k'_z - \frac{\gamma_z}{\beta}k_\theta)^2$$

May have zeros!   Depends on twist angles.

There are no solutions (to the linearized EOM), for a certain subspace of the flat sector.

The one-loop correction includes a factor from integrating out the radial field:

$$\prod_{k_\theta \geq 2} \prod_{k'_y, k'_z} \frac{1}{(k_y^2 + k_z^2)^{1/2}}$$

due to bulk diffeos

Singularities, resulting from the zeros above,
will persist if we consider a refinement of the triangulation.

Appears also in BFR version.

What have we learned?

• Boundary theory describing boundary gravitons has geodesic distance as field variable.

(Could have expected something involving areas of minimal surfaces as suggested by entanglement based bulk reconstruction proposals.)

Boundary theories very similar in 3D and in 4D.

➡Suggest a general mechanism.

• Hamilton-Jacobi functional / amplitudes have simpler structure for terms dominating at large radius

➡Suggests an approximation scheme for quantum gravity amplitudes.

Many directions to go

Fully non-perturbative description of flat space:

Use 2-categorical state sum. Describes a 3D Membrane embedded in (flat) 4D space.

[Baratin, Freidel 14]

Radial refinement and holographic renormalization: [wip]

Consider a refinement of bulk triangulation and find change of effective action.
Provides a new avenue to define (one-loop) path integral measure.
Connect to holographic renormalization.

Generalized boundaries:

Generalize to other backgrounds and boundary and bulk topologies.
Develop approximation scheme for Hamilton-Jacobi functionals/ quantum gravity amplitudes around "simple" boundaries.

More wide open questions:

Relation to BMS4?
Black hole entropy?

Summary

Aim:
Concrete model for holography, allowing for finite boundaries and explicit bulk reconstruction, connecting perturbative and non-perturbative frameworks.

In 3 and 4 dimensions we have a holographic description for flat/topological sector:
Can this be used in 4D as a starting point for expansion of full theory around flat sector?

Explore connection of quantum gravity to (a new kind of) topological theory:
Many connections to recent developments in condensed matter (topological phases).