On the continuum limit of spin foams

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(Perimeter Institute)

ILQGS, Nov 28 2023

Overview

- The need for a refinement limit: diffeomorphism symmetry?
- Perfect discretizations
- Consistent boundary formalism and coarse graining algo's
- Renormalization flow in diff-invariant theories
- Continuum limit for spin foams: perturbative results

Papers starting 2008/2009: w/ Seth Asante, Benjamin Bahr, Johanna Borissova, Clement Delcamp, Hal Haggard, Philipp Hoehn, Marc Geiller, Mercedes Martin-Benito, Jose Padua-Arguelles, Sebastian Steinhaus and others.

Recent review: Asante, BD, Steinhaus 2022, Handbook of Quantum Gravity

Diffeomorphism symmetry:
a guiding principle for quantization

- Ensures correct number of propagating degrees of freedom / correct dynamics
- Ensures constraint implementation
- Diffeomorphism symmetry in the discrete ensure discretization independence
- Diffeomorphism symmetry in the discrete resolve discretization ambiguities and artifacts

But:

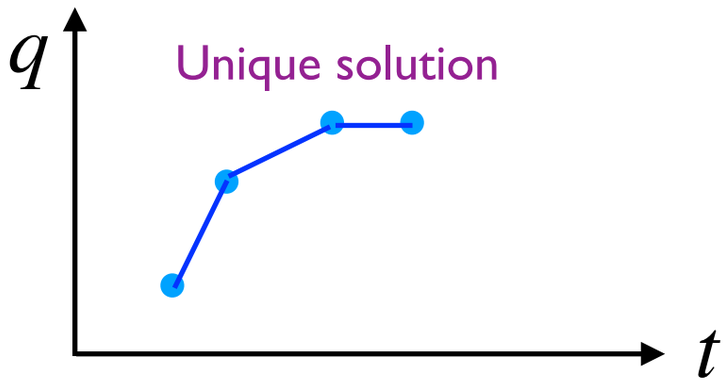
- Discretizations used as regulator for quantization, or arises as consequence of discretization
- Discretizations typically break diffeomorphism symmetry
- Exceptions: TQFT's, (0+1)D systems (with appropriate discretization)

Diffeomorphism symmetry in (0+1)D systems

Reparametrized particle in a potential

$$S_d = \sum_{n=0}^{N-1} \left(\frac{q_{n+1} - q_n}{t_{n+1} - t_n} \right)^2 (t_{n+1} - t_n) - V\left(\frac{q_n + q_{n+1}}{2}\right)(t_{n+1} - t_n)$$

Variables: q_n, t_n



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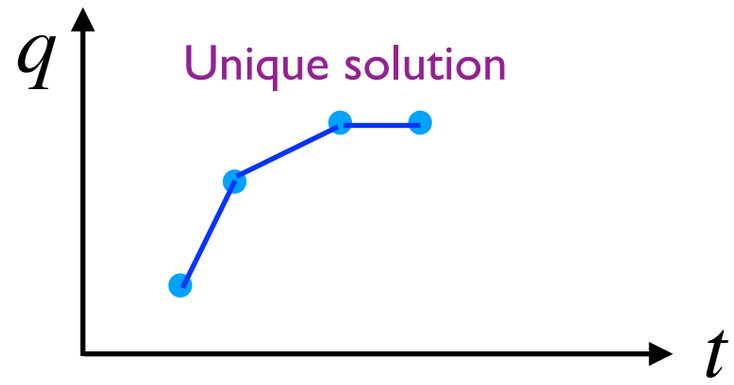

Reparametrization invariance broken.
Hessian evaluated on solution has a large eigenvalue and a small eigenvalue.

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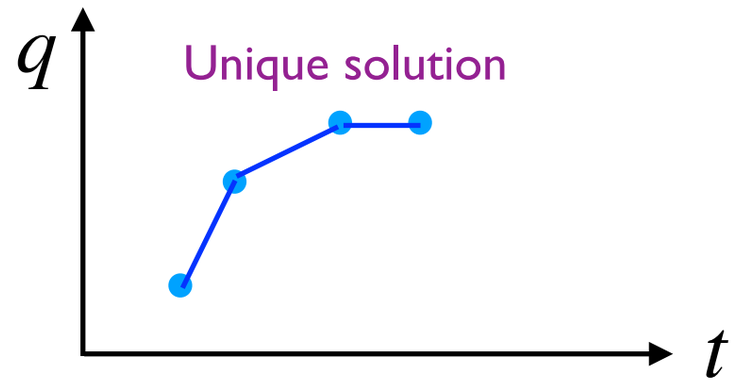

Unique solution

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Perfect discretization/ action

$$S_{perf} = \sum_{n=0}^{N-1} S_{HJ}(q_n, q_{n+1}\,; t_n, t_{n+1})$$

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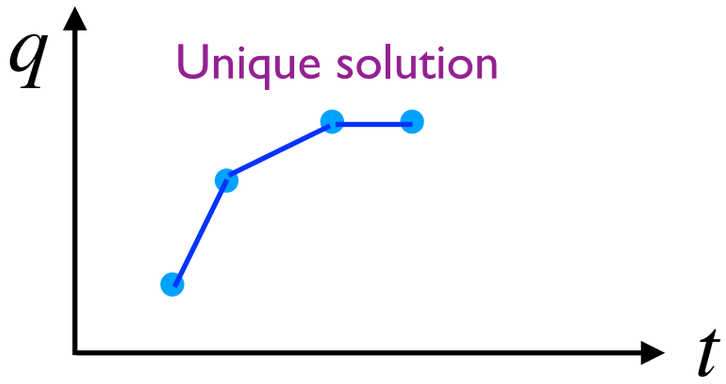

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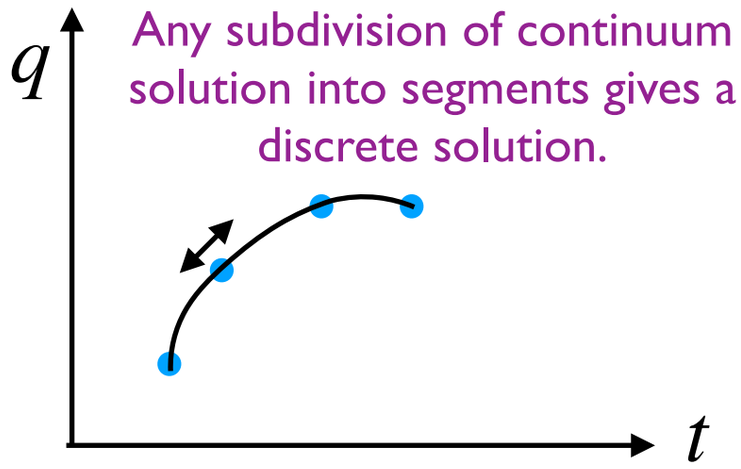

Reparametrization invariance is restored.
Hessian has a null direction.

Diffeomorphism symmetry in (0+1)D systems

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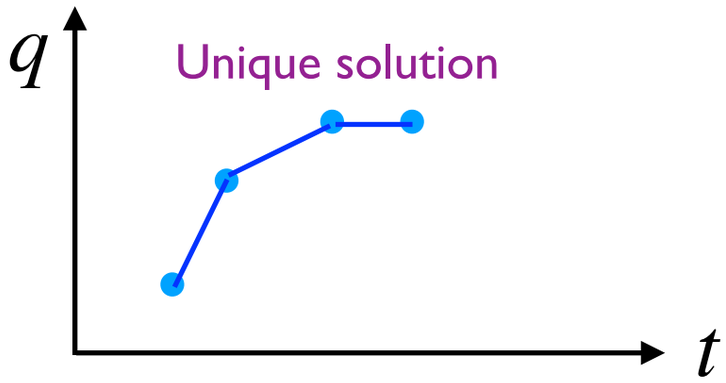

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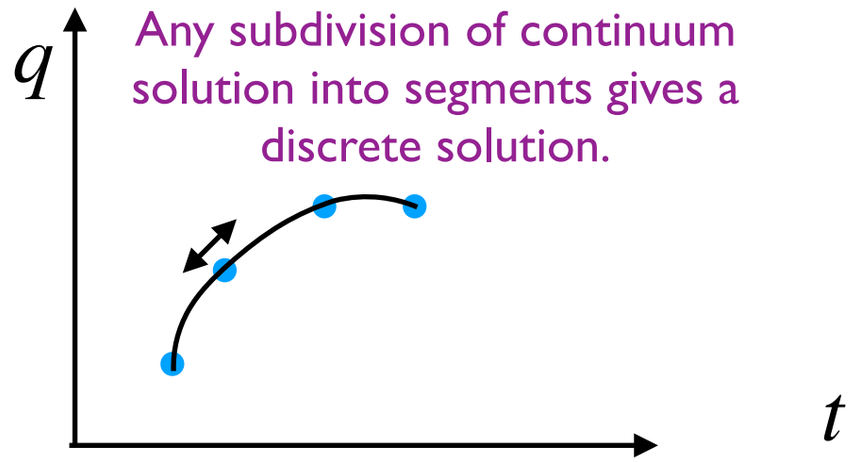

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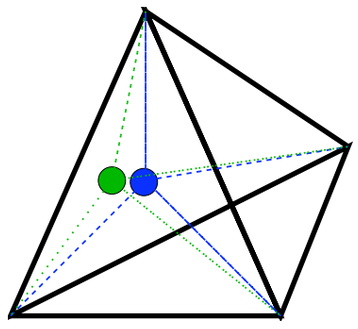
Reproduces/ mirrors perfectly continuum dynamics at arbitrary scales.

Continuum limit becomes trivial.

Diffeomorphism symmetry in Regge gravity

3D Regge, $\Lambda = 0$



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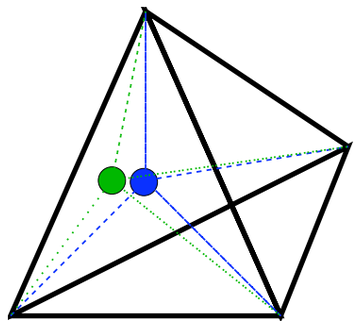

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Diffeomorphism symmetry

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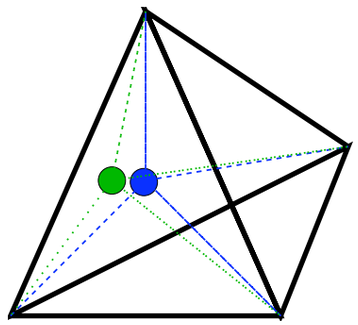

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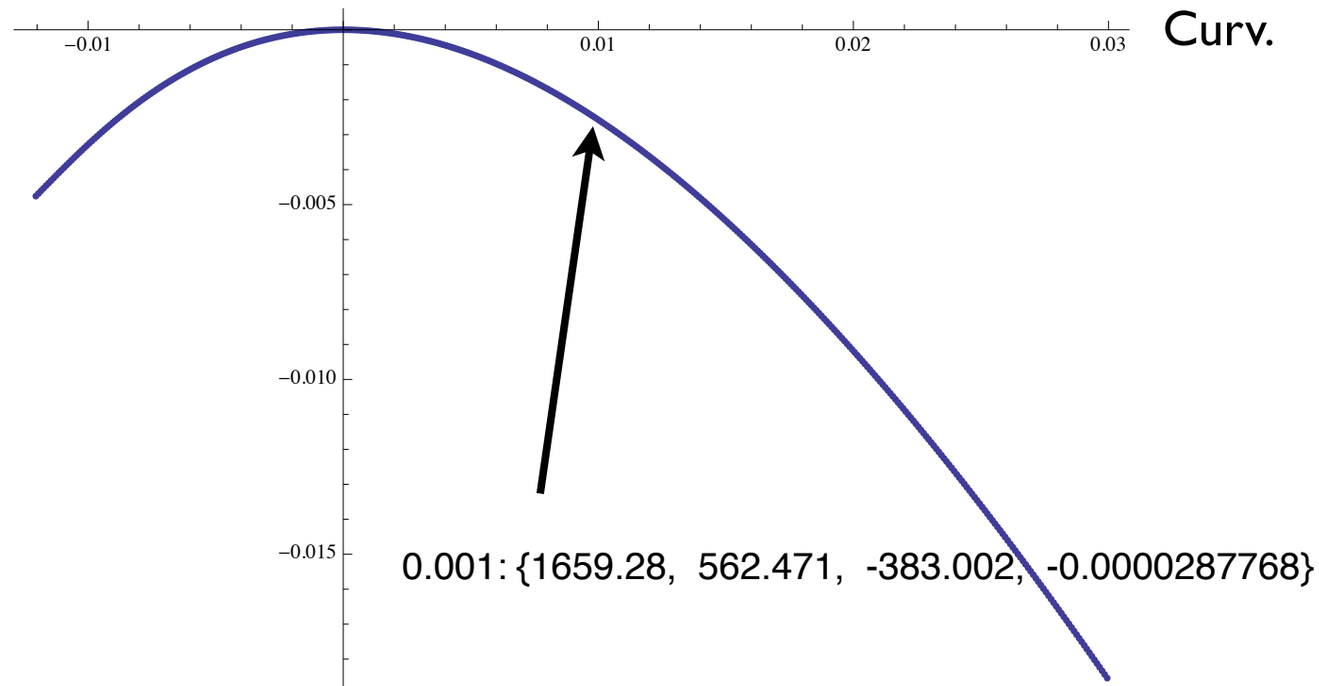
4D Regge, $\Lambda = 0$

[Bahr, BD 09]



Lowest eigenvalue
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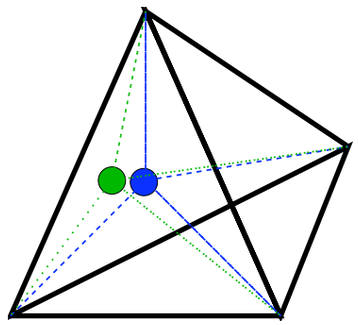

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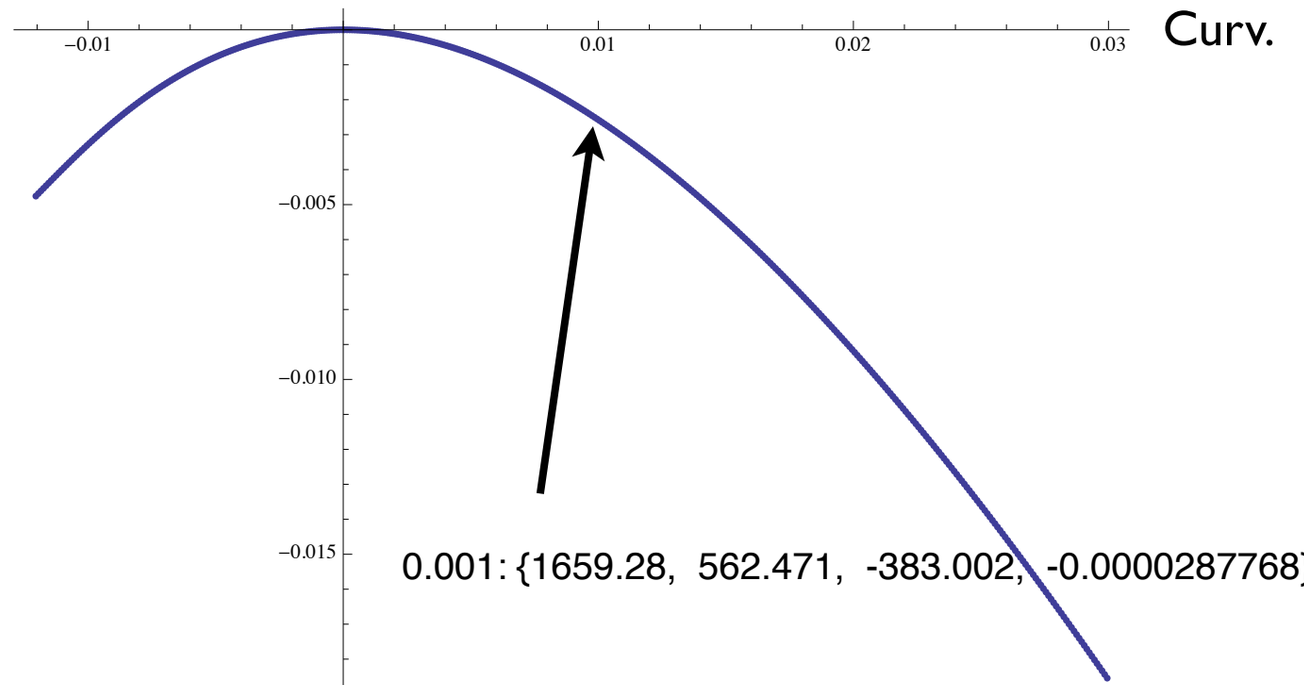

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Diffeomorphism symmetry is broken.

Expect that only solutions with small edge lengths
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Diffeomorphism symmetry in Regge gravity

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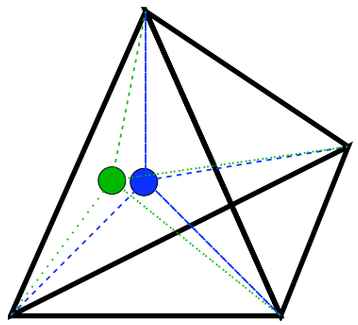

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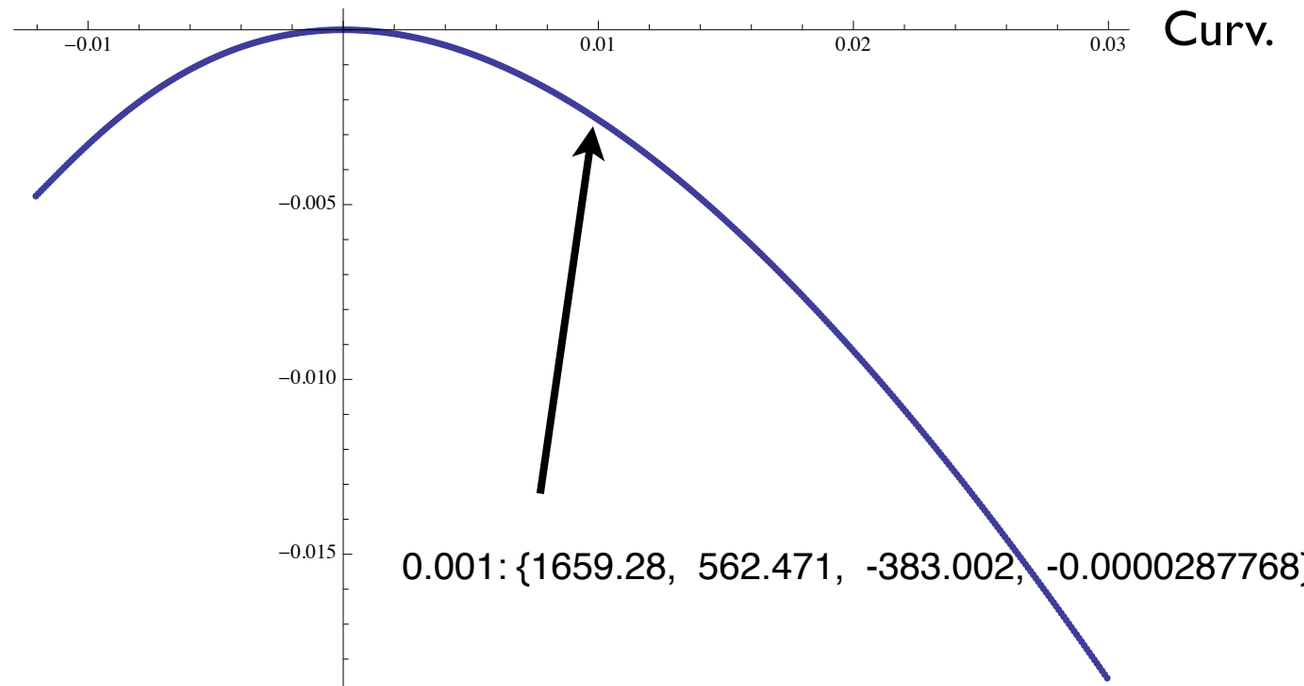

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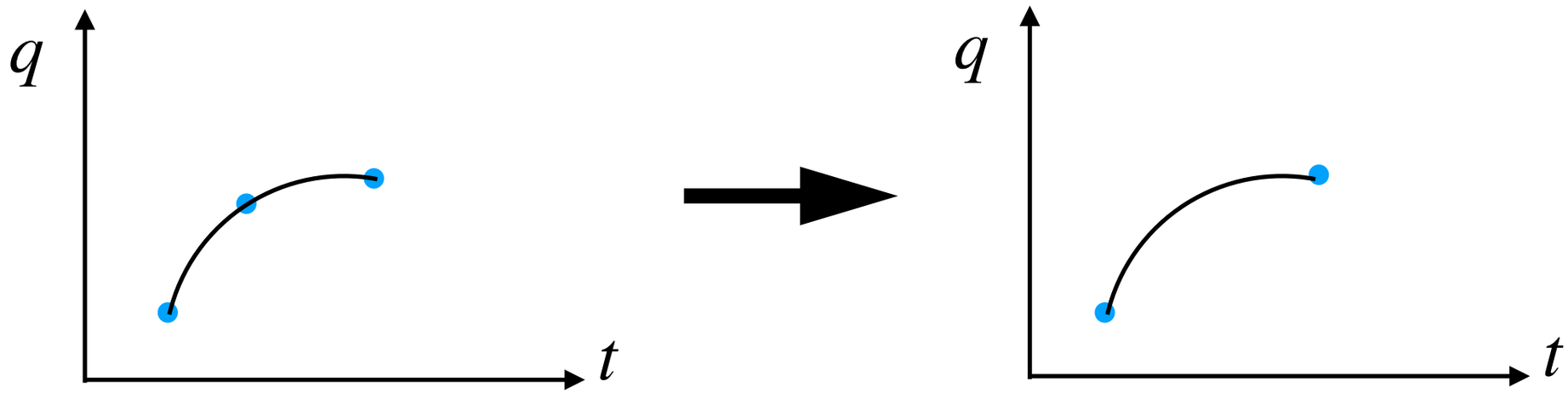
Issue: integrate over configurations with arbitrarily large edge lengths in path integral.

Construct discretizations with diffeomorphism symmetry?

How to construct perfect discretizations?

$$S_{HJ}(q_0, q_2\,; t_0, t_2) = \mathrm{extr}_{q_1}\left(S_{HJ}(q_0, q_1\,; t_0, t_1) + S_{HJ}(q_1, q_2\,; t_1, t_2)\right)$$



Perfect actions are invariant under changes of the discretization.

How to construct perfect discretizations?

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The Hamilton-Jacobi function is a fixed point of the renormalization flow:

$$S'(q_0, q_2\,; t_0, t_2) = \mathsf{extr}_{q_1,t_1}\left(S(q_0, q_1\,; t_0, t_1) + S(q_1, q_2\,; t_1, t_2)\right)$$

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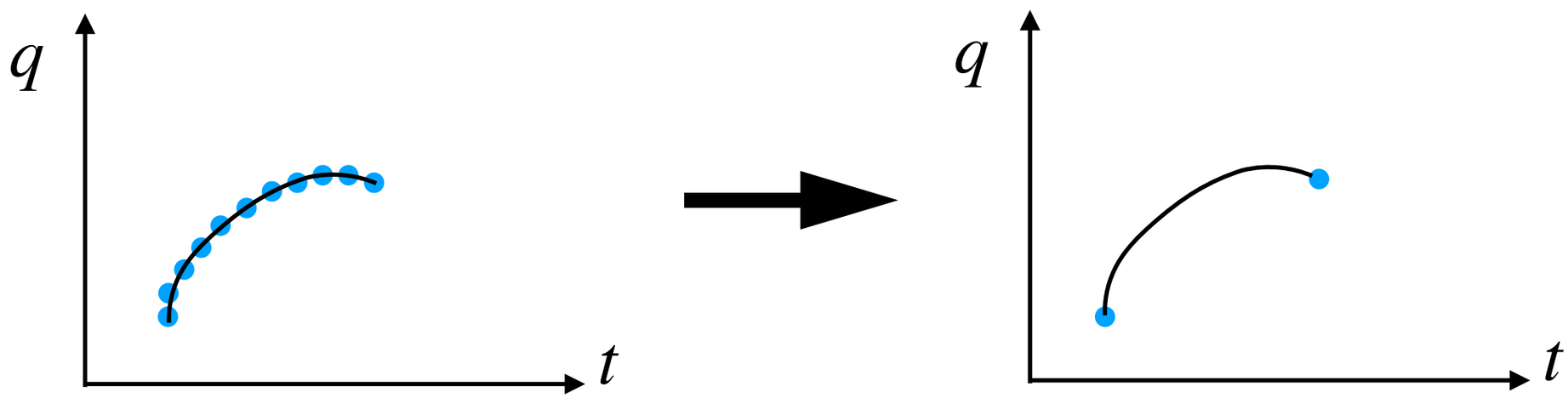
Can be applied in practice, perturbatively, in quantum theory.

[Bahr, BD 2009,
BD, Steinhaus 2011,
…
Asante, BD 2021]

Amounts to solving the theory.

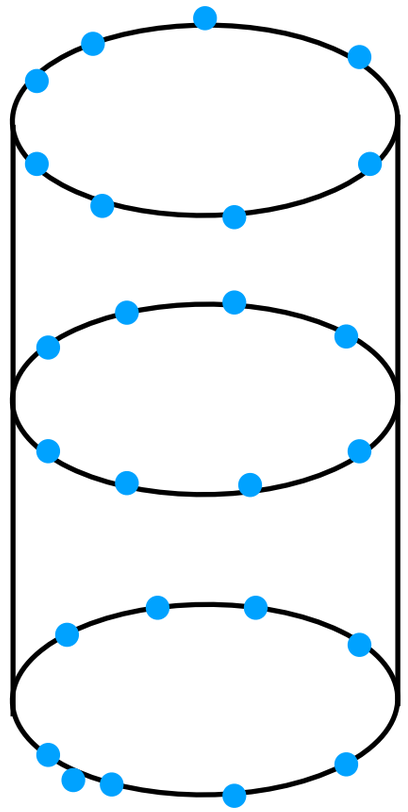


Applications

- (0+1)D systems - anharmonic oscillator: path integral solution via fixed point equations.
  [Bahr, BD, Steinhaus 2011]
- Regge gravity: homogeneously curved simplices as fixed point solutions. [Bahr, BD 2009]
- Regge path integral (3D): Unique, triangulation invariant (one-loop) measure, including $\pi/4$ phase shift.
  [BD, Steinhaus 2011, Borissova, BD 2023]
- Regge path integral (4D): Measure, invariant under 5-1 moves, has to be non-local.
  [BD, Kaminski, Steinhaus 2014]
- Free lattice field theories (w/ gauge symmetries): construction of perfect discretizations: non-local [Bahr, BD, He 2010]
- 4D linearized gravity: construction of perfect non-local discretization and one-loop measure
  [Asante, BD 2021]
- Restricted spin foams: fixing face weights. [Bahr, Steinhaus 2016]

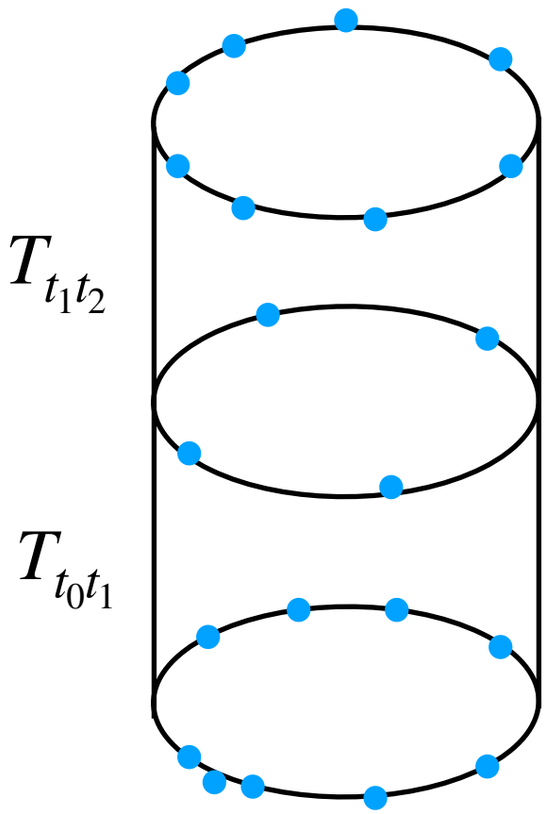
Why do non-topological diffeomorphism symmetric discretizations have to be non-local?

Final data



Initial data

Should give the same evolution.



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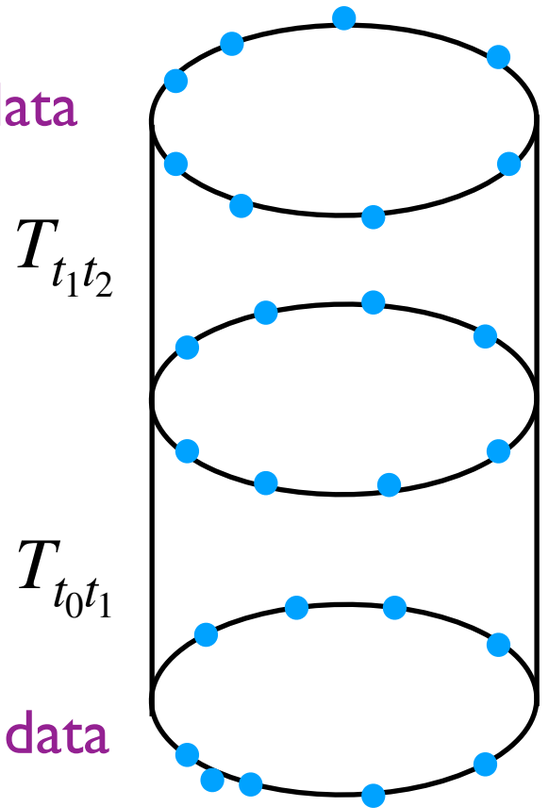

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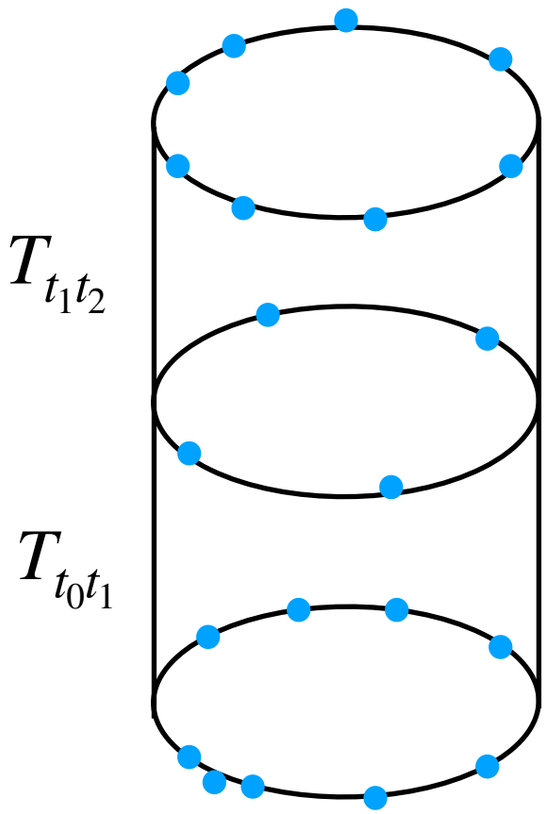

Less data

Non-local action: data of more than one slice are necessary to determine next time slice.

How to avoid non-local amplitudes?

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⇒The consistent boundary formalism.

[BD 2012, BD 2014]

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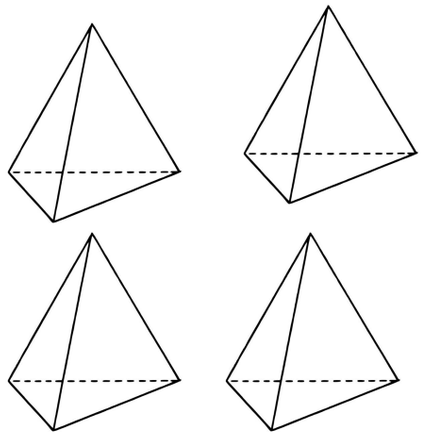
Shift of perspective:

Gluing simplest building blocks
(carrying minimal boundary data)



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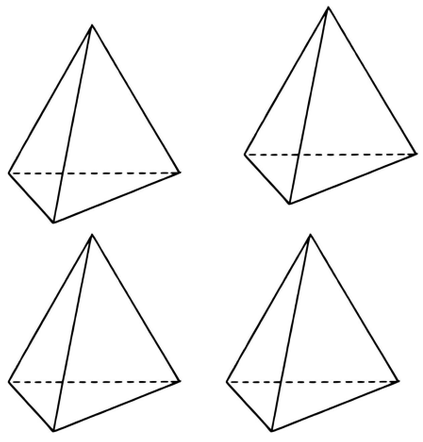
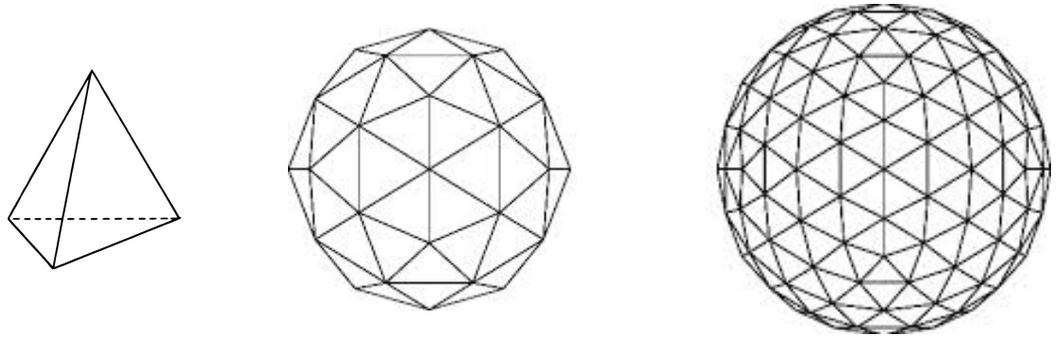


Partially ordered set of building blocks
(with respect to amount of bdry data)



Need consistency relations between amplitudes for these building blocks: Renormalization flow.

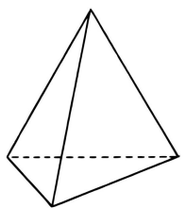
Cylindrically consistent dynamics

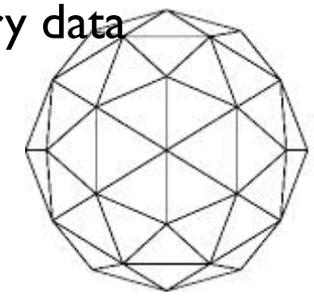
[BD 2012]

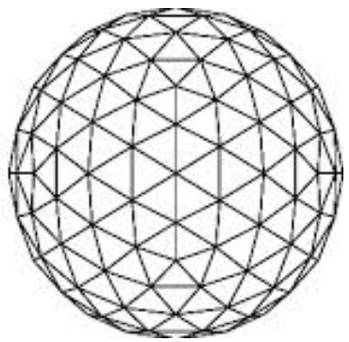
embedding coarse
boundary data
into finer boundary data

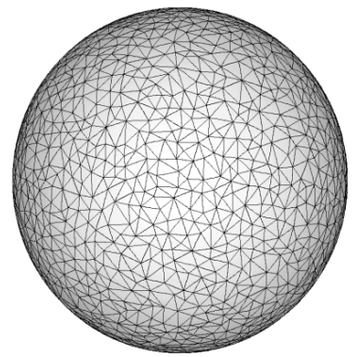
Amplitude for
very fine data.

Amplitude for
coarse data.









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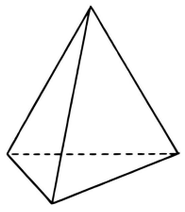
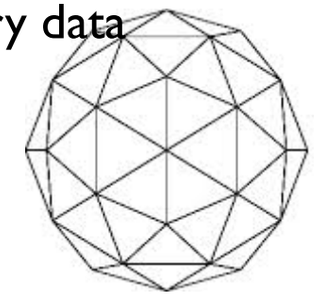
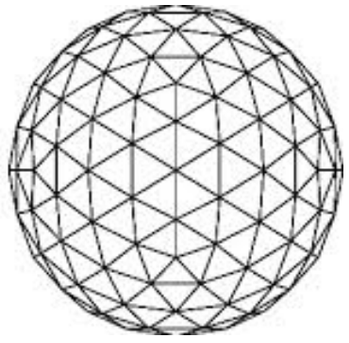
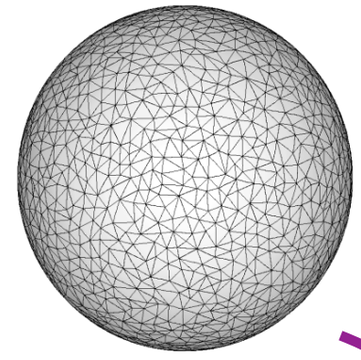








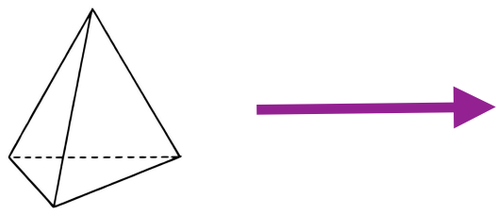
Find same amplitude again.

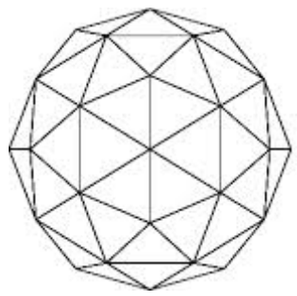
Pull back amplitude to coarse data,
using embedding maps.

Constructing a cylindrically consistent dynamics

1. Start with amplitude for simplest building block.

2. Defines amplitude for more complicated boundary via gluing principle.





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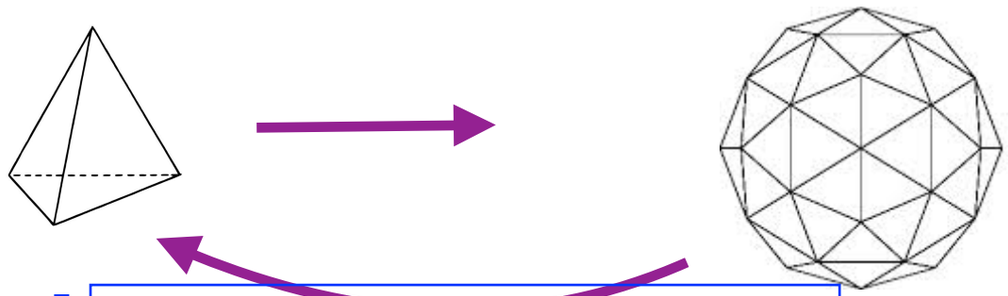
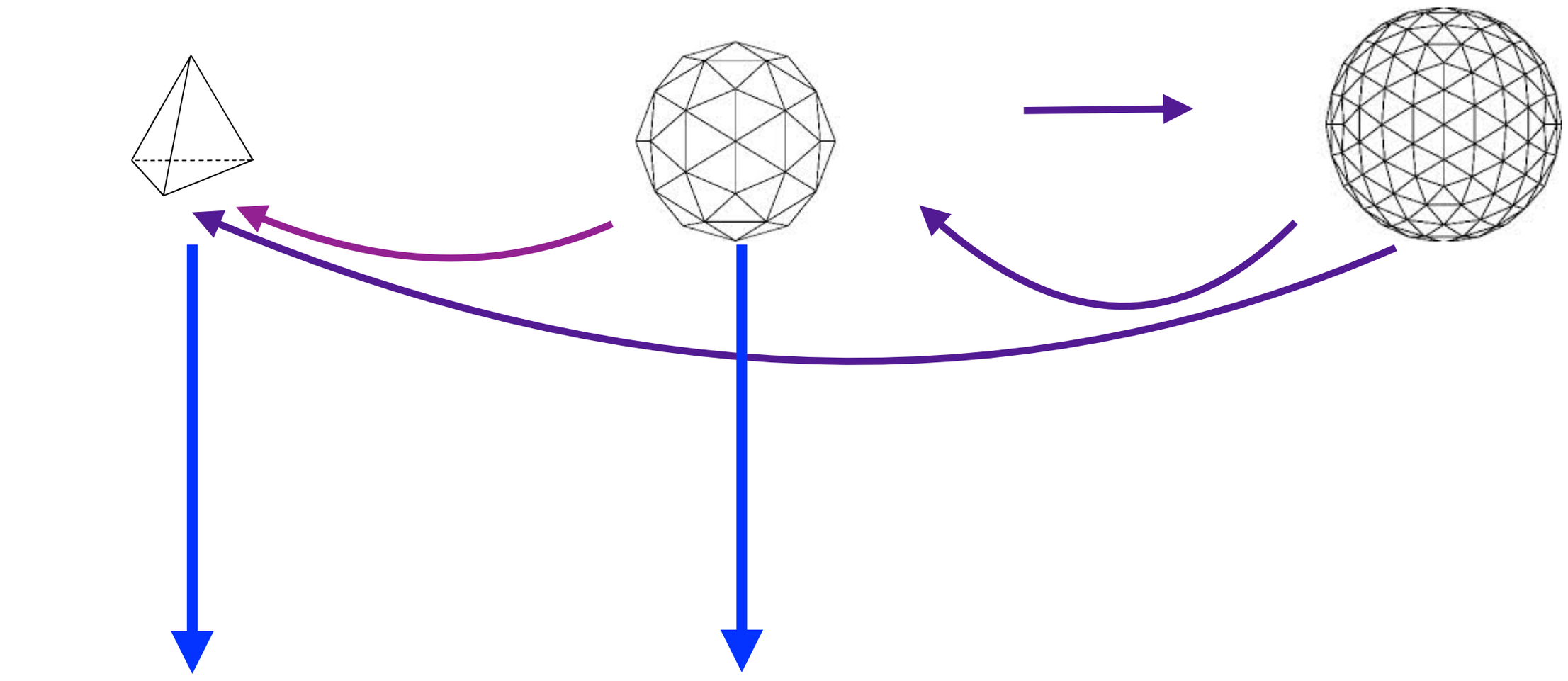


3. Iterative coarse graining via tensor network algorithm, determines effective amplitude and dynamically preferred embedding maps.

Improved amplitude

Constructing a cylindrically consistent dynamics

4. Take more and more complicated boundaries into account.



Improved amplitude

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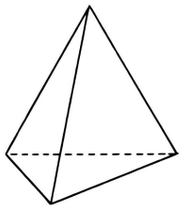
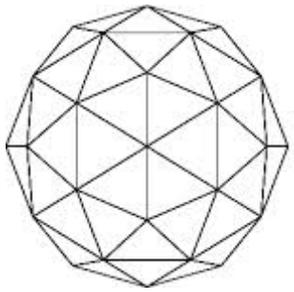
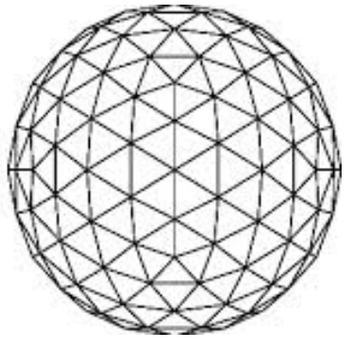
Constructing a cylindrically consistent dynamics

Seed: Initial (inconsistent) amplitudes







Improved amplitude

Improved amplitude

Improved amplitude

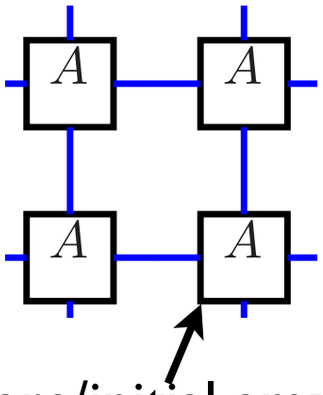
Determine consistent amplitudes with more and more complicated boundaries.

Amplitude changes across all scales!

The renormalization trajectory is encoded in the consistent family of amplitudes.

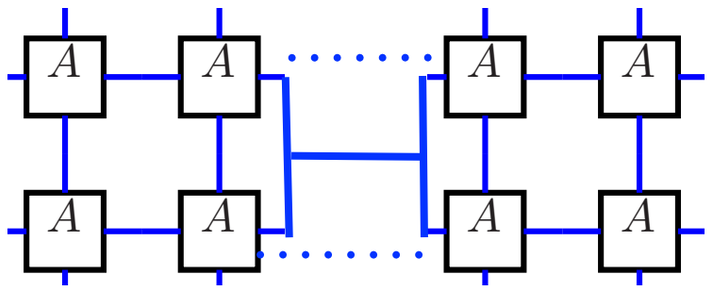
Constructing the continuum limit is a small part of this construction.

Tensor network renormalization methods

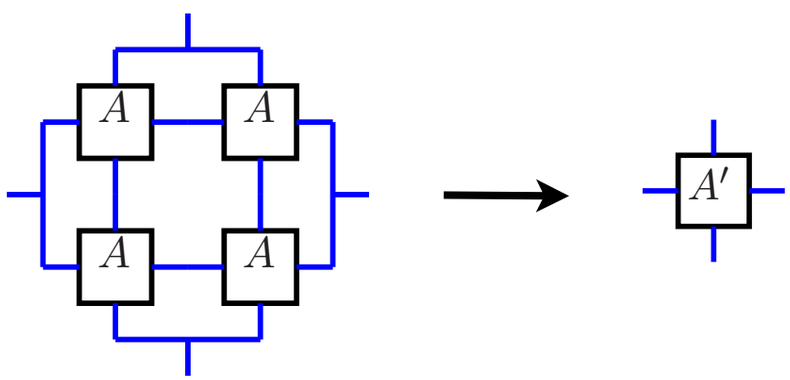


bare/initial amplitude
depending on four variables

Glue building blocks to block with more boundary data.



Find a truncation (embedding map) that would minimize the error as compared to full summation (dotted lines). Use singular value decomposition, keeping only the largest singular values.
Leads to field redefinition, and ordering of fields into more and less relevant.



Use embedding maps to define coarse grained amplitude with the initial number of boundary variables.

Iterate. Find fixed point.
Repeat with more boundary data.

Applications

- (q-deformed) spin net / intertwiner models, analogues to spin foams in 2D

  [BD, Eckert, Martin-Benito 2011, BD, Martin-Benito, Schnetter 2013, BD, Martin-Benito, Steinhaus 2013, BD, Schnetter, Seth, Steinhaus 2016]

- Decorated tensor networks: first working algorithm for 3D gauge theories

  [BD, Mizera, Steinhaus 2014]

- 3D spin foams (with analogue simplicity constraints): new algorithm and phase diagram

  [Delcamp, BD 2016]

- Tensor network algorithms with fusion basis for (q-deformed) gauge models: captures torsion

  - still the leading algorithm

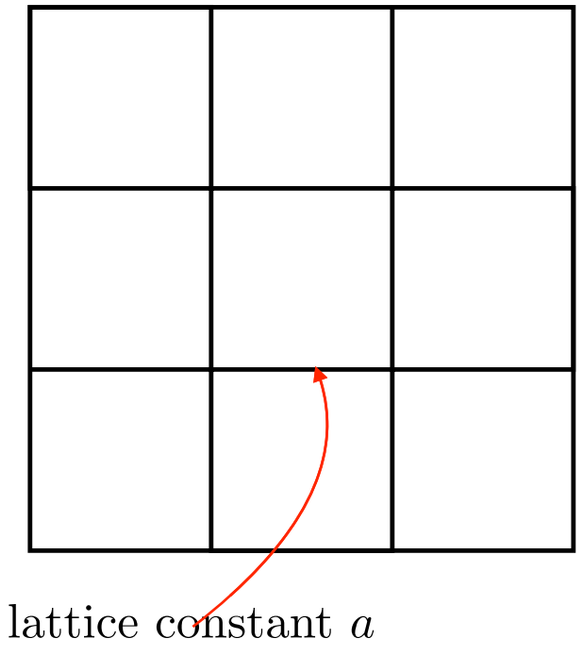
  [Cunningham, BD, Steinhaus 2020]

- Coupling matter to intertwiner models

  [Steinhaus 2015]

We do not have fundamental amplitudes

In lattice gauge theory



- define model at scale a
- compute effective action at larger scales
- in particular: find beta-functions:
  couplings as functions of scale

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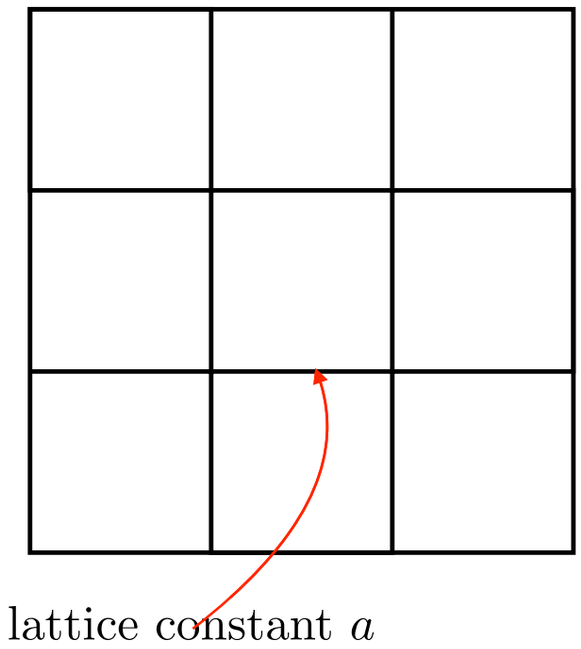

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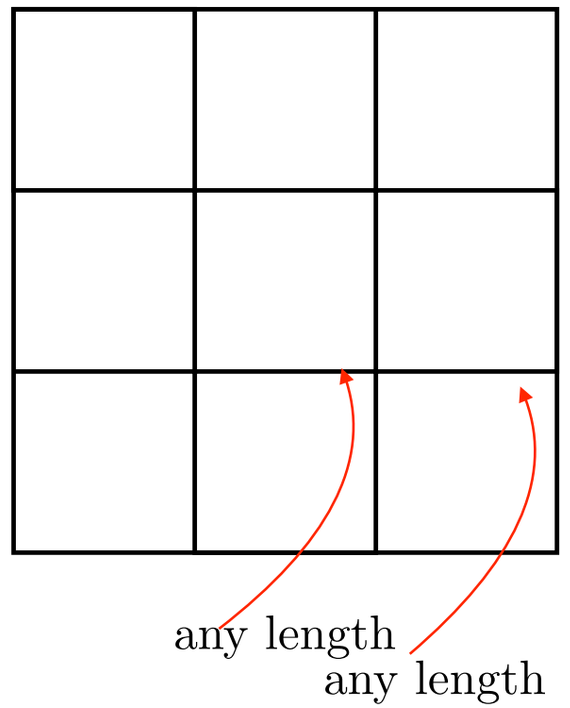
with a ‘dynamical lattice’



- edges can assume any length
- need to define physics on all (inhomogeneous) length scales
- need to know beta-functions to define consistent dynamics

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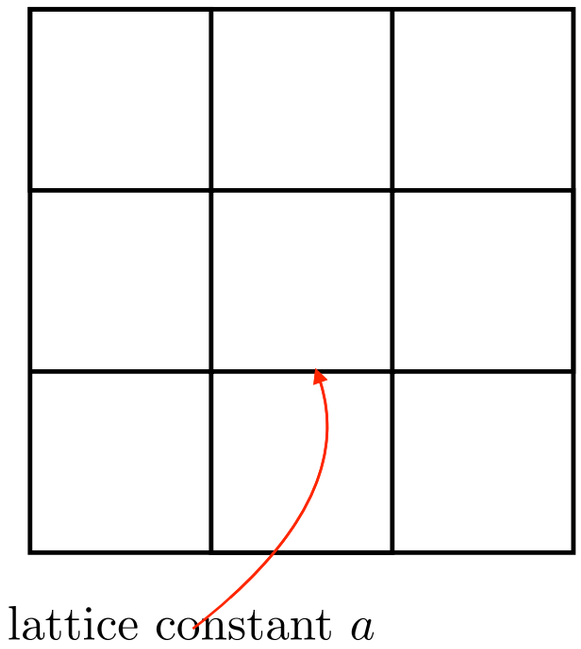

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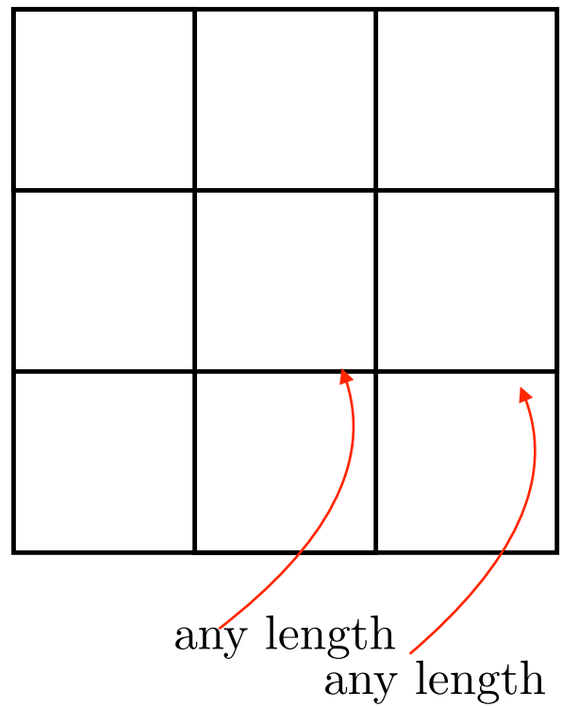

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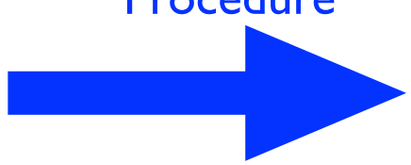
Initial “seed” amplitudes
(Guess of a dynamics over all scales)

Iterative coarse graining
Procedure



Improved amplitudes
(over all scales)

Lattice form factors

$$S \sim f(\text{lengths})(\phi_i - \phi_{i+1})^2 + g(\text{lengths})(\phi_i + \phi_{i+1})^2$$

Need to construct (discrete) metric dependent coupling "constants", which:

-take renormalization flow into account

-are consistent for arbitrarily irregular lattices

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Need to construct (discrete) metric dependent coupling "constants", which:

-take renormalization flow into account

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It is impossible to guess consistent amplitudes,
rather one needs to construct them via iterative coarse graining flow.

The consistent boundary formalism allows to do so,
and identifies a dynamically preferred truncation scheme.

On the continuum limit of 4D spin foams.

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Assume spin foams admit a phase, which at larger scales does lead to smooth geometries.
E.g. in-between Ashtekar-Lewandowski vacuum (degenerate geometry)
and BF vacuum (BF-flat or BF homogeneously curved geometry).

[BD, Geiller 2014,
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Construct perturbative continuum limit: do we obtain (linearized) gravity?

Effective spin foams allow to obtain an answer (and more).

Effective spin foams

[Asante, BD, Haggard PRL 2020]

[Asante, BD, Padua-Arguelles 2021]

Captures key construction principles of spin foams.
But much more amenable to calculations and much more transparent.

$$Z = \sum_{\text{discr.areas}} \exp\left(\frac{\imath}{\ell_P^2} S_{\text{ARegge}}(\{A_t\})\right) \prod_{\text{tetra}} G(\{A_t\})$$

Oscillating factor with Area Regge action

(motivated by higher gauge theory)

Gaussian factors peaked on constraints.
Deviation proportional to $\ell_P \sqrt{\gamma \, \text{Area}}$
(Determined from anomaly/ non-commutativity.)

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Action:

$$S = S_{\text{ARegge}} - \imath \sum_{\tau} \ln G$$

What is the lattice continuum limit for the Area Regge action?
(Open question since the 90's. Widely assumed to not lead to GR. Original flatness problem.)

Do the constraint terms change anything?

Continuum limit of effective spin foams

[BD 2021,
BD, Kogios 2022]

- Construct (linearized) Area Regge action on infinite regular lattice.
- Classify variables according to their scaling in the Hessian (in particular mass/ massless)
- Compute series expansion of (effective) action in lattice constant

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What is the lattice continuum limit for the (linearized) Area Regge action?

- Despite having 50 or 100 or more variables per lattice site: only length degrees of freedom are massless.
- The continuum limit is given by (linearized) general relativity.
- Next to leading order in lattice constant: Weyl square term.
- Arises from an effective area metric (which has 20 components).

Although Area Regge calculus leads to an extension of the configuration space from length to area metrics, only length metric dof's are massless.
We thus obtain general relativity in the limit.

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from length to area metrics, only length metric dof's are massless.
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Do the constraint terms change anything?

- Do only affect massive degrees of freedom: turn mass parameter complex (and BI-parameter dependent).
- Universality: Form of constraint implementation (in which models differ) does not matter for continuum limit.

Effective continuum action for spin foams

Can we obtain such an action (EH+Weyl-squared) directly from the continuum?

Use modified Plebanski theory framework.

[Krasnov 2008+; Freidel 2008]

Modify further.

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Results: [BD, Borissova 2022]

- Derivation of action for area metrics from (modified) Plebanski action.
- Integrating out additional area metric dof's (linearized):

$$L_{eff} = L_{EH} - \frac{1}{4}\frac{1}{\Box - M(\gamma)^2}\text{Weyl}^2$$

Ghost-free!

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3 coupling constants G, Λ, γ: as needed in Asymptotic Safety, CDT, EDT.

γ is an anisotropy parameter as in CDT (appears in space-like area spectrum, but not in time-like).

Summary

- Diffeomorphism symmetry in the discrete ensures discretization independence. Continuum limit becomes trivial.
- Perfect actions mirror continuum dynamics, but are non-local.
- Consistent boundary formalism: allows to construct a renormalization trajectory using a dynamically preferred truncation scheme
- Tensor networks provide algorithms for consistent boundary formalism.

Recent review: Asante, BD, Steinhaus 2022, In: Handbook of Quantum Gravity

Summary

- Diffeomorphism symmetry in the discrete ensures discretization independence. Continuum limit becomes trivial.
- Perfect actions mirror continuum dynamics, but are non-local.
- Consistent boundary formalism: allows to construct a renormalization trajectory using a dynamically preferred truncation scheme
- Tensor networks provide algorithms for consistent boundary formalism.
- Effective spin foams allow perturbative continuum limit.
- Surprise: Find (linearized) general relativity at leading order. Resolves flatness problem.
- Weyl-squared term as a correction, arising from extension from length to area metrics.

$$L_{eff} = L_{EH} - \frac{1}{4}\frac{1}{\Box - M(\gamma)^2}\mathrm{Weyl}^2$$

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Diffeomorphism symmetry in (0+1)D systems

Work with action/ variational principle.

$$S = \int_{t_i}^{t_f} L(q, \dot{q})\, dt$$

"Reparametrize" | Add t as independent variable

$$S_{\text{rep}} = \int_{s_i}^{s_f} L\left(q, \frac{dq/ds}{dt/ds}\right) \frac{dt}{ds} ds$$

1D diffeomorphism symmetry:

$$(q(s), t(s)) \longrightarrow (q(f(s)), t(f(s)))$$

is a solution → is a solution

Multitude of solutions for fixed boundary data

t_n are a fixed choice of discrete points on the time axis:

$$S_d = \sum_{n=0}^{N-1} L\left(\frac{q_n + q_{n+1}}{2}, \frac{q_{n+1} - q_n}{t_{n+1} - t_n}\right)(t_{n+1} - t_n)$$

Replacing graph $q(t)$ by piecewise straight curve.

For arbitrary choices of s_n:

$$S_d = \sum_{n=0}^{N-1} L\left(\frac{q_n + q_{n+1}}{2}, \frac{q_{n+1} - q_n}{t_{n+1} - t_n}\right)(t_{n+1} - t_n)$$

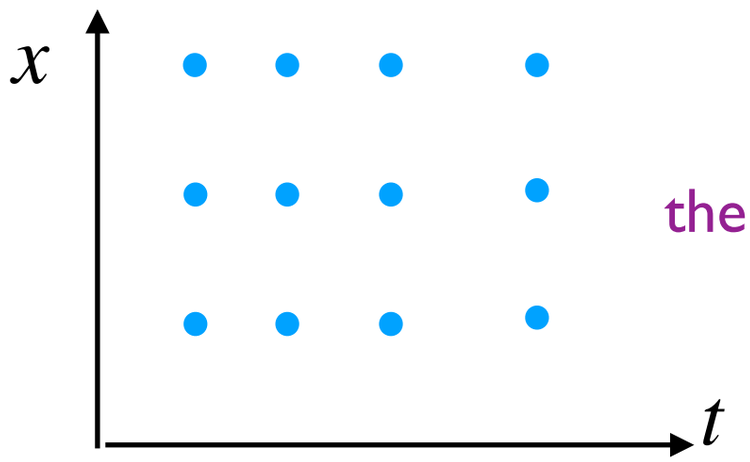
Here we vary also t_n !

1D diffeomorphism symmetry?

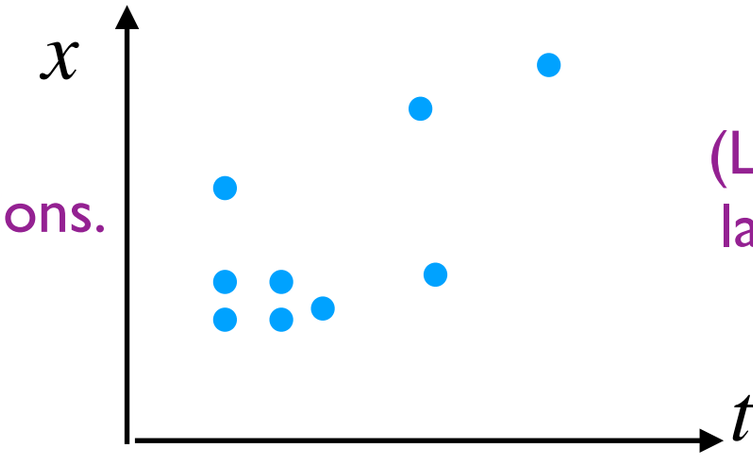
Do we find multitude of solutions for fixed boundary data?

Higher dimensional systems

Diffeomorphism symmetry for discrete 2D systems

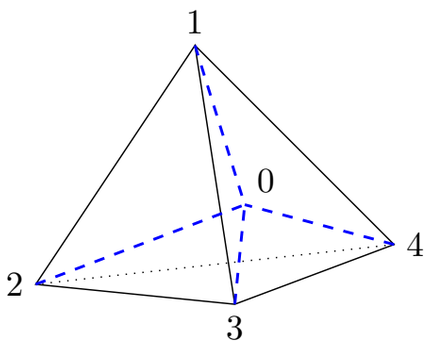


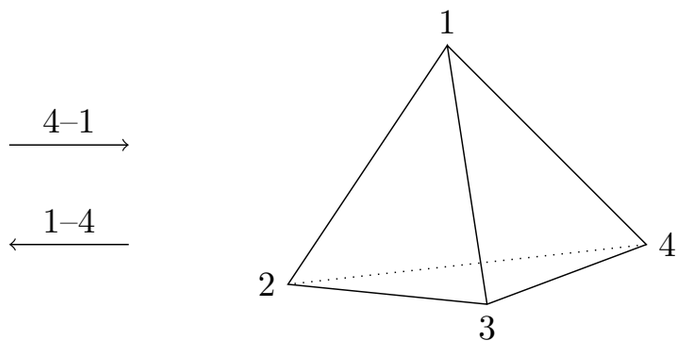
Should give
the same predictions.

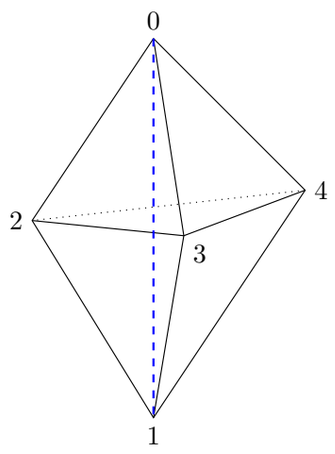


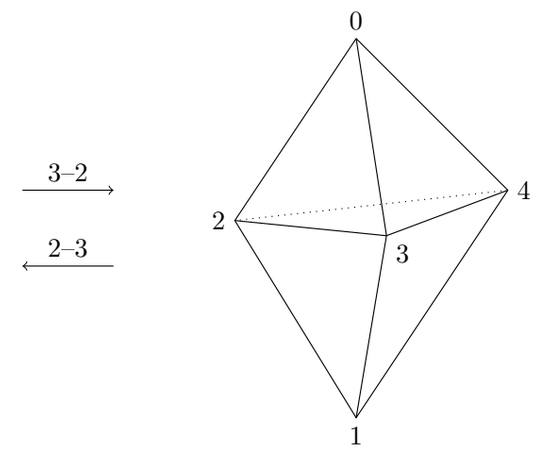
(Locally) variable
lattice constant.

Discretization independence: (eg variables on edges)









Satisfied by 3D Regge calculus: a discretization of gravity.

[BD, Steinhaus 2011,
Borissova, BD 2023]