ILQGS Panel: The future of spin foams

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Concentrate on universal features of Spin Foams

The good:

Areas are fundamental and have discrete spectrum.
(3D angles can be seen as auxiliary variables.)

Convergence with other quantum gravity approaches.

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and in the algebra of secondary simplicity constraints (gluing/ shape matching) parametrized by γ.

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The bad (?):

The anomaly is responsible for the "flatness problem".

[Bonzom 08, … Conrady, Hellmann-Kaminski, Han, Engle-Kaminski-Han, …, Gozzini 21]

[Asante, BD, Haggard 2020]

Applies to all models where

a) areas are independent variables
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Numerical proof that this problem occurs in EPRL in "standard classical limit". [Gozzini 21]

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One can thus consider this issue in a model, which captures the above key ingredients of spin foams.

Effective spin foams models: Seconds on a laptop instead of weeks on HPC.
Transparent encoding of the dynamics, in particular the constraints.

[Asante, BD, Haggard 2020, Asante, BD, Padua-Arguelles 2021]

Concentrate on universal features

[Asante, BD, Haggard 20]

In the discrete:
First proof that Regge equations of motion are reproduced for examples with inner edge.
By computing the full non-perturbative partition function and expectation values of observables.
Requires $\gamma \sim < 0.1$, but this allows for large deficit angles, consistent with Regge dynamics.

Consistent with semi-classical results by Han 2013, Han, Huang, Liu, Qu 2021:
-bound on γ
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[BD 21]

In the continuum:

Leveraging that non-perturbative results show good approximation by saddle point analysis near flat space:

Perturbative analysis.

Study of linearized Area Regge action (+ $\frac{i}{\gamma}$ constraints^2) on hypercubical lattice:

- leads to linearized Einstein Hilbert action (zeroth order in the lattice constant)
- a correction, which is of fourth order in the lattice constant, of six order in derivatives and quadratic in (derivative of) curvature

Continuum limit resolves flatness problem even without explicit implementation of gluing constraints. Even for Barrett-Crane.

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- Non-metric degrees of freedom are getting very massive in the continuum limit: likely to extend to higher order perturbations
- Different SF models differ in whether and how the non-metric degrees of freedom are suppressed.
- But are suppressed by dynamics in continuum limit anyway — even without constraints.
- Emergence of universality in continuum limit.

Needed: "Effective" continuum action for spin foams (Area Regge action).
Corrections to gravitational dynamics due to anomaly/ extended configuration space.

Concentrate on universal features for quantum gravity
Lorentzian path integrals

Spin foams: one of the few approaches based on Lorentzian path integral.
However, due to high numerical demands of EPRL/FK hardly explored issues important for Lorentzian path integrals.
Using effective spin foams: Encountered issue of how to deal with integrations over infinite domains.

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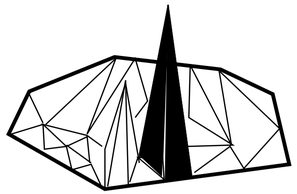

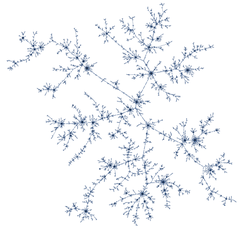

- How to compute path integrals with highly oscillating amplitude and (practically) unbounded integration range?
- What happens with the conformal factor problem of Euclidean quantum gravity (which killed almost all lattice approaches)?
- Key to understand non-perturbative continuum limit.
- What configurations to sum over: Allow causally irregular configurations? Allow topology change in time? Appearance of Euclidean configurations?

[Han, Liu]

[BD, Gielen, Schander: Causally irregular configurations appear even in the simplest cosmological example.]

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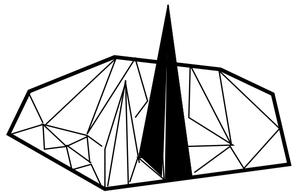

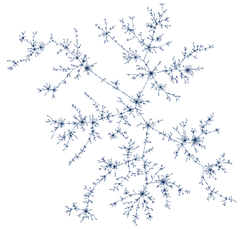

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[Feldbrugge, Lehners Turok;
Han, Huang, Liu, Qu, Wan;
Asante, BD, Jia, Padua-Arguelles: to appear]

[Asante, BD, Jia,
Padua-Arguelles:
to appear]

- Picard Lefshetz methods
- In simplest examples: make integrals (quickly) convergent. Conformal factor rotated to suppressing Euclidean branch.
- Can allow for causally irregular configurations: With Picard-Lefshetz methods - indications that these will be always suppressed.

  - Convergence with Causal Dynamical Triangulations: causally irregular configurations suppressed
  - Convergence with results from quantum cosmology with PL-integration

There is a lot to explore for Lorentzian path integrals.
Need effective numerical models and methods.

[Asante, BD, Padua-Arguelles 2021]

Universal Summary

- Concentrate on universal features of spin foams.
  Areas as fundamental degrees of freedom.

- Amplitudes of any model will change under coarse graining flow: universal features will survive. [Asante, Bahr, BD, Steinhaus, …]

- Concentrate on universal features of quantum gravity. Consider issues which are also of interest to other approaches.
  E.g. the Lorentzian path integral.

Need effective numerical models and methods.

Thank you!

Effective Spin Foam Models

[Asante, BD, Haggard PRL 2020]

Effective Spin Foam models:

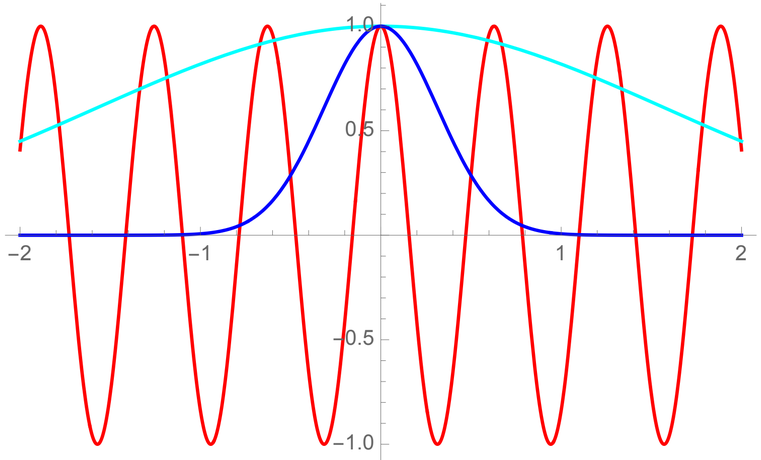
- much much more amenable to numerical investigations: seconds on laptop compared to weeks on HPC
- 3D angles are already integrated out.
  The number of angle variables is 5 x #(4-simplices). Integrating them out beforehand saves a lot of computational resources.
- much more transparent encoding of the dynamics, in particular with regard to gluing constraints
- form can be motivated from higher gauge theory (as opposed to gauge theory)

$$Z = \sum_{\text{discr.areas}} \exp\left(\frac{\imath}{\ell_P^2} S_{\text{Regge}}(\{A_t\})\right) G(\{A_t\})$$

Oscillating factor with Area Regge action
(motivated by higher gauge theory)

Gaussian factor peaked on constraints.
Deviation proportional to $\ell_P\sqrt{\gamma\,\text{Area}}$
(Determined from anomaly/ non-commutativity.)

Oscillating factor and **Gaussians**
for two different γ



Naive $\ell_P \to 0$ limit: Oscillations win over Gaussian. Constraints are not 'visible'.
Leading to flatness problem for spin foams.

To avoid washing out of Gaussians we need (via naive estimation):

$$\sqrt{\gamma}\frac{\sqrt{\text{Area}}}{\ell_P}\text{curv}_t \leq \mathcal{O}(1)$$

[Han, Asante-BD-Haggard]

A proper semi-classical regime does also require a small Barbero-Immirzi parameter.
(It indeed is an anomaly parameter.)

Explicit tests

Explicit computation of partition functions and expectation values for small triangulations

[Asante, BD, Haggarrd CQG 20]

Triangulations with inner edge:

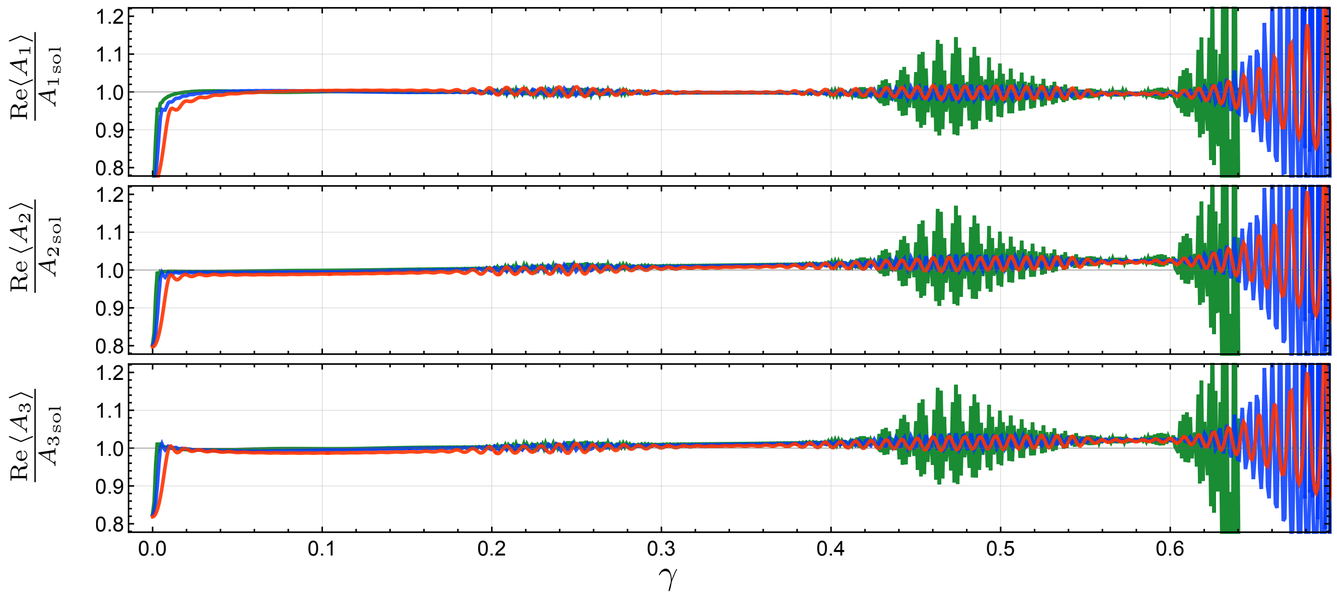
Computed via an exact evaluation of the path integral with observable insertions expectation values of various geometric observables.

We showed that we do have a regime of scale, γ and curvature, for which the correct equations of motions are implemented.

First explicit proof that spin foams can implement the correct equations of motions.

(With finite, non-vanishing curvature angles, finite but small γ.)

For curvature angle $\epsilon \sim -1$ and areas $\sim \gamma \ell_P^2 \lambda,\ \lambda = 100{,}200{,}400$



Found that semi-classical regime can be even larger than suggested by naive bound, in particular if curvature (per triangle) is small.

Instabilities in expectation values: non-perturbative effects, resulting from interplay of discrete spectra and constraint implementation.

For all tested curvatures: $\gamma \sim 0.1$ defines semi-calssical regime.

Discrete dynamics: the Barbero-Immirzi parameter is not a free parameter.

Scaling of the Hessian

- λ and Λ both give the lattice constant, but originate from different sources. Λ counts derivatives.

$$H_{(ab)(ab)} \sim \frac{1}{\lambda^2} \times$$

	(a_1)	(a_2)	(b_1)	(b_2)
(a_1)	Λ^2	0	Λ^3	0
(a_2)	0	Λ^0	Λ^1	Λ^1
(b_1)	Λ^3	Λ^1	Λ^0	0
(b_2)	0	Λ^1	0	Λ^0

- Correction from integrating out the zeta-variables.

$$\Gamma_{(a)(a)} = -H_{(a)(b)} \cdot H^{-1}_{(b)(b)} \cdot H_{(b)(b)}$$

- $H_{(b)(b)}$ is invertible (perturbatively without inverse derivatives)!
  (This is where we might have needed the G-matrix: but it is not necessary here!)

- With this scaling we already know that:

$$\Gamma_{(a)(a)} \sim \frac{1}{\lambda^2} \times$$

	(a_1)	(a_2)
(a_1)	Λ^6	Λ^4
(a_2)	Λ^4	Λ^2

- That is the correction for metric sector scales with a_L^4.
- Integrating out the spurious (a2) variables leads to even higher order corrections.