Causal Structure in Spin-foams

Eugenio Bianchi & Pierre Martin-Dussaud

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PennState

QISS THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

Causal Structure in Spin-foams

Is causality a fundamental or an emergent property of space-time?

Causal Structure in Spin-foams

Is causality a fundamental or an emergent property of space-time?

Plan:

kinematics of general relativity

+discrete

+surface d.o.f.

+dynamics

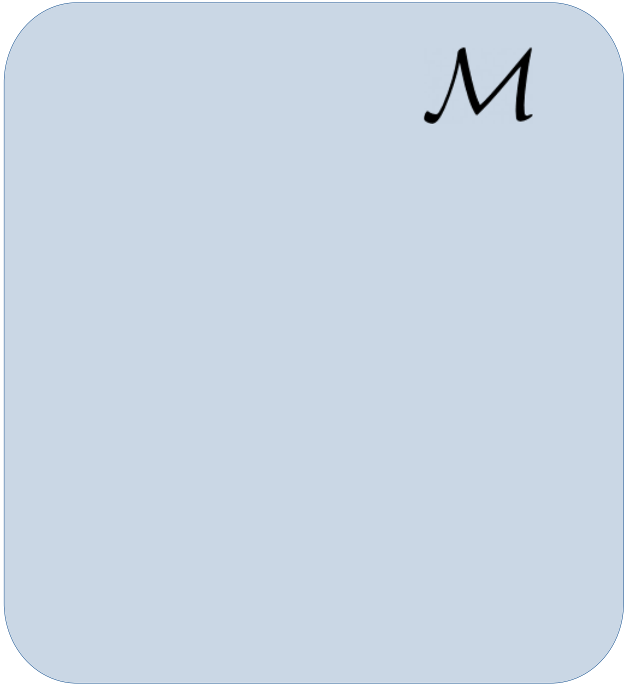
+quantum: path-integral

spinfoams (crash-course)

causal spinfoams

kinematics of general relativity

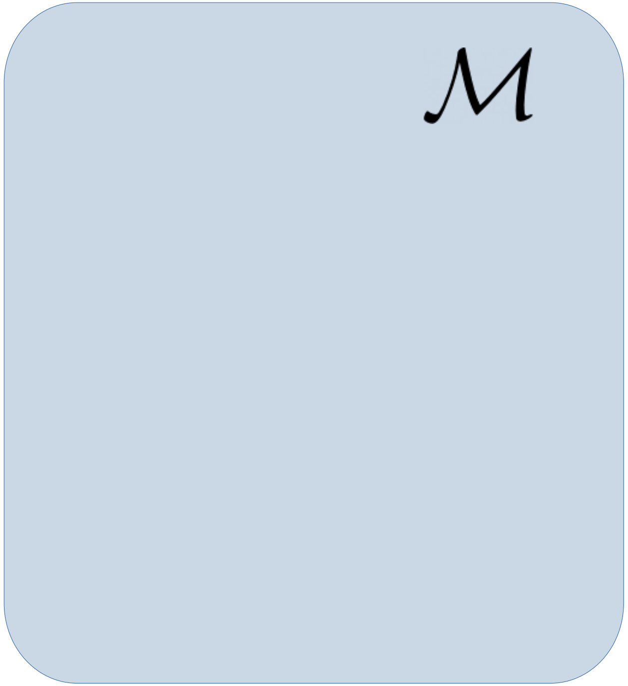
manifold $+$ lorentzian metric g



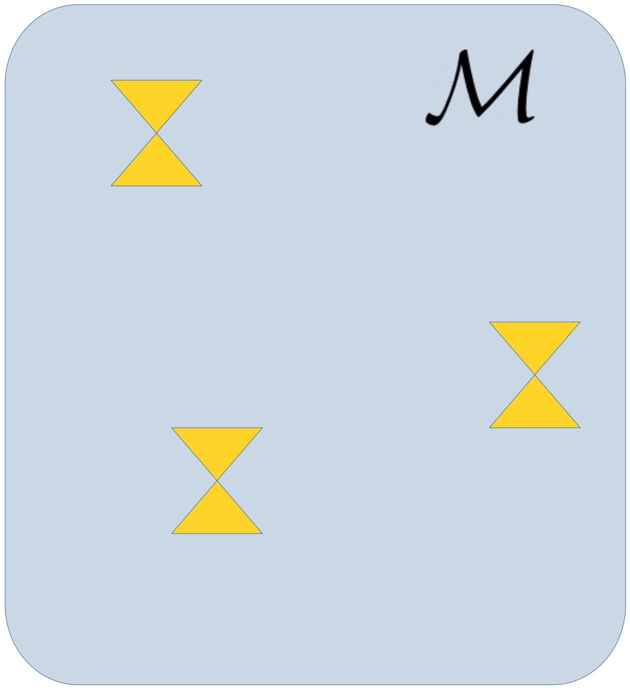
kinematics of general relativity

manifold + lorentzian metric g

signature $(\eta, -\eta, -\eta, -\eta)$ $\qquad \eta = \pm 1$



kinematics of general relativity

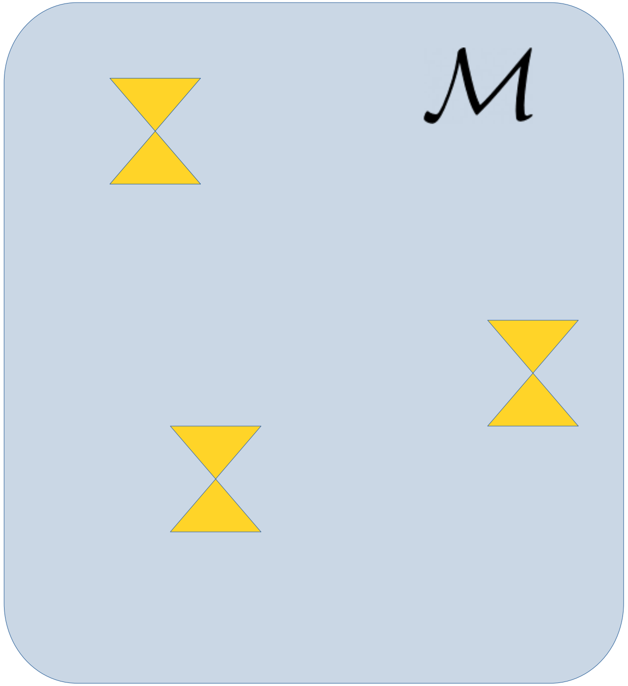


manifold + lorentzian metric g

signature $(\eta, -\eta, -\eta, -\eta)$ $\eta = \pm 1$

causal structure = light-cone structure

kinematics of general relativity



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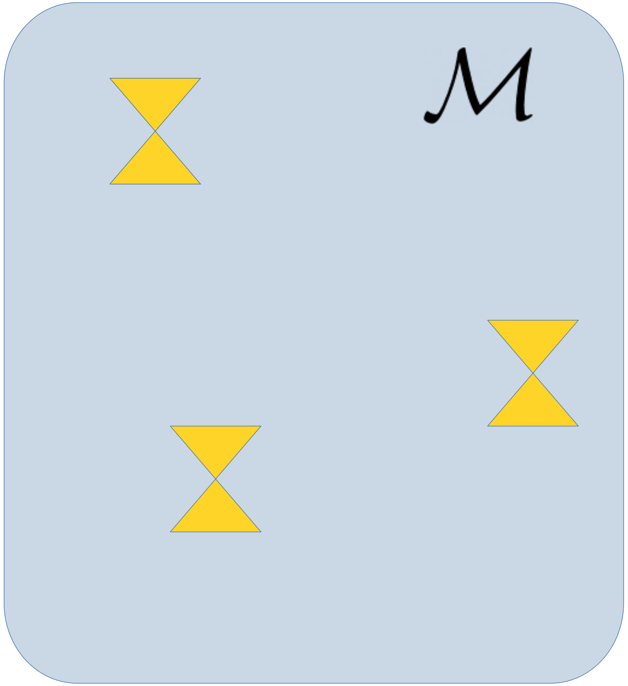
causal structure = light-cone structure

= equivalence classes on $T_x M$

$$\operatorname{sign} g(u,u) = \begin{cases} \eta & \text{Time-like} \\ 0 & \text{Null} \\ -\eta & \text{Space-like} \end{cases}$$

kinematics of general relativity



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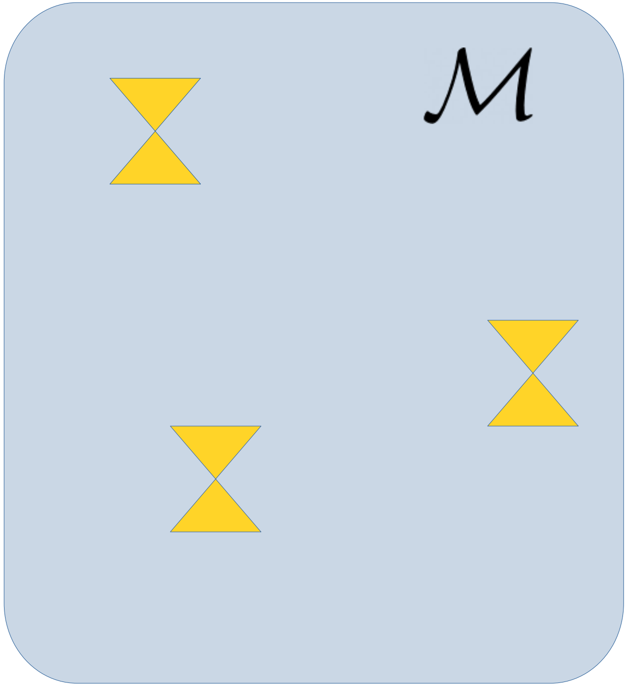
No time arrow!

kinematics of general relativity



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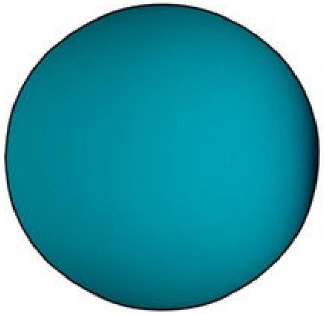
No time arrow!

Malament's theorem (1977):

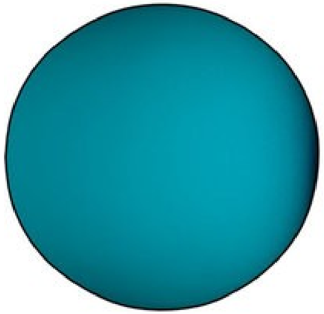
Metric = Causal structure + Conformal factor $\Omega(x)$

+discrete



2-dimensional
differentiable manifold

+discrete



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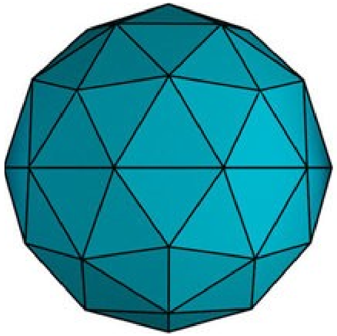
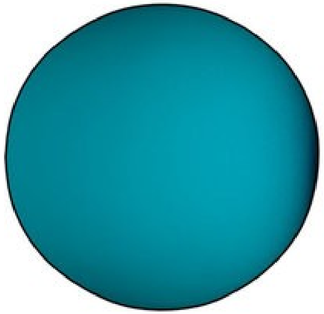


complex of triangles

+discrete



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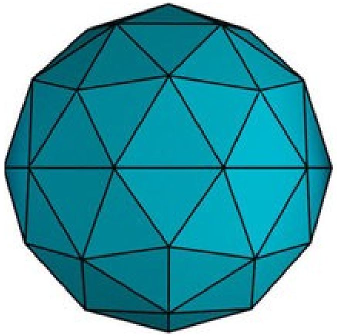

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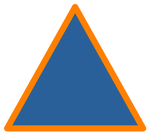
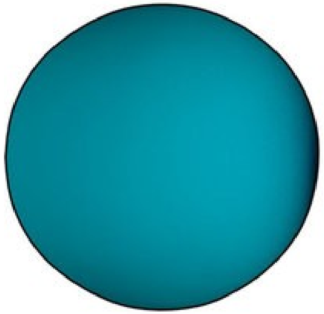


Triangle
Segment
Point

+discrete



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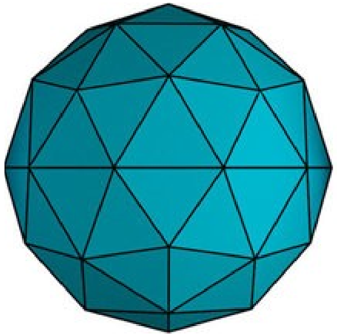

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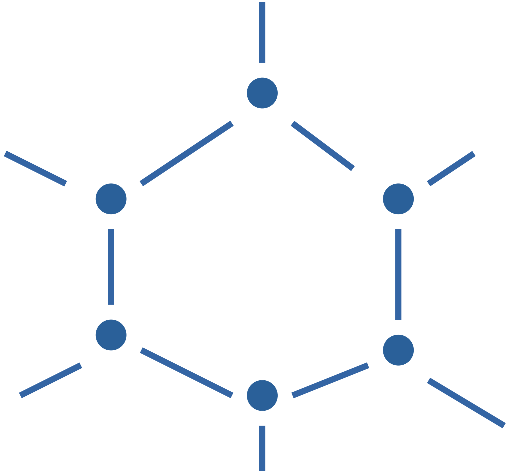
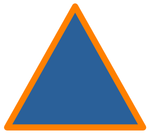


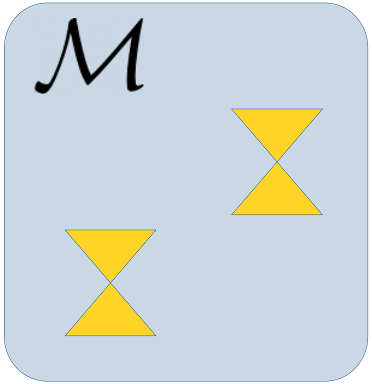
dual skeleton



Triangle
Segment
Point

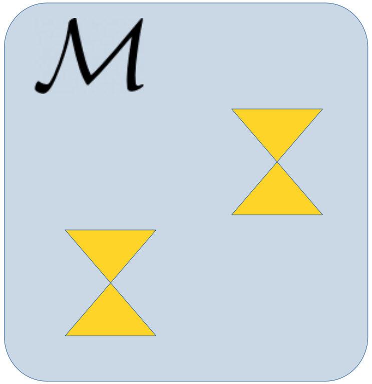
Vertex
Link
Face

+discrete



4-dimensional differentiable manifold with lorentzian metric

+discrete



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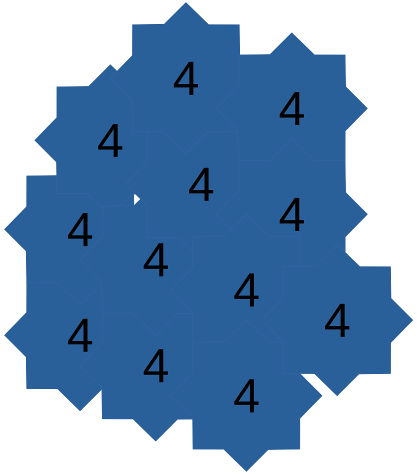
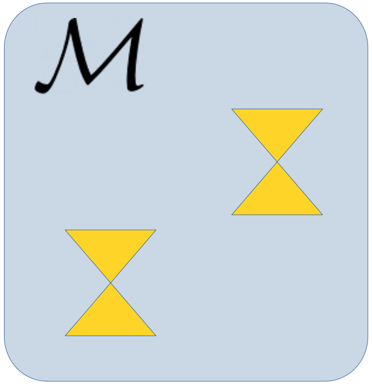


complex of 4-simplices (with space-like tetrahedra)

+discrete



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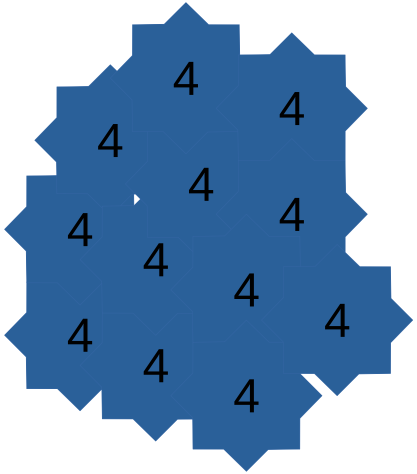

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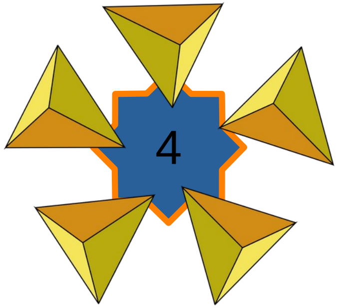
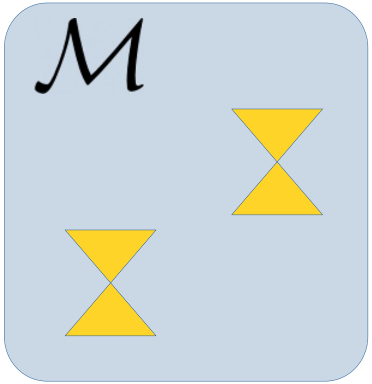


4-simplex
Tetrahedron
Triangle
Segment
Point

+discrete



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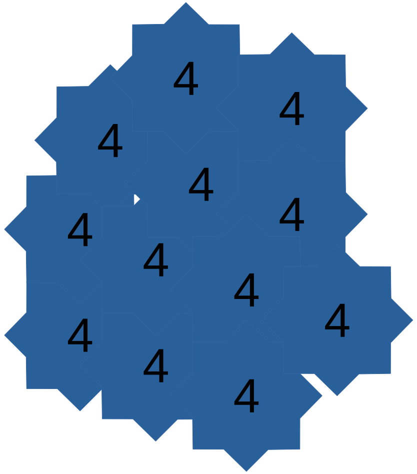

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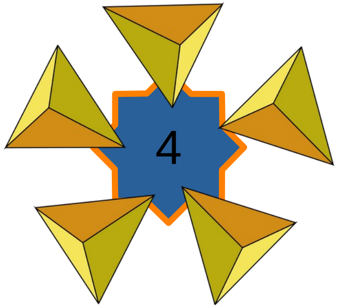
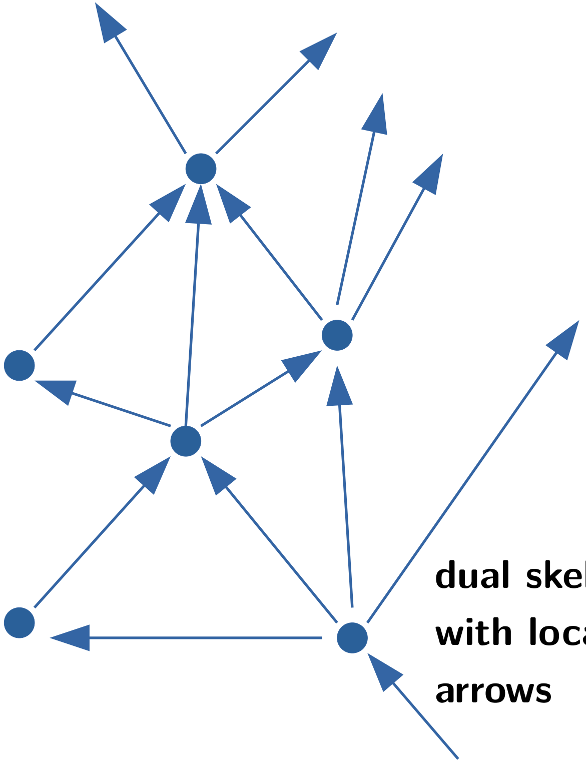


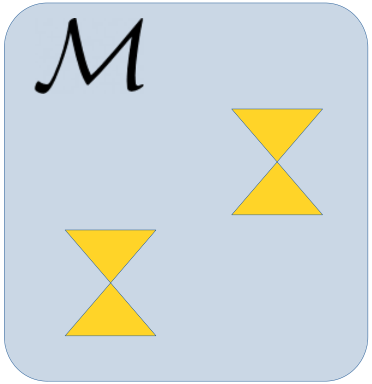
4-simplex
Tetrahedron
Triangle
Segment
Point



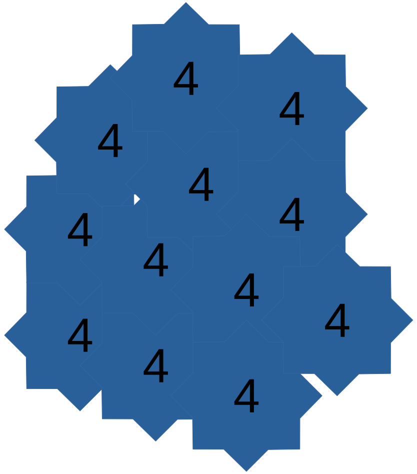
dual skeleton with local time arrows

Vertex
Link
Face
...
...

+discrete



4-dimensional differentiable manifold with lorentzian metric



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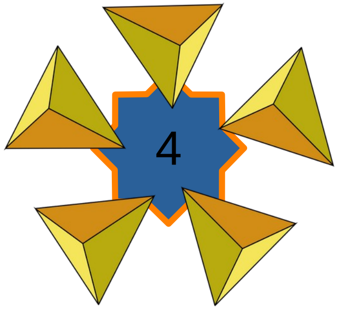
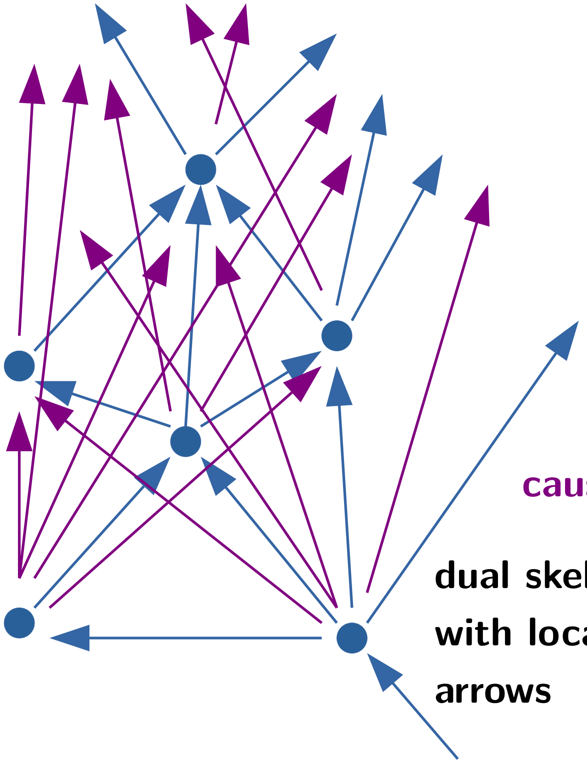


4-simplex
Tetrahedron
Triangle
Segment
Point

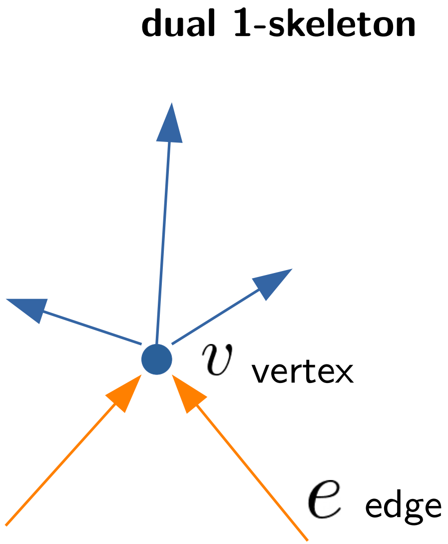


causal set

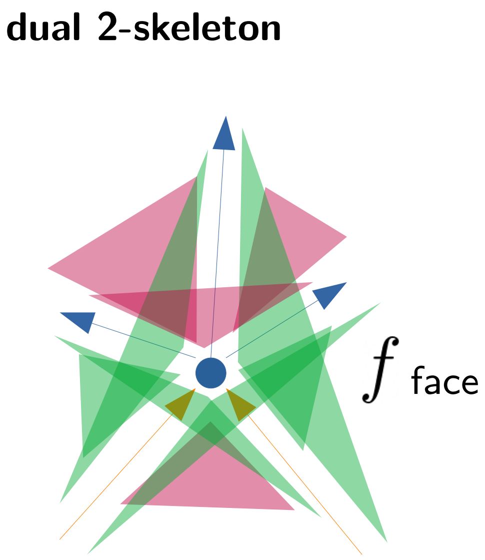
dual skeleton with local time arrows

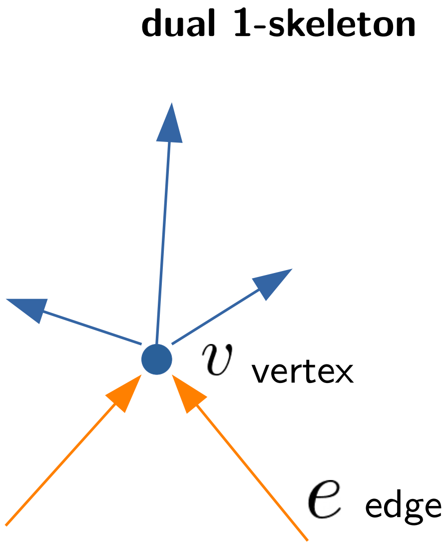
Vertex
Link
Face
...
...

+surface d.o.f.



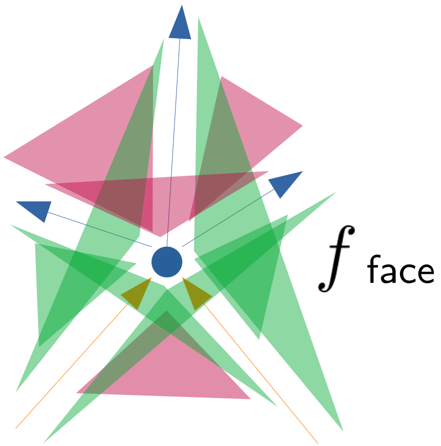
+surface d.o.f.



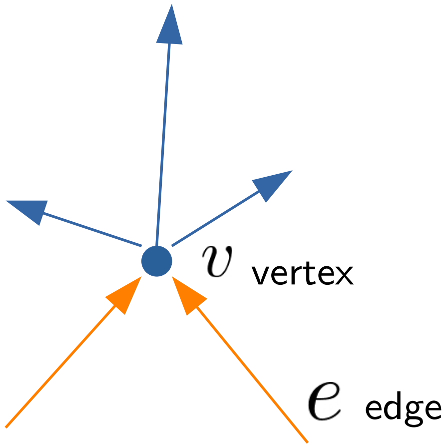


+surface d.o.f.

dual 2-skeleton



dual 1-skeleton

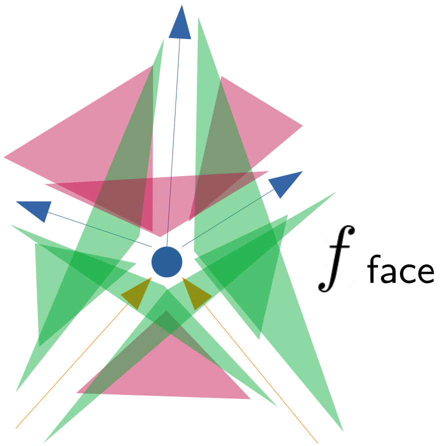


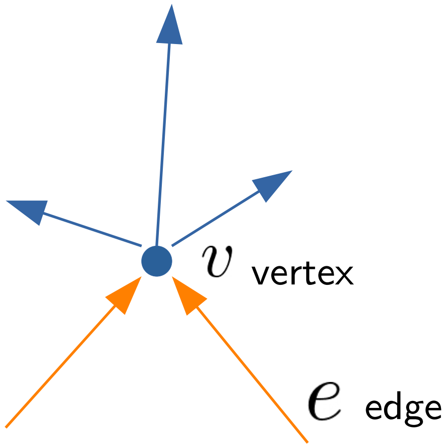
$$\varepsilon_v(e) = \begin{cases} -1 & \text{incoming (past)} \\ 1 & \text{outgoing (future)} \end{cases}$$

dual 2-skeleton

+surface d.o.f.

dual 1-skeleton





5 half-edges

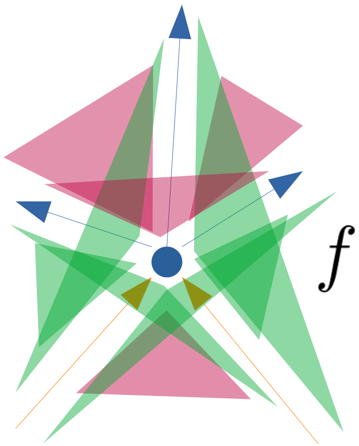
$$\boxed{\varepsilon_{v_1}(e) = -\varepsilon_{v_2}(e)}$$

Causal constraint

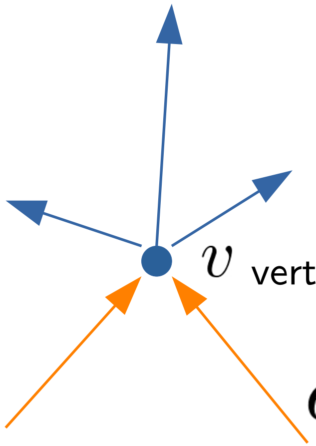
$$\varepsilon_v(e) = \begin{cases} -1 & \text{incoming (past)} \\ 1 & \text{outgoing (future)} \end{cases}$$

+surface d.o.f.

dual 2-skeleton



dual 1-skeleton



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$$\boxed{\varepsilon_{v_1}(e) = -\varepsilon_{v_2}(e)}$$

Causal constraint

$$\varepsilon_v(f) = \begin{cases} \eta & \text{co-chronal (time)} \\ -\eta & \text{anti-chronal (space)} \end{cases}$$

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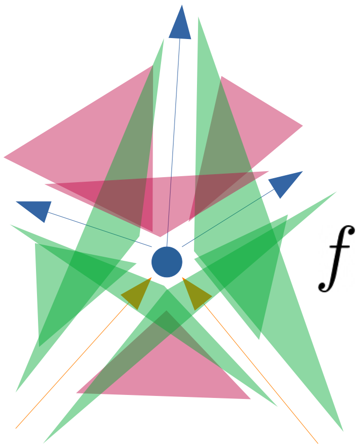
+surface d.o.f.

dual 2-skeleton



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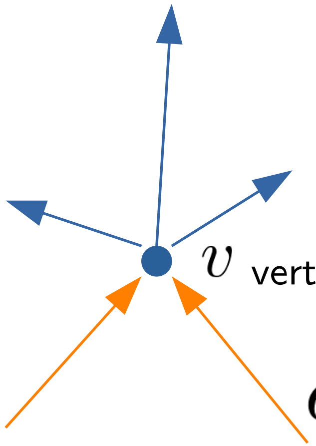

5 half-edges

$$\boxed{\varepsilon_{v_1}(e) = -\varepsilon_{v_2}(e)}$$

Causal constraint

$$\boxed{\varepsilon_v(f) = \eta \varepsilon_v(e_1)\varepsilon_v(e_2)}$$

1:2

No time-arrow

$$\varepsilon_v(f) = \begin{cases} \eta & \text{co-chronal (time)} \\ -\eta & \text{anti-chronal (space)} \end{cases}$$

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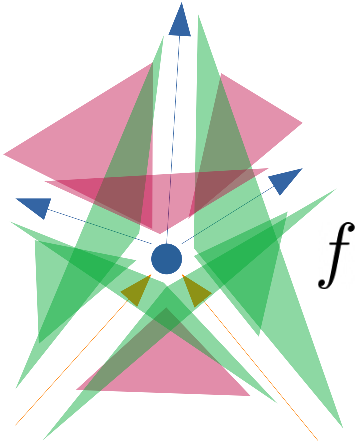
+surface d.o.f.

dual 2-skeleton



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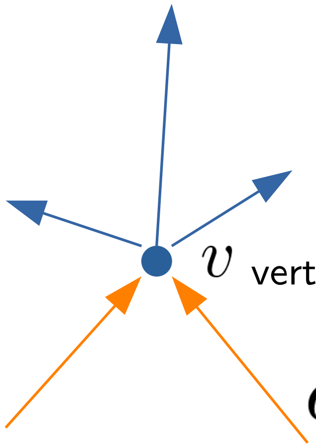

10 wedges

$$\prod_{f\in\text{cycle}} \varepsilon_v(f) = \eta^{\#\text{cycle}}$$

Causal constraint

5 half-edges

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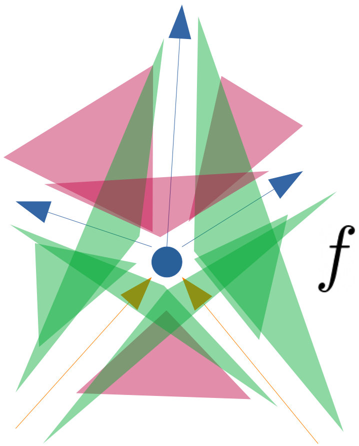
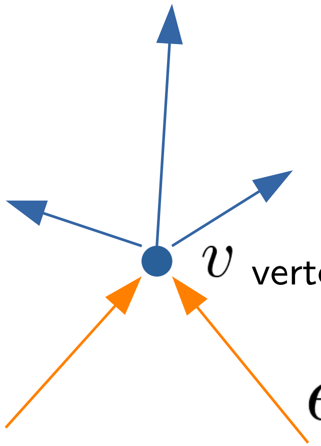


The causal structure of a 2-complex can be encoded by assigning an orientation (a sign) either to the edges, or to the wedges, satisfying some "causal constraints".

dual 1-skeleton



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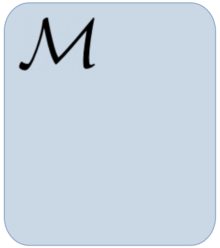
+dynamics



Palatini action

$$S(g_{\mu\nu}, \Gamma) = \int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$$

metric

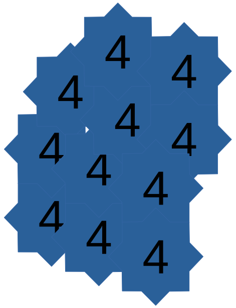
affine connection

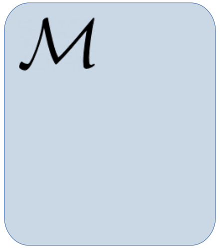
+dynamics

[Barrett 94']

1st order Regge action

$$S[l_s,\theta_{t\sigma},\mu_\sigma] = \sum_\sigma \sum_{t|t\in\sigma} A_t(l_s)\theta_{t\sigma} + \sum_\sigma \mu_\sigma \det \gamma_\sigma$$





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affine connection

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1st order Regge action

sum over 4-simplices

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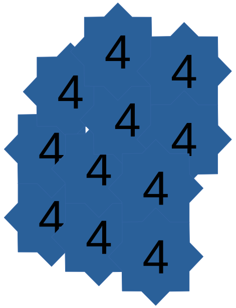
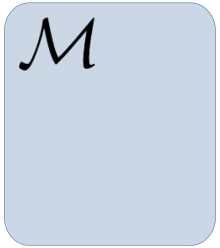
sum over wedges





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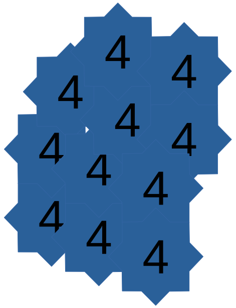
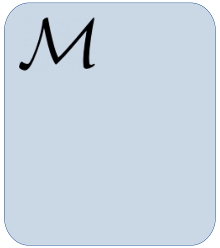
Lagrange multiplier

closure constraint

sum over wedges





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Lagrange multiplier

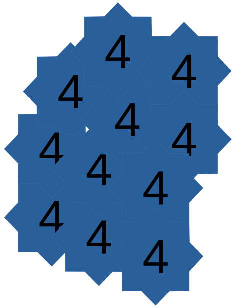
closure constraint

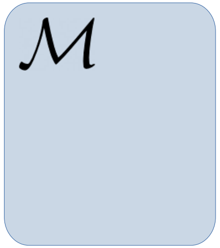
length of segments

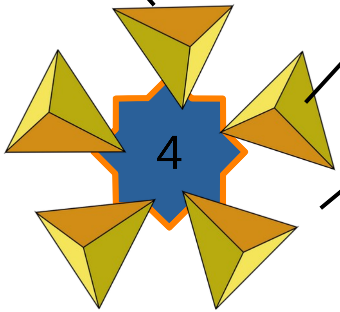
sum over wedges

dihedral angle between tetrahedra

area of triangles







Palatini action

$$S(g_{\mu\nu}, \Gamma) = \int d^4x \sqrt{-g} g^{\mu\nu} R_{\mu\nu}(\Gamma)$$

metric

affine connection

+dynamics

[Barrett 94']

1st order Regge action

$$S[l_s, \theta_{t\sigma}, \mu_\sigma] = \sum_\sigma \sum_{t|t\in\sigma} A_t(l_s)\theta_{t\sigma} + \sum_\sigma \mu_\sigma \det \gamma_\sigma$$

sum over 4-simplices

length of segments

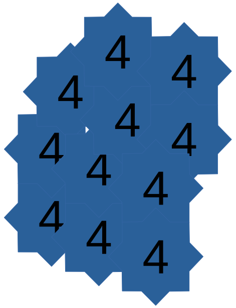
Lagrange multiplier

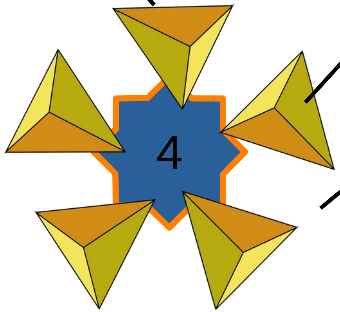
sum over wedges

area of triangles

dihedral angle between tetrahedra

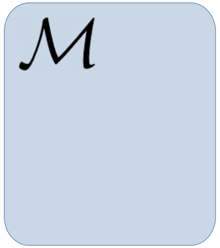
closure constraint





$$\boxed{\operatorname{sign}\theta_{t\sigma} = \varepsilon_{t\sigma}}$$

Orientation structure



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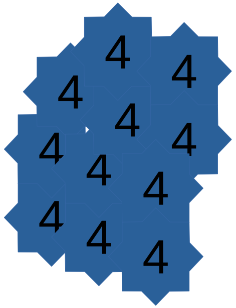

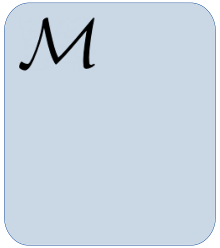

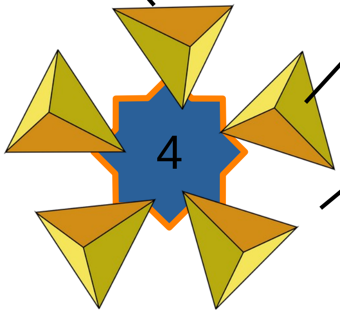

$$\boxed{\operatorname{sign} \theta_{t\sigma} = \varepsilon_{t\sigma}}$$

Orientation structure

Equations of motion

$$\boxed{\prod_{f\in\text{cycle}} \varepsilon_v(f) = \eta^{\#\text{cycle}}}$$

Causal structure

Palatini action

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metric

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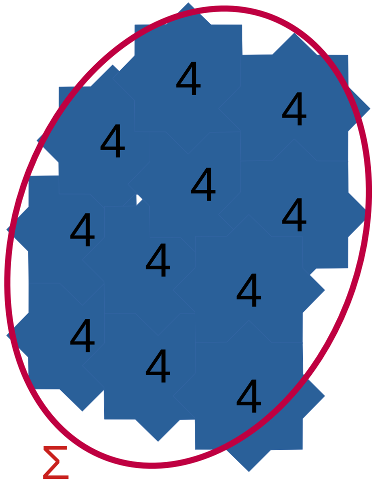
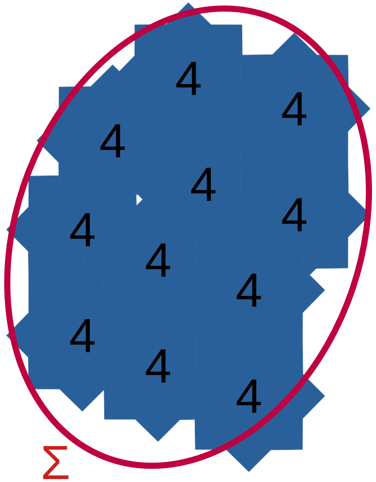


+quantum: path integral

$$\mathcal{A}(l_\Sigma, \theta_\Sigma) = \int [dl_s][d\theta_{t\sigma}] \prod_\sigma \delta(\det \gamma_\sigma)\, e^{\frac{i}{\hbar}\sum_{t\in\sigma} A_t \theta_{t\sigma}}$$



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$$\varepsilon = \pm 1 \qquad \rho > 0$$

$$\varepsilon\rho$$

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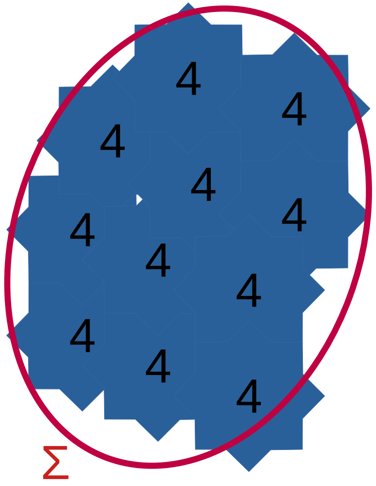

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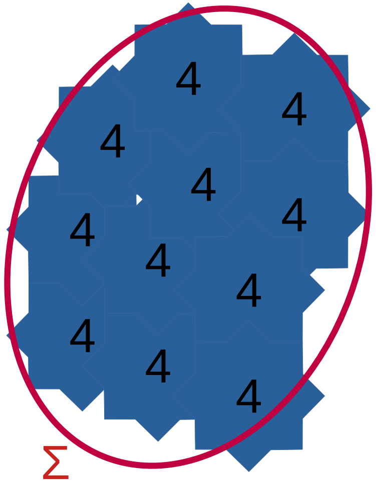

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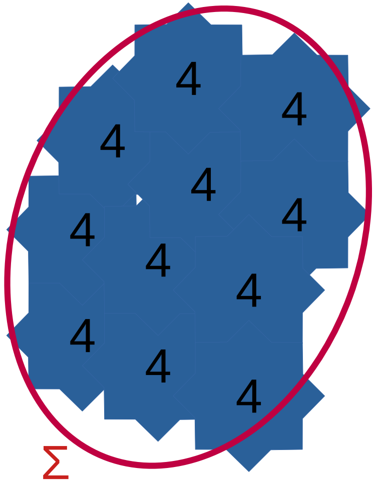
1. This sum is made over all possible configurations of ε assigned to wedges, including both causal and non-causal (spurious) configurations.

$$\boxed{\prod_{f\in\text{cycle}} \varepsilon_v(f) = \eta^{\#\text{cycle}}}$$



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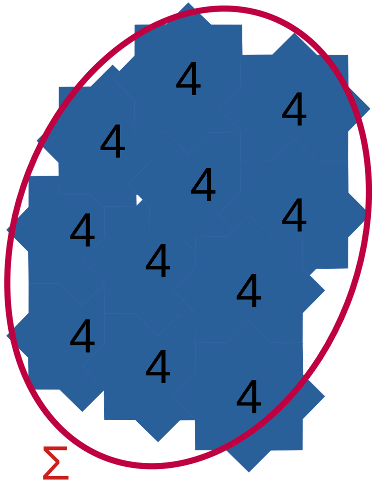

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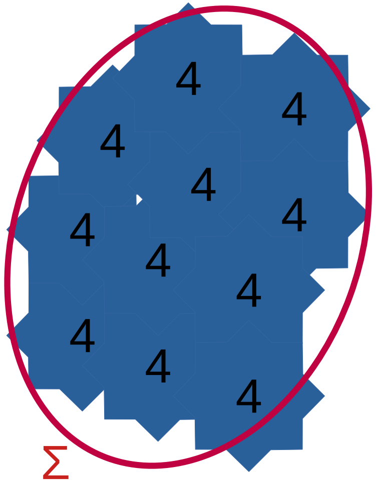

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4. The choice of including the spurious terms or not depends on what we want to compute: a projector on the physical Hilbert space or a causal propagator?

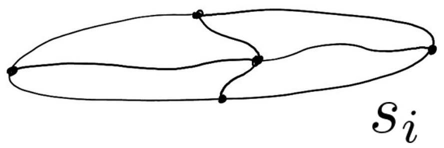
Feynman propagator: $(\Box + m^2)\ W = \delta$

Hadamard function: $(\Box + m^2)\ W = 0$

[Teitelboim 82']
[Livine, Oriti 02']

Spinfoams

(EPRL)

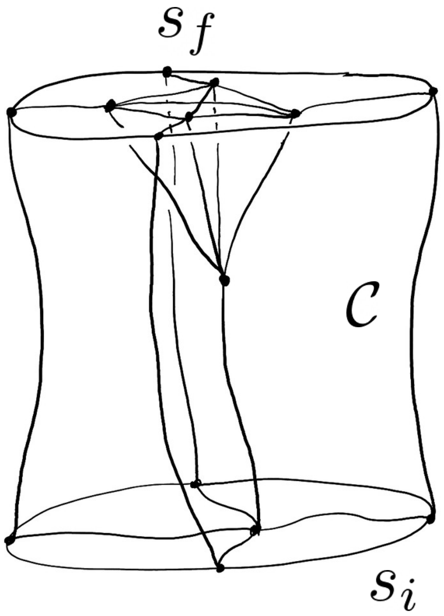


Spin-network:

quantum geometry of space

Spinfoams

(EPRL)



Spin-foam:
quantum
geometry of
space-time

Spin-network:
quantum geometry of space

Spinfoams

(EPRL)

Transition amplitude

$$W(s_i, s_f) = \sum_{\mathcal{C}} W_{\mathcal{C}}(s_i, s_f)$$

sum over 2-complexes

Spin-foam:
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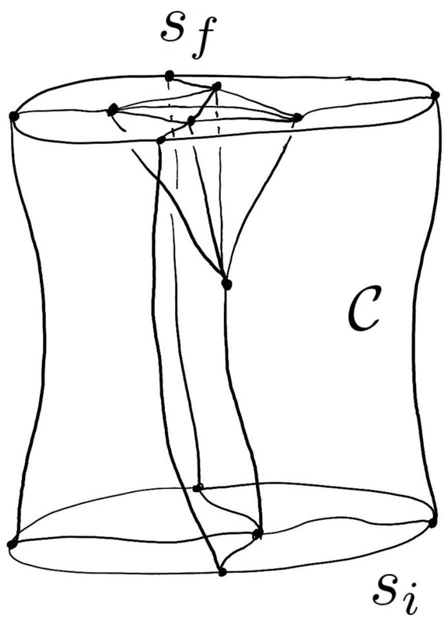

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$$W_{\mathcal{C}}(s_i, s_f) = \int_{SU(2)} [dh_w] \prod_f \delta(\prod_{w \in f} h_w) \prod_v A_v(h_w)$$

wedge variable ∈ SU(2)

face

vertex

Spin-foam:
quantum
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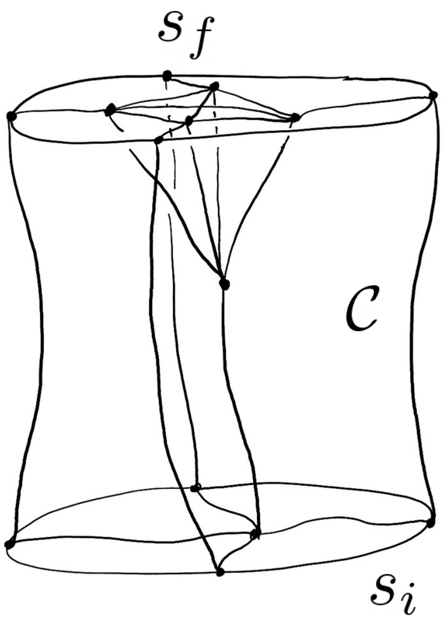

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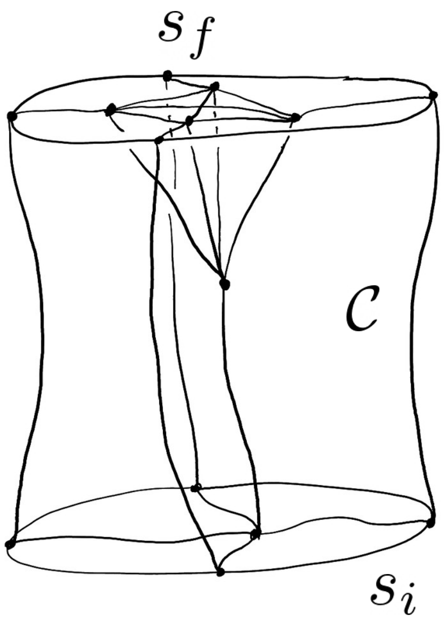

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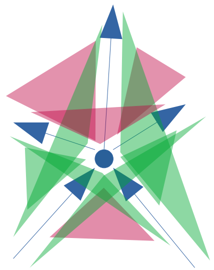
Vertex amplitude

$$A_v(h_w) = \int_{SL_2(\mathbb{C})} [dg_e] \prod_{w \in v} K(h_w, g_{s_w} g_{t_w}^{-1})$$

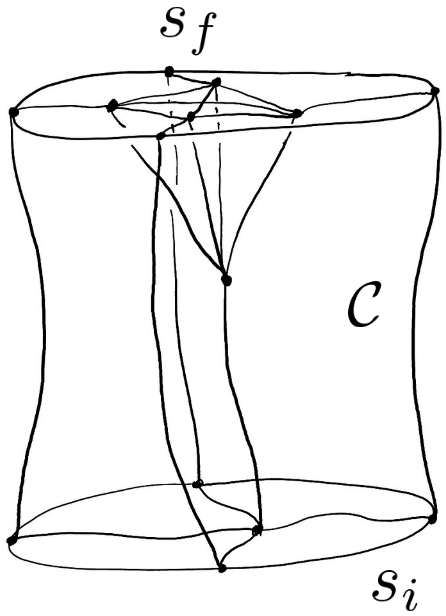
edge variable ∈ $SL_2(C)$

wedges



Spinfoams

(EPRL)



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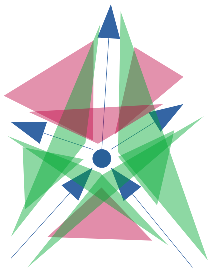

Wedge amplitude

$$K(h, g) = \sum_j \frac{(2j+1)^4}{\pi^3} \int [d\zeta][dz'][dz''] \, \langle z' | h^\dagger | z'' \rangle^{2j} \; \mathcal{B} \, e^{iS_\gamma}.$$

Causal spinfoams

"wedge action"

$$S_\gamma = \gamma j \log \frac{\|g^T \zeta\|^2}{\|\zeta\|^2}$$

(EPRL)

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sum over spins

integrations over CP^1

Causal spinfoams

(EPRL)

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integrations over CP[1]

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semi-classical limit

$$\theta_w$$

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Vertex amplitude

$$A_v = \sum_{\substack{[\varepsilon_w] \\ \text{causal } \eta=1}} A_v^\varepsilon + \sum_{\substack{[\varepsilon_w] \\ \text{causal } \eta=-1}} A_v^\varepsilon + \sum_{\substack{[\varepsilon_w] \\ \text{spurious}}} A_v^\varepsilon$$

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causal constraint

signature

$(\eta, -\eta, -\eta, -\eta)$

Causal spinfoams

(EPRL)

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sum over spins

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θ_w

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semi-classical limit

e^{iS_R} $\quad$ e^{-iS_R} $\quad$ 0

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Causal spinfoams

(EPRL)

Spin-foam amplitude

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Causal spinfoams

(EPRL)

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Causal spinfoam: propagator of spin-networks (time-evolution)

Full EPRL spinfoam: projector on the physical states (Hamiltonian constraint)

Causal spinfoams

[Christodoulou, Längvik, Riello, Röken, Rovelli, 13']

(EPRL)

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[Livine, Oriti 03']

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$$W_{\mathcal{C}} = \sum_{\substack{[\varepsilon_w] \\ \text{causal } \eta=1}} W_{\mathcal{C}}^{\varepsilon} + \sum_{\substack{[\varepsilon_w] \\ \text{causal } \eta=-1}} W_{\mathcal{C}}^{\varepsilon} + \sum_{\substack{[\varepsilon_w] \\ \text{spurious}}} W_{\mathcal{C}}^{\varepsilon}$$

Causal spinfoam: propagator of spin-networks (time-evolution)

Full EPRL spinfoam: projector on the physical states (Hamiltonian constraint)

[Livine, Oriti 03'] For the Barret-Crane model:

$$K^p(x_1,x_2) = \frac{2\sin(\beta(x_1,x_2)\,p/2)}{p\sinh\beta(x_1,x_2)} = \frac{1}{p\sinh\beta(x_1,x_2)} \sum_{\varepsilon=\pm 1} \varepsilon\, e^{i\varepsilon\beta(x_1,x_2)\,p/2}$$

[Engle 13']

[Immirzi, Gupta, Smolin, Markopoulou...]

Causal spinfoams

[Christodoulou, Längvik, Riello, Röken, Rovelli, 13']

(EPRL)

Some spurious terms are divergent.

Spin-foam amplitude

$$W_{\mathcal{C}} = \sum_{\substack{[\varepsilon_w] \\ \text{causal } \eta=1}} W_{\mathcal{C}}^{\varepsilon} + \sum_{\substack{[\varepsilon_w] \\ \text{causal } \eta=-1}} W_{\mathcal{C}}^{\varepsilon} + \sum_{\substack{[\varepsilon_w] \\ \text{spurious}}} W_{\mathcal{C}}^{\varepsilon}$$

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[Engle 13'] *"Proper vertex amplitude"*

$$A_v^{(+)}(\{k_{ab},\psi_{ab}\}) := (-1)^{\Xi} \int_{\mathrm{SL}(2,\mathbb{C})^5} \delta(X_4) \prod_a \mathrm{d}X_a \prod_{a<b} \alpha\left(X_a \mathcal{I}\psi_{ab}, X_b \mathcal{I}\, \Pi_{ba}\left(\{X_{ab}\}\right) \psi_{ba}\right)$$

[Immirzi, Gupta, Smolin, Markopoulou...]

Conclusion

- Causality exhausts almost all of the information content of the gravitational field (Malament's theorem).
- Causality can be represented by arrows on the edges of the dual skeleton.
- Causality can also be encoded on the wedges (more variables) provided a new "causal constraint" is satisfied.
- This causal constraint can be derived from the equations of motion.
- In the path-integral, one can restrict or not to causal histories only, depending on what one wants to compute, a projector or a propagator.
- Same thing with spinfoams.

Eugenio Bianchi & Pierre Martin-Dussaud
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