The loop quantum gravity vertex: A proposal

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Motivation: correcting BC

1. Problem with non-diagonal components of the graviton propagator (i.e., components not of the form $\langle \Omega \mid h^{aa}(x) h^{bb}(y) \mid \Omega \rangle$). Seems to be due to the BC vertex not depending on intertwiners. More precisely: In BC model, intertwiner degrees of freedom are frozen out by strong imposition of simplicity constraints in quantum theory ($\hat{C}_i \psi = 0$). (Will be discussed later)

2. But simplicity constraints are second class (i.e., $\{C_i, C_j\} \not\approx 0$). Thus they shouldn't be imposed strongly *anyway*. (Will be discussed later)

3. More precisely: beyond non-triviality of the intertwiners, it would be nice if the intertwiners of the spin-foam model exactly matched those of LQG. Would have exact matching of covariant and Hamiltonian approaches.

The philosophy in attacking these issues is the following. By attempting to correct the basic error in (2.), we hope the problem in (1.) will be solved. But how does one impose second class constraints *correctly* in quantum theory? There are many proposals for this, and all suffer ambiguities. Use expectation (3.) to guide us in addressing (2.).

Classical Theory

GR as a constrained BF theory

Holst-BF action

$$S_{Holst-BF} = \int \mathrm{tr}\left[B \wedge F + \frac{1}{\gamma_{HBF}} ({}^\star B) \wedge F\right]$$

Simplicity constraint:

$$\mathrm{tr}\left[{}^\star B \wedge B\right]_{abcd} = \mathcal{V} \epsilon_{abcd}$$

where $\mathcal{V} := \frac{1}{4!}\epsilon^{abcd}\mathrm{tr}[{}^\star B \wedge B]_{abcd}$, with the ϵ's normalized by $\epsilon_{0123} = \epsilon^{0123} = 1$.

Two classes of solutions:

$$B = {}^\star(e \wedge e) \qquad \text{or} \qquad B = e \wedge e$$

Substitution into $S_{Holst-BF}$ yields (modulo coeff.) the *Holst action*:

$$S_{Holst} = \int \mathrm{tr}\left[{}^\star(e \wedge e) \wedge F + \frac{1}{\gamma_H} e \wedge e \wedge F\right]$$

with the two sectors respectively corresponding to

$$\gamma_H = \gamma_{HBF} \qquad \text{or} \qquad \gamma_H = \frac{1}{\gamma_{HBF}}.$$

Note: No topological sector. Just two possible γ_H-sectors. $\gamma_H = \gamma_{BI} \equiv \gamma$.

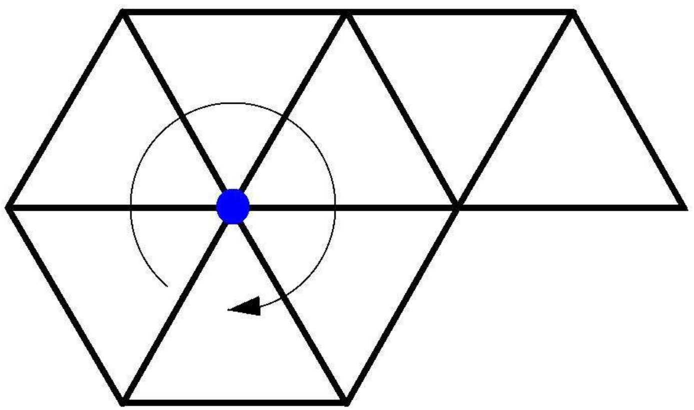
The discretization

Regge triangulation

Introduce triangulation Δ of space-time $\mathcal{M}$ by (oriented) 4-simplices:

triangulation components	(dual to)	symbol
4-simplices	(vertices)	v
tetrahedra	(edges)	t (or e)
triangles	(faces)	f

Geometry is flat on each 4-simplex ("Regge geometry"). Curvature is concentrated on the "bones" f. Around each f we have a "link" of alternating 4-simplices and tetrahedra.



The basic discrete variables

can be introduced and motivated as follows.

1. Introduce a geometry g on $\mathcal{M}$ that is flat on each 4-simplex.
2. In each 4-simplex v, fix a reference point $p(v)$, and in each tetrahedron t, fix a reference point $p(t)$. For each 4-simplex v and tetrahedron t therein, let
$$(V_{vt})^a{}_b : T_{p(t)}\mathcal{M} \to T_{p(v)}\mathcal{M}$$
denote parallel transport from $p(t)$ to $p(v)$ within v, as determined by g. V_{vt} is unambiguous because g is flat in v.
3. For each v, define a tetrad $e(v)^I_a$ at $p(v)$ such that $g_{ab}|_{p(v)} = e(v)^I_a e(v)_{Ib}$. Likewise for each t, define a tetrad $e(t)^I_a$ at $p(t)$ such that $g_{ab}|_{p(t)} = e(t)^I_a e(t)_{Ib}$.
4. For each v and t therein, define the matrix $(V_{vt})^I{}_J = (V_{tv}^{-1})^I{}_J$ by $e(v)^I_b (V_{vt})^b{}_a = (V_{vt})^I{}_J e(t)^J_a$. The fact that $g_{cd}|_{p(v)} (V_{vt})^c{}_a (V_{vt})^d{}_b = g_{ab}|_{p(t)}$ implies $(V_{vt})^I{}_J \in SO(4)$.
5. For each t, because of local flatness of g, $e(t)^I_a$ can be consistently extended to all of t by parallel transport via g. Using this cotetrad field on t, for each triangle f in t, define
$$B_f(t)^{IJ} := \int_f {}^\star(e(t)^I \wedge e(t)^J).$$

For each triangle f and each pair of tetrahedra $t, t' \in \mathrm{Link}(f)$,

$$U_f(t,t') := V_{tv_1} V_{v_1 t_1} V_{t_1 v_2} \cdots V_{v_n t'}$$

where the product is around the link in the clock-wise direction from t' to t.

Constraints on the variables

1. $U_f(t,t') B_f(t') = B_f(t) U_f(t,t') \qquad \forall\ f$ and $t, t' \in \mathrm{Link}(f)$
2. (closure) $\sum_{f \in t} B_f(t) = 0 \qquad \forall\ t$
3. (discrete simplicity constraints)

$$\begin{aligned}
&\text{(i)} && C_{ff} := && \tfrac{1}{4}\mathrm{tr}\left[({}^\star B_f(t)) B_f(t)\right] \approx 0 && \forall f \\
&\text{(ii)} && C_{ff'} := && \tfrac{1}{4}\mathrm{tr}\left[({}^\star B_f(t)) B_{f'}(t)\right] \approx 0 && \forall f, f' \in t \\
&\text{(iii)} && && \mathrm{tr}\left[({}^\star B_f(v)) B_{f'}(v)\right] \approx \pm 12 V(v) &&
\end{aligned}$$

$\forall f, f' \in v$ not in the same t

(1.) will be imposed prior to varying the action (next slide). **(2.)** will be dictated by the action in quantum theory, **(3i.),(3ii.)** will be imposed separately in quantum theory. **(3iii.)** is automatically satisfied when the rest of the constraints are satisfied, due to our choice of variables.

Discrete action

($U_f(t) := U_f(t,t)$, holonomy around the full link, starting at t.)

$$\begin{aligned} S_{disc} &= \frac{1}{2}\sum_{f\in int\Delta} \mathrm{tr}\left[B_f(t)U_f(t) + \frac{1}{\gamma_{HBF}}{}^\star B_f(t)U_f(t)\right] \\ &+ \frac{1}{2}\sum_{f\in\partial\Delta} \mathrm{tr}\left[B_f(t)U_f(t,t') + \frac{1}{\gamma_{HBF}}{}^\star B_f(t)U_f(t,t')\right] \end{aligned}$$

Boundary/3-slice variables: $B_f(t) \in \mathfrak{so}(4)$, $U_f(t,t') \in SO(4)$.

Symplectic structure:

$$J_f(t) = B_f(t) + \frac{1}{\gamma_{HBF}}{}^\star B_f(t)$$

In the limits $\gamma_{HBF} \ll 1$ and $\gamma_{HBF} \gg 1$,

$$\gamma_{HBF} \ll 1 \quad \rightsquigarrow \quad J = \frac{1}{\gamma_{HBF}}{}^\star B \qquad\qquad \gamma_{HBF} \gg 1 \quad \rightsquigarrow \quad J = B$$

Self-dual structure of $SO(4)$

Group and Lie algebra decomposition

$$SO(4) = SU(2) \times SU(2)/\mathbb{Z}_2 \qquad \mathfrak{so}(4) = \mathfrak{su}(2)_+ \oplus \mathfrak{su}(2)_-$$

Generators

For any orthonormal basis $\{n^I, v_1^I, v_2^I, v_3^I\}$ of $\mathbb{R}^4$, one can define

$$J_\pm^i := \frac{1}{2}({}^\star J \pm J)_{IJ} v_i^I n^J \qquad \Rightarrow \qquad [J_\pm^i, J_\pm^j] = \epsilon^{ij}{}_k J_\pm^k$$

Casimir operators

$$\tfrac{1}{4}\hat{J}^{IJ}\hat{J}_{IJ} = \hat{J}_+^i \hat{J}_{+i} + \hat{J}_-^i \hat{J}_{-i} \qquad \mathit{Spect.} = \{j^+(j^+ + 1) + j^-(j^- + 1)\}$$

$$\tfrac{1}{8}\epsilon_{IJKL}\hat{J}^{IJ}\hat{J}^{KL} = \hat{J}_+^i \hat{J}_{+i} - \hat{J}_-^i \hat{J}_{-i} \qquad \mathit{Spect.} = \{j^+(j^+ + 1) - j^-(j^- + 1)\}$$

Representations

$SO(4)$ rep'ns decompose into $SU(2)$ rep'ns. They are labelled by (j^+, j^-):

$$\begin{aligned} D^{(j^+,j^-)}(g^+, g^-)^{\mathcal{A}\mathcal{A}'}{}_{\mathcal{B}\mathcal{B}'} &= D^{j^+}(g^+)^{\mathcal{A}}{}_{\mathcal{B}} D^{j^-}(g^-)^{\mathcal{A}'}{}_{\mathcal{B}'} \\ &= D^{j^+}(g^+)^{A_1 \cdots A_{2j+}}{}_{B_1 \cdots B_{2j+}} D^{j^-}(g^-)^{A'_1 \cdots A'_{2j-}}{}_{B'_1 \cdots B'_{2j-}} \\ &= D^{j^+}(g^+)^{(A_1 \cdots A_{2j+})}{}_{(B_1 \cdots B_{2j+})} D^{j^-}(g^-)^{(A'_1 \cdots A'_{2j-})}{}_{(B'_1 \cdots B'_{2j-})} \end{aligned}$$

Quantum theory: Kinematics
Hilbert space associated with a 3-slice

Switch to the dual, 2-complex picture, Δ^*. For each 3-surface Σ intersecting no vertices of Δ^*, let $\gamma_\Sigma := \Sigma \cap \Delta^*$. Hilbert space associated with Σ:

$$\mathcal{H}_\Sigma = L^2\left(SO(4)^{|E(\gamma_\Sigma)|}\right)$$

Let $\hat{J}_f(t)^{IJ}$ denote the right-inv. vect. fields, determined by the basis J^{IJ} of $\mathfrak{so}(4)$, on the copy of $SO(4)$ associated with the link $l = f \cap \Sigma$ determined by f, with orientation such that the node $n = t \cap \Sigma$ is the source of l.

Then

$$\hat{B}_f(t) := \left(\frac{\gamma_{HBF}^2}{\gamma_{HBF}^2 - 1}\right)\left(\hat{J}_f(t) - \frac{1}{\gamma_{HBF}}{}^\star\hat{J}_f(t)\right)$$

Gauge-invariant Hilbert space

$$\mathcal{H}_\Sigma^G := SO(4)\text{-gauge-invariant subspace of } \mathcal{H}_\Sigma.$$

Is the solution to closure (Gauss) constraint. $\mathcal{H}_\Sigma^G$ has as an orthonormal basis the $SO(4)$ spin-networks:

$$\Psi_{j_l^+, j_l^-, I_n^+, I_n^-}(g_l^+ g_l^-) \equiv \langle g_l^+ g_l^- \,|\, j_l^+, j_l^-, I_n^+, I_n^- \rangle$$

$$:= \left(\bigotimes_l D^{(j_l^+)}(g_l^+) \cdot \bigotimes_n I_n^+ \right) \left(\bigotimes_l D^{(j_l^-)}(g_l^-) \cdot \bigotimes_n I_n^- \right).$$

History of quantum states: spin-foams

A spin-foam is a history of a spin-network. Hence a 2-complex in space-time with faces labelled by spins and edges labelled by intertwiners.

Diagonal quantum simplicity constraints

$$\hat{C}_{ff} = \frac{1}{4}\epsilon_{IJKL}\hat{B}_f(t)^{IJ}\hat{B}_f(t)^{KL} = (\hat{B}_f^+)^i(\hat{B}_f^+)_i - (\hat{B}_f^-)^i(\hat{B}_f^-)_i$$

Assume $\gamma_{HBF} \ll 1$ or $\gamma_{HBF} \gg 1$. Then

$$\hat{C}_{ff} \propto (\hat{J}_f^+)^i(\hat{J}_f^+)_i - (\hat{J}_f^-)^i(\hat{J}_f^-)_i$$

So that

$$\hat{C}_{ff}\Psi = 0 \qquad \Rightarrow \qquad j_f^+ = j_f^- .$$

More complicated for intermediate γ_{HBF}.

What is the correspondence between $j_{SO(4)} \equiv j_+ = j_-$ here and j in LQG? We have the germ of an argument that $j_{LQG} = 2j_{SO(4)}$ from looking at the large j limit of the area operators (tricky part: bringing in γ, Newton constant, and numerical factors correctly).

Off-diagonal quantum simplicity constraints

These are constraints on the intertwiners. They are second class constraints ($\{C_{f_1 f_2}, C_{f_1 f_3}\} \not\approx 0$), therefore, they should not be imposed strongly ($\hat{C}_{f f'} \Psi = 0$), but weakly in some sense. Inspired by the Gupta-Bleuler formalism, we seek to find a space $\mathcal{H}_{phys}$ such that

$$\langle \phi \mid \hat{C}_{f f'} \mid \psi \rangle = 0 \qquad \forall \phi, \psi \in \mathcal{H}_{phys}.$$

This is not sufficient to determine $\mathcal{H}_{phys}$. We will use our desire to have isomorphism with LQG Hilbert space to guide us, as well as look for an alternative operator equation to use.

Reformulation of the off-diagonal simplicity constraints

$\gamma_{HBF} = \gamma$ ($B = {}^\star e \wedge e$) sector. For each tetrahedron t,

$$\exists e(t)^I_a \text{ in } t \text{ s.t. } \left(B_f(t) = \int_f e(t)^I \wedge e(t)^J \qquad \forall f \in t \right)$$

iff

$$\exists n^I \text{ s.t. } \left(n_I B_f(t)^{IJ} = 0 \qquad \forall f \in t \right)$$

In particular, if $n^I = (1, 0, 0, 0)$, the latter condition becomes $B_f(t)^{0i} = 0$.

$\gamma_{HBF} = \gamma^{-1}$ ($B = e \wedge e$) sector. For each tetrahedron t,

$$\exists e(t)^I_a \text{ in } t \text{ s.t. } \left(B_f(t) = \int_f ({}^\star e(t) \wedge e(t))^{IJ} \qquad \forall f \in t \right)$$

iff

$$\exists n^I \text{ s.t. } \left(\epsilon_{IJKL} n^J B_f(t)^{KL} = 0 \qquad \forall f \in t \right)$$

In particular, if $n^I = (1, 0, 0, 0)$, the latter condition becomes $B_f(t)^{ij} = 0$.

This reformulation allows distinction between the two γ-sectors.

Reformulation for quantum theory

Let $J_f^i := (J_f^+)^i + (J_f^-)^i = -\frac{1}{2}\epsilon^i{}_{jk}J^{jk}$.

	$\gamma_{HBF} \ll 1$	$\gamma_{HBF} \gg 1$
$\gamma_{HBF} = \gamma$ $(B = {}^\star e \wedge e)$ sector	$(\gamma \ll 1)$ $B_f^{ij} = 0$ $J_f^{0i} = 0$ $J_f^{IJ} J_{fIJ} = 2J_f^i J_{fi}$	$(\gamma \gg 1)$ $B_f^{ij} = 0$ $J_f^{ij} = 0$ $J_f^i J_{fi} = 0$
$\gamma_{HBF} = \gamma^{-1}$ $(B = e \wedge e)$ sector	$(\gamma \gg 1)$ $B_f^{0i} = 0$ $J_f^{ij} = 0$ $J_f^i J_{fi} = 0$	$(\gamma \ll 1)$ $B_f^{0i} = 0$ $J_f^{0i} = 0$ $J_f^{IJ} J_{fIJ} = 2J_f^i J_{fi}$

Thus, two possible equations to impose in quantum theory, corresponding to two limits of the *physical* γ.

- Let H denote the $SO(3)$ subgroup of $SO(4)$ generated by J^i_f. Under the action of H, the (j,j) representation becomes a $j \otimes j$ representation, whence it decomposes

$$j \otimes j = 0 \oplus 1 \oplus \cdots \oplus 2j.$$

- Let $\mathcal{H}_{j,j}$ denote the carrying space for the spin (j,j) $SO(4)$ rep'n, and $\mathcal{H}_j$ denote the carrying space for the spin j $SU(2)$ rep'n. The choice of $\{\hat{J}^i_f\}$, equiv to choice of H, thus gives a decomp.

$$\mathcal{H}_{(j,j)} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \cdots \oplus \mathcal{H}_{2j}.$$

- At a node with adjacent spins j_1, j_2, j_3, j_4, the $SO(4)$ intertwiner space is

$$\mathcal{I}^{(\vec{j},\vec{j})}_{SO(4)} = \mathrm{Inv}_{SO(4)} \left(\mathcal{H}_{(j_1,j_1)} \otimes \cdots \otimes \mathcal{H}_{(j_4,j_4)}\right)$$

Consider the subspaces

$$\begin{aligned} \mathcal{I}_1 &:= \mathrm{Inv}_{SO(4)} \left(\mathcal{H}_0 \otimes \mathcal{H}_0 \otimes \mathcal{H}_0 \otimes \mathcal{H}_0\right) \subset \mathcal{I}^{(\vec{j},\vec{j})}_{SO(4)} \\ \mathcal{I}_2 &:= \mathrm{Inv}_{SO(4)} \left(\mathcal{H}_{2j} \otimes \mathcal{H}_{2j} \otimes \mathcal{H}_{2j} \otimes \mathcal{H}_{2j}\right) \subset \mathcal{I}^{(\vec{j},\vec{j})}_{SO(4)} \end{aligned}$$

Note $\mathcal{I}_1 = \{\text{BC intertwiner}\}$.
On both $\mathcal{I}_1$ and $\mathcal{I}_2$, $\hat{C}_{ff'}$ has vanishing matrix elements.

Let $\hat{C}_4 := \frac{1}{4}\hat{J}_f^{IJ}\hat{J}_{fIJ}$ and $\hat{C}_3 := \hat{J}_f^i\hat{J}_{fi} = \frac{1}{2}\hat{J}_f^{ij}\hat{J}_{fij}$.

On $\mathcal{I}_1$ we have

$$\hat{C}_3\psi = 0.$$

Is precisely the simplicity constraint for the $\gamma \gg 1$ case.

$\mathcal{I}_2$ can be obtained by first solving

$$\left(\sqrt{2\hat{C}_4 + \hbar^2} - \sqrt{\hat{C}_3 + \hbar^2/4} - \hbar/2\right)\psi = 0 \qquad (1)$$

to obtain a space $\tilde{\mathcal{I}}_2$. $\mathcal{I}_2$ is then $\mathrm{Inv}_{SO(4)}\left(\tilde{\mathcal{I}}_2\right)$. Note (1) yields

$$2C_4 - C_3 = 0 \qquad (2)$$

in the classical limit. Taking the $SO(4)$ invariant part of $\tilde{\mathcal{I}}_2$ can be interpreted as requiring (2) to hold only with respect to *some* choice of n^I:
Is precisely the simplicity constraint for the $\gamma \ll 1$ case.

Thus, $\mathcal{I}_1$ solves simplicity constraint for $\gamma \gg 1$,
and $\mathcal{I}_2$ solves simplicity constraint for $\gamma \ll 1$.

$\mathcal{I}_1$ is the BC intertwiner, leading to BC model.
What is the new space $\mathcal{I}_2$, and to what spin-foam model does it lead?

Embedding of LQG Hilbert space into $SO(4)$ theory Hilbert space.

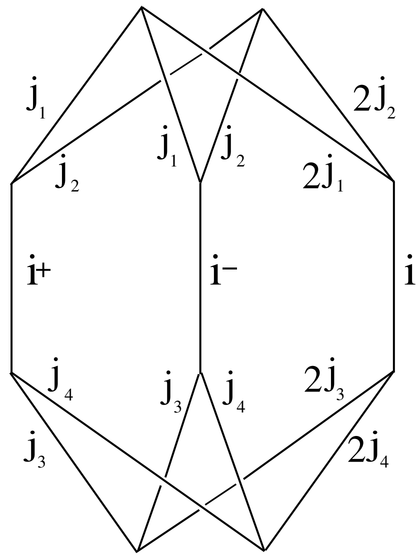
$$\mathcal{I}^{2\vec{j}}_{SO(3)} \subset \mathcal{H}_{2j_1} \otimes \cdots \otimes \mathcal{H}_{2j_4} \subset \mathcal{H}_{(j_1,j_1)} \otimes \cdots \otimes \mathcal{H}_{(j_4,j_4)}$$

Let $P_{SO(4)}$ denote group averaging on $\mathcal{H}_{(j_1,j_1)} \otimes \cdots \otimes \mathcal{H}_{(j_4,j_4)}$. Then

$$f := P_{SO(4)}\big|_{\mathcal{I}^{2\vec{j}}_{SO(3)}} : \mathcal{I}^{2\vec{j}}_{SO(3)} \to \mathcal{I}^{(\vec{j},\vec{j})}_{SO(4)}.$$

In terms of the usual intertwiner basis based on a ‘virtual link’:

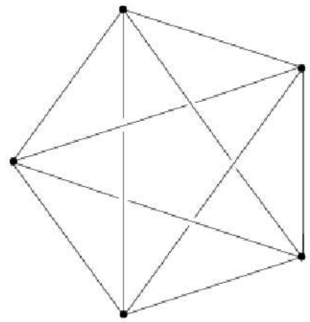
$$f \mid i\rangle = \sum_{i^+ i^-} f^i_{i^+ i^-} \mid i^+ i^-\rangle, \qquad f^i_{i+i-} =$$



$\Big|_{\text{Triv. conn.}}$

Finally, define $f : \mathcal{H}_{LQG} \to \mathcal{H}_{SO(4)}$ in terms of spin networks: spins mapping as $2j \mapsto (j,j)$, and intertwiners mapping as $I \mapsto f(I)$.

Vertex amplitude



Find vertex amplitude by evaluating the amplitude for a single 4-simplex v. 10 $B_f(t)$'s and 10 $U_f(t,t')$ are the boundary variables. For each pair $t, t' \in v$, there is a unique f between them, and we write $B_{tt'} = B_f(t)$ and $U_{tt'} = U_f(t,t')$. Let $\Pi_f(t)^{IJ} := J_f(t)^{IJ} = B_f(t) + \frac{1}{\gamma_{HBF}} {}^\star B_f(t)$. Then

$$A[B_{tt'}] = \int dV_{vt} \; e^{i\sum Tr[\Pi_{tt'} V_{tv} V_{vt'}]}.$$

Transforming to the conjugate variables gives

$$\begin{aligned} A[U_{tt'}] &= \int dB_{tt'} e^{-i\sum Tr[\Pi_{tt'} U_{tt'}]} \; A[B_{tt'}] \\ &= \left|\frac{\partial \vec{B}}{\partial \vec{\Pi}}\right| \int d\Pi_{tt'} e^{-i\sum Tr[\Pi_{tt'} U_{tt'}]} \; A[B_{tt'}] \\ &= \left|\frac{\partial \vec{B}}{\partial \vec{\Pi}}\right| \int dV_{vt} \prod_{tt'} \delta(U_{tt'} V_{t'v} V_{vt}). \end{aligned}$$

This is the amplitude. We transform back to the spin network basis, using the $SO(4)$ spin network functions, $\Psi_{j^\pm_{tt'}, i^\pm_t}(U_{tt'})$, at the same time dropping the physically irrelevant constant $\left|\frac{\partial \vec{B}}{\partial \vec{\Pi}}\right|$:

$$\begin{aligned} A[j^{\pm}_{tt'}, i^{\pm}_t] &= \int dU_{tt'} \, \Psi_{j^{\pm}_{tt'}, i^{\pm}_t}(U_{tt'}) \; A[U_{tt'}] \\ &= \int dV_{vt} \;\; \Psi_{j^{\pm}_{tt'}, i^{\pm}_t}(V_{tv} V_{vt'}) \\ &= \;\; 15 j_{SO(4)}(j^{+}_{tt'}, j^{-}_{tt'}; i^{+}_t, i^{-}_t). \end{aligned}$$

Combining with the constraints and the embedding f, we obtain the $SO(3)$ LQG spin-foam model with partition function

$$Z_{\mathrm{GR}} = \sum_{l_f, i_e} \prod_f (\dim \tfrac{l_f}{2})^2 \;\prod_v A(l_f, i_e)$$

with $l_f \in \mathbb{Z}$ and

$$A(l_f, i_e) = 15 j_{SO(4)}\Big(\tfrac{l_f}{2}, \tfrac{l_f}{2}; f(i_e)\Big) = \sum_{i^+_e, i^-_e} 15 j_{SO(4)}\Big(\tfrac{l_f}{2}, \tfrac{l_f}{2}; i^+_e, i^-_e\Big) \prod_{e \in v} f^{i_e}_{\;i^+_e i^-_e} \quad .$$

(where we have switched notation $t \to e$)

Relation to other models

Livine and Speziale have proposed to solve the simplicity constraints weakly, using $SU(2)$ *coherent states*.

In brief, Livine and Speziale use $SU(2)$ coherent states to construct $SO(4)$ coherent states and then impose the simplicity constraints on the *labels* of the coherent states.

The models one obtains with the two different methods in the two different γ limits are

	$\gamma \ll 1$	$\gamma \gg 1$
Casimir operator equation	The LQG vertex	Barrett-Crane model
Coherent states	The LQG vertex	Freidel-Krasnov-Livine-Speziale model

Conclusions

Summary of key points

Regarding the LQG vertex spin-foam model:

1. Off-diagonal simplicity constraints are second class: therefore they have been imposed weakly. Intertwiner degrees of freedom freed.
2. New intertwiner degrees of freedom may solve problem with the graviton propagator.
3. Only spin-foam model so far with boundary states/3-slice states matching those of LQG.
4. Alternative way to derive the model using coherent states, as proposed by Livine and Speziale.
5. Can perhaps be viewed as corresponding to $\gamma \ll 1$.

Possible danger: $\gamma \ll 1$ may indicate risk of falling into topological sector.

On the other hand, for FKLS model: The relation to LQG is not clear.

In short, we are only beginning to understand these alternatives to the BC model — and the most important test, calculating the graviton propagator, has not yet been done.

For both the LQG-vertex model, and the FKLS model, *a GFT formulation has been found* (Freidel and Krasnov). However, there are

Many more tasks/questions

1. Intermediate (i.e., finite and non-negligible) γ?
   (FK give a view on this, and a corresponding proposal.)
2. More than 4-valent nodes?
3. Lorenztian formulation of the present model?
4. Finite amplitude result for these new models, for fixed triangulation?
5. Graviton propagator from these new models?

Relevant new literature

[1] E., Pereira, and Rovelli 2007 The loop-quantum-gravity vertex-amplitude, `arXiv:0705.2388`

[2] E., Pereira, and Rovelli 2007 Flipped spinfoam vertex and loop gravity, `arXiv:0708.1236`

[3] Livine and Speziale 2007 A new spinfoam vertex for quantum gravity, `arXiv:0705.0674`

[4] Freidel and Krasnov 2007 A new spin foam model for 4d gravity, `arXiv:0708.1595`

[5] Livine and Speziale 2007 Consistently solving the simplicity constraints for spinfoam quantum gravity, `arXiv:0708.1915`