# THE PRINCIPLE OF RELATIVE LOCALITY

Laurent Freidel

ILQG March 2011

G. Amelino-Camelia, J. Kowalski-Glikman, L. Smolin

hep-th:1101.0931, ...

Look around

What do you see?
What do you directly experience?

Do you see space? spacetime?
Do you experience directly being in a spacetime?

We don't really see spacetime, we see momentum space...

As naive observers we do not directly observe spacetime points

we do not directly observe events macroscopically displaced from us

Our most fundamental measurements are all about energy-momentum quanta we absorb and emit

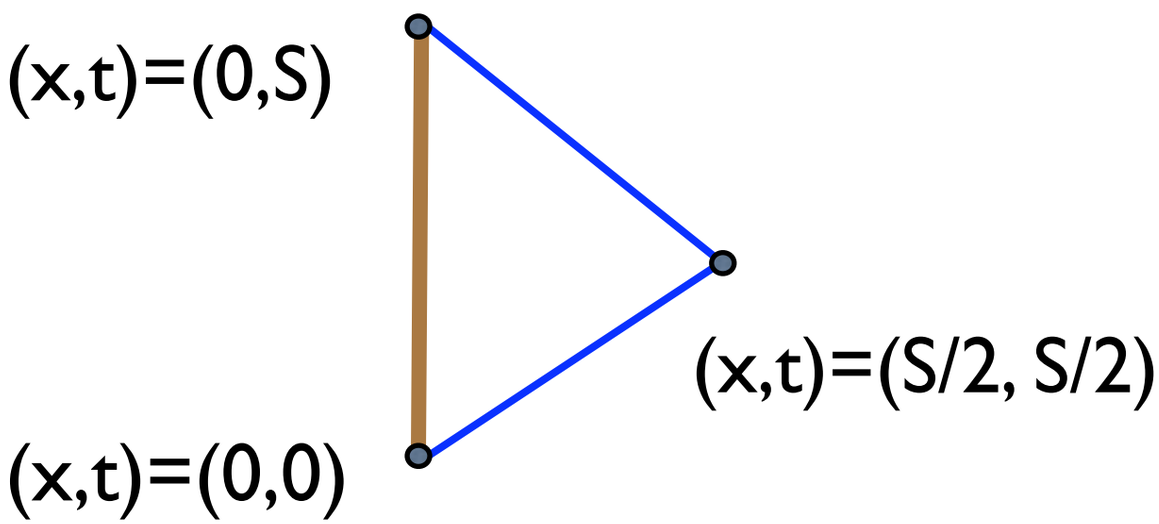
We see photons arriving with different momenta and energies at different angles

Physics happens in phase space. We do not have to assume that the projection from phase space to space time is trivial.

# Absolute locality

Spacetime is constructed by inference from energy and momenta measurement

e.g. Einstein procedure of photon exchange to give coordinates to distant events via momentum space measurements




One fundamental hypothesis is that the energy of the probe we use is inessential.

This is the absolute locality hypothesis. Why?

Is it a low energy approximation?

# Absolute locality

As we are going to see the absolute locality hypothesis is equivalent to the assumption that momentum space is a linear manifold.

→ The notion of locality is related to hypothesis about the geometry of momentum space

What if momentum space is a non linear manifold?

Do we still all infer the same spacetime?
Do we still infer the same spacetime at different energies?

Introducing the possibility for momentum space to be non linear allows us to propose a framework in which locality is relaxed in a controlled manner

# Quantum gravity limit

In this work we propose a framework in which we can relax in a controlled manner the notion of locality

What could be the motivation? Quantum gravity

We work in a regime where we neglect both quantum mechanics and gravity $\hbar$ and $G_{Newton}$ are neglected while

$\hbar, G \to 0$

$$\sqrt{\frac{\hbar}{G_{Newton}}} = m_p$$

is held fixed

In this limit there is a fundamental energy scale

$\longrightarrow$ one expects a deformation of momentum space

# Geometry of momentum space

Operational point of view: a local observer is equipped with a calorimeter and a clock.

From her measurement she concludes that each isolated system possesses 4 conserved quantities: Energy momentum space $\mathcal{P}$

She can realise two type of measurements:

One particle measurements: measurement of the mass and of the kinetic energy

Multi particle measurements: scattering processes, interactions, merging.

# Geometry of momentum space

One postulates that single particle measurements determine the geometry of $\mathcal{P}$

$\mathcal{P}$ is a lorentzian metric manifold with an origin 0

The mass is interpreted as the timelike distance from the origin

$$D^2(p) \equiv D^2(p,0) = m^2.$$

The kinetic energy defines the geodesic spacelike distance between a particle $p$ at rest and a particle $p'$ of identical mass $D(p) = D(p') = m$

$$D^2(p,p') = -2mK$$

from these measurements we can reconstruct the metric on $\mathcal{P}$

$$dk^2 = h^{ab}(k)dk_a dk_b$$

# Geometry of momentum space

When we define operationally momentum space $\mathcal{P}$

we use one type of calorimeter,
chosing another calorimeter will amount to a redefinition

$$p \to p' = \phi(p)$$

The theory has to be invariant under diffeomorphism on momentum space.

$\longleftrightarrow$ calorimeter regauging

# Geometry of momentum space

In the multiple particle case we should have a rule to associate a total momenta to the combination of particles

We postulate that there exists a composition of momenta

$$(p,q) \to p'_a = (p \oplus q)_a$$

More complicated interaction processes are build up by iteration of this composition e.g $(p \oplus q) \oplus k$

We do not assume that it is linear or commutative or associative

Outgoing momenta can be turned in ingoing momenta: there is an operation $p \to \ominus p$

satisfying $(\ominus p) \oplus p = 0$

We also ask that $(\ominus p) \oplus (p \oplus k) = k,$ Left Loop

kikkawa, sabinin, L.F

# Geometry of momentum space

The composition rule defines an affine connection on $\mathcal{P}$

One first notes that under a diffeomorphism $p \to p' = \phi(p)$

The operator

$$\boxed{U(p)^b_a \equiv \partial^b_q (p \oplus q)_a|_{q=0}}$$

transforms as a map from $T_0 P$ to $T_p(P)$

It can be interpreted as a paralell transport operator

One additional hypothesis: mono alternaticity $p_{N+1} = p_N \oplus p, \quad p_1 = p$

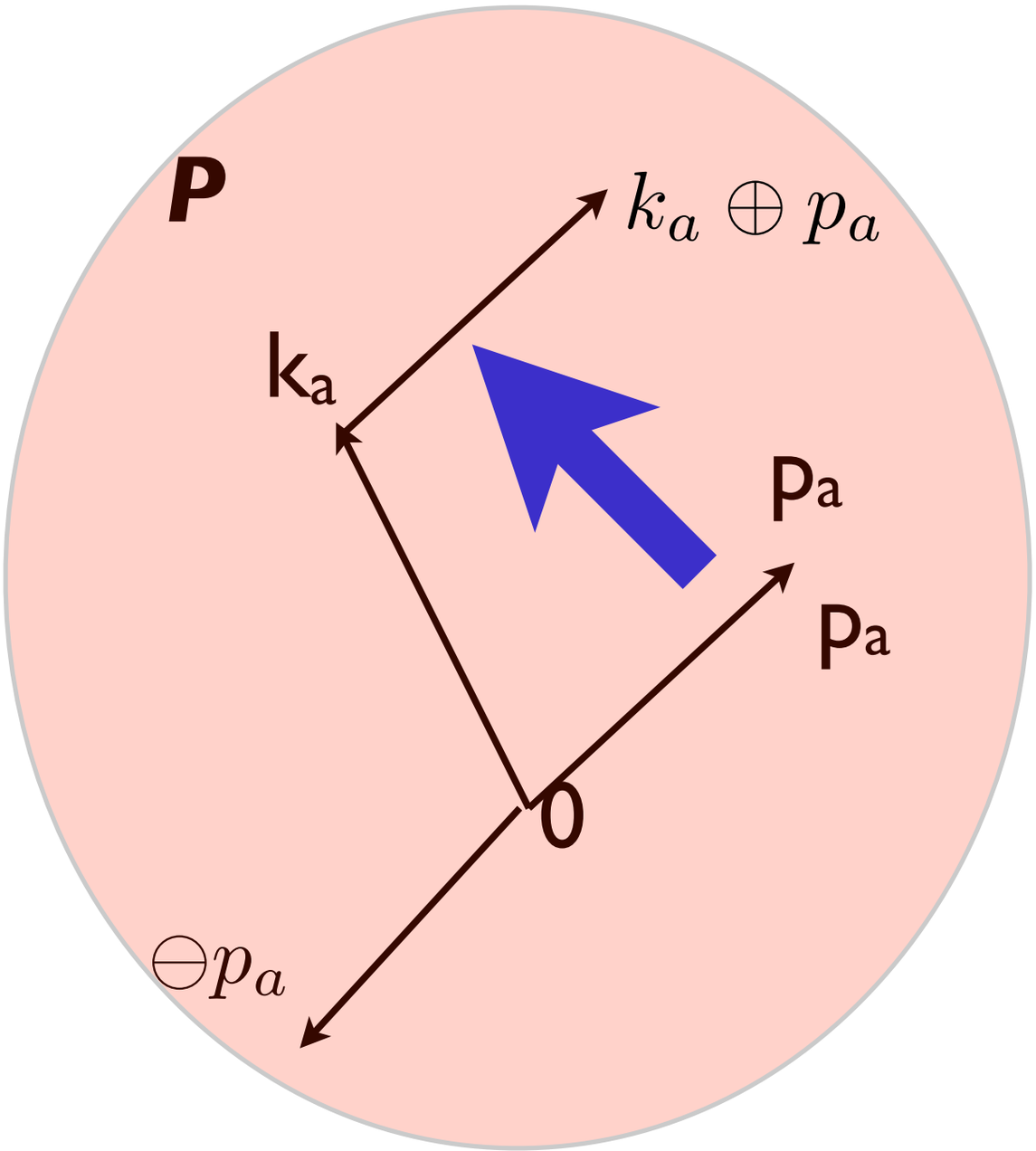
$$p_{N+1} \oplus q = p_N \oplus (p \oplus q)$$

geodesic motion

# Geometry of momentum space

Momenta combine into interactions: The rule:

$$(k, q) \to k'_a = k_a \oplus q_a$$




can be thought as a rule for combining geodesics on a curved manifold, so it defines a connection or parallel transport.

$$\begin{aligned} k_a \oplus dp_a &= k_a + U(k)^b_a dp_b \\ &= k_a + dp_a + \Gamma^{bc}_a k_b dp_c + \end{aligned}$$

# Geometry of momentum space

The composition rules defines an affine connection on $\mathcal{P}$

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p \oplus q)_c|_{q,p=o} = -\Gamma_c^{ab}(0)$$

transforms as an affine connexion

Torsion measures non commutativity

$$-\partial_p^a \partial_q^b [(p \oplus q)_c - (q \oplus p)_c]|_{p,q=0} = T_c^{ab}(0)$$

What about associativity?

# Geometry of momentum space

The composition rules defines an affine connection on $\mathcal{P}$

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p \oplus q)_c|_{q,p=o} = -\Gamma^{ab}_c(0)$$

transform as an affine connexion

Curvature measures non associativity

$$2\frac{\partial}{\partial p_{[a}}\frac{\partial}{\partial q_{b]}}\frac{\partial}{\partial k_c}\left((p \oplus q) \oplus k - p \oplus (q \oplus k)\right)_d|_{q,p,k=o} = R^{abc}{}_d(0)$$

To define the connection away from 0 we "translate" the addition

using the left translation operator $L_k(p) \equiv k \oplus p$

# Three aspects of geometry, which can be measured:

$$p_a \oplus q_a = p_a + q_a + \Gamma_a^{bc} p_b q_c + \ldots$$

- Torsion: measures non-commutativity of interactions.

$$T_a^{bc} = \Gamma_a^{bc} - \Gamma_a^{cb}$$

- Curvature: measures non-associativity of interactions.

$$R^{abc}{}_d = \partial^a \Gamma_d^{bc} - \partial^b \Gamma_d^{ac} + \Gamma\Gamma$$

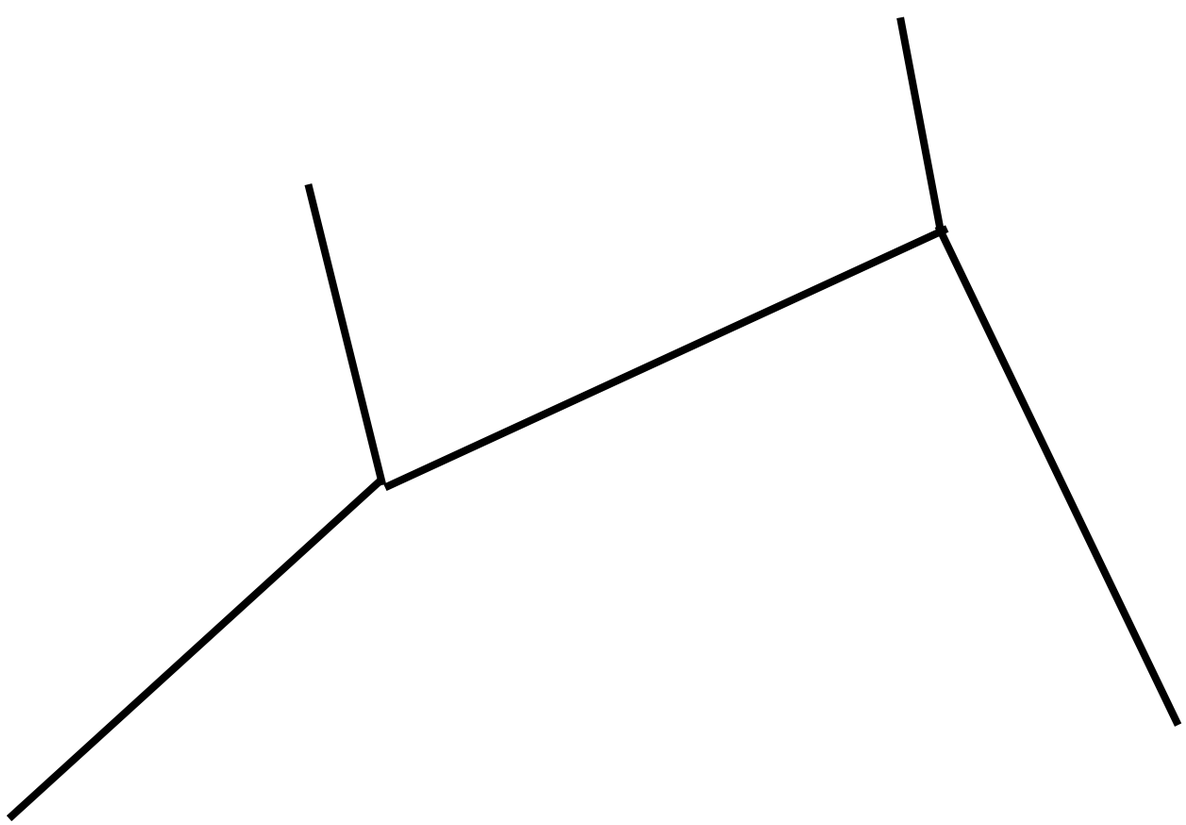
- Non-metricity: if the connection defined by interactions is not the metric connection defined from propagation.

$$N^{abc} = \nabla^a g^{bc}$$

# Dynamics

- Spacetime emerges from the dynamics on momentum space.
- In our limit, we study first classical particle dynamics
- Each process has an action principle

$$S^{process} = \sum_{trajectories,I} S_I^{free} + \sum_{interactions,\alpha} S_\alpha^{int}$$



# Emergence of space-time

Spacetime is an auxiliary concept that emerges from the dynamics of particles

Free particle dynamics

$$S = \int_{-\infty}^{0} (x^a \dot{k}_a - N\mathcal{C}(k))$$

conjugate part coordinates

mass shell

$$\{x_I^a, k_b^J\} = \delta_b^a \delta_I^J \qquad \mathcal{C}^J(k) \equiv D^2(k) - m_J^2.$$

*The free particle action makes no reference to a metric for spacetime. Spacetime geometry is inferred from the geometry of momentum space.*

in the usual case the metric is flat and $D(k)^2 = k_0^2 - \vec{k}^2$

# Worldline action

The variation of the worldline action gives

bulk eom

$$\dot{k}_a^J = 0$$

$$\dot{x}_J^a = \mathcal{N}_J \frac{\delta \mathcal{C}^J}{\delta k_a^J}$$

simplified by Riemannian normal coordinates

$$\mathcal{C}^J(k) = 0$$

$$\mathcal{C}(k) = k_0^2 - k_i^2 - m^2 \; :$$

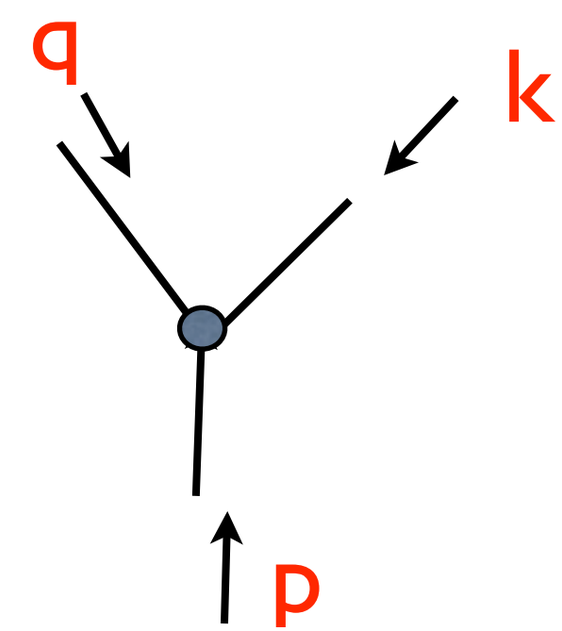
# Emergence of space-time

The interaction imposes the conservation law

$$S^{int} = \mathcal{K}(k(o))_a z^a$$

Lagrange multiplier

z becomes the location of the interaction:
the interaction coordinate




e.g. $\mathcal{K}_a = (p \oplus (q \oplus k))_a$

# Worldline action

The variation of the worldline action gives

boundary eom

$$x^a_J(0) = z^b \frac{\delta \mathcal{K}_b}{\delta k^J_a}$$

particle coordinates

interaction coordinates

$$x^a_J(0) = z^a - z^b \sum_{L \in \mathcal{J}(J)} C_{J,L} \Gamma^{ac}_b k^L_c + \ldots$$

if z=0 then x=0 : interaction is local for an observer at the origin

For a distant observer there is a dispersion $\Delta x \approx |z||\Gamma|k$

Locality is relative

# Two kinds of spacetime coordinates

$$x^a = z^b U(k)^a_b$$

particle coordinates

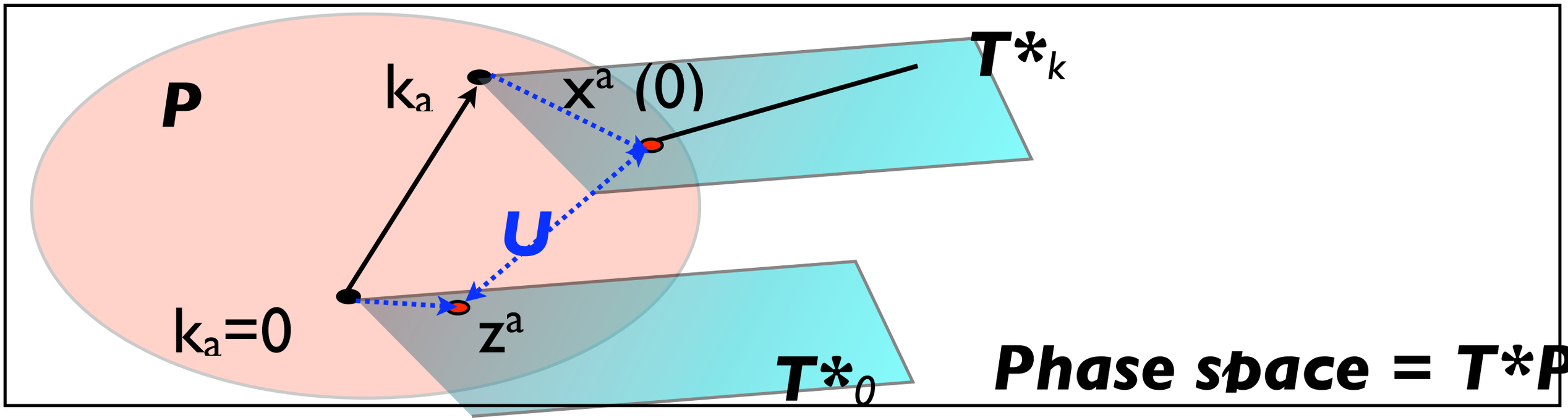
interaction coordinates

$$U(k)^b_a = \frac{\delta K_a}{\delta k_b}$$

Paralell transport operator

$T^*_k P$

$T^*_0 P$




No canonical projection from phase space to space time. $T^*P \neq M \times P$

each particle carries its own momentum dependent spacetime related by parallel transport to the interaction spacetime

# Two kinds of spacetime coordinates

$$\boxed{x^a = z^b U(k)^a_b}$$

$$U(k)^b_a = \frac{\delta K_a}{\delta k_b}$$

particle coordinates

interaction coordinates

Paralell transport operator

$T^*_k P$

$T^*_0 P$

If the conservation law is linear then U =1 and x=z
the interaction is local

If the conservation law is non linear the interaction is only relatively local i-e x=0 if z=0

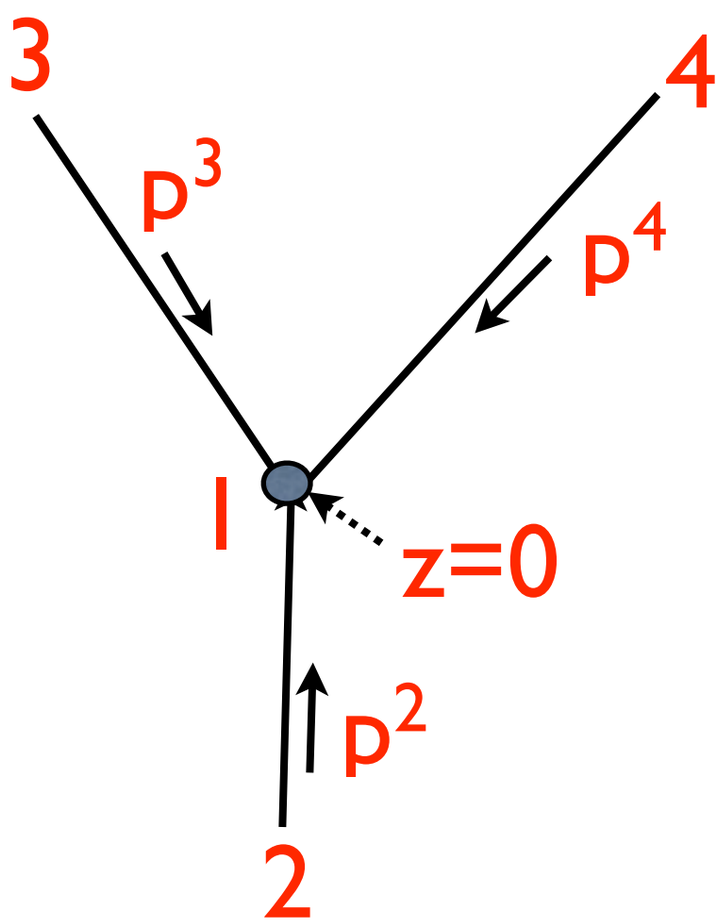
x is a commutative coordinate

z is a non commutative coordinate

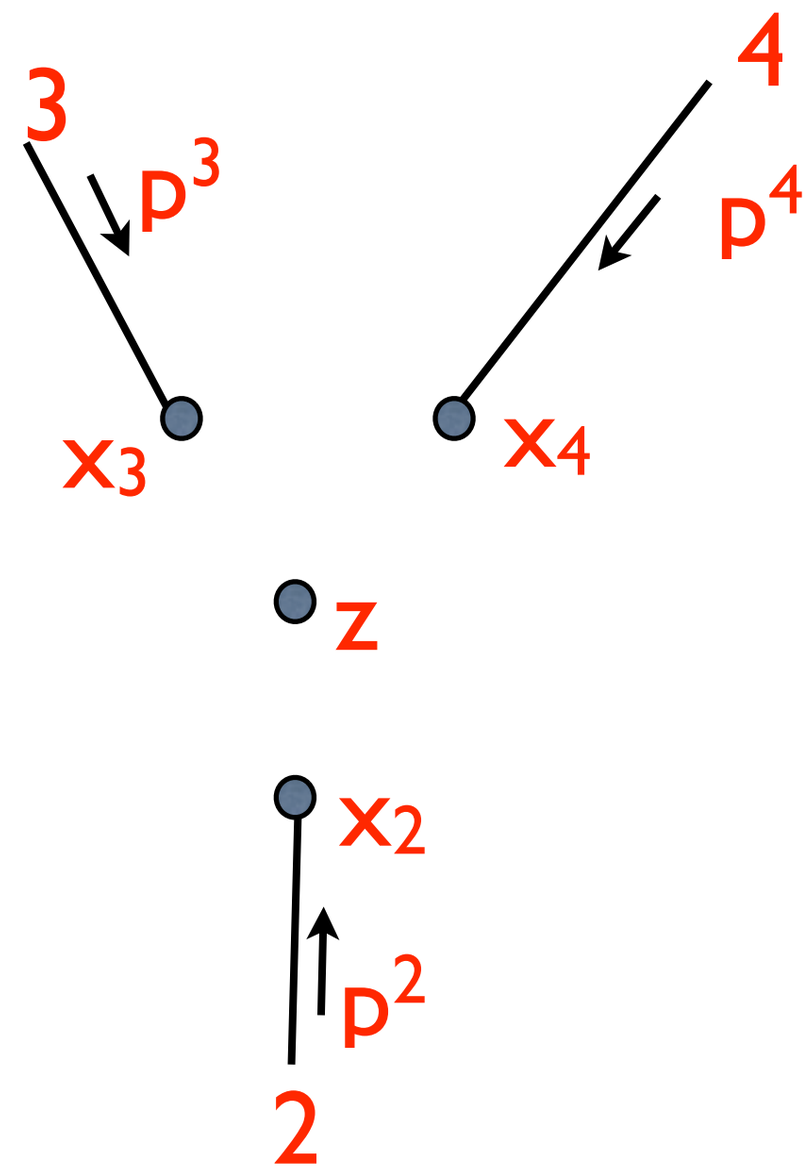
$$\{z^a, z^b\} = T^{ab}_d z^d + R^{abc}{}_d p_c z^d + \cdots$$

Vertex looks local to local observers for which z=0

Vertex looks non-local to distant observers




local observer




distant observer

$$\delta x^a_I = \pm\{b^c \mathcal{K}_c, x^a_I\} = b^a + \Gamma^{ac}_b b^a p^I_c + \ldots$$

# Specialising the geometry

## The correspondence principle

Special relativity describe accurately all phenomena involving momenta smaller than a mass scale m to be determined exp.

$\longrightarrow$ Torsion is of order 1/m curvature of order 1/m^2

## The dual equivalence principle

The algebra of interactions is independent on the nature ( color, charge) of the particles

$\longrightarrow$ Torsionless if no modification of statistics

## Maximal symmetry

Momentum space has as many symmetries as flat space time

$\longrightarrow$ its an AdS or dS spacetime with radius of curvature $M_p$
connexion is metric

# Experimental test

*Theorists propose but experiments decide.*

The geometry of momentum space should be measured rather than assumed

A new phenomenological sets of questions opens up

Two types of search: theoretical or purely phenomelogical

Given the maximally symmetric model find a clean measure of the dual cosmological constant

Test the 4 principles:
Torsion, non metricity, Lorentz invariance, homogeneity

# Experimental test

Measure of the curvature of momentum space

A thomas precession analogy motivated by Girelli, Livine

A system in orbit ( electron ,part at the LHC)
encloses a loop in momentum space at each period of revolution

The localisation of the orbiting particle will be shifted with respect to the particle at rest, it experiences a boost

$$N_i = \frac{\Delta A_{cd}}{m_P^2} R^{cda}{}_i p_a \approx \frac{\Delta A_{cd}}{m_P^2} m R^{cd0}{}_i$$

small displacement in space and time, cumulative.

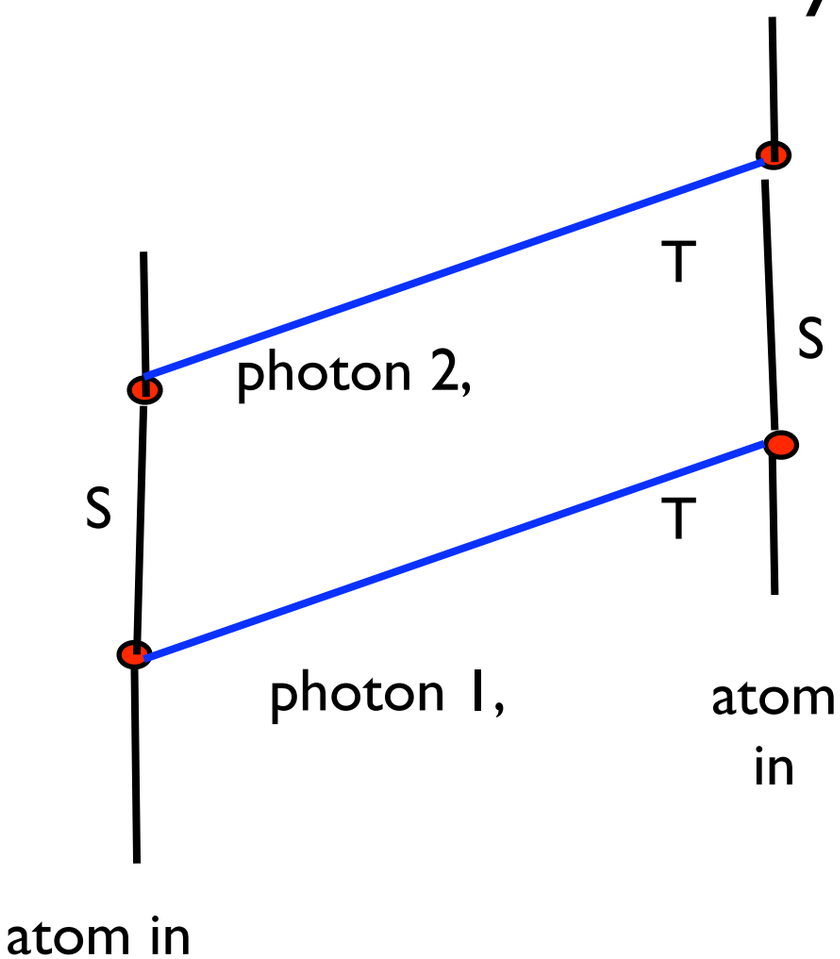
It pulls itself by the bootstraps!

# Gamma ray exp

The process of localising a distant object is momentum dependent

The experiment: a distant star emits two photons
One of low energy and one of high energy

If the photons are emitted at the same time for an observer local to the star are they observed arriving at the same time by us ?




Note that in Riemannian normal coordinates the speed of light is constant

$$D(p) = \eta^{ab} p_a p_b$$

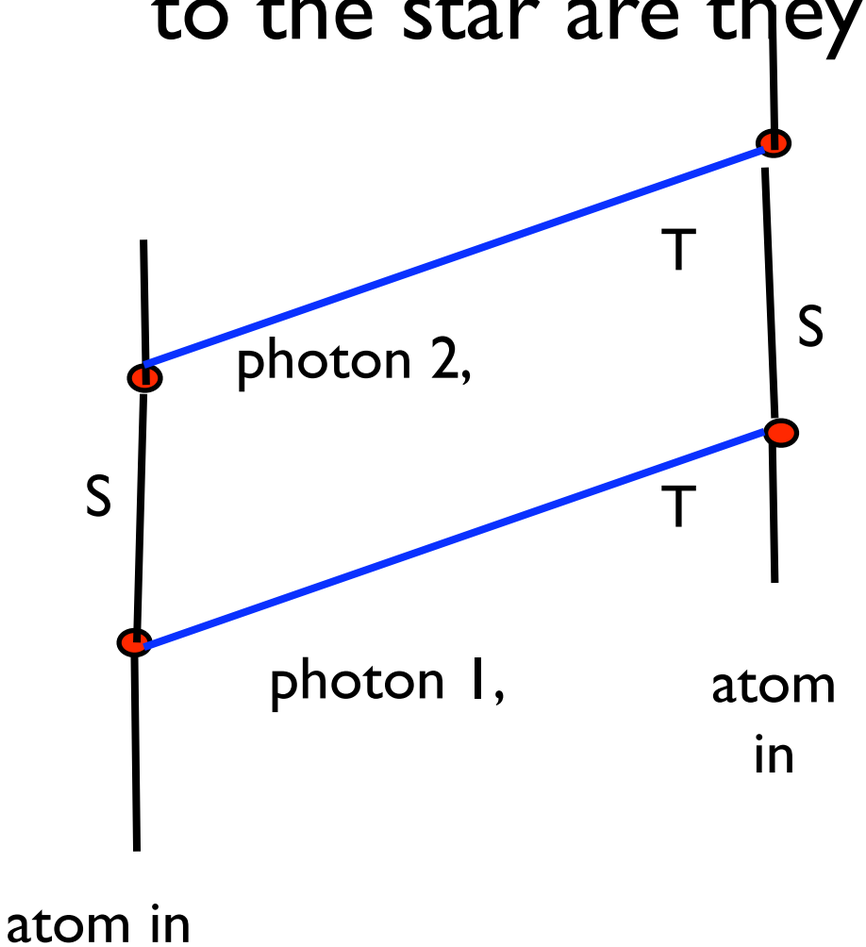
$$\partial^b g^{bc}|_{p=0} = 0 \rightarrow \Gamma = T + N$$

# Gamma ray exp

The process of localising a distant object is momentum dependent

The experiment a distant star emits two photons
One of low energy and one of high energy

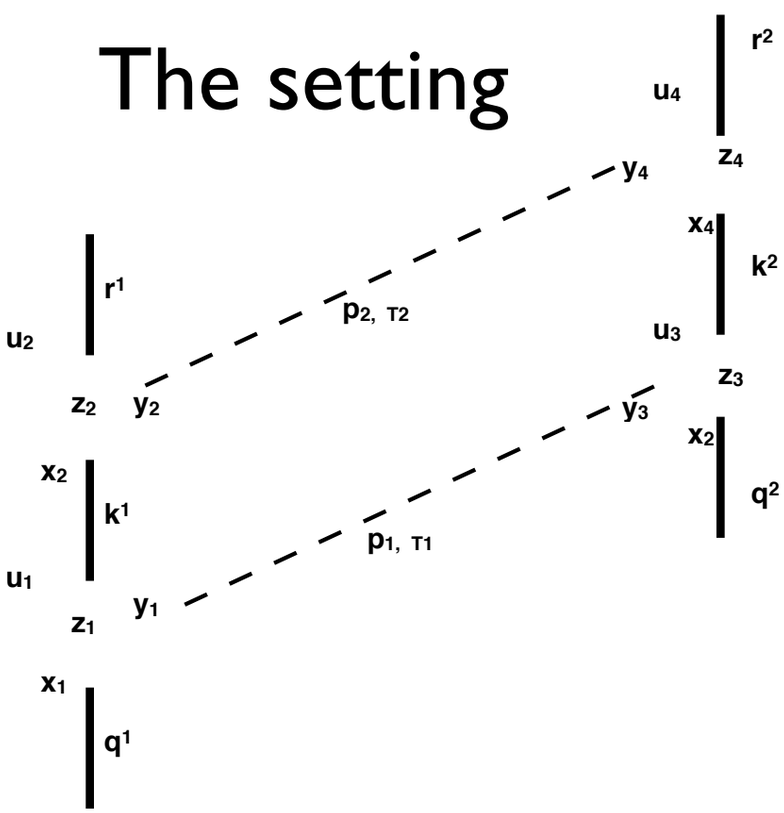
If the photons are emitted at the same time for an observer local to the star are they observed arriving at the same time by us ?




NO !
even if the photons have the same speed

# Gamma ray exp

The setting




Idea of the derivation: compute from two different perspective the transport from z1 to z4 using the parallel transport

Results:

The leading order effect is due to non-metricity.

$$S_2 - S_1 = -T\Delta E N^{+++}$$

For a metric connection the next effect is due to the torsion

no time delay $\Delta S = 0$ but

photons of different energies appear to come from different locations

$$\Delta\theta = \frac{1}{2}(E_1 + E_2)\sqrt{T_-^{+a}\eta_{ab}T_-^{+b}}$$

dual gravitational lensing

# Soccer ball issue

If one modifies the law of addition of momenta with a scale $m_P$ how come we do not see strange effects for soccer balls?

$$p_1 \oplus p_2 = p_1 + p_2 + \frac{1}{m_P} T(p_1, p_2) + \cdots$$

The main point is that the effective mass scale for the interaction of two soccer balls of size N is N $m_P$

# Soccer ball issue

The argument is as follows:
Supose that two rigid bodies 1, 2 both composed of N particles interact such that each atom of 1 exchanges a photon with an atom of 2

$$p^1_{in} = p^1_{out} \oplus k \quad k \oplus p^2_{in} = p^2_{out}$$

Lets also assume that all atoms of 1, 2 have the same momenta

$$P_{in} = N p_{in} \qquad P_{out} = N p_{out} \qquad p \oplus p = 2p$$

normal coord.

$$P_{tot} = N(p^1_{in} \oplus p^2_{pin}) = N(p^1_{out} \oplus p^2_{out})$$

$$= P_1 + P_2 - \frac{1}{N m_P}\Gamma(P_1, P_2) + \cdots$$

$$m_P \to N m_p$$

There is no soccer ball prob !

# Conclusion

New framework in which we can relax the notion of absolute locality in a controlled manner

Momentum space possess a non trivial geometry (metric, connection) that can and should be probed experimentally

Under general principles a preferred class of momentum space geometries can be proposed

Many interesting and extremely surprising experimental consequences

Interesting new math: connection between algebra and geometry

No soccer ball problem