Effective Dynamics of Bianchi-I spacetime in LQC: Kasner transitions and inflationary scenario

Brajesh Gupt
(based on arXiv:1205.6763v2 and work to appear with Parampreet Singh)
Louisiana State University

International Loop Quantum Gravity Seminar

March 26 , 2013

Why study Bianchi models?

- Anisotropic spacetime introduces more degrees of freedom compared to isotropic spacetime
- Much richer physics due to non-vanishing Weyl scalar
- Classically the anisotropic shear scalar in Bianchi-I model varies as $\sigma^2 \propto a^{-6}$. Singularity can also take place due to diverging anisotropic shear as $a \to 0$
- According to Belinskii-Khalatnikov-Lifshitz (BKL) behavior, during a generic approach to a spacelike singularity, each point transits from one Bianchi-I type universe to another Bianchi-I type (Kasner transition), giving rise to Mixmaster behavior

Loop quantum cosmology of Bianchi-I spacetime

$$ds^2 = -dt^2 + a_1^2\, dx^2 + a_2^2\, dy^2 + a_3^2\, dz^2$$

In the classical theory, approach to singularity can be classified as (Doroshkevich, Ellis, Jacobs, MacCallum, Thorne ...)

- **Point or Isotropic singularity:** $a_1,\ a_2,\ a_3 \to 0$.
- **Barrel:** $a_1 \to \mathrm{const},\ a_2,\ a_3 \to 0$
- **Pancake:** $a_1 \to 0,\ a_2,\ a_3 \to \mathrm{const}$
- **Cigar:** $a_1 \to \infty,\ a_2,\ a_3 \to 0$

Quantization performed by (Ashtekar, Wilson-Ewing(09)). Earlier approaches to quantization developed by Bojowald, Chiou, Date, Martin-Benito, Mena Marugan, Pawlowski, Szulc, Vandersloot

- Classical singularity resolved
- Resolution of all physical singularities studied in the effective dynamics (Singh (11))
- Physics of effective dynamics studied: big bang is replaced by bounce (Artymowski, Cailleteau, Chiou, Lalak, Maartens, Singh, Vandersloot)

Questions:

Kasner Transitions:

- What is the relation between the geometrical nature of spacetime in pre and post bounce regime?
- Are there transitions from one type to other?
- Are some transitions favored over others? If yes, depending on what?

Inflation:

- Does anisotropy prevent inflation?
- How does LQC modify the dynamics and the amount of inflation?
- How is the amount of inflation affected as compared to the isotropic spacetime?

Main results:

Kasner Transitions:

- Kasner transitions are seen in Bianchi-I spacetime for the first time, a feature not present in classical theory
- Transitions are not random, there turns out to be a "selection rule"
- Depending on the anisotropy and matter content some transitions are favored over others

Inflation:

- Inflation takes place irrespective of the initial anisotropic shear
- Non-trivial dependence of amount of inflation on the initial shear scalar
- Modification in the initial value of inflaton field as compared to isotropic spacetime to generate a given amount of inflation

Plan of the talk:

- Review of Kasner solution
- Review of effective dynamics of Bianchi-I
- Kasner transitions with perfect fluid with $P = \rho$
- Inflation in Bianchi-I spacetime with quadratic potential $V(\phi) = m^2\phi^2/2$

Kasner solution: classical theory

Vacuum:

$$a_i \propto t^{k_i} \quad \text{such that} \quad k_1 + k_2 + k_3 = 1; \quad k_1^2 + k_2^2 + k_3^2 = 1 \tag{1}$$

Stiff matter, $w = P/\rho = 1$:

$$a_i \propto t^{k_i} \quad \text{such that} \quad k_1 + k_2 + k_3 = 1; \quad k_1^2 + k_2^2 + k_3^2 = 1 - k^2 \tag{2}$$

where k_i are Kasner exponents and k is a constant.

Point	$k_1,\ k_2,\ k_3 > 0$
Barrel	$k_1 = 0,\ k_2, k_3 > 0$
Pancake	$k_1, k_2 = 0,\ k_3 > 0$
Cigar	$k_1 < 0,\ k_2, k_3 > 0$

$0 \leq w < 1$:

- Close to singularity, behaves like vacuum for all $0 \leq w < 1$
- In the future asymptotic limit
  - $a_i \propto t^{2/3}$ for Dust ($w = 0$)
  - $a_i \propto t^{1/2}$ for Radiation ($w = 1/3$)

Bianchi-I: Effective dynamics

Effective Hamiltonian: (Chiou, Vandersloot; Ashtekar, Wilson-Ewing)

$$\mathcal{H}_{\rm eff} = -\frac{1}{8\pi\gamma^2 V}\left(\frac{\sin(\bar\mu_1 c_1)}{\bar\mu_1}\frac{\sin(\bar\mu_2 c_2)}{\bar\mu_2} p_1 p_2 + \text{cyclic terms}\right) + \mathcal{H}_{matt}, \qquad (1)$$

where

$$\bar\mu_1 = \lambda\sqrt{\frac{p_2 p_3}{p_1}}, \quad \bar\mu_2 = \lambda\sqrt{\frac{p_1 p_3}{p_2}} \quad \bar\mu_3 = \lambda\sqrt{\frac{p_2 p_1}{p_3}} \quad \text{and} \quad \lambda^2 = 4\sqrt{3}\pi\gamma\ell_{\rm Pl}^2$$

.

triad and connection: $p_1 = a_2 a_3 \quad \& \quad \text{classically}, \quad c_1 = \gamma \dot{a}_1 \qquad (2)$

Hamilton's equation of motion: $\frac{\dot p_1}{p_1} = \frac{1}{\gamma\lambda}\left[\sin(\bar\mu_2 c_2) + \sin(\bar\mu_3 c_3)\right]\cos(\bar\mu_1 c_1) \qquad (3)$

Energy Density: vanishing of the Hamiltonian constraint gives:

$$\rho = \frac{1}{8\pi G\gamma^2\lambda^2}\left[\sin(\bar\mu_1 c_1)\sin(\bar\mu_2 c_2) + \text{cyclic terms}\right] \le \rho_{\rm crit} = 0.41\rho_{\rm Pl} \qquad (4)$$

Expansion scalar:

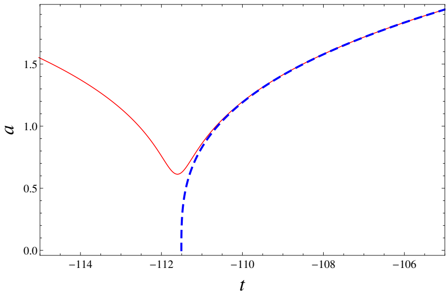
$$\theta = \frac{1}{2}\left(\frac{\dot p_1}{p_1} + \frac{\dot p_2}{p_2} + \frac{\dot p_3}{p_3}\right) \le \theta_{\rm max} = \frac{3}{2\gamma\lambda} \qquad (5)$$

Shear scalar:

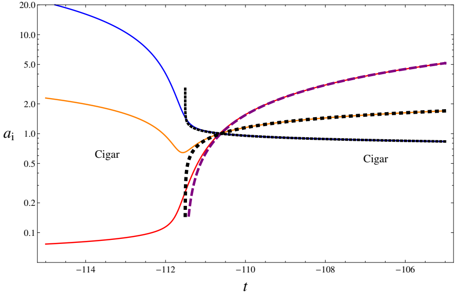
$$\sigma^2_{\rm max} = \frac{10.125}{\gamma^2\lambda^2} = \frac{11.57}{\ell^2_{\rm Pl}} \qquad (6)$$

Kasner transition: Stiff matter, $P = \rho$

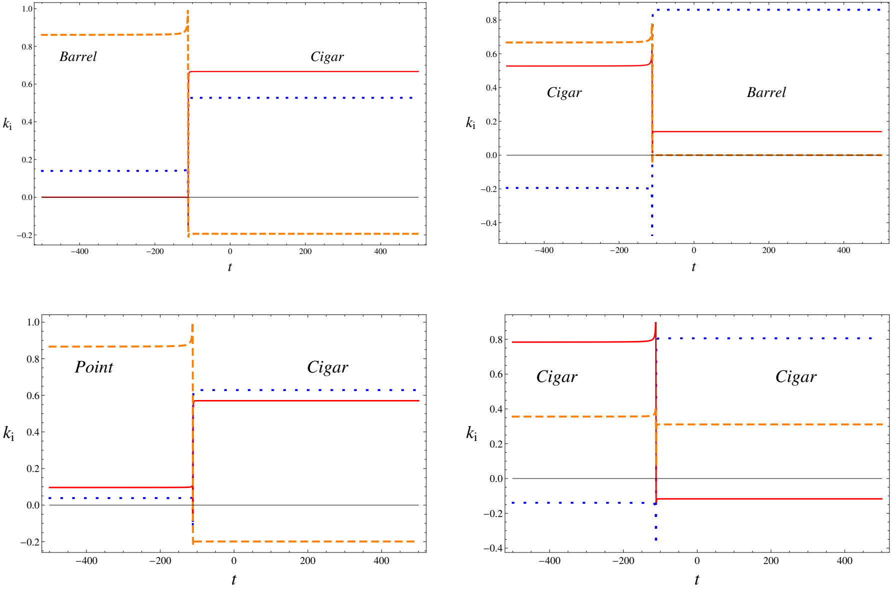
- Classical trajectory undergoes singularity
- The mean scale factor $a = (a_1 a_2 a_3)^{1/3}$ in LQC bounces.



- The directional scale factors undergo Kasner transition across the bounce
- Transitions depend on anisotropy present in the spacetime



Stiff matter, $w = 1$



Kasner transition for $w = 1$

$0 < \|\delta\| < \frac{1}{2}$	$\|\delta\| = \frac{1}{2}$	$\frac{1}{2} < \|\delta\| < \frac{1}{\sqrt{3}}$	$\|\delta\| = \frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}} < \|\delta\| < 1$
P $\leftrightarrow$ P	P $\leftrightarrow$ P	P $\leftrightarrow$ P	P $\not\leftrightarrow$ P	P $\not\leftrightarrow$ P
B $\not\leftrightarrow$ P	B $\leftrightarrow$ P	B $\leftrightarrow$ P	B $\not\leftrightarrow$ P	B $\not\leftrightarrow$ P
C $\not\leftrightarrow$ P	C $\not\leftrightarrow$ P	C $\leftrightarrow$ P	C $\leftrightarrow$ P	C $\leftrightarrow$ P
B $\not\leftrightarrow$ B	B $\not\leftrightarrow$ B	B $\not\leftrightarrow$ B	B $\leftrightarrow$ B	B $\not\leftrightarrow$ B
B $\not\leftrightarrow$ C	B $\not\leftrightarrow$ C	B $\not\leftrightarrow$ C	B $\not\leftrightarrow$ C	B $\leftrightarrow$ C
C $\not\leftrightarrow$ C	C $\not\leftrightarrow$ C	C $\not\leftrightarrow$ C	C $\not\leftrightarrow$ C	C $\leftrightarrow$ C

where $|\delta| = \sqrt{\frac{3\sigma^2}{2\theta^2}}$.

- Depending on the value of δ some transitions are favored over others.
- In the low anisotropy regime only Point-Point transition takes place
- Cigar-Cigar transition only happens in the large anisotropy regime.

Inflation

- **Inflation:** a phase of accelerated expansion in the early universe
  Widely studied and explored in the classical theory. (Albrecht, Barrow, Guth, Linde, Rothman, Steinhardt, Steigman, Turner...)
- Does anisotropy prevent inflation? Barrow and Turner (81); Steigman and Turner (83)
- Quantum theory of gravity is required (Rothman & Madsen (85), Rothman & Ellis(86)); Anisotropy helps attain more inflation (Maartens, Sahni and Saini (01))
- Inflation in LQC in isotropic spacetime...(Ashtekar, Pawlowski, Singh to appear)

  Isotropic inflation in effective theory studied in detail by Ashtekar & Sloan (09) establishing its inevitability (99.99%); other aspects also studied by Corichi & Karami (10)

Inflation in Bianchi-I spacetime

Generalized Friedmann equation: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{1}{6}\sigma^2$ (1)

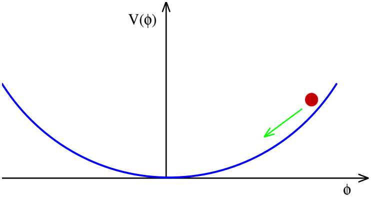
Raychaudhuri equation: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) - \frac{1}{3}\sigma^2$ (2)

where a is the mean scale factor.

Conservation equation: $\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$ (3)

$$\rho = \dot{\phi}^2/2 + V, \quad P = \dot{\phi}^2/2 - V, \quad V = m^2\phi^2/2 \qquad (4)$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\dot{\phi}^2 - V(\phi)\right) - \frac{1}{3}\sigma^2 \qquad (5)$$



Slow-roll: $V(\phi) \gg \dot{\phi}^2$

$\ddot{a} > 0$

Anisotropy ⇒ Enhanced Hubble friction ⇒ Fast KE decay ⇒ Arrival of slow roll

Amount of inflation: Classical theory

Number of efoldings: $N = \ln\left(\frac{a_{t_{\rm off}}}{a_{t_{\rm on}}}\right) \propto \phi_{\rm on}^2$

$|\dot\phi(0)| = (2\cdot 10^{-5},\, 2\cdot 10^{-3},\, 2\cdot 10^{-2}, 0.1)\, m_{\rm Pl}^2$, $\phi(0) = 3.14\, m_{\rm Pl}$ and $\epsilon_{\rm J}^2 = \sigma^2/4\pi G\rho$

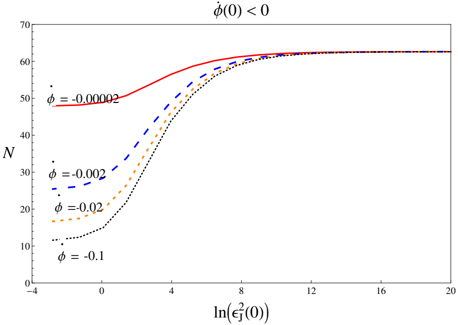


Figure: slow-roll starts at a higher value of ϕ as $\sigma^2(0)$ increases

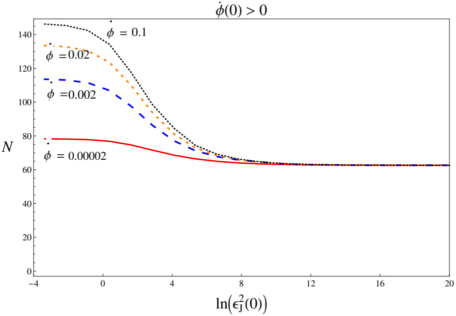
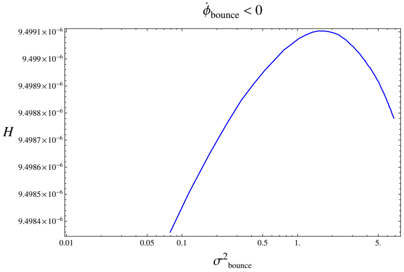


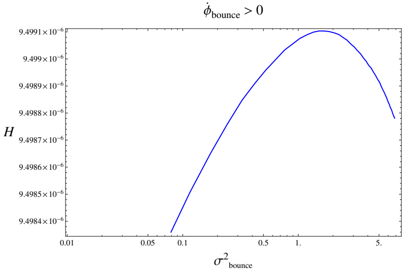
Figure: slow-roll starts at a lower value of ϕ as $\sigma^2(0)$ increases

Depending on the initial condition on $\dot\phi$ non-zero anisotropy, monotonically, either enhances or reduces N in the classical theory.

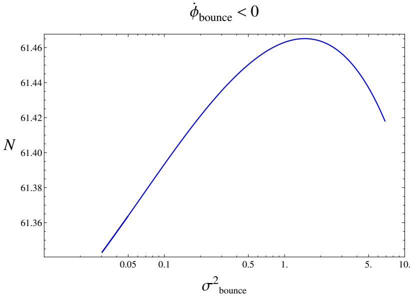
Bianchi-I inflation in LQC: amount of inflation

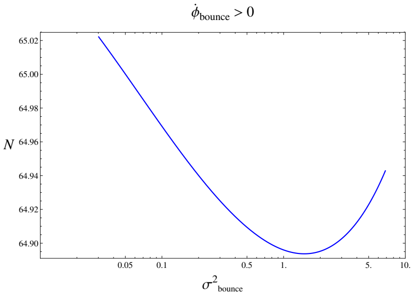
Hubble rate:





Number of e-foldings:





Maximum number of e-foldings (N) for $\phi(0) = 3.14\, m_{\rm Pl}$ various $\dot\phi(0)$

$\dot\phi(0)$	-0.2	-0.02	-0.002
N	42.7503	60.3810	62.3939

$\dot\phi(0)$	0.2	0.02	0.002
N	71.8433	65.0221	62.8843

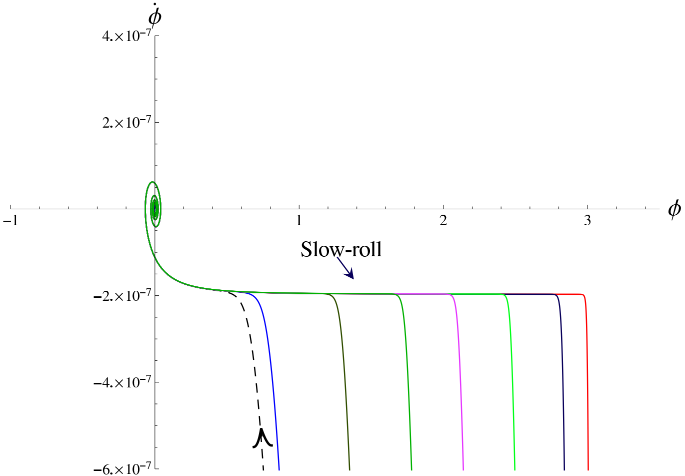
Isotropic LQC: $\rho_{\rm bounce} = \rho_{\rm crit} \Rightarrow \dot\phi(0) = -0.905\, m_{\rm Pl}^2$

$$N_{\rm iso} = 3.12 \quad \text{for} \quad \phi(0) = 3.14\, m_{\rm Pl} \tag{1}$$

In comparison to the isotropic spacetime, Bianchi-I spacetime generates more e-foldings for the same initial value of the inflaton at the bounce.

In the isotropic spacetime, for $N \approx 60$, $\phi(0) \approx 5.50 m_{\rm Pl}$ for an inflaton which is rolling down.
Bianchi-I spacetime widens the window of value of $\phi(0)$ to generate a given number of e-foldings.

Phase portrait: LQC



All trajectories meet the slow roll curve in their future evolution.

Slow-roll is an attractor for all these solutions in Bianchi-I spacetime.

Summary

- There are Kasner transitions across the bounce in Bianchi-I spacetime
- These transitions follow a pattern and depending on anisotropy and matter content some of them are favored- "*selection rule*"
- Inflation takes place irrespective of the initial anisotropic shear
- Anisotropy may enhance or reduce the amount of inflation depending on the initial conditions on the inflaton velocity
- Bianchi-I spacetime widens the window of the value of inflaton at the bounce, for a given number of e-foldings