# Covariant Effective Dynamics and Nonsingular Black Holes

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ILQGS panel discussion on black holes in LQG

- The covariant $\bar{\mu}$-scheme effective dynamics of sperical symmetric LQG.
- The full effective spacetime of the BH-to-WH transition.

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# Topic 1: Covariant mubar-scheme effective dynamics

New $\bar{\mu}$-scheme effective Hamiltonian of spherical symmetry LQG featured by the covariance [HM, Hongguang Liu 2022]

$$\mathbf{H}_{\text{eff}}^{(\text{cov})} = \int \mathrm{d}x \, \frac{\sqrt{E^x} E^\varphi}{2G\Delta} \left[\sin^2\left(\zeta_2/2\right) - 4\sin^2\left(\zeta_1/2\right)\right] + \frac{1}{4G\sqrt{E^x}} \left( -\frac{2E^x E^{x\prime} E^{\varphi\prime}}{E^\varphi} + \frac{4E^x E^{x\prime\prime} + E^{x\prime 2}}{2E^\varphi} - 2E^\varphi \right)$$

$$\zeta_1 \equiv \frac{2\sqrt{\Delta}\sqrt{E^x}}{E^\varphi} K_x + \frac{\sqrt{\Delta}}{\sqrt{E^x}} K_\varphi, \quad \zeta_2 \equiv \frac{4\sqrt{\Delta}\sqrt{E^x}}{E^\varphi} K_x$$

1. The canonical variables are $E^x, E^\varphi$ and their conjugate momenta $K_x, K_\varphi$

   sperical symm metric: $$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{E^\varphi(x,t)^2}{E^x(x,t)} \mathrm{d}x^2 + E^x(x,t)\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right).$$

2. In terms of LQG "$\bar{\mu}$-holonomies"

   $$h_x = e^{2i\bar{\mu}_x K_x}, \quad \bar{\mu}_x = \frac{\sqrt{\Delta}\sqrt{E^x}}{E^\varphi}; \qquad h_\varphi = e^{i\bar{\mu}_\varphi K_\varphi}, \quad \bar{\mu}_\varphi = \frac{\sqrt{\Delta}}{\sqrt{E^\varphi}}$$

   $\mathbf{H}_{\text{eff}}^{(\text{cov})}$ reduces to the ADM Hamiltonian when $\Delta \to 0$.

3. $\mathbf{H}_{\text{eff}}^{(\text{cov})}$ is a true physical Hamiltonian rather than a constraint. $E^x, E^\varphi, K^x, K^\varphi$ are canonical coordinates of the reduced phase space

   The effective dynamics: $\dot{f} = \{f, \ \mathbf{H}_{\text{eff}}^{(\text{cov})}\}$, (PDE on 1+1 dimensions)

4. The effective dynamics is generally covaraint. The Hamiltonian can be derived from a covariant Lagrangian.

Early debate on the covariace of effective dynamics in LQG: [Gambini, Olmedo, Pullin 2022; Bojoward, Brahma, Reyes 2020-2022; Tibrewala 2013; $\cdots$]

# Mimetic-gravity effective Lagrangian

The Lagrangian behind the effective Hamiltonian is the mimetic-gravity Lagrangian with certain higher derivative interactions.

$$S\left[g_{\mu\nu},\phi,\lambda\right]=\frac{1}{8\pi G}\int_{\mathscr{M}_4}\mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R}^{(4)}+L_\phi\left(\chi_1,\chi_2\right)+\lambda\left(\nabla^\mu\phi\nabla_\mu\phi+1\right)\right]$$

Field contents:

$$g_{\mu\nu}:\ \text{gravity},\quad \phi:\ \text{mimetic scalar},\quad \lambda:\ \text{lagrangian multiplier}$$

$$\chi_1=\Box\phi,\quad \chi_2=\phi_{\mu\nu}\phi^{\mu\nu},\quad \phi_{\mu\nu}=\nabla_\mu\nabla_\nu\phi$$

Chamseddine, Mukhanov 2016
Ben Achour, Lamy, Liu, Noui 2017
MH, Liu 2022

1. $L_\phi$ contains the higher derivative interactions. $L_\phi\to 0$ goes back to gravity coupled to non-rotational dust, a standard setup for deparametrized gravity.
2. The scalar $\phi$ is the clock field that defines the internal time of the system.
3. By the foliation with constant-$\phi$ slices, the Hamiltonian analysis of the mimetic-gravity Lagrangian (with spherical symmetry) gives the covariant $\bar{\mu}$-scheme Hamiltonian $\mathbf{H}_{\text{eff}}^{(\text{cov})}$.
4. The Hamiltonian effective dynamics $\dot{f}=\{f,\ \mathbf{H}_{\text{eff}}^{(\text{cov})}\}$ is equivalent to the mimetic-gravity EOMs $\delta S=0$. The effective Hamiltonian $\mathbf{H}_{\text{eff}}^{(\text{cov})}$ generates the time translation w.r.t the internal time $\phi$.
5. The dual role played by $\phi$: (1) serving as the clock field, and (2) the higher derivative interaction $L_\phi$ results in the $\bar{\mu}$-scheme polymerization (holonomy corrections).

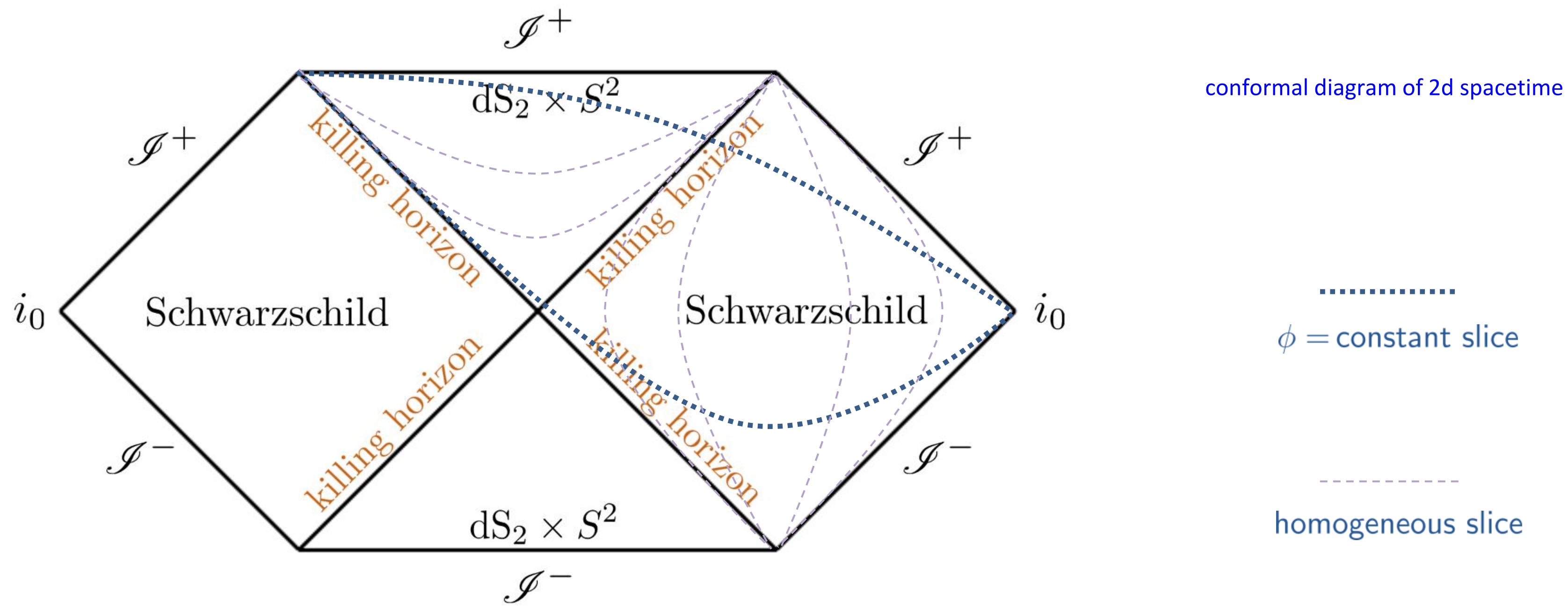
The $\bar{\mu}$-scheme effective dynamics (of spherical symmetric LQG) is generally covariant, as is manifest at the level of Lagrangian.

It suggests the mimetic-gravity Lagrangian to be the effective Lagrangian of LQG at least in the spherical symmetric setup.

# Non-singular black hole solution from the covariant effective dynamics

- Assuming a global killing symmetry (timelike outside the killing horizon)
- Imposing asymptotic Schwarzschild boundary condition far away from the black hole.

The solution of effective equation:



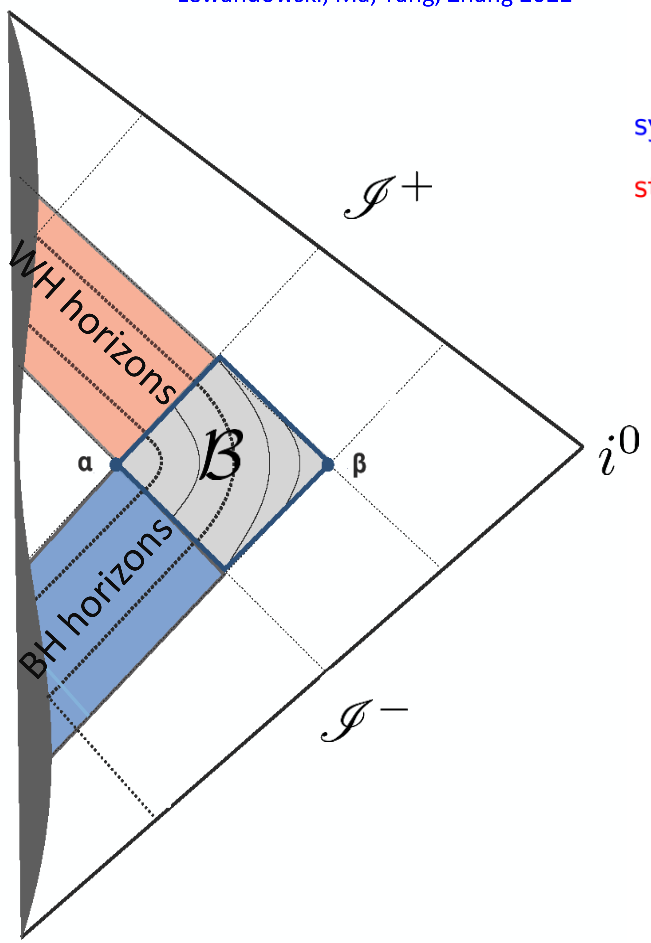

1. The spacetime is singularity-free. The future infinity of the black hole approaches asymptotically $\mathrm{dS}_2 \times S^2$ (Nariai geometry).
2. The spacetime has complete $\mathscr{I}^+$, containing $\mathscr{I}^+$ of $\mathrm{dS}_2$.
3. There is no event horizon. But there are killing horizons. The holonomy correction is negligible at the killing horizon.

To be understood: How to go beyond the killing symmetry and derive the dynamical effective spacetime?

# Topic 2: A full effective spacetime of the black-hole-to-white-hole transition

MH, Rovelli, Soltani, to appear

Lewandowski, Ma, Yang, Zhang 2022




The spacetime includes the gravitational collapse of a 'star', bounce, BH-to-WH transition, and a single asymptotic region

The construction of the geometry involves the junction conditon with the star, killing symmetry, and the interpolation in $\mathcal{B}$. The effective equation is not used outside the star.

- The gravitational collapse: homogeneous and pressureless star, governed by LQC effective eqn and with symmetric bounce.
- Outside the star, a regular effective metric covers the entire vacuum spacetime and gives the BH-to-WH transition.
- In particular, there is a regular metric in the region $\mathcal{B}$, where the BH-like horizon transits to the WH-like horizon.
- Outside the star, there is killing symmetry everywhere except in the region $\mathcal{B}$.
- The killing symmetry is broken in the dynamical region $\mathcal{B}$. But there is a regular metric inside.

## Conclusion and open questions

On one hand, we have the effective dynamics that is generally covariant. It is an 1+1 dimensional field theory containing rich dynamical information.

On the other hand, we have the full spacetime of the BH-to-WH transition (without using the effective equations). The $\mathcal{B}$ region is dynamical and relates to quantum effect, but it can have a regular effective metric inside, so it should be within the regime where the effective dynamics is valid.

How to derive the full picture of BH-to-WH transition from effective dynamics?

How to implement the Hawking radiation and backreaction?

Thanks for your attension!