Quantum theory from information inference principles

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ILQGS
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based on:
PH (to appear hopefully soon)
PH, C. Wever (to appear hopefully soon too)

Physics and information theory

Tools from information theory proved useful in concrete physical situation to help understand and interpret physical phenomena

- BH entropy
- thermalization
- quantum information
- ....

Can apply tools to given theories/problems $\Rightarrow$ growing number of applications

But: can concepts from information theory tell us something deeper about the structure of physical theories? Can they be used in the architecture of physical theories?

idea:

(im-)possibility of information theoretic tasks $\Leftrightarrow$ particular structure of theory

Operational approaches and information theory

Shall follow an operational approach to physics

⇒ consider relations among systems and observers

advantage: only speak about what an observer has access to and not about how the universe 'really' is (relations among observers)

disadvantage: unobservable 'realist' structure can facilitate global description (spacetime)

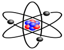
⇒ old and ubiquitous debate 'operationalism vs realism' in physics

⇒ clearly, no resolution here, but ask:

How much can an operational and information theoretic approach teach us about physics?

Which structures can we deduce?

Operational implications: from single to many observers

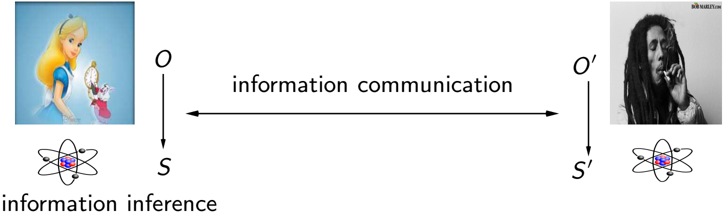


O

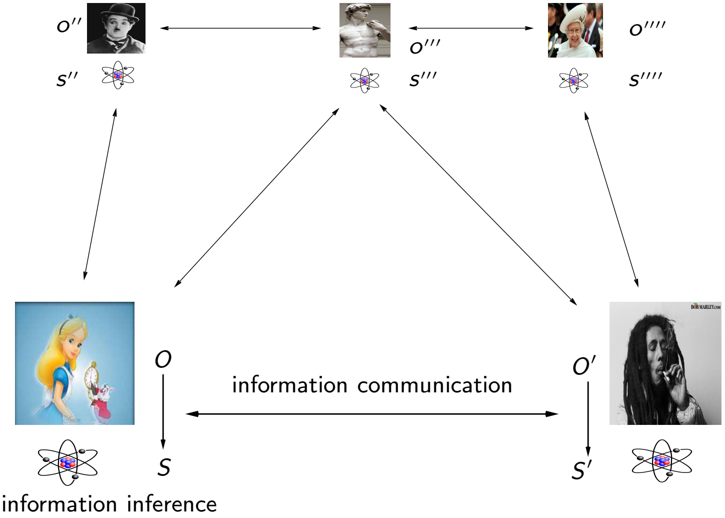
S

information inference

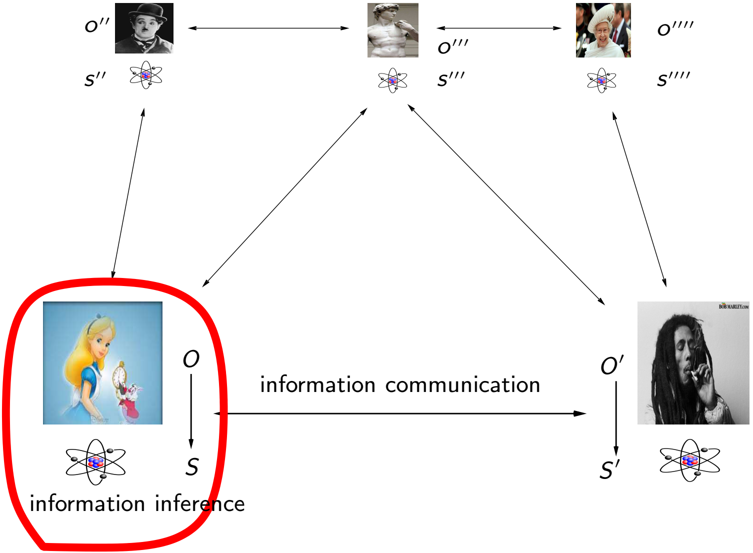
Operational implications: from single to many observers



Operational implications: from single to many observers

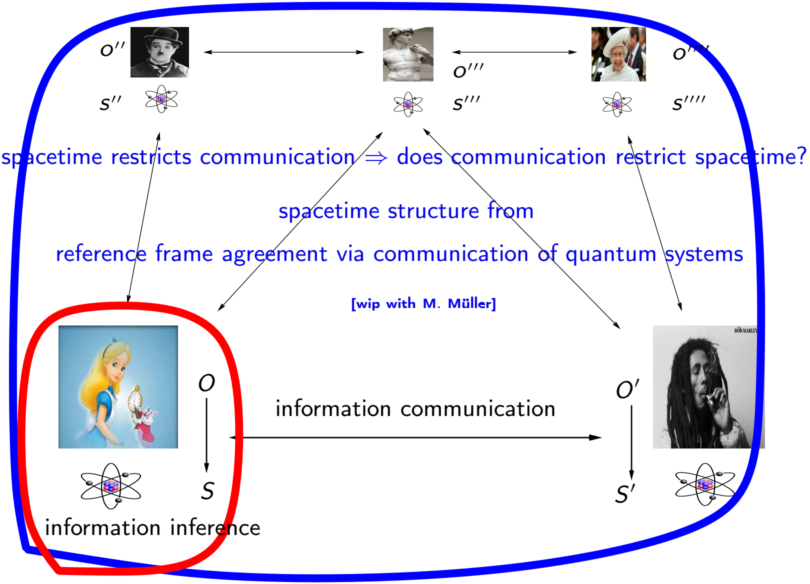


Operational implications: from single to many observers



from information inference to quantum theory ⇒ THIS TALK!

Operational implications: from single to many observers



from information inference to quantum theory ⇒ THIS TALK!

Disclaimer

Limits to operational information theoretic approach:

1. finite systems, finitely many observers, clear separation between observers and systems $\Rightarrow$ approximation

2. will only obtain ‘skeleton’ of theories (state spaces, transformations), but not the ‘flesh’ (concrete Hamiltonian, action, etc) rendering it a ‘living’ theory.

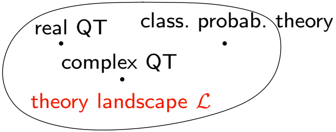
Nevertheless: novel perspective on architecture of physical theories

What is a (re-)construction of QT?

axiomatization of QT with some basic set of postulates

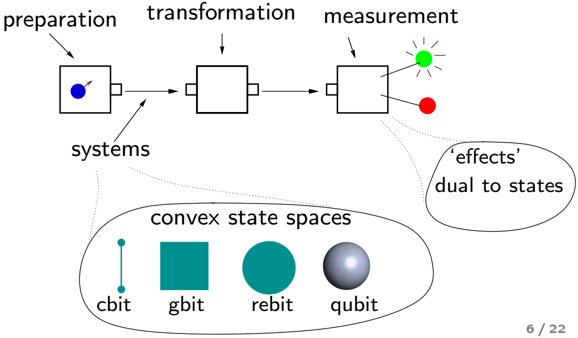
1. define landscape $\mathcal{L}$ of theories within which axioms can be formulated
2. which physical statements characterize QT within $\mathcal{L}$?

$\Rightarrow$ derive quantum state spaces, operations,...



usually: $\mathcal{L}$ = 'generalized probability theories' (GPT)

- operational axioms, but primacy on probability and info inference not explicit
- wave of QT reconstructions within GPT framework
  ['01-'14 Hardy, Masanes, Müller, Brukner, Dakic, D'Ariano, Chiribella, Perinotti......]



Why a (re-)construction of QT?

1. Give operational sense to usual textbook axioms (why $\mathcal{H}$, $\otimes$, $\mathbb{C}$,...?)
2. Better understand QT within larger context
3. why or why not QT in its present form a fundamental theory
4. Often voiced: will clarify interpretation of QT [Rovelli, Fuchs,...]
   $\Rightarrow$ hope thus far not realized (e.g., GPTs interpretationally neutral)

Why another (re-)construction of QT?

QT as framework for information inference [Rovelli, Zeilinger, Brukner, Fuchs, Spekkens,......]
$\Rightarrow$ derive with primacy on information inference

advantage: 1. 'simpler' axioms on relation between O and S
2. emphasizes information inference and close to Relational QM [for RQM see Rovelli, Smerlak]

disadvantage: landscape $\mathcal{L}$ smaller than for GPTs

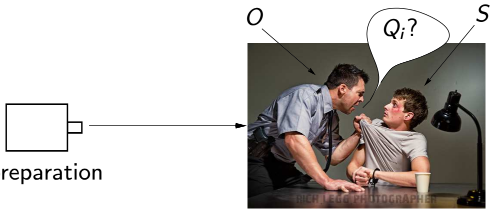
$\Rightarrow$ novel perspective, new 'coordinates' on theory space

Outline for the remainder

Table of contents

Specifying the landscape of inference theories

Observer O interrogating system S with *binary* questions Q_i, $i = 1, \ldots$



preparation

interrogation

- each Q_i non-trivial 1-bit question (info measure later)
- O has tested identical S sufficiently often to 'know' set Σ of all possible answer statistics
- Bayesian viewpoint: for specific S, O assigns probabilities p_i to Q_i accord. to his info about
  1. Σ
  2. particular S
- p_i encode all O can say about $S \Rightarrow$ state of S (rel. to O): collection of p_i
  $\Rightarrow$ state space: Σ (to be convex)

Specifying the landscape of inference theories

- assume: $\exists$ state of 'no information' $p_i = \frac{1}{2}\ \forall\, i \Rightarrow$ call *totally mixed state*
- Q_i, Q_j are:

independent — if, relative to totally mixed state of S, answer to only Q_i gives O no information about answer to Q_j (and vice versa) $\Rightarrow p(Q_i, Q_j) = p_i \cdot p_j$ factorizes

compatible — if O may know answers to both simultaneously $\Rightarrow p_i, p_j$ can be simultaneously $0, 1$

complementary — if knowledge of Q_i disallows O to know Q_j at the same time (and vice versa) $\Rightarrow p_i = 0, 1$, then $p_j = 1/2$

- assumption: state parametrized by max. set of pairwise indep. Q_i

$$\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ \vdots \\ p_{D_{\boldsymbol{N}}} \end{pmatrix}, \quad p_i \text{ prob. that } Q_i = 1$$

- Specker's principle: n Q_i pairwise compatible $\Rightarrow$ mutually compatible

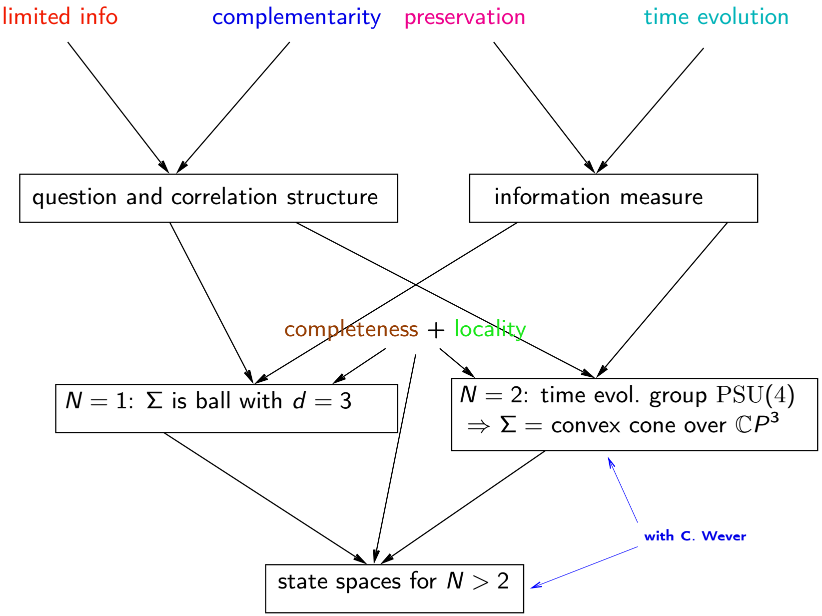
Postulates for a system of N qubits

(LI + C motivated from Rovelli, Zeilinger, Brukner)

- LI: (limited information) "O can acquire maximally $N \in \mathbb{N}$ independent bits of information about S at the same time."
  $\exists\, Q_i,\ i = 1, \ldots, N$ (mutually) independent compatible
- C: (complementarity) "O can always get up to N new (independent) bits of information about S. Whenever O asks a new question he experiences no net loss of information."
  $\exists\, Q_i',\ i = 1, \ldots, N$ independent compatible but $Q_i, Q'_{j=i}$ complementary
- CO: (completeness) Any $\vec{P}_{O \to S}$ permissible, s.t. info in $\vec{P}_{O \to S}$ compatible with LI and C
- P: (preservation) "O's total amount of information about S preserved between interrogations".
- T: (time evolution) Time evolution of $\vec{P}_{O \to S}$ continuous
- LO: (locality) "Info inference is local: O can determine $\vec{P}_{O \to S}$ for a composite system by asking only questions to its components."

Claim: Σ is space of $2^N \times 2^N$ density matrices over $(\mathbb{C}^2)^{\otimes N}$ and states evolve unitarily

Strategy



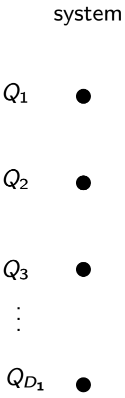
limited info
complementarity
preservation
time evolution
question and correlation structure
information measure
completeness + locality
$N = 1$: Σ is ball with $d = 3$
$N = 2$: time evol. group $\mathrm{PSU}(4)$
$\Rightarrow \Sigma =$ convex cone over $\mathbb{C}P^3$
with C. Wever
state spaces for $N > 2$

Compatibility and independence structure of questions

$N = 1$: only individual Q_i, $i = 1, \ldots, D_1 \Rightarrow D_1 =?$ (know $D_1 \geq 2$)

$N = 2$: $2D_1$ individual Q_i

vertex: individual question Q_i

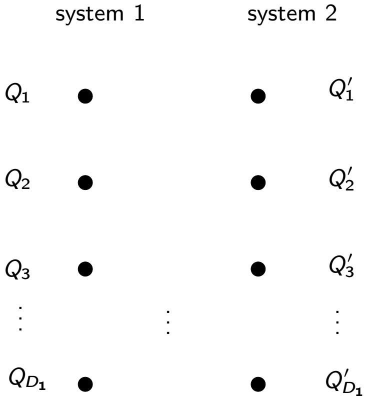


Compatibility and independence structure of questions

$N=1$: only individual Q_i, $i=1,\ldots,D_1 \Rightarrow D_1 =?$ (know $D_1 \geq 2$)

$N=2$: $2D_1$ individual Q_i

vertex: individual question Q_i, Q'_j



Compatibility and independence structure of questions

$N = 1$: only individual Q_i, $i = 1, \ldots, D_1 \Rightarrow D_1 =?$ (know $D_1 \geq 2$)

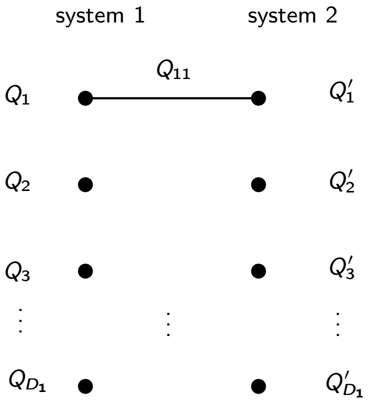
$N = 2$: $2D_1$ individual Q_i + D_1^2 composite questions:

$Q_{ij} := Q_i \leftrightarrow Q_j'$ "Are answers to Q_i and Q_j' the same?"

+ ???

<u>vertex:</u> individual question Q_i, Q_j'

<u>edge:</u> composite question Q_{ij}



Compatibility and independence structure of questions

$N = 1$: only individual Q_i, $i = 1, \ldots, D_1 \Rightarrow D_1 =?$ (know $D_1 \geq 2$)

$N = 2$: $2D_1$ individual Q_i + D_1^2 composite questions:

$Q_{ij} := Q_i \leftrightarrow Q'_j$ "Are answers to Q_i and Q'_j the same?"

+ ???

vertex: individual question Q_i,Q'_j

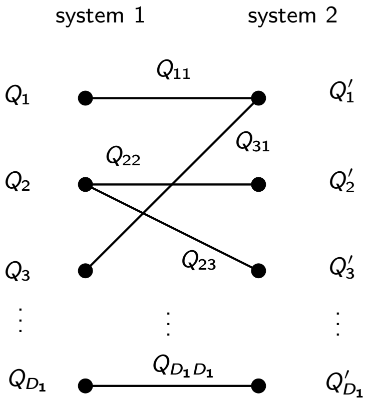
edge: composite question Q_{ij}

show: Q_{ij}

1. pairwise indep.
2. complementary if corresp. edges intersect (e.g., Q_{11}, Q_{31})
3. compatible if corresp. edges non-intersecting (e.g., Q_{11}, Q_{22})

$\Rightarrow$ entanglement: > 1 bit in Q_{ij}

[see also Brukner, Zeilinger]

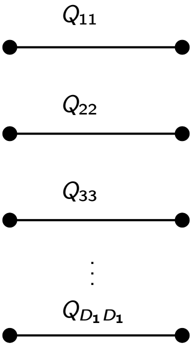


What is the dimension of the Bloch sphere?

Logical argument from $N = 2$ case:

- Q_{ii}, $i =, \dots, D_1$ pairwise independent, compatible
- O can acquire answers to all D_1 composites Q_{ii} simultaneously (Specker)
- LI: O cannot know more than $N = 2$ independent bits about S

$\Rightarrow$ answers to any two Q_{ii} determine answers to all other Q_{jj}



- e.g., truth table for any three Q_{ii} ($a \neq b$):
  $\Rightarrow Q_{33} = Q_{11} \leftrightarrow Q_{22}$ or $\neg(Q_{11} \leftrightarrow Q_{22})$

Q_{11}	Q_{22}	Q_{33}
0	1	a
1	0	a
1	1	b
0	0	b

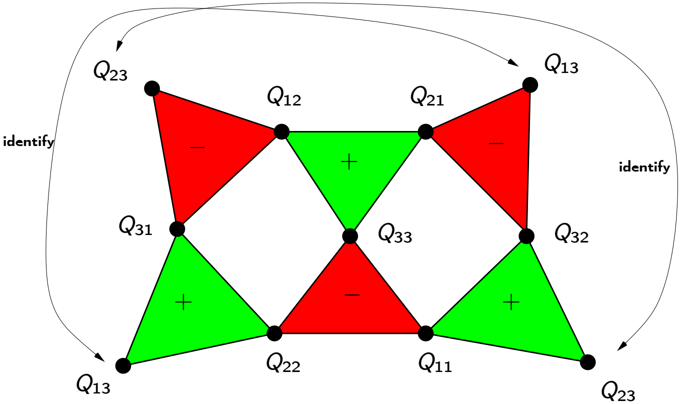
$\Rightarrow$ holds for all compatible sets of Q_{ij}:
$2 \leq D_1 \leq 3$

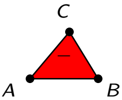
$\Rightarrow$ # DoFs: 15 if $D_1 = 3$; 9 if $D_1 = 2$

Correlation structure for qubits ($N = 2$ and $D_1 = 3$)

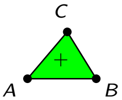
Compatibility structure of Qs $\Rightarrow$ correlation structure for 2 qubits in QT

Q, Q' compatible if connected by edge, otherwise complementary





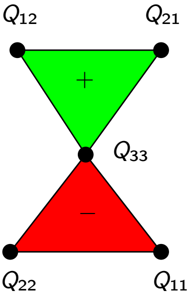
$\Leftrightarrow$ odd correlation
$A = \neg(B \leftrightarrow C)$,
etc...



$\Leftrightarrow$ even correlation
$A = B \leftrightarrow C$,
etc...

Correlation structure for rebits ($N = 2$ and $D_1 = 2$)

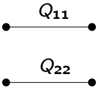
similarly for 2 rebits



key difference rebits vs. qubits:

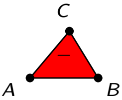
$Q_{33} = \neg(Q_{11} \leftrightarrow Q_{22})$

- non-local ($\nexists\ Q_3, Q_3'$)

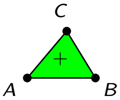


$\Rightarrow$ violates locality

$\Rightarrow$ henceforth: $D_1 = 3$



$\Leftrightarrow$ odd correlation
$A = \neg(B \leftrightarrow C)$,
etc...



$\Leftrightarrow$ even correlation
$A = B \leftrightarrow C$,
etc...

Information measure

recall: state of S relative to O:

$$\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ \vdots \\ p_{D_N} \end{pmatrix}, \quad p_i \text{ prob. that } Q_i = 1, Q_i \text{ indep.}$$

preservation and time evolution (+ operational cond.) imply:

1. reversible time evolution $T \in$ some 1-param. group

$$\vec{P}_{O\to S}(t) = T(t) \cdot \vec{P}_{O\to S}(0) \qquad (1)$$

2. O's info about Q_i $\alpha_i = (2p_i - 1)^2 \Rightarrow O$'s total info about S:

$$I_{O\to S} = ||2\vec{P}_{O\to S} - \vec{1}||^2 = \sum_{i=1}^{D_N} (2p_i - 1)^2 \qquad (2)$$

[from different perspective also proposed by Brukner, Zeilinger]

3. $\{$all possible time evolutions$\} \subset \mathrm{SO(D_N)}$

$\Rightarrow$ info $I_{O\to S}$ ‘conserved charge’ of time evol.

$N = 1$ and the Bloch ball

argued before: $D_1 = 3 \Rightarrow$ have: $\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$

- **pure states:**

$$I_{O\to S} = (2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 = 1\,\texttt{bit} \qquad (3)$$

- **mixed states:**

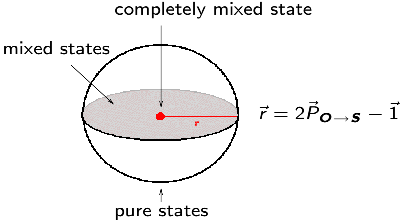
$$0\,\texttt{bit} < (2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 < 1\,\texttt{bit} \qquad (4)$$

- **completely mixed state:**

$$(2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 = 0\,\texttt{bit} \qquad (5)$$

using completeness axiom:

1. Bloch sphere ✓
2. $\{\text{all time evolutions } T\} = \mathrm{SO}(3)$ ✓

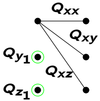


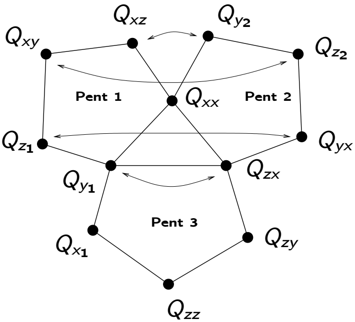
$N = 2$: time evol. group $\mathrm{PSU}(4)$ and $\mathbb{C}P^3$ (very non-trivial!!!) [with C. Wever]

from before: $D_2 = 15$

- $\exists$ 6 max. complementary sets of 5 Qs, e.g.

Pent 1$= \{Q_{xx}, Q_{xz}, Q_{xy}, Q_{z_1}, Q_{y_1}\} =$





- 'conserved info charges' for pure states:

  $I_{O\to S}(\text{Pent } 1) = \alpha_{xx}+\alpha_{xy}+\alpha_{xz}+\alpha_{y_1}+\alpha_{z_1} = 1$

- e.g., info swap Pent 1 $\leftrightarrow$ Pent 2 leaves 'charges' invar.

1. 15 such swaps $\Rightarrow$ define the 15 generators of $\mathfrak{su}(4) \simeq \mathfrak{so}(6) \simeq \mathfrak{psu}(4)$
2. get: evol. group $\mathrm{PSU}(4)$ as in QT ($\rho_{4\times4} \mapsto U\rho_{4\times4}U^\dagger$, $U \in \mathrm{SU}(4)$) ✓
3. get: space of pure states $\mathbb{C}P^3 \Rightarrow$ all states cone over $\mathbb{C}P^3$ as in QT ✓

The case for $N > 2$ [with C. Wever]

easier!!! $\Rightarrow$ $N = 2$ case contains non-trivial part

permit: group of time evol. contains pairwise qubit unitaries

get:

1. time evol. group $\mathrm{PSU}(2^{\mathrm{N}})$ as in QT (pairwise unitaries generate all unitaries [Harrow]) ✓
2. pure quantum state space $\mathbb{C}P^{2^{\boldsymbol{N}}-1}$ contained in pure state space permitted by axioms ✓

still show: other 'solutions' to axioms are diff., but equiv. reps of QT

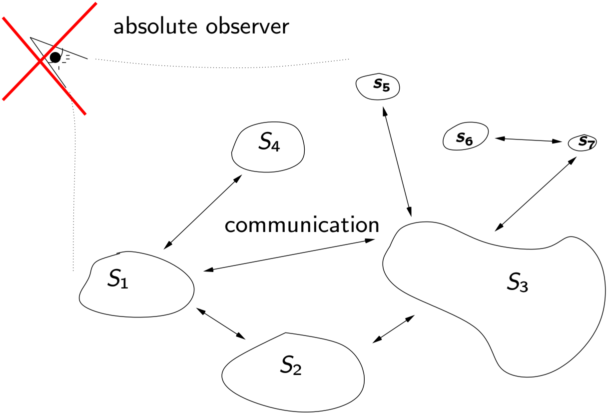
Conclusions

quantum theory is a framework for information inference

quantum theory *is* beautiful!

An operational alternative to the ‘wave function of the universe’

quantum state as state of information also in cosmology/gravity?



- no absolute observer
- universe as information exchange network of subsystems/subregions
- each subsystem assigns state to rest of network, but absence of a global state (‘self-reference problem’)
- realized in concrete toy model: elliptic-dS **[Hackl, Neiman ’14]**