

Intrinsic Time Quantum Geometrodynamics (ITQG)

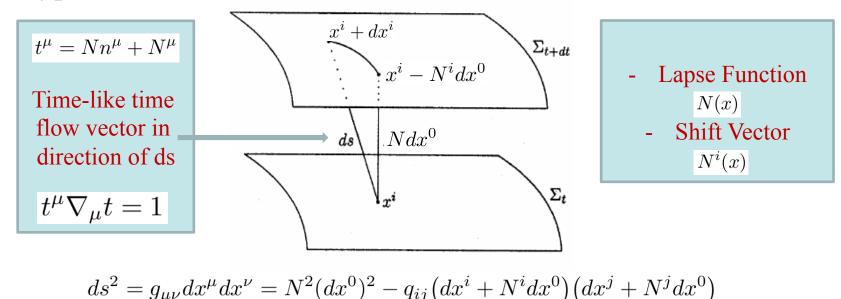
- New Approach to classical and quantum gravity
- Main References
 - Prog. Theor. Exp. Phys. 2015, 083E01
 - Gr-qc: 1707.02720
- Collaborators
 - Chopin Soo (National Cheng Kung University, Taiwan)
 - Hoi Lai Yu (Academica Sinica, Taiwan)
 - Eyo E. Ita (US Naval Academy): On sabbatical with University of South Africa (UNISA)

Outline of talk

- Review Hamiltonian formulation of GR and its challenges/difficulties/issues
- Heuristic Procedure to reduce Phase Space
 - Define intrinsic time
 - extract Physical D.O.F. via gauge-fixing path integral
 - Emergent Lapse
- The Physical Hamiltonian and intrinsic time evolution
- Quantum Mechanics of Gravity
- Summary

3+1 Decomposition of space-time

• Foliate 4D space-time ($\Sigma_t \times R$ topology) into 3D spatial hypersurfaces of simultaneity



- Line element from solution of Einstein equations (a-posteriori)
 - Lapse and Shift are known functions describing a particular observer
- Problem: we do not have an "a-priori" general solution to EFE

Hamiltonian formulation of GR

• Totally constrained system $q_{ij}(x), \tilde{\pi}^{ij}(x)$

$$S = \int dt \int_{\Sigma} d^3x \left(\widetilde{\pi}^{ij} \frac{\partial q_{ij}}{\partial t} - N^i H_i - NH \right) + B dry. \ terms$$

- First class constraints
 - Diffeomorphism constraint:

$$H_i = -2\nabla_j \widetilde{\pi}_i^j$$

– Hamiltonian constraint:

$$H = \beta^2 \widetilde{\pi}^2 - \bar{H}^2$$

$$\widetilde{\pi} = q_{ij}\widetilde{\pi}^{ij}$$

$$\bar{H}^2 = G_{ijkl}\widetilde{\pi}^{ij}\widetilde{\pi}^{kl} + V(q_{ij})$$

$$\beta_{GR} = \frac{1}{\sqrt{6}}$$

$$V_{GR}(q_{ij}) = -\frac{1}{(2\kappa)^2}(R - 2\Lambda_{eff})$$

• Hamiltonian: Lie-drags fields along time flow vector

$$\int_{\Sigma} d^3x (N^i(x)H_i(x) + N(x)H(x)) \qquad t^{\mu} = Nn^{\mu} + N^{\mu}$$

Some Challenges

- Foliation (observer) dependence of time flow vector field
 - Lapse and shift in general unknown since EFE solution in general unknown: do not have an a-priori space-time.
 - Vanishing Hamiltonian generates foliation-dependent (arbitrary) evolution of fields
 - Problem of time, and issues with WDW equation
- Constraints algebra not a true Lie algebra of 4D space-time diffeomorphisms $\{\vec{H}(N), \vec{H}(M)\} = \vec{H}(\mathcal{L}_{\vec{N}}\vec{M});$
 - Structure functions $\{\vec{H}[\vec{N},H(N)] = H(\mathcal{L}_{\vec{N}}N); \\ \{H(N),H(M)\} = -\vec{H}\big(q^{ij}\big(N\partial_j M M\partial_j N\big)\big).$
- It is desirable in the theory of gravitation to:
 - Obtain the reduced phase space of the theory for Physical D.O.F.
 - Perform a complete quantization consistent with axioms of Quantum Mechanics and Quantum Field Theory

Root Cause of Difficulties

- Prevailing Guidance
 - "I am led to the conclusion that four dimensional spacetime covariance cannot be a symmetry of the natural world" Paul Dirac
 - "There is no such thing as a 4-geometry in quantum geometrodynamics." "Spacetime is a Concept of Limited Validity" John A. Wheeler
- Proposal: Intrinsic Time Quantum Geometrodynamics (ITQG)
 - Abandon the call for four dimensional space-time covariance altogether
 - Paradigm Shift from 4DdI to 3DdI combined with intrinsic time evolution
 - Provides a solution to all of these problems in one stroke

Procedure to Reduce Phase Space

- Start from Einstein-Hilbert ADM action
 - Note all occurrences of time t are at this stage foliation-dependent, and we have constraints

$$S = \int dt \int_{\Sigma} d^3x \left[\widetilde{\pi}^{ij} \frac{\partial q_{ij}}{\partial t} + 2N_i \nabla_j \widetilde{\pi}^{ij} - N(\beta^2 \widetilde{\pi}^2 - \bar{H}^2) \right]$$

Define ITQG Phase space variables

$$q_{ij} = q^{1/3}\bar{q}_{ij}; \quad \widetilde{\pi}^{ij} = q^{-1/3}\bar{\pi}^{ij} + \frac{1}{3}q^{ij}\widetilde{\pi}$$

 $q = \det(q_{ij})$

 $\det(\bar{q}_{ij}) = 1$

Unimodular spatial 3-metric

Traceless Momentum variable

- Clean canonical separation of action
 - Must remove derivatives from momentum terms

$$S = \int dt \int_{\Sigma} d^3x \left[\bar{\pi}_{\underline{i}j}^{ij} \frac{\partial \bar{q}_{ij}}{\partial t} + \tilde{\pi}_{\underline{j}} \frac{\partial \ln q^{1/3}}{\partial t} + 2N_i \nabla_j \left(q^{-1/3} \bar{\pi}^{ij} + \frac{1}{3} q^{ij} \tilde{\pi} \right) - N \left(\beta^2 \tilde{\pi}^2 - \bar{H}^2 \right) \right]$$

RPS Procedure Continued

- Integrate by parts
 - Decouples terms linear in momentum from constraints to combine with canonical structure

$$S = \int_{\partial \Sigma} d^2x \left(\frac{2}{3} n_i N^i \widetilde{\pi} + 2q^{-1/3} n_i N_j \overline{\pi}^{ij} \right) + \int dt \int_{\Sigma} d^3x \left[\widetilde{\pi} \left(\frac{\partial \ln q^{1/3}}{\partial t} - \frac{2}{3} \nabla_i N^i \right) \right.$$

$$\left. + \overline{\pi}^{ij} \left(\frac{\partial \overline{q}_{ij}}{\partial t} - q^{-1/3} \left(\nabla_i N_j + \nabla_j N_i - \frac{2}{3} q_{ij} \nabla_k N^k \right) \right) - N \left(\beta^2 \widetilde{\pi}^2 - \overline{H}^2 \right) \right]$$

$$= Boundary \ terms + \int dt \int_{\Sigma} \left[\widetilde{\pi} \left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{N}} \right) \ln q^{1/3} + \overline{\pi}^{ij} \left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{N}} \right) \overline{q}_{ij} - N \left(\beta^2 \widetilde{\pi}^2 - \overline{H}^2 \right) \right].$$

- Diffeomorphism constraints are gone
 - Hodge decomposition for volume element

$$\delta \ln q^{1/3} = \delta T + \nabla_i \delta Y^i$$

Diffeomorphism scalar

Gauge invariant (PHYSICAL) part. Defines: INTRINSIC TIME T

Diffeomorphism (gauge part)

Physical Interpretation

- Intrinsic time
 - Collapses multi-fingered time into a single parameter global time T $T - T_{now} = \frac{2}{3} \ln \left(\frac{V}{V_{now}} \right)$

$$\delta T = \frac{2}{3}\delta \ln V \qquad V = \int_{\Sigma} d^3x \sqrt{q(x)}$$

- Equal to logarithmic change in 3-volume
- Spatial volume of the universe is universal clock for all observers and fields: preserves causality as long as universe is expanding (compact as well)
- Classical and Quantum dynamics in T will be:
 - Independent of the foliation: gauge invariant
 - The same for ALL observers in $\Sigma = \Sigma_T \sim \Sigma(V)$

Returning to the action

$$S = \int dt \int_{\Sigma} \left[\widetilde{\pi} \left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{N}} \right) \ln q^{1/3} + \overline{\pi}^{ij} \left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{N}} \right) \overline{q}_{ij} - N \left(\beta^2 \widetilde{\pi}^2 - \overline{H}^2 \right) \right]$$

Rearrange Hodge decomp.

$$\frac{\partial \ln q^{1/3}}{\partial t} - \frac{2}{3} \nabla_i N^i = \left(\frac{\partial}{\partial t} - \mathcal{L}_{\vec{N}} \right) \ln q^{1/3} = \frac{\partial T}{\partial t}$$

• But also

Foliationdependent evolution along time vector

$$\frac{\partial}{\partial t} - \mathcal{L}_{\vec{N}} = \frac{\partial}{\partial T}$$

Subtract off the gauge-variant part parallel to 3-space

Gauge-invariant, PHYSICAL evolution normal to 3-space in INTRINSIC TIME

• Extend to fields

$$\delta \bar{q}_{ij} = \delta \bar{q}_{ij}^{Phys} + \delta \bar{q}_{ij}^{Gauge}$$

$$rac{\partial ar{q}_{ij}}{\partial t} - \mathcal{L}_{ec{N}} rac{ar{q}_{ij}}{\partial T} = rac{\partial ar{q}_{ij}}{\partial T}$$

Gauge-invariant D.O.F. no diffeomorphism constraints, only Hamiltonian constraint remaining

Implement Hamiltonian constraint

Back to the action

$$S = \int dt \int_{\Sigma} \left[\widetilde{\pi} \frac{\partial \ln q^{1/3}}{\partial T} + \overline{\pi}^{ij} \frac{\partial \bar{q}_{ij}}{\partial T} - N(\beta^2 \widetilde{\pi}^2 - \bar{H}^2) \right]$$

- Recapitulation:
 - Starting from a totally constrained system with foliation-dependent evolution and four constraints per point, we have reduced it to a system with physical evolution and one constraint per point
- Equation of motion for lapse
 - Yields algebraic equation for $\tilde{\pi} = q_{ij}\tilde{\pi}^{ij}$

$$\frac{\delta S}{\delta N(x)} = H(x) = \beta^2 \widetilde{\pi}^2(x) - \overline{H}^2(x) = 0.$$

Bar(H) will play the role of the (Physical) Hamiltonian density on the reduced phase space of ITQG

Phase space path integral

So the quantization of gravity to its reduced phase space amounts to evaluation of the path integral

$$\int D\mu \exp\left[i\int dt \int_{\Sigma} d^3x \left(\bar{\pi}^{ij} \frac{\partial \bar{q}_{ij}}{\partial T} + \tilde{\pi} \frac{\partial \ln q^{1/3}}{\partial T}\right)\right] \prod_{x \in \Sigma} \delta(H(x))\delta(\chi(x)) \operatorname{Det}\{\chi(x), H(x)\} \Theta\left(\tilde{\pi}(x) + \frac{\bar{H}(x)}{\beta}\right). \tag{45}$$

In the path integral we have inserted a theta function, which is needed to propagate positive energy solutions forward in time. To implement the delta functionals we split the measure into traceless and trace parts as

$$D\mu = \prod_{i,j} \prod_{x \in \Sigma} \delta \bar{\pi}^{ij}(x) \delta \bar{q}_{ij}(x) \prod_{x} \delta \ln q^{1/3}(x) \delta \tilde{\pi}(x), \tag{46}$$

and integrate over the trace parts $\delta \ln q^{1/3}(x) \delta \widetilde{\pi}(x)$. Since there is only one gauge-fixing condition per point, then the determinant in (45) reduces to just one function per point

- Procedure (Applied at each spatial point)
 - Constraint to be implemented: Hamiltonian constraint

Quadratic WDW eqn. **Admits EXPANDING** and CONTRACTING universes

$$\delta(H(x)) = \delta\Big(\beta^2 \widetilde{\pi}^2 - \bar{H}^2\Big) = \frac{1}{2\beta \bar{H}(x)} \Big(\delta(\widetilde{\pi}(x) - \frac{\bar{H}(x)}{\beta}) + \delta(\widetilde{\pi}(x) + \frac{\bar{H}(x)}{\beta})\Big)$$

Gauge-fixing condition: Volume expands

$$\frac{\text{Makes}}{\partial \ln q^{1/3}} = 1$$

$$\frac{\partial \ln q^{1/3}}{\partial T} = 1$$
 $\chi(x) = \ln q^{1/3}(x,T) - \ln q(x,T_0) - T$

Requires that positive energy solution corresponds to expansion of the Universe with intrinsic time T (Hodge decomposition)

Determinant: Must be nonzero

$$\{\chi, H\} = \{\ln q^{1/3} - T - \ln q_0^{1/3}, \beta^2 \widetilde{\pi}^2 - \bar{H}^2\} \bigg|_{H=0} = 2\beta^2 \widetilde{\pi} = 2\beta \bar{H}$$

Determinant always positive for positive energy (gauge accessible)

Path integral continued

• Issues with quadratic WDW equation

- Quadratic in Momentum: propagates positive and negative energy solutions forward in time, which implies both expanding and contracting universe: Observational evidence implies only expanding
- Lack of conserved probability density: devoid of quantum mechanical interpretation
- Rectified by Theta function
- Singular operator products and Operator-ordering issues rectified by momentric variables (later)

In the path integral we have inserted a theta function, which is needed to propagate positive energy solutions forward in time. To implement the delta functionals we split the measure into traceless and trace parts as

$$D\mu = \prod_{i,j} \prod_{x \in \Sigma} \delta \bar{\pi}^{ij}(x) \delta \bar{q}_{ij}(x) \prod_{x} \delta \ln q^{1/3}(x) \delta \tilde{\pi}(x), \tag{47}$$

and integrate over the trace parts $\delta \ln q^{1/3}(x)\delta \widetilde{\pi}(x)$. Since there is only one gauge-fixing condition per point, then the determinant in (46) reduces to just one function per point. Substituting contraint and gauge-fixing delta functionals, and the functional determinant into the path integral there is a perfect cancellation of all factors contributing to the path integration measure and we are left with

$$\int \prod_{i,j} \prod_{x \in \Sigma} \delta \bar{\pi}^{ij}(x) \delta \bar{q}_{ij}(x) \exp \left[i \int \delta T \int_{\Sigma} d^3x \left(\bar{\pi}^{ij} \frac{\partial \bar{q}_{ij}}{\partial T} - \frac{\bar{H}}{\beta} \right) \right]. \tag{48}$$

Whatever degrees of freedom the theory starts with, are preserved under Physical evolution in T. So, without loss of generality, one can use the Physical D.O.F. as the starting point in the action, and study their classical evolution and quantization with respect to the intrinsic time T.

End result

- Intrinsic time quantum geometrodynamics
 - Driven by a Physical, diffeomorphism-invariant
 Hamiltonian that generates evolution of the physical degrees of freedom with respect to intrinsic time T
 - System is totally unconstrained

So we can take, as the starting point for intrinsic time gravity, the action based on the physical degrees of freedom

$$S = \int \delta T \int_{\Sigma} d^3 x \bar{\pi}^{ij} \frac{\partial \bar{q}_{ij}}{\partial T} - \int dt H_{Phys}, \tag{50}$$

with physical Hamiltonian

$$H_{Phys} = \int_{\Sigma} d^3x \frac{\bar{H}(x)}{\beta}.$$
 (51)

We can take \bar{q}_{ij} , $\bar{\pi}^{ij} = \bar{q}_{ij}^{Phys}$, $\bar{\pi}_{Phys}^{ij}$, with no constraints and no gauge-fixing. There is no gauge-group volume to take into account, and the path integration measure of reduces to

$$D\mu_{Phys} = \prod_{i,j} \prod_{x \in \Sigma} \delta \bar{\pi}_{TT}^{ij}(x) \delta \bar{q}_{ij}^{TT}(x). \tag{52}$$

Emergent Lapse

• Formula (Prog. Theor. Exp. Phys. (2014) 013E01) Soo, Yu

$$Ndt = \frac{\delta \ln q^{1/3} - \mathcal{L}_{\vec{N}dt} \ln q^{1/3}}{(4\beta \kappa \bar{H}/\sqrt{q})} = \frac{\delta T}{(4\beta \kappa \bar{H}/\sqrt{q})}$$

- Same as a-posteri obtained from Einstein Equations driven by H
- Expanding Volume

$$q(x,T) = q(x,T_0)e^{3(T-T_0)}$$

• Intrinsic time correlates with proper time on clocks of observers in space-times solving Einstein Equations, as well as with nontrivial evolution of the quantum state.

ITQG Revisited

Traceless momentric

$$\bar{\pi}_{j}^{i} = q^{1/3} q_{jm} (\widetilde{\pi}^{im} - \frac{1}{3} q^{im} \widetilde{\pi}), \ \bar{q}_{ij} = q^{-1/3} q_{ij}$$

Unimodular metric

Fundamental commutation relations

$$[\bar{q}_{ij}(x), \bar{q}_{kl}(y)] = 0$$

$$[\bar{q}_{ij}(x), \bar{\pi}_l^k(y)] = i\hbar \bar{E}_{l(ij)}^k \delta(x - y)$$

$$[\bar{\pi}_j^i(x), \bar{\pi}_l^k(y)] = \frac{i\hbar}{2} \left(\delta_j^k \bar{\pi}_l^i - \delta_l^i \bar{\pi}_j^k \right) \delta(x - y)$$

Traceless projector (Vielbein for supermetric)

$$\bar{E}_{j(kl)}^{i} = \frac{1}{2} \left(\delta_{k}^{i} \bar{q}_{jl} + \delta_{l}^{i} \bar{q}_{jk} \right) - \frac{1}{3} \delta_{j}^{i} \bar{q}_{kl} \qquad \bar{E}_{n(ij)}^{m} \bar{E}_{m(kl)}^{n} = \bar{G}_{ijkl}$$

- Meaning of CR:
 - Momentric generates SL(3,R) transformations of the metric which preserves its positivity and unimodularity (self-adjoint operators)
 - Momentric CR is the Lie algebra of SU(3)

The Hamiltonian

Physical Hamiltonian

Weight one density

$$H_{Phys} = \int_{\Sigma} d^3x \frac{\bar{H}(x)}{\beta}$$

- Generator of evolution in intrinsic time T
- Diffeomorphism invariant (3DdI)
- The free theory

Kinetic operator under square root is SU(3) Casimir (Laplacian).

$$\bar{H}(x) = \sqrt{\bar{\pi}_i^j \bar{\pi}_j^i}$$

POSITIVE, SELF **ADJOINT Hamiltonian** density in either case

The theory with interactions

$$\bar{H}(x) = \sqrt{Q^{\dagger}{}_i^j Q_j^j + q \mathcal{K}} = \sqrt{\bar{\pi}_i^j \bar{\pi}_j^i + \mathcal{V}[q_{ij}]} ~~ \text{Potential term under square root}$$

- Similarity transformation Nonhermitian $Q_i^i = e^W \bar{\pi}_i^i e^{-W}$

$$Q_j^i = e^W \bar{\pi}_j^i e^{-W}$$

- W any 3DdI functional of unimodular metric

Emergence of Einstein's GR

• Combination of Chern-Simons and 3D Einstein-Hilbert terms

Coupling Darameters

$$W_T = gW_{CS} - \alpha W_{EH} = \frac{g}{4} \int \tilde{\epsilon}^{ijk} \left(\Gamma^l_{im} \partial_j \Gamma^m_{kl} + \frac{2}{3} \Gamma^l_{im} \Gamma^m_{jn} \Gamma^n_{kl} \right) d^3x - \alpha \int \sqrt{q} R d^3x$$

- Expansion

$$Q_j^i = e^{W_T} \bar{\pi}_j^i e^{-W_T} = \frac{\hbar}{i} \bar{E}_{j(mn)}^i \frac{\delta}{\delta \bar{q}_{mn}} - i\alpha\hbar \sqrt{q} \bar{R}_j^i + ig\hbar \tilde{C}_j^i$$

Hamiltonian density

Traceless 3D Ricci tensor

Cotton-York tensor

$$\bar{H} = \sqrt{Q_i^{\dagger j}Q_j^i + q\mathcal{K}} = \sqrt{\bar{\pi}_i^j\bar{\pi}_j^i + \hbar^2\big(g\tilde{C}_i^j - \alpha\sqrt{q}\bar{R}_i^j\big)\big(g\tilde{C}_j^i - \alpha\sqrt{q}\bar{R}_j^i\big) - i\alpha\hbar[\bar{\pi}_i^j,\bar{R}_j^i] + q\mathcal{K}}$$

Singular commutator term

The Hamiltonian (continued)

Commutator term

$$-i\alpha\hbar\sqrt{q}[\bar{\pi}_j^i(x),\bar{R}_i^j(x)] = [Q_i^{\dagger j}(x),Q_j^i(x)] = -\frac{5}{6}\alpha\hbar^2\lim_{y\to x} \left(-\nabla_x^2 + R\right)\delta(x-y)$$

- Heat Kernel regularization of coincident limit
 - Carried out in gr-qc/1707.0272 (Soo, Yu, Ita)
 - Identify renormalized finite parameters phenomenologically $\alpha = \frac{72(4\pi\epsilon)^{3/2}}{65\hbar^2(2\kappa)^2}; \ \ \mathfrak{K} = \frac{2\Lambda_{eff}}{(2\kappa)^2} + \frac{5\alpha\hbar^2}{32\pi^{3/2}\epsilon^{5/2}}$
- Hamiltonian density after regulator removal
 - Einstein Hilbert potential term emerges

$$\bar{H}(x) = \sqrt{\bar{\pi}_i^j \bar{\pi}_j^i + \hbar^2 g^2 \tilde{C}_i^j \tilde{C}_j^i - \frac{q}{(2\kappa)^2} (R - 2\Lambda_{eff})}$$

Recall that this is SELF-ADJOINT and POSITIVE DEFINITE

Quantum Mechanics of Gravity.

First-order Schrodinger equation

$$i\hbar\frac{\delta\Psi}{\delta T} = \hat{H}_{Phys}\Psi \end{tabular} \begin{tabular}{l} \label{eq:wavefunction} \label{eq:wavefunction} \label{eq:wavefunction} \label{eq:wavefunction} \label{eq:hys} \end{tabular}$$

-Unitary intrinsic time evolution

$$\Psi[\bar{q},T]=\mathfrak{T} \exp\left[-\frac{i}{\hbar}\int_{T_0}^T H_{Phys}(T')\delta T'\right]\Psi[\bar{q},T_0]$$
 3DdI Intrinsic time ordering

- Proposal for Ground state at Big-Bang (q=0) $\Psi[\bar{q}, T_0] = e^{gW_{CS}[\bar{q}]}$
 - -Chern-Simons state: Partition function is well-defined in metric representation (implies the state is normalizable)
 - -Saddle-point is conformally flat S^3 classical geometry
 - -Probability is conserved
- Turns Einstein's GR into renormalizable QFT
 - –Due to higher derivative terms $\tilde{C}_i^j \tilde{C}_j^i$ (Horava)

Summary

- Symmetry of ITQG is 3DdI combined with intrinsic time evolution
- Intrinsic time provides a foliation-independent, observer-independent time for classical and quantum evolution, consistent with the expansion of the universe
- Constrained system with foliation-dependent quantization equivalent (for expanding universe) to reduced phase space foliation-independent quantization
- Physical Degrees of freedom imply transparent classical and quantum dynamics

Questions?

- Thank you for your attention
- Contact email address
 - ita@usna.edu
 - Will have a separate email address for UNISA
- Future directions
 - Initiative with USUHS on Capstones