

# Intrinsic Time Quantum Geometrodynamics (ITQG)

Eyo Eyo Ita

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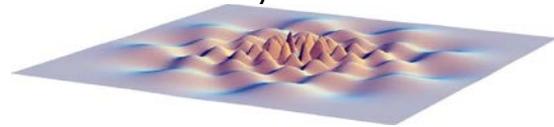


# Outline of Talk

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- Geometric Decomposition
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  - Momentric variables
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# Introduction

- Major unsolved problem in theoretical physics since 1916 is the consistent unification of gravitation with quantum mechanics.
- Einstein's theory of general relativity (GR) describes gravitation as a manifestation of the geometry of four-dimensional spacetime.  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- Quantum Mechanics describes physics on small scales where the wave-particle duality of matter becomes manifest.
- Both theories are unparalleled in accuracy and precision as confirmed by experiment. But all known attempts to reconcile them with each other have failed



# Axioms of Quantum Mechanics

- Axiom 1: The state of a quantum mechanical system is completely specified by a wavefunction  $\Psi(\mathbf{r}, t)$

$$\int_{-\infty}^{\infty} \Psi^*(\mathbf{r}, t) \Psi(\mathbf{r}, t) d\tau = 1$$

- Axiom 2: To every observable  $a$  in classical mechanics there corresponds a linear, Hermitian operator  $\hat{A}$  in quantum mechanics.

$$\hat{A}\Psi = a\Psi$$

- Axiom 3: In any measurement of the observable associated with operator  $\hat{A}$  only the eigenvalues  $a$  can be observed
- Axiom 4: Expectation value of the observable corresponding to  $\hat{A}$  is given by

$$\langle A \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{A} \Psi d\tau$$

- Wavefunction evolves in time according to a Schrodinger equation

$$\hat{H}\Psi(\mathbf{r}, t) = i\hbar \frac{\partial \Psi}{\partial t}$$

- Field theory applies QM to systems with infinite number of d.o.f.
- Would like to apply these axioms to GR as a field theory

# Canonical Gravity

- Performing a 3+1 decomposition leads to the realization that GR is a totally constrained system with a vanishing Hamiltonian
- Spacetime covariance implies 4 first class constraints: 3 diffeomorphism and one Hamiltonian constraint
- According to Paul Dirac, first class constraints are the generators of gauge transformations (unphysical)
- Quantization requires implementation of spacetime covariance at a quantum level- leading to problems:
  - Constraints fail to reproduce algebra of spacetime diffeos
  - Problem of time (Wheeler-DeWitt Equation)

# Root cause for difficulties

- Wheeler DeWitt Equation (unsolved since 1960)

$$\hat{H}\Psi[q_{ij}] = \left[ \frac{1}{2\sqrt{q}} G_{ijkl} \hat{\pi}^{ij} \hat{\pi}^{kl} - \sqrt{q} R \right] \Psi[q_{ij}] = 0$$

- Problem of time (vanishing Hamiltonian)
- Singular Operator products in field theory
- Lack of a positive definite, conserved probability density
- General relativity is perturbatively nonrenormalizable as a relativistic quantum field theory
- Tension between Einstein's spacetime covariance and unitarity
- Nonperturbative canonical approach fails to reproduce classical algebra of spacetime diffeomorphisms at the quantum level

# Intrinsic Time Quantum Geometrodynamics (ITQG)

- Need a time which reconciles unitary evolution of wavefunction of the universe with time intervals measured by observers using clocks in spacetimes solving the Einstein equations
- 4D Spacetime covariance is only a classical concept: cannot be implemented at a quantum level
- Paradigm shift from quantum 3D spatial covariance combined with intrinsic time evolution provides a consistent quantization of gravity
- Resolves all of these difficulties in one stroke
- Published by Eyo Ita, Chopin Soo, Ho-Lai Yu
  - Prog. Theor. Exp. Phys. 2015, 083E01 (8 pages)

# Geometrodynamical Decomposition

- ADM Canonical one form

$$\Theta = \int_{\Sigma} d^3x \tilde{\pi}^{ij}(x) \delta q_{ij}(x)$$

- Canonical pair  $(\ln q^{1/3}, \tilde{\pi} = q_{ij} \tilde{\pi}^{ij})$

- Remaining DOF  $\bar{E}_{j(mn)}^i = \frac{1}{2}(\delta_m^i \bar{q}_{jn} + \delta_n^i \bar{q}_{jm}) - \frac{1}{3} \delta_j^i \bar{q}_{mn}$

– Unimodular spatial 3-metric:  $(\bar{q}_{ij} = q^{-1/3} q_{ij})$

– Traceless Momentric:  $(\tilde{\pi}_j^i = q^{1/3} \bar{E}_{j(mn)}^i \tilde{\pi}^{mn})$

- Hodge decomposition for compact manifolds without boundary

$$\delta \ln q^{1/3} = \delta T + \frac{2}{3} \nabla_i \delta N^i$$

3DDI Intrinsic time interval

$\mathcal{L}_{\delta N} \ln q^{1/3}$  (gauge)

# The quantum theory

- Schrodinger Equation

$$\Psi[\bar{q}, T] = U(T, T_0)\Psi[\bar{q}, T_0]$$

$$i\hbar \frac{\delta\Psi}{\delta T} = H_{Phys} \Psi$$

– 3DDI Evolution in T

$$U(T, T_0) = \mathbb{T} \left\{ \exp \left[ -\frac{i}{\hbar} \int_{T_0}^T H_{Phys}(T') \delta T' \right] \right\}$$

- Physical Hamiltonian

$$H_{Phys} = \int_{\Sigma} \frac{\bar{H}(x)}{\beta} d^3x$$

*Weight One density (3DDI)*

$$\bar{H} = \sqrt{\bar{\pi}_i^j \bar{\pi}_j^i + \mathcal{V}[q_{ij}]}$$

$$\beta = \frac{1}{\sqrt{6}}$$

$$\mathcal{V} = -\frac{q}{(2\kappa)^2} [R - 2\Lambda_{eff}]$$

*For GR case*

- Confers upon gravity a Quantum Mechanical Interpretation (conserved probability, etc)

# Commutation Relations

- Commutation relations

$$[\bar{q}_{ij}(x), \bar{q}_{kl}(y)] = 0, \quad [\bar{q}_{ij}(x), \hat{\pi}_l^k(y)] = i\hbar \bar{E}_{l(ij)}^k \delta(x - y)$$

$$[\hat{\pi}_j^i(x), \hat{\pi}_l^k(y)] = \frac{i\hbar}{2} (\delta_j^k \hat{\pi}_l^i - \delta_l^i \hat{\pi}_j^k) \delta(x - y)$$

- Features
  - Self-adjoint, traceless momentric operator
  - Preserves metric positivity and unimodularity
  - Inherent SU(3) Lie algebra structure regulates the theory
  - Kinetic operator is the quadratic Casimir

# Momentric Variables

- SU(3) Generators

$$T^A(x) = \frac{1}{\hbar\delta(0)} (\lambda^A)_i^j \hat{\pi}_j^i(x)$$

*Gell - Mann matrices*

$$[T^A(x), T^B(y)] = if_C^{AB} T^C \frac{\delta(x-y)}{\delta(0)}$$

- Schrodinger Representation

$$\begin{aligned} \frac{\hbar}{i} (\lambda^A)_j^i \bar{E}_{i(mn)}^j \frac{\delta}{\delta \bar{q}_{mn}(x)} \langle \bar{q} | \prod_y |l^2, C, I, m_3, m_8\rangle_y \\ = \frac{\hbar\delta(0)}{2} \langle \bar{q} | \prod_y |l^2, C, I, m_3, m_8\rangle_y \end{aligned}$$

- State Labels

*Laplacian* :  $l^2 = T^A T^A$

*Isospin* :  $I = \sum_{B=1}^3 T^B T^B$

*Cartan subalgebra* :  $m_3, m_8$

*Determinant* :  $C = d_{ABC} T^A T^B T^C$

# The General Potential

- Intrinsic time dependence

$$\ln\left[\frac{q(x, T)}{q(x, T_{now})}\right] = 3(T - T_{now}) = 2\ln(V/V_{now})$$

- The Hamiltonian

$$\bar{H}(x) = \sqrt{\pi_i^j \pi_j^i + \mathcal{V}} = \sqrt{\pi_i^{j\dagger} \pi_j^i + \mathcal{V}}$$

- Potential

- Most general weight-2, semi-positive definite object with dimension six operators
- For the “free” theory  $\mathcal{V} = 0$
- At classical level

$$\mathcal{V}[q_{ij}] = \frac{\delta W}{\delta q_{ij}} E_{n(ij)}^m E_{m(kl)}^n \frac{\delta W}{\delta q_{kl}} = \bar{W}_n^m \bar{W}_m^n$$

# The Interacting Theory

- Interactions introduced via similarity transformation of momentric operator
  - Nonhermitian
  - Generate unitarily inequivalent  $SL(3,R)$  representation

$$\hat{Q}_j^i = e^W \hat{\pi}_j^i e^{-W} = \frac{\hbar}{i} \bar{E}_{j(mn)}^i \left[ \frac{\delta}{\delta \bar{q}_{mn}} - \frac{\delta W}{\delta \bar{q}_{mn}} \right] = \frac{\hbar}{i} \bar{E}_{j(mn)}^i \frac{\delta}{\delta \bar{q}_{mn}} + i\hbar \bar{W}_j^i$$

- Attributes of Hamiltonian

- (semi) positive definite

- Self-adjoint

- 3DDI

- Not intrinsic time reversal invariant

- Admit extensions of GR (UV completeness)

$$\bar{H} = \sqrt{\hat{Q}_i^j \hat{Q}_j^i}$$

# Phenomenological Proposition

- Ingredients for Potential

$$W = \frac{g}{4} \int \tilde{\epsilon}^{ijk} (\bar{\Gamma}_{im}^l \partial_j \bar{\Gamma}_{kl}^m + \frac{2}{3} \bar{\Gamma}_{im}^l \bar{\Gamma}_{jn}^m \bar{\Gamma}_{kl}^n) d^3x + b \int_{\Sigma} d^3x \sqrt{q} R = \frac{g}{4} I_{CS}[q_{ij}] + b I_{EH}$$

*Chern - Simons*

*Einstein - Hilbert*

- Separate T dependence

$$\tilde{W}_j^i = \sqrt{q} (\Lambda' + a' q^{-1/3} \bar{q}^{kl} \bar{R}_{kl}) \delta_j^i + b' \sqrt{q} q^{-1/3} \bar{q}^{ik} \bar{R}_{kj} + g \hbar \tilde{C}_j^i + (\partial_i \ln q \text{ terms})$$

*Traceless Ricci*

*Cotton - York tensor*

- General Procedure

- Extract intrinsic time dependence of Hamiltonian
- Construct ground state for different eras
- CY-EH potential provides normalizable state (UV complete Partition function)

# The Early Universe

- Small volume  $T - T_{now} \rightarrow -\infty$

- Era of Cotton-York dominance

$$\bar{H} = \sqrt{\hat{\pi}_i^j \hat{\pi}_j^i + g^2 \hbar^2 \tilde{C}_i^j \tilde{C}_j^i}$$

- ZPE is absent  $[\hat{\pi}_j^i, \tilde{C}_i^j] = 0$

- Initial state of the universe

- Minimum Energy ( $\bar{\pi}_j^i = \tilde{C}_j^i = 0$ ): Conformally flat, junction condition
- Realizes Robertson-Walker Big Bang (S3 geometry)
- Compatible with Penrose hypothesis (vanishing Weyl curvature)
- Primordial gravitational waves ( $1/k^3$  dependence of 2-point correlation functions)

- Emergence of Einstein-Hilbert Gravity

# The late Universe

- Large volume  $T - T_{now} \rightarrow \infty$ 
  - Einstein-Hilbert and cosmological terms dominate the potential
  - Preserves advantages of Hamiltonian

$$\bar{H} = \sqrt{\hat{\pi}_i^j \hat{\pi}_j^i + \hbar^2 (g\tilde{C}_i^j + b\sqrt{q}\bar{R}_i^j)(g\tilde{C}_j^i + b\sqrt{q}\bar{R}_j^i)} + [\hat{\pi}_j^i, ib\hbar\sqrt{q}\bar{R}_i^j]$$

- Emergence of Einstein-Hilbert Gravity
  - ZPE from Heat Kernel regularization
  - Suppress Non-GR terms

$$[\hat{\pi}_j^i, ib\sqrt{q}\hbar\bar{R}_i^j] = -\frac{5}{12}b\hbar^2\delta(0)\sqrt{q}\left(5R - \frac{9}{\epsilon}\right)$$

- Einstein's GR dominates at low curvatures and long wavelengths, 4D symmetry not fundamental

# Additional Features

- Underlying SU(3) structure regulates the theory

$$H_{Phys} = \hbar \int \sqrt{(Q^A)^\dagger Q^A} \frac{\delta(0)}{\sqrt{2\beta}} d^3x, \quad Q^A = e^{-W} T^A(x) e^W$$

- Planck's constant cancels out from S.E.

- Dimensionless CR

$$[\bar{q}_{ij}(x), \bar{q}_{kl}(y)] = 0 \quad [\bar{q}_{ij}(x), T^A(y)] = \frac{i}{2} \left( (\lambda^A)_i^k \bar{q}_{kj} + (\lambda^A)_j^k \bar{q}_{ki} \right) \frac{\delta(x-y)}{\delta(0)}$$

$$[T^A(x), T^B(y)] = i f_C^{AB} T^C \frac{\delta(x-y)}{\delta(0)}$$

- Quantum Geometroynamics Redux

- SL(3,R) transformations modulo spatial diffeomorphisms generated by

$$U_{Phys}[\alpha] = e^{-(i/\hbar) \int_{\Sigma} (\alpha_{TT})_i^j \bar{\pi}_j^i d^3x}$$

# Summary

- ITQG features a physical Hamiltonian, which generates unitary evolution of the state wrt intrinsic time. Solution to the problem of time
- First-order Schrodinger equation provides conserved probability, together with diffeomorphism-invariant intrinsic time ordering, which confers upon ITQG the usual quantum mechanical interpretation
- The Hamiltonian reproduces Einstein's GR as a particular case of a wider class of theories, with rich and phenomenologically interesting extensions
- New set of fundamental commutation relations with an underlying group theoretical interpretation, without Planck's constant, is discovered

# Future Research/Questions?

- Combine with Standard Model
- Search for Primordial gravitational waves (BICEP, Planck, etc.)
- Classical theory of CY-EH potential
- CMB Predictions
  - Contact info: [ita@usna.edu](mailto:ita@usna.edu)