Intrinsic Time Quantum Geometrodynamics (ITQG)

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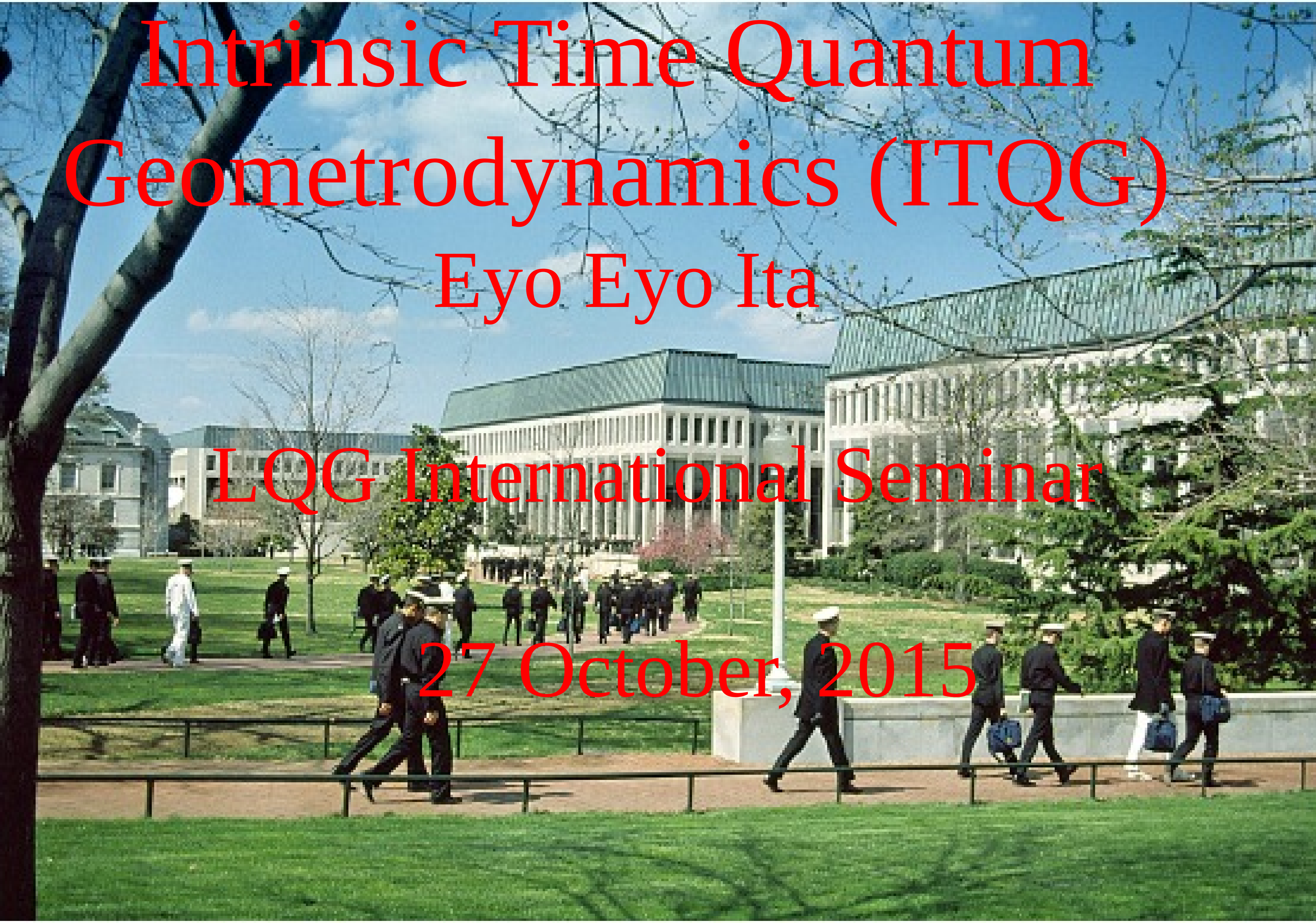

Outline of Talk

- Introduction
- Axioms of Quantum Mechanics/Canonical Gravity
- Intrinsic Time Quantum Geometrodynamics (ITQG)
- Geometric Decomposition
  - The quantum theory/Commutation relations
  - Momentric variables
- General Potential
  - Interacting theory
- Phenomonological Proposition
  - The Early Universe
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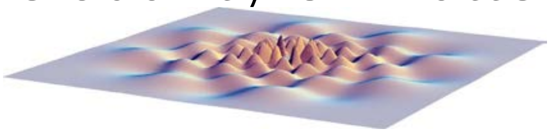
- Additional Features
- Summary

Introduction

- Major unsolved problem in theoretical physics since 1916 is the consistent unification of gravitation with quantum mechanics.
- Einstein's theory of general relativity (GR) describes gravitation as a manifestation of the geometry of four-dimensional spacetime. $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- Quantum Mechanics describes physics on small scales where the wave-particle duality of matter becomes manifest.



- Both theories are unparalleled in accuracy and precision as confirmed by experiment. But all known attempts to reconcile them with each other have failed

Axioms of Quantum Mechanics

- Axiom 1: The state of a quantum mechanical system is completely specified by a wavefunction $\Psi(\mathbf{r},t)$

$$\int_{-\infty}^{\infty}\Psi^*(\mathbf{r},t)\Psi(\mathbf{r},t)d\tau = 1$$

- Axiom 2: To every observable a in classical mechanics there corresponds a linear, Hermitian operator $\hat{A}$ in quantum mechanics.

$$\hat{A}\Psi = a\Psi$$

- Axiom 3: In any measurement of the observable associated with operator $\hat{A}$ only the eigenvalues a can be observed
- Axiom 4: Expectation value of the observable corresponding to $\hat{A}$ is given by $<A>=\int_{-\infty}^{\infty}\Psi^*\hat{A}\Psi d\tau$
- Wavefunction evolves in time according to a Schrodinger equation $\hat{H}\Psi(\mathbf{r},t) = i\hbar\frac{\partial\Psi}{\partial t}$
  - Field theory applies QM to systems with infinite number of d.o.f.
  - Would like to apply these axioms to GR as a field theory

Canonical Gravity

- Performing a 3+1 decomposition leads to the realization that GR is a totally constrained system with a vanishing Hamiltonian
- Spacetime covariance implies 4 first class constraints: 3 diffeomorphism and one Hamiltonian constraint
- According to Paul Dirac, first class constraints are the generators of gauge transformations (unphysical)
- Quantization requires implementation of spacetime covariance at a quantum level- leading to problems:
  - Constraints fail to reproduce algebra of spacetime diffeos
  - Problem of time (Wheeler-DeWitt Equation)

Root cause for difficulties

- Wheeler DeWitt Equation (unsolved since 1960)

$$\widehat{H}\Psi[q_{ij}] = \left[\frac{1}{2\sqrt{q}}G_{ijkl}\widehat{\pi}^{ij}\widehat{\pi}^{kl} - \sqrt{q}R\right]\Psi[q_{ij}] = 0$$

  - Problem of time (vanishing Hamiltonian)
  - Singular Operator products in field theory
  - Lack of a positive definite, conserved probability density
- General relativity is perturbatively nonrenormalizable as a relativistic quantum field theory
- Tension between Einstein's spacetime covariance and unitarity
- Nonperturbative canonical approach fails to reproduce classical algebra of spacetime diffeomorphisms at the quantum level

Intrinsic Time Quantum Geometrodynamics (ITQG)

- Need a time which reconciles unitary evolution of wavefunction of the universe with time intervals measured by observers using clocks in spacetimes solving the Einstein equations
- 4D Spacetime covariance is only a classical concept: cannot be implemented at a quantum level
- Paradigm shift from quantum 3D spatial covariance combined with intrinsic time evolution provides a consistent quantization of gravity
- Resolves all of these difficulties in one stroke
- Published by Eyo Ita, Chopin Soo, Ho-Lai Yu
  - Prog. Theor. Exp. Phys. 2015, 083E01 (8 pages)

Geometrodynamic Decomposition

- ADM Canonical one form

$$\Theta = \int_{\Sigma} d^3x \, \widetilde{\pi}^{ij}(x)\delta q_{ij}(x)$$

- Canonical pair ($\ln q^{1/3}$, $\widetilde{\pi} = q_{ij}\widetilde{\pi}^{ij}$)
- Remaining DOF $\bar{E}^i_{j(mn)} = \frac{1}{2}(\delta^i_m \bar{q}_{jn} + \delta^i_n \bar{q}_{jm}) - \frac{1}{3}\delta^i_j \bar{q}_{mn}$
  - Unimodular spatial 3-metric: ($\bar{q}_{ij} = q^{-1/3} q_{ij}$)
  - Traceless Momentric: ($\bar{\pi}^i_j = q^{1/3}\bar{E}^i_{j(mn)}\widetilde{\pi}^{mn}$)
- Hodge decomposition for compact manifolds without boundary

$$\delta \ln q^{1/3} = \delta T + \frac{2}{3}\nabla_i \delta N^i$$

δT: *3DDI Intrinsic time interval*

$\frac{2}{3}\nabla_i \delta N^i$: $\mathcal{L}_{\delta\vec{N}} \ln q^{1/3}$ *(gauge)*

The quantum theory

- Schrodinger Equation

$$\Psi[\bar{q}, T] = U(T, T_0)\Psi[\bar{q}, T_0]$$

$$i\hbar\frac{\delta\Psi}{\delta T} = H_{Phys}\Psi$$

  - 3DDI Evolution in T

$$U(T, T_0) = \mathrm{T}\Big\{\exp\Big[-\frac{i}{\hbar}\int_{T_0}^{T} H_{Phys}(T')\delta T'\Big]\Big\}$$

- Physical Hamiltonian

$$H_{Phys} = \int_{\Sigma}\frac{\bar{H}(x)}{\beta}d^3x \qquad \bar{H} = \overbrace{\sqrt{\bar{\pi}^j_i\bar{\pi}^i_j + \mathcal{V}[q_{ij}]}}^{Weight\ One\ density\ (3DDI)}$$

$$\beta = \frac{1}{\sqrt{6}} \qquad \mathcal{V} = -\frac{q}{(2\kappa)^2}[R - 2\Lambda_{eff}] \qquad \textit{For GR case}$$

- Confers upon gravity a Quantum Mechanical Interpretation (conserved probability, etc)

Commutation Relations

- Commutation relations

$$[\bar{q}_{ij}(x), \bar{q}_{kl}(y)] = 0, \quad [\bar{q}_{ij}(x), \hat{\bar{\pi}}^k_l(y)] = i\hbar \bar{E}^k_{l(ij)}\delta(x-y)$$

$$[\hat{\bar{\pi}}^i_j(x), \hat{\bar{\pi}}^k_l(y)] = \frac{i\hbar}{2}\big(\delta^k_j \hat{\bar{\pi}}^i_l - \delta^i_l \hat{\bar{\pi}}^k_j\big)\delta(x-y)$$

- Features
  - Self-adjoint, traceless momentric operator
  - Preserves metric positivity and unimodularity
  - Inherent SU(3) Lie algebra structure regulates the theory
  - Kinetic operator is the quadratic Casimir

Momentric Variables

- SU(3) Generators

$$T^A(x) = \frac{1}{\hbar\delta(0)}(\lambda^A)^j_i \hat{\bar{\pi}}^i_j(x)$$

$Gell-Mann\ matrices$

$$[T^A(x), T^B(y)] = i f^{AB}_C T^C \frac{\delta(x-y)}{\delta(0)}$$

- Schrodinger Representation

$$\begin{aligned}\frac{\hbar}{i}(\lambda^A)^i_j \bar{E}^j_{i(mn)} \frac{\delta}{\delta\bar{q}_{mn}(x)} \langle\bar{q}| \prod_y |l^2, C, I, m_3, m_8\rangle_y \\ = \frac{\hbar\delta(0)}{2} \langle\bar{q}| \prod_y |l^2, C, I, m_3, m_8\rangle_y\end{aligned}$$

- State Labels

$$Laplacian:\ l^2 = T^A T^A \qquad Isospin:\ I = \sum_{B=1}^{3} T^B T^B$$

$$Cartan\ subalgebra:\ m_3, m_8 \qquad Determinant:\ C = d_{ABC} T^A T^B T^C$$

The General Potential

- Intrinsic time dependence

$$\ln\left[\frac{q(x,T)}{q(x,T_{now})}\right] = 3(T - T_{now}) = 2\ln(V/V_{now})$$

- The Hamiltonian

$$\bar{H}(x) = \sqrt{\pi^j_i \pi^i_j + \mathcal{V}} = \sqrt{\pi^{j\,\dagger}_i \pi^i_j + \mathcal{V}}$$

- Potential
  - Most general weight-2, semi-positive definite object with dimension six operators
  - For the “free” theory $\mathcal{V} = 0$
  - At classical level

$$\mathcal{V}[q_{ij}] = \frac{\delta W}{\delta q_{ij}} E^m_{n(ij)} E^n_{m(kl)} \frac{\delta W}{\delta q_{kl}} = \bar{W}^m_n \bar{W}^n_m$$

The Interacting Theory

- Interactions introduced via similarity transformation of momentric operator
  - Nonhermitian
  - Generate unitarily inequivalent SL(3,R) representation

$$\hat{Q}^i_j = e^W \hat{\bar{\pi}}^i_j e^{-W} = \frac{\hbar}{i}\bar{E}^i_{j(mn)}\left[\frac{\delta}{\delta \bar{q}_{mn}} - \frac{\delta W}{\delta \bar{q}_{mn}}\right] = \frac{\hbar}{i}\bar{E}^i_{j(mn)}\frac{\delta}{\delta \bar{q}_{mn}} + i\hbar \bar{W}^i_j$$

- Attributes of Hamiltonian
  - (semi) positive definite
  - Self-adjoint
  - 3DDI
  - Not intrinsic time reversal invariant
  - Admit extensions of GR (UV completeness)

$$\bar{H} = \sqrt{\hat{Q}^{\dagger}{}^j_i \hat{Q}^i_j}$$

Phenomenological Proposition

- Ingredients for Potential

$$W = \frac{g}{4}\int \tilde{\epsilon}^{ijk}(\bar{\Gamma}^{l}_{im}\partial_j\bar{\Gamma}^{m}_{kl} + \frac{2}{3}\bar{\Gamma}^{l}_{im}\bar{\Gamma}^{m}_{jn}\bar{\Gamma}^{n}_{kl})d^3x + b\int_{\Sigma} d^3x\sqrt{q}R = \frac{g}{4}I_{CS}[q_{ij}] + bI_{EH}$$

$Chern - Simons$ $Einstein - Hilbert$

- Separate T dependence

$$\tilde{W}^i_j = \sqrt{q}(\Lambda' + a' q^{-1/3}\bar{q}^{kl}\overline{R}_{kl})\delta^i_j + b'\sqrt{q}q^{-1/3}\bar{q}^{ik}\overline{\overline{R}}_{kj} + g\hbar\tilde{C}^i_j + (\partial_i \ln q \ terms)$$

$Traceless\ Ricci$ $Cotton - York\ tensor$

- General Procedure
  - Extract intrinsic time dependence of Hamiltonian
  - Construct ground state for different eras
  - CY-EH potential provides normalizable state (UV complete Partition function)

The Early Universe

- Small volume $T - T_{now} \rightarrow -\infty$
  - Era of Cotton-York dominance

$$\bar{H} = \sqrt{\hat{\bar{\pi}}^j_i \hat{\bar{\pi}}^i_j + g^2 \hbar^2 \tilde{C}^j_i \tilde{C}^i_j}$$

  - ZPE is absent $[\hat{\bar{\pi}}^i_j, \tilde{C}^j_i] = 0$
- Initial state of the universe
  - Minimum Energy ($\bar{\pi}^i_j = \tilde{C}^i_j = 0$): Conformally flat, junction condition
  - Realizes Robertson-Walker Big Bang (S3 geometry)
  - Compatible with Penrose hypothesis (vanishing Weyl curvature)
  - Primordial gravitational waves (1/k^3 dependence of 2-point correlation functions)
- Emergence of Einstein-Hilbert Gravity

The late Universe

- Large volume $T - T_{now} \rightarrow \infty$
  - Einstein-Hilbert and cosmological terms dominate the potential
  - Preserves advantages of Hamiltonian

$$\bar{H} = \sqrt{\hat{\bar{\pi}}^j_i \hat{\bar{\pi}}^i_j + \hbar^2 (g\tilde{C}^j_i + b\sqrt{q}\bar{R}^j_i)(g\tilde{C}^i_j + b\sqrt{q}\bar{R}^i_j) + [\hat{\bar{\pi}}^i_j, ib\hbar\sqrt{q}\bar{R}^j_i]}$$

- Emergence of Einstein-Hilbert Gravity
  - ZPE from Heat Kernel regularization
  - Suppress Non-GR terms

$$[\hat{\bar{\pi}}^i_j, ib\sqrt{q}\hbar\bar{R}^j_i] = -\frac{5}{12} b\hbar^2 \delta(0)\sqrt{q}\left(5R - \frac{9}{\epsilon}\right)$$

- Einstein's GR dominates at low curvatures and long wavelengths, 4D symmetry not fundamental

Additional Features

- Underlying SU(3) structure regulates the theory

$$H_{Phys} = \hbar \int \sqrt{(Q^A)^\dagger Q^A} \frac{\delta(0)}{\sqrt{2}\beta} d^3x, \;\; Q^A = e^{-W} T^A(x) e^W$$

- Planck's constant cancels out from S.E.
  - Dimensionless CR

$$[\bar{q}_{ij}(x), \bar{q}_{kl}(y)] = 0$$

$$[\bar{q}_{ij}(x), T^A(y)] = \frac{i}{2}\Big((\lambda^A)^k_i \bar{q}_{kj} + (\lambda^A)^k_j \bar{q}_{ki} \Big) \frac{\delta(x-y)}{\delta(0)}$$

$$[T^A(x), T^B(y)] = i f^{AB}_C T^C \frac{\delta(x-y)}{\delta(0)}$$

- Quantum Geometrodynamics Redux
  - SL(3,R) transformations modulo spatial diffeomorphisms generated by

$$U_{Phys}[\alpha] = e^{-(i/\hbar) \int_\Sigma (\alpha_{TT})^j_i \bar{\pi}^i_j d^3x}$$

2 DOF

Summary

- ITQG features a physical Hamiltonian, which generates unitary evolution of the state wrt intrinsic time. Solution to the problem of time
- First-order Schrodinger equation provides conserved probability, together with diffeomorphism-invariant intrinsic time ordering, which confers upon ITQG the usual quantum mechanical interpretation
- The Hamiltonian reproduces Einstein's GR as a particular case of a wider class of theories, with rich and phenomenologically interesting extensions
- New set of fundamental commutation relations with an underlying group theoretical interpretation, without Planck's constant, is discovered

Future Research/Questions?

- Combine with Standard Model
- Search for Primordial gravitational waves (BICEP, Planck, etc.)
- Classical theory of CY-EH potential
- CMB Predictions
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