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Quantum Reference Frames in Spacetime

Viktoria Kabel

International LQG Seminar, 19.09.2023

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Quantum Reference Frames in Spacetime

Viktoria Kabel

based on joint work with

Č. Brukner, E. Castro-Ruiz, A.-C. de la Hamette, and W. Wieland

arXiv: 2112.11473, 2302.11629

Motivations

Operationalism

In practice, any reference frame is a **physical system**, most often one that should ultimately be described by **quantum** physics.

The **problem of time** evokes the need for a reference frame in order to describe evolution.

Quantum Gravity

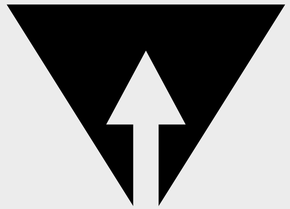
Gauge Theory

Reference frames allow to construct **gauge-invariant observables** through dressings.

First-Principles Approach

Idea: Take principles of known theories such as quantum theory, general relativity, and QFT and try to push them as far as possible.

- ✦ linearity & the superposition principle
- ✦ relativity of position, momenta, directions, …
- ✦ symmetries of the equations of motion

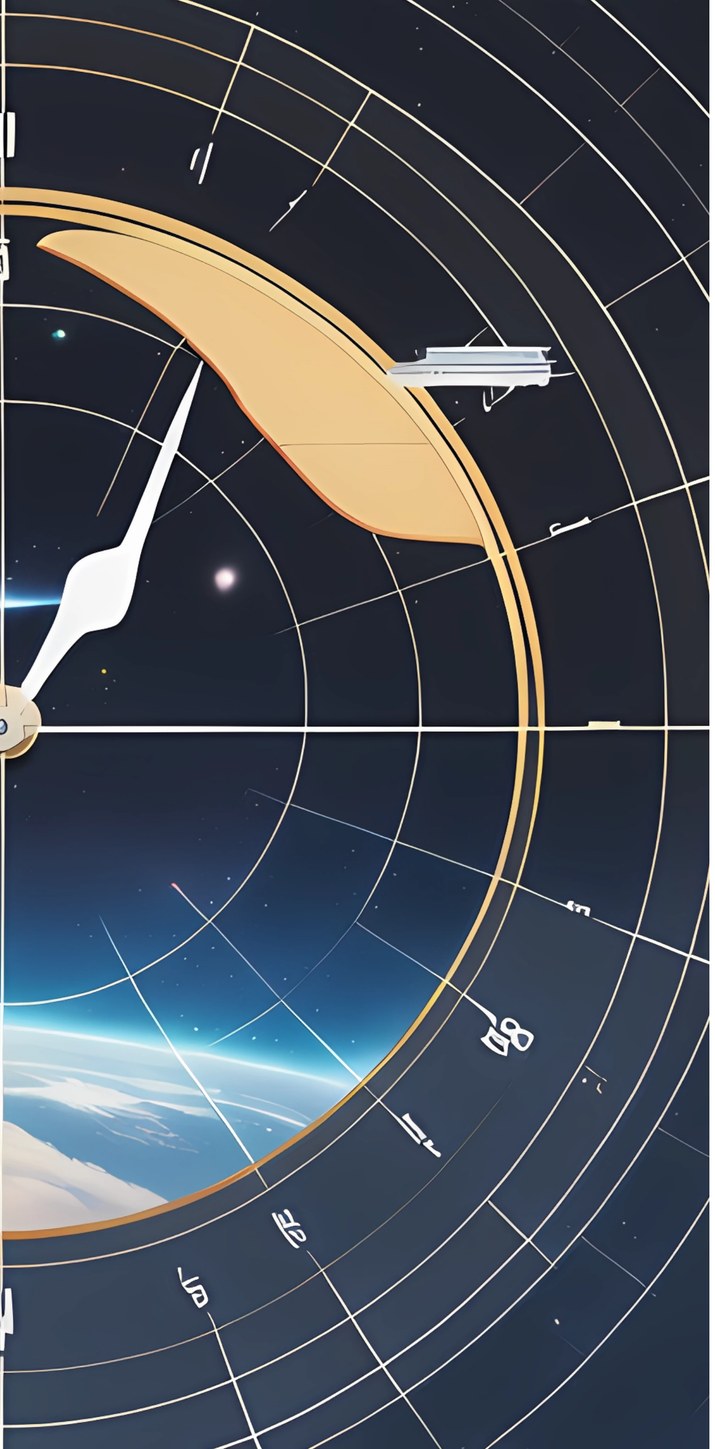


Bottom-Up Approach

Quantum Reference Frames

Top-Down Approach

As such, it is **complementary** to top-down approaches to quantum gravity, which aim to find a more fundamental theory from which the known theories can be derived.



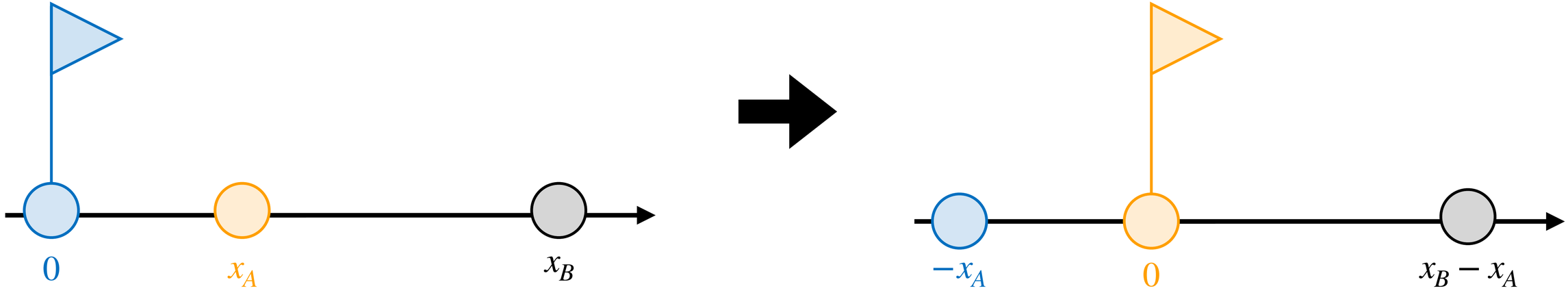
Outline

- ✦ **Introduction to Quantum Reference Frames**
- ✦ **Quantum Reference Frames & Gravity**
  - ▸ Massive Objects in Superposition
- ✦ **Gravity & Quantum Reference Frames**
  - ▸ QRFs at the Boundary of Spacetime
- ✦ **Outlook & Connections**

Reference Frames & Symmetries

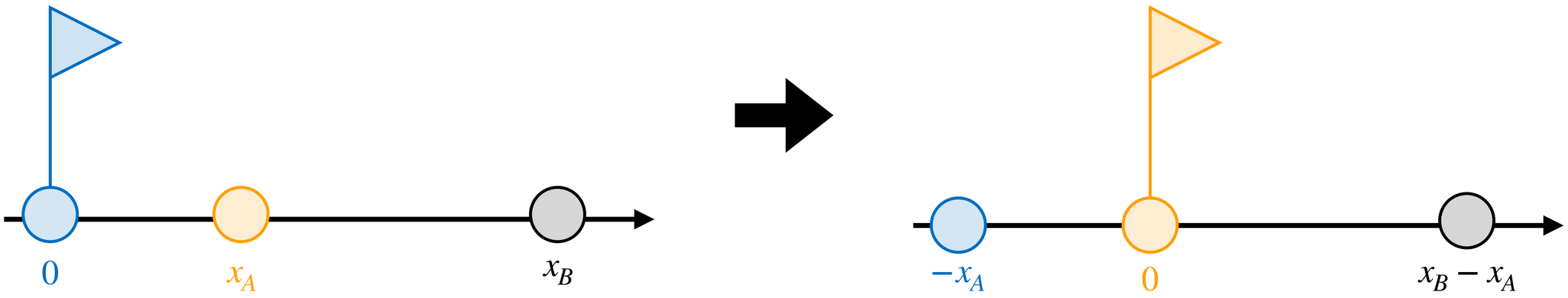
Classical Physics

- ✦ Example: invariance under translations.
- ✦ Corresponds to free choice of position reference frame.



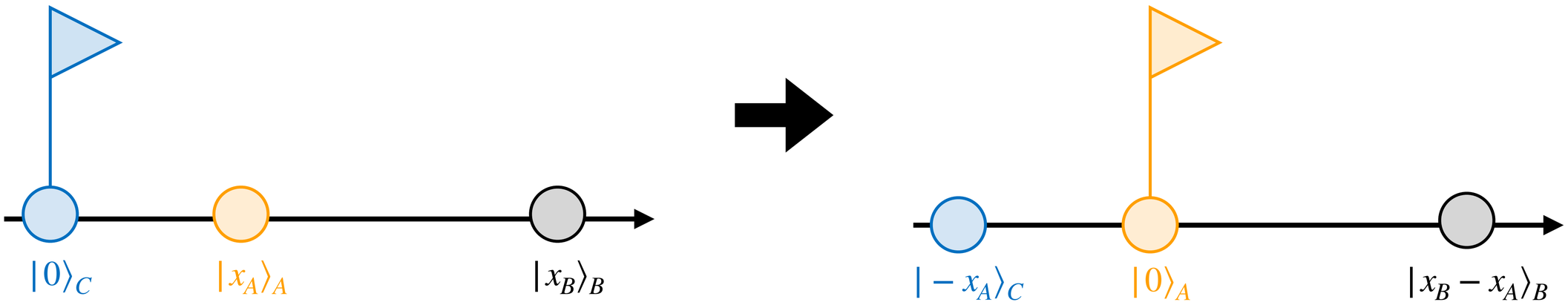
Reference Frames & Symmetries

Formalism



Quantum Reference Frames

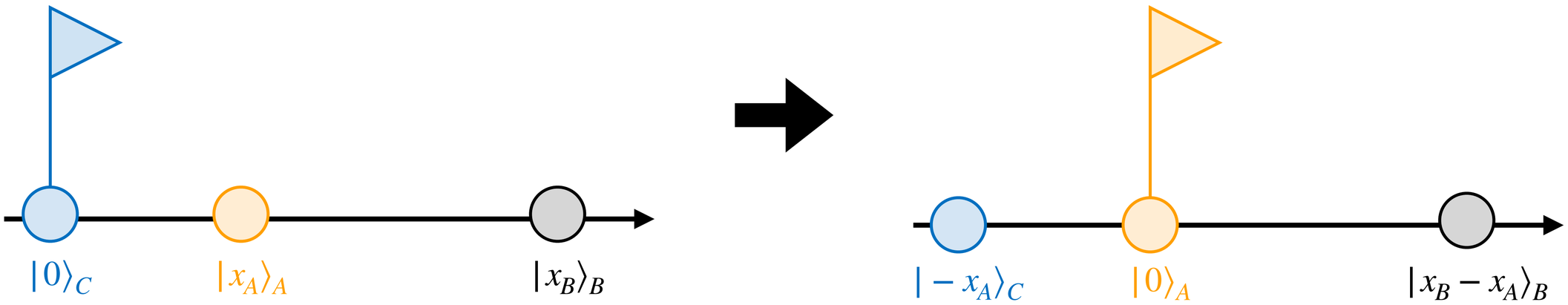
Formalism



Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

Formalism



$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

$$\in \mathscr{H}^{(C)}_{ABC} = \mathscr{H}^{(C)}_{A} \otimes \mathscr{H}^{(C)}_{B} \otimes \mathscr{H}^{(C)}_{C}$$

$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A |-x_A\rangle_C |x_B - x_A\rangle_B$$

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Quantum Reference Frames

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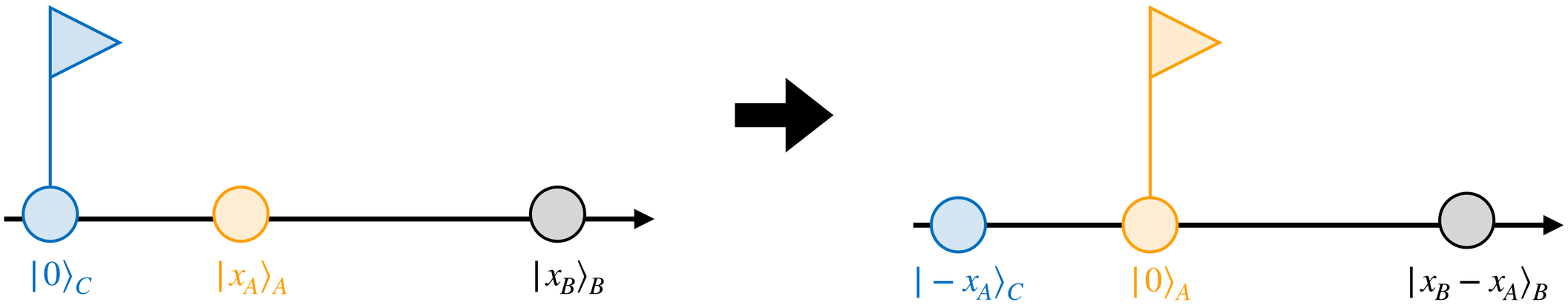

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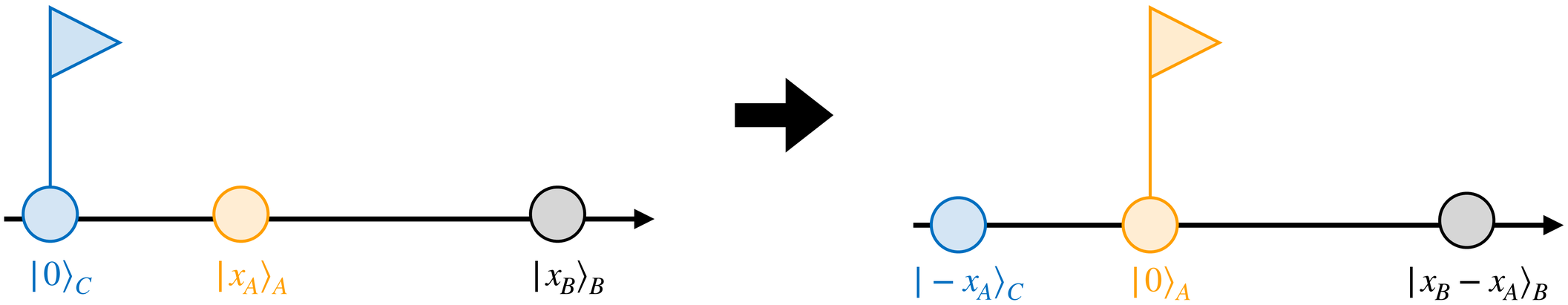
$$= \mathscr{P}_{CA} |0\rangle_C |x_A\rangle_A \boxed{e^{\frac{i}{\hbar} x_A \hat{p}_B} |x_B\rangle_B}$$

Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

Formalism



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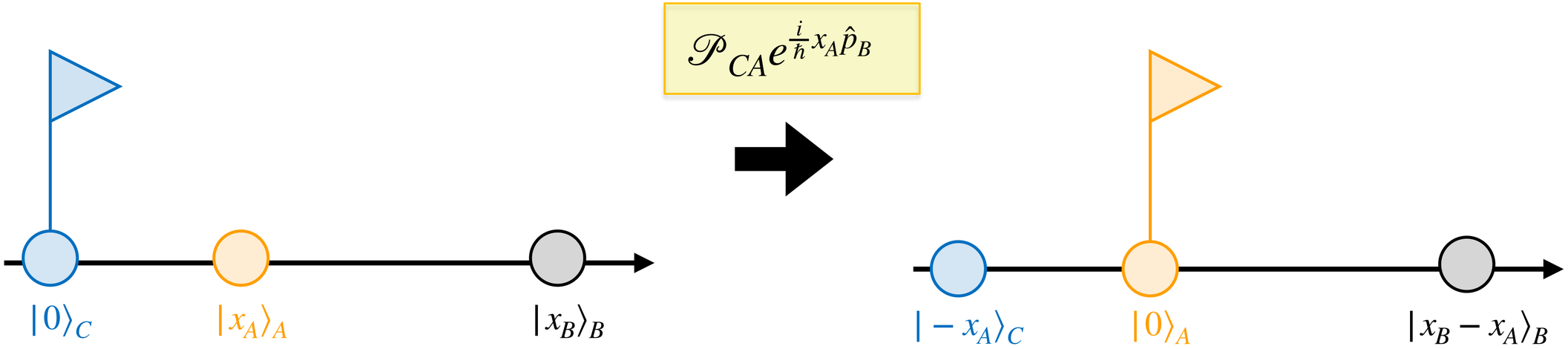
$$\boxed{\mathscr{P}_{CA} = SWAP_{CA} \circ \int |-x_A\rangle\langle x_A|_A \, dx_A}$$

Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

Formalism



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Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

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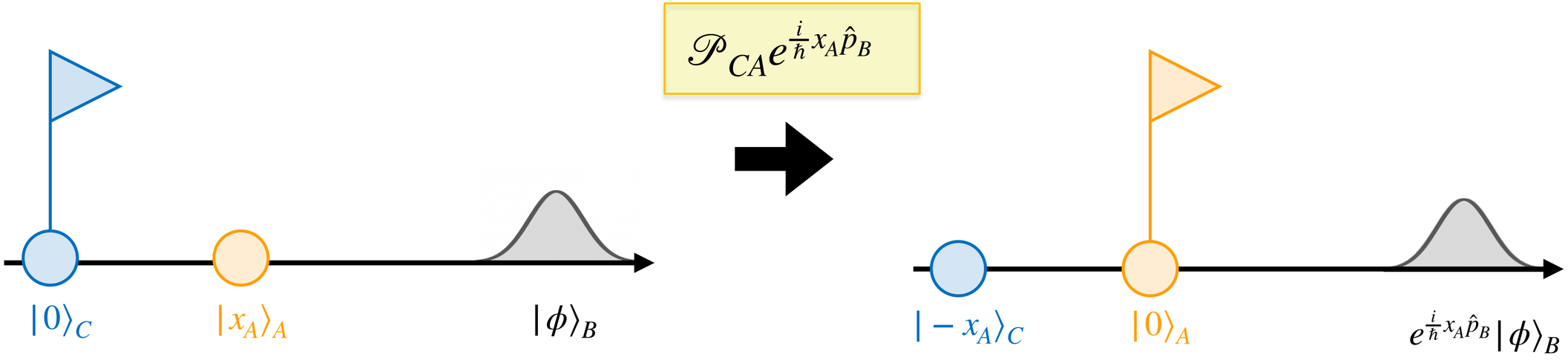

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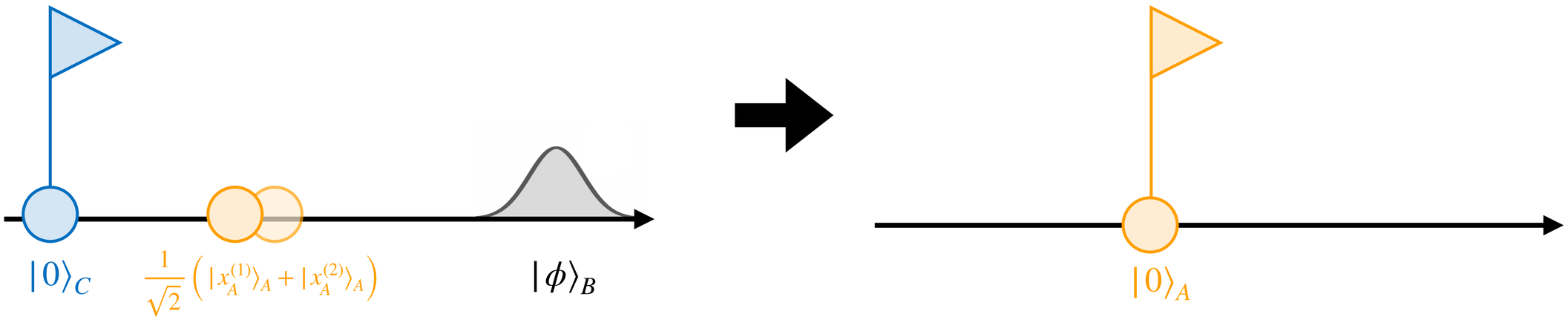
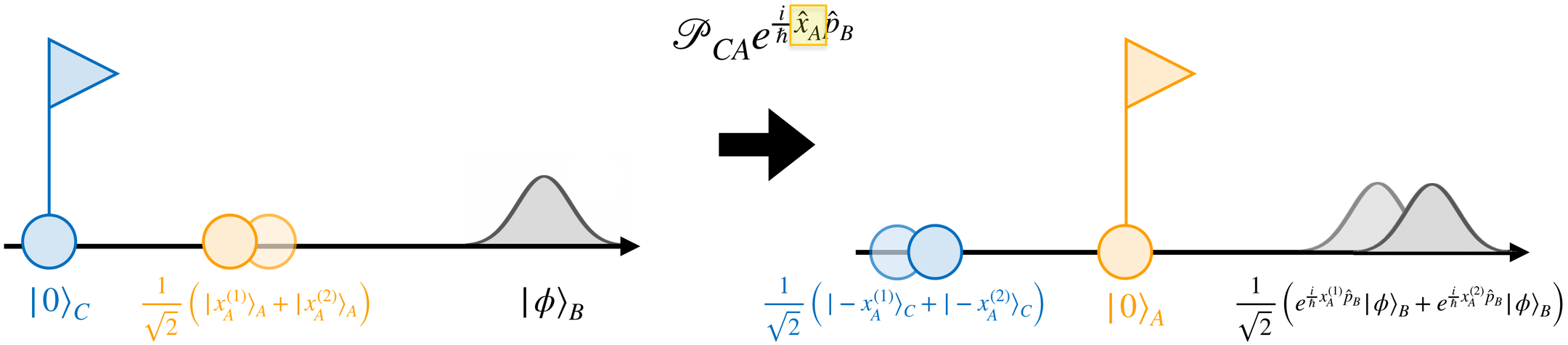


$$|\psi\rangle_{ABC}^{(C)} = |0\rangle_C \frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

Formalism



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$$= \mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}|0\rangle_C \frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

Quantum controlled translation

$$\mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

Giacomini, Castro-Ruiz, and Brukner (2019)

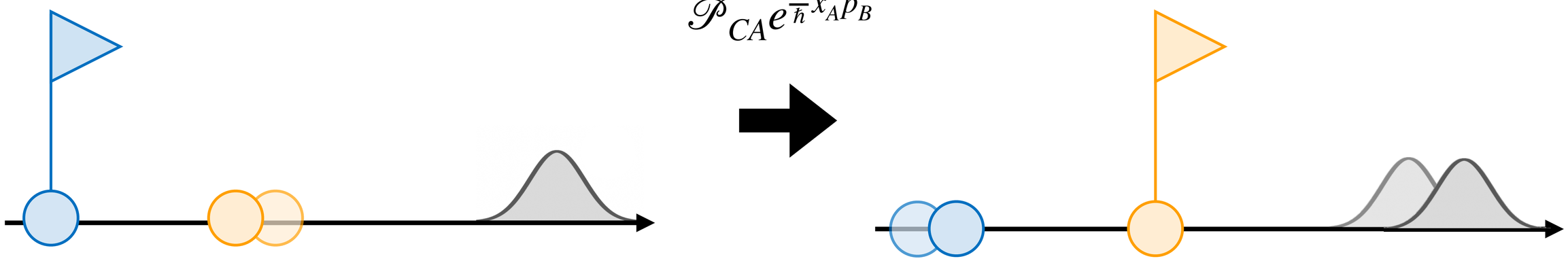
Quantum Reference Frames & Quantum Symmetries

Quantum Physics

- ✦ Generalisation to **quantum reference frames**.
- ✦ Example: invariance under **superpositions of translations**.

frame dependence of superposition & entanglement

$$\mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$



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Quantum Reference Frames & Quantum Symmetries

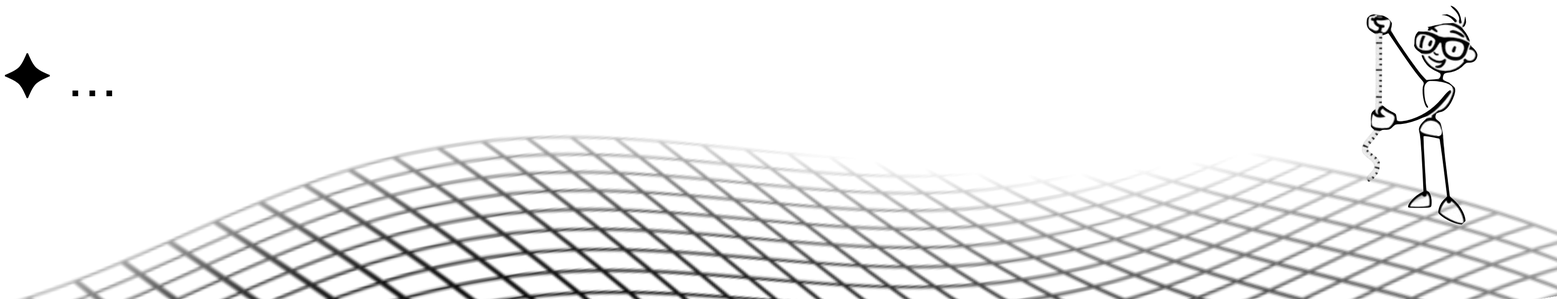
The General Idea

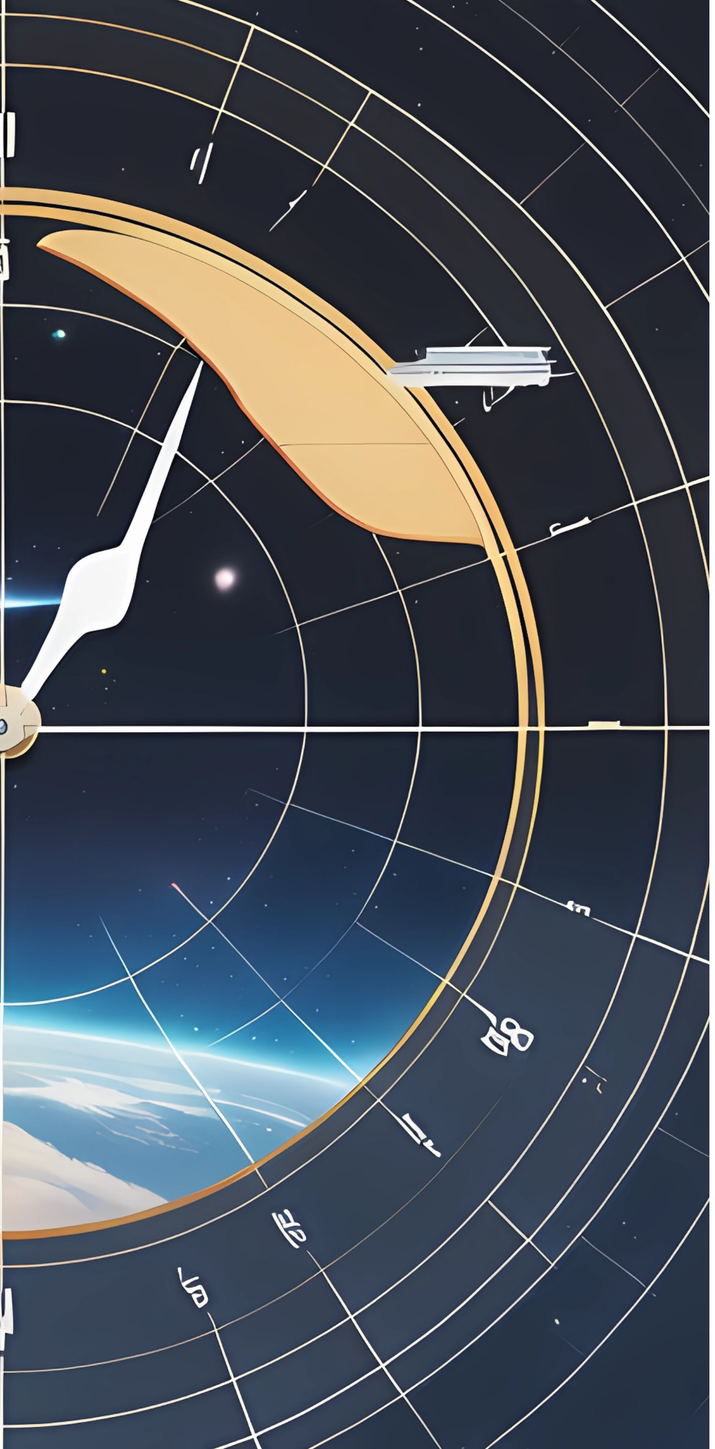
Symmetries of *known* physical theories + **Linearity of quantum theory** → **Quantum Symmetries**

Invariance under quantum-controlled transformations.

Examples

- ✦ Translations, Galilei group
- ✦ Spin rotations
- ✦ Conformal Transformations
- ✦ Lorentz boosts
- ✦ Asymptotic symmetries
- ✦ …





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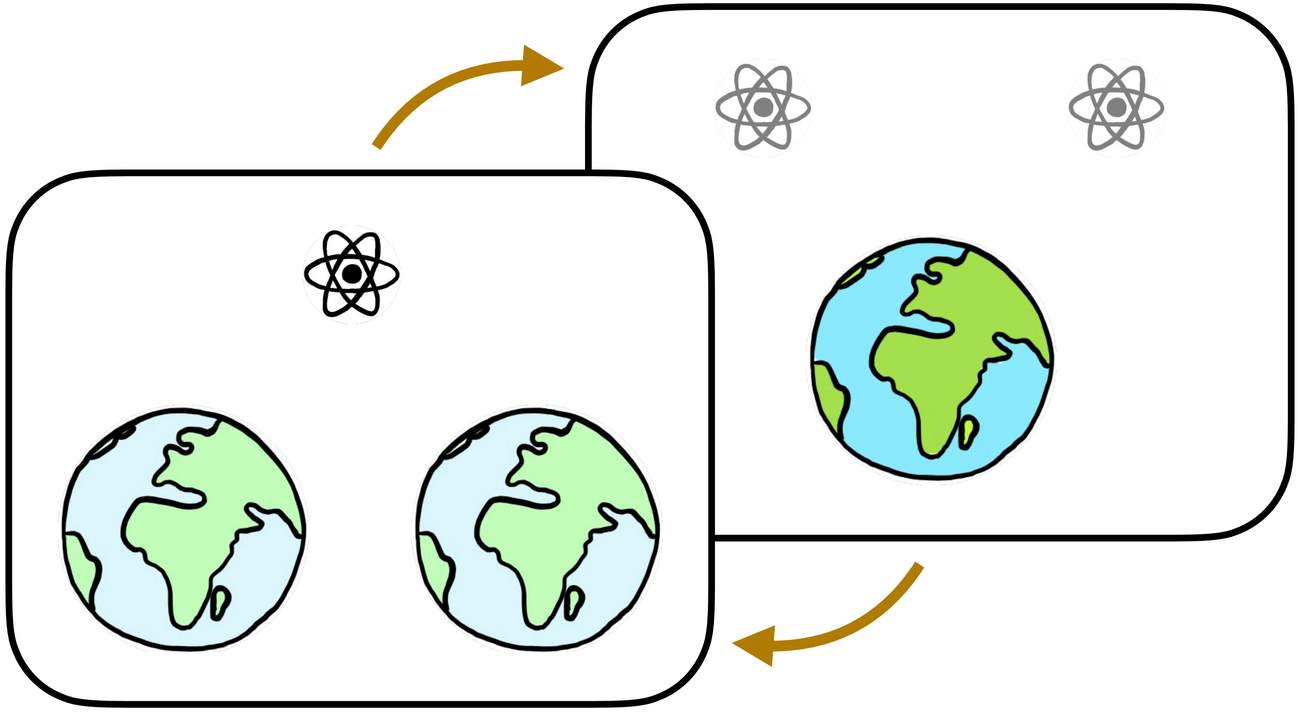
Quantum Reference Frames & Gravity

Massive Objects in Superposition



Goal: Describe motion of test particle in presence of a gravitational source in superposition.

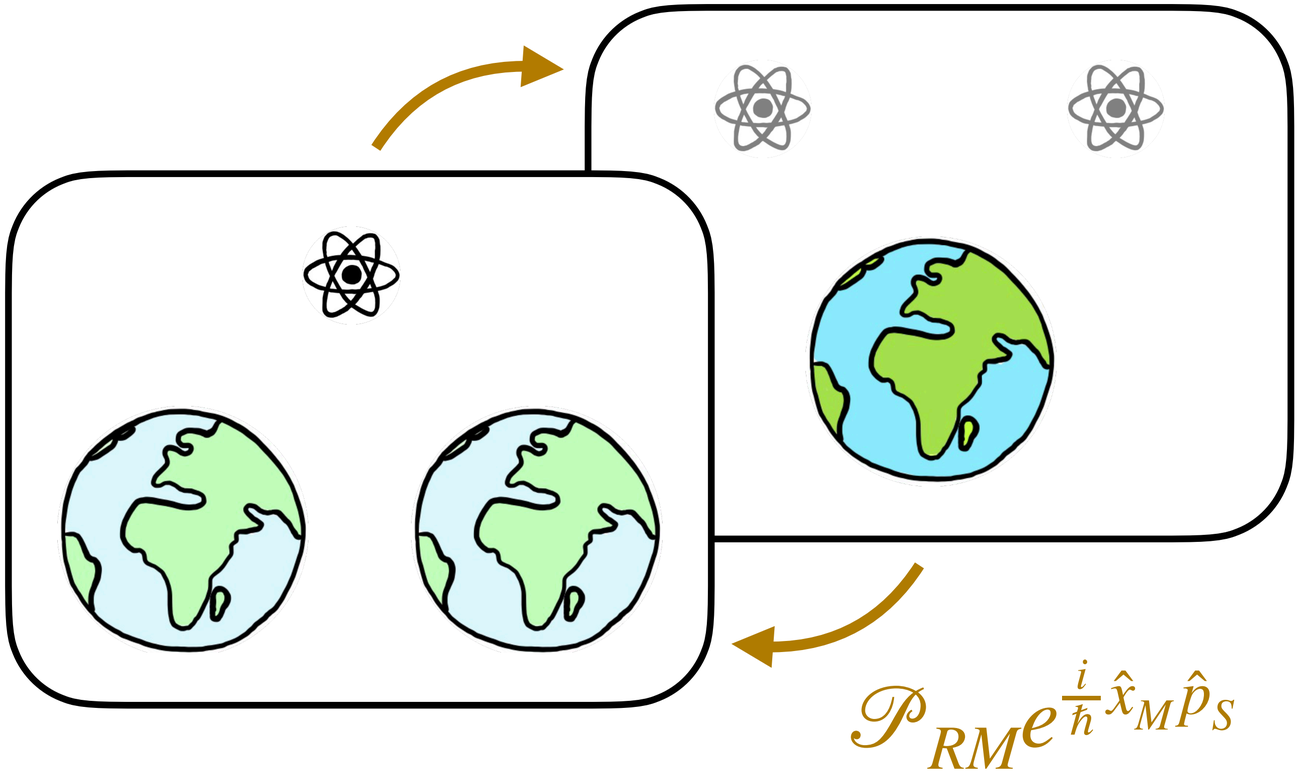
de la Hamette, VK, Castro-Ruiz, and Brukner (2023)

Quantum Reference Frames & Gravity

Massive Objects in Superposition

Strategy

✦ Change into QRF in which the gravitational source is definite.



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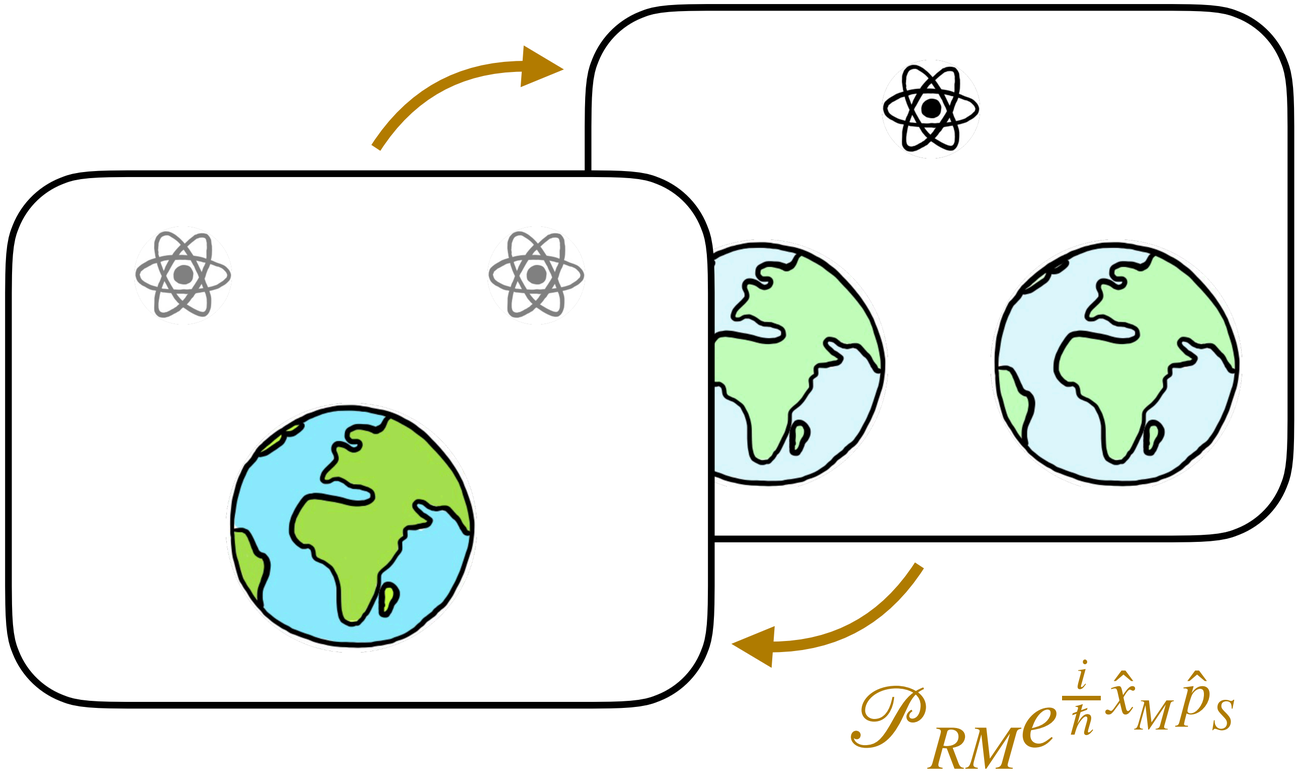


Quantum Reference Frames & Gravity

Massive Objects in Superposition

Strategy

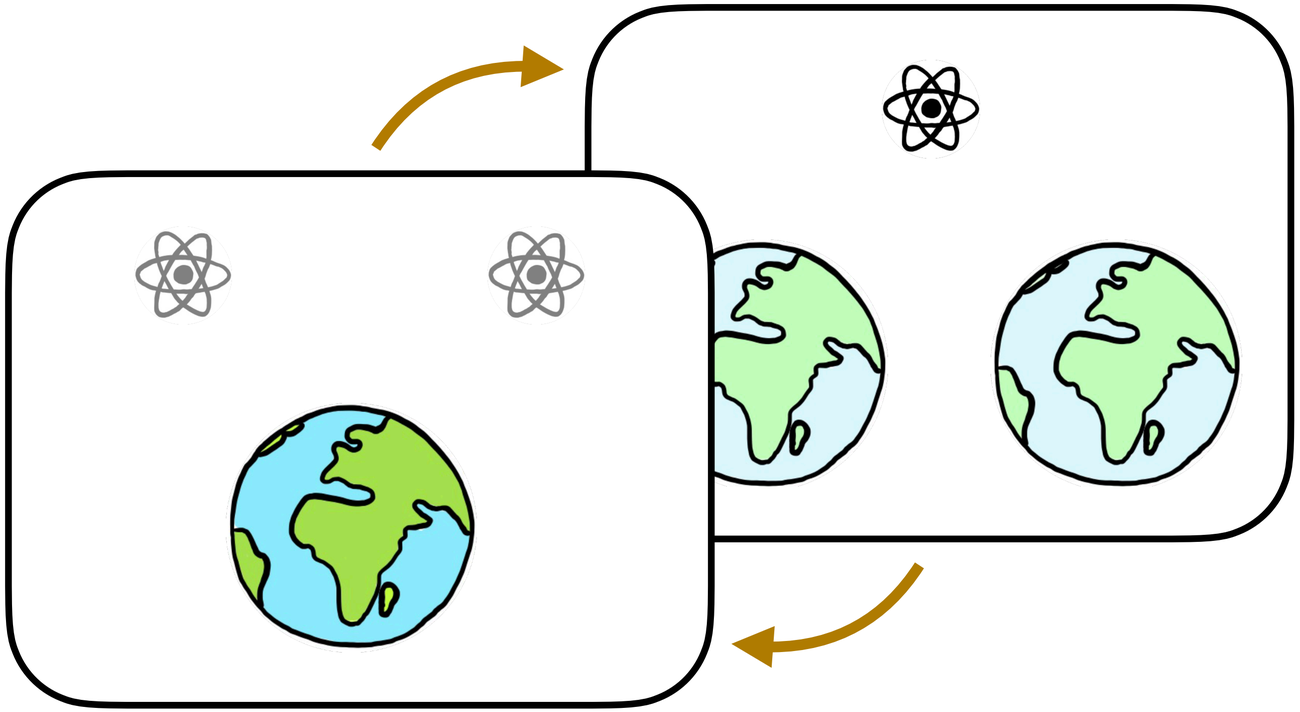
- ✦ Change into QRF in which the gravitational source is definite.
- ✦ Solve problem in the new reference frame.

geodesic motion

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

quantum phase

$$\Phi^{(i)} = \int_{A^{(i)}}^{B^{(i)}} m_S\sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$



Quantum Reference Frames & Gravity

Massive Objects in Superposition

Strategy

- ✦ Change into QRF in which the gravitational source is definite.
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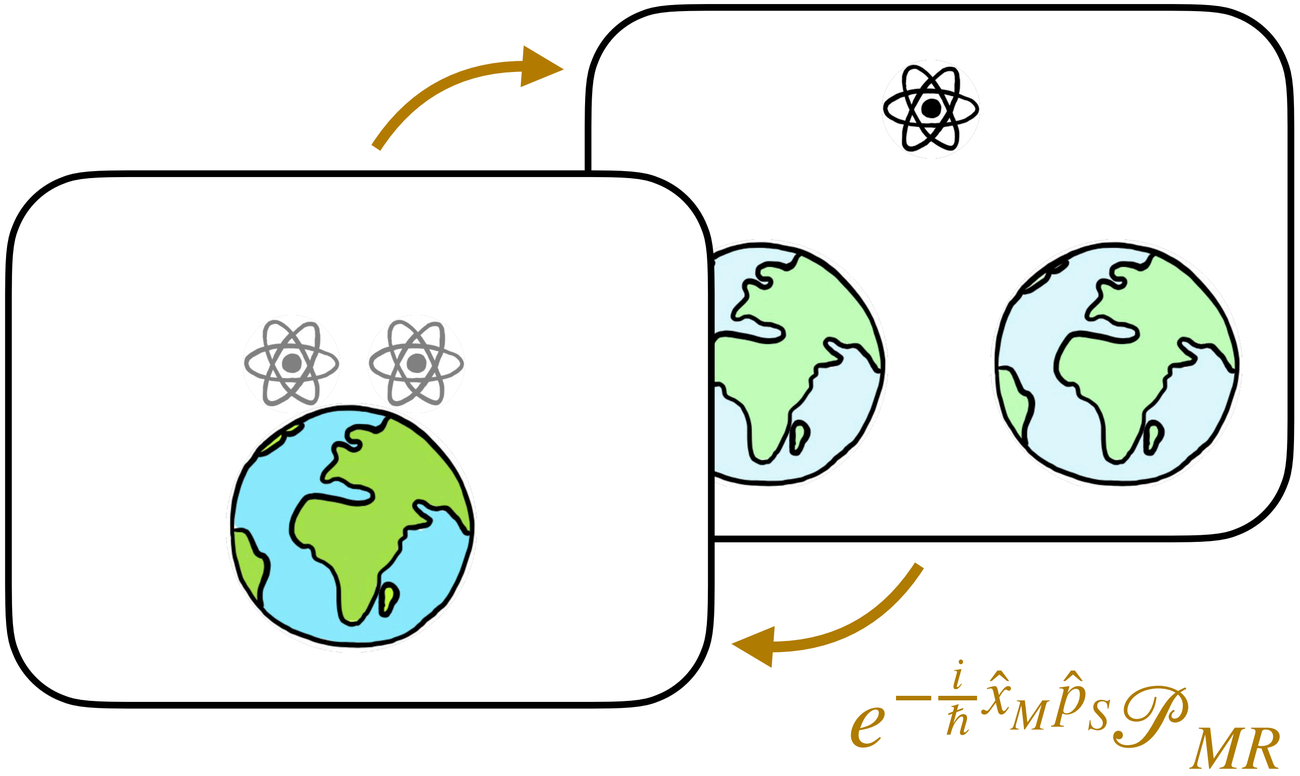
- ✦ Transform back to infer the dynamics *assuming that the change of QRF is a symmetry of the equations of motion*.



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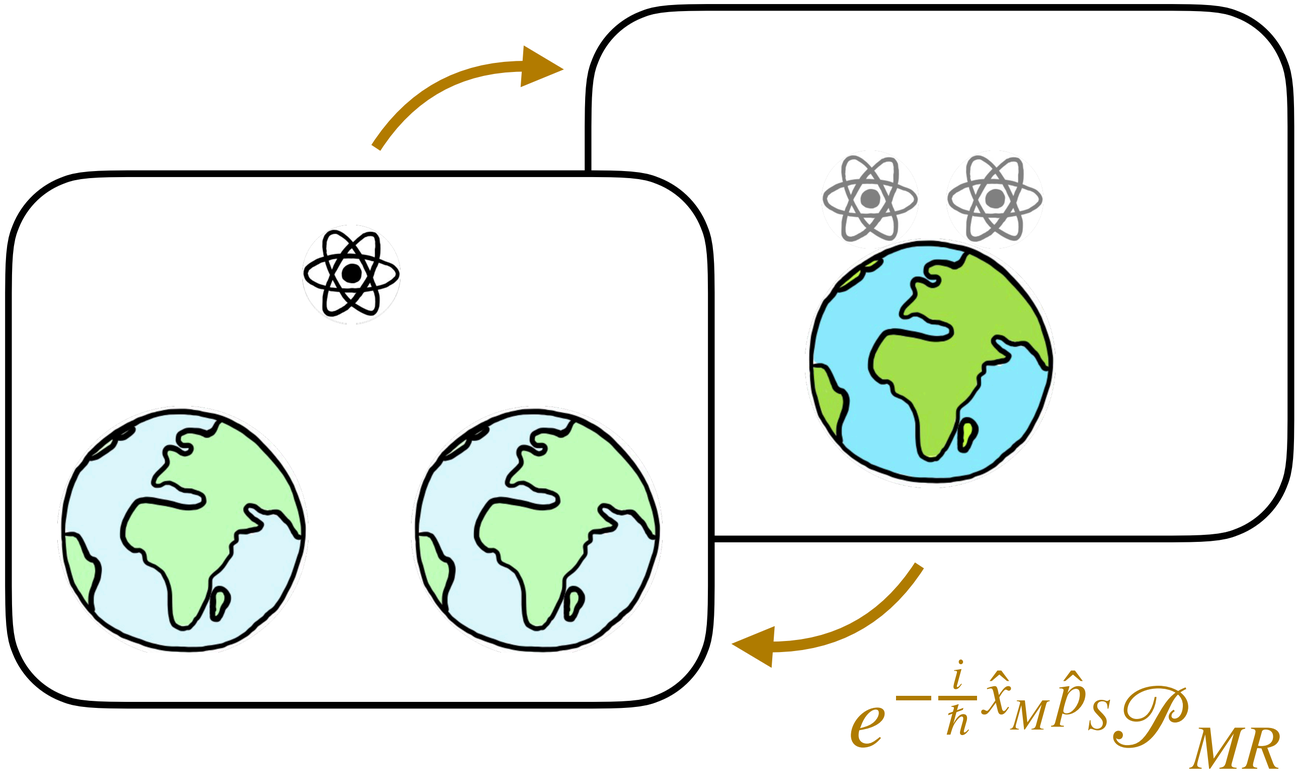


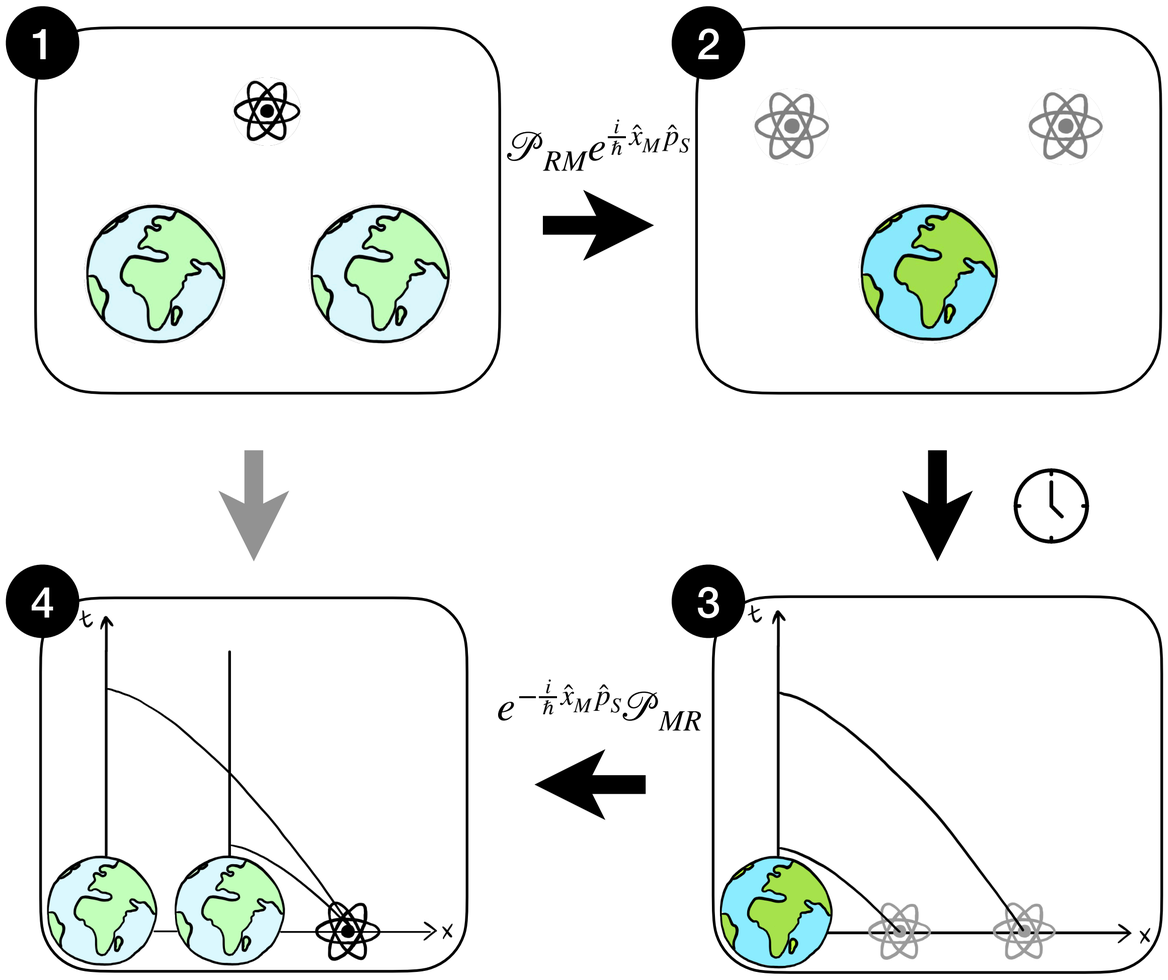
"extended symmetry principle"

Quantum Reference Frames & Gravity

Summary

Implications

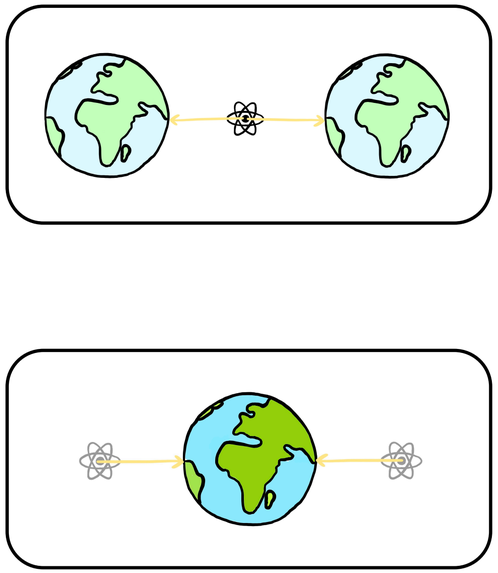
- ✦ Concrete predictions while staying **agnostic** about the quantum nature of the gravitational field.
- ✦ Coherence check for the **quantum nature of the gravitational field** sourced by a massive object in superposition.



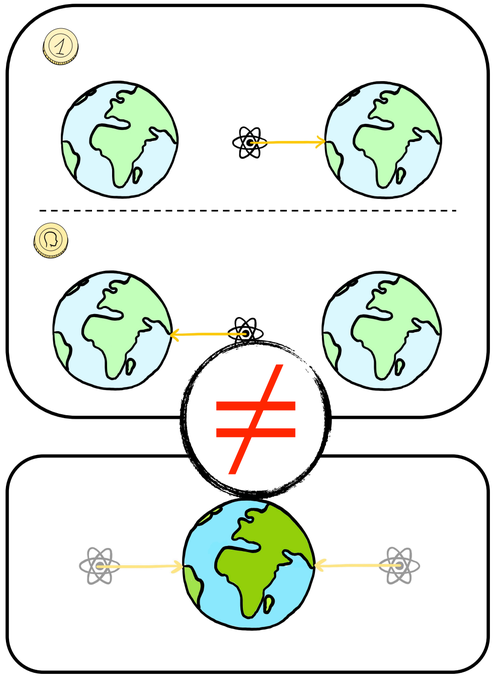
de la Hamette, VK, Castro-Ruiz, and Brukner (2023)

Quantum Reference Frames & Gravity

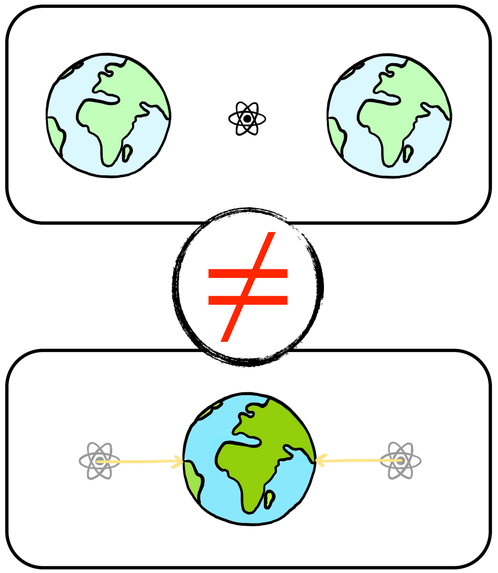
Comparison between Approaches



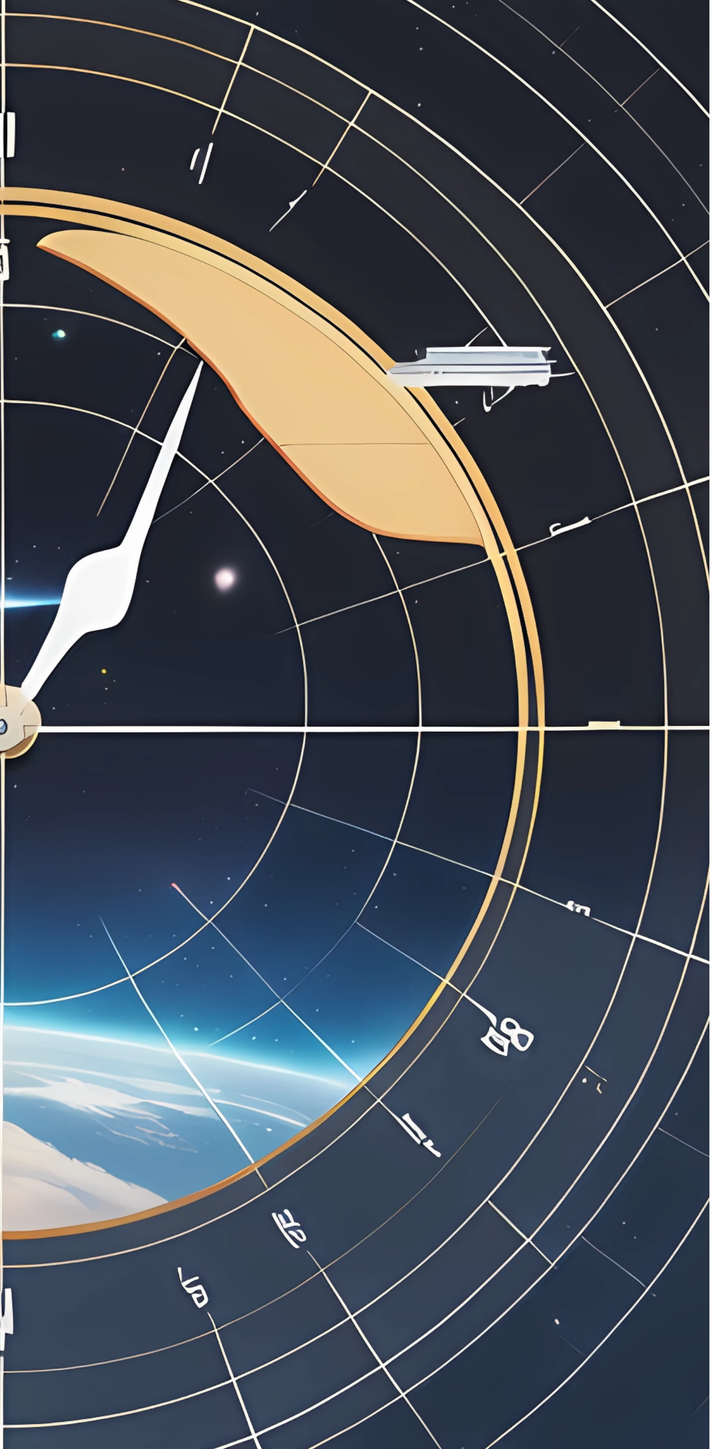
Extended Symmetry Principle



Collapse Models



Semi-Classical Gravity



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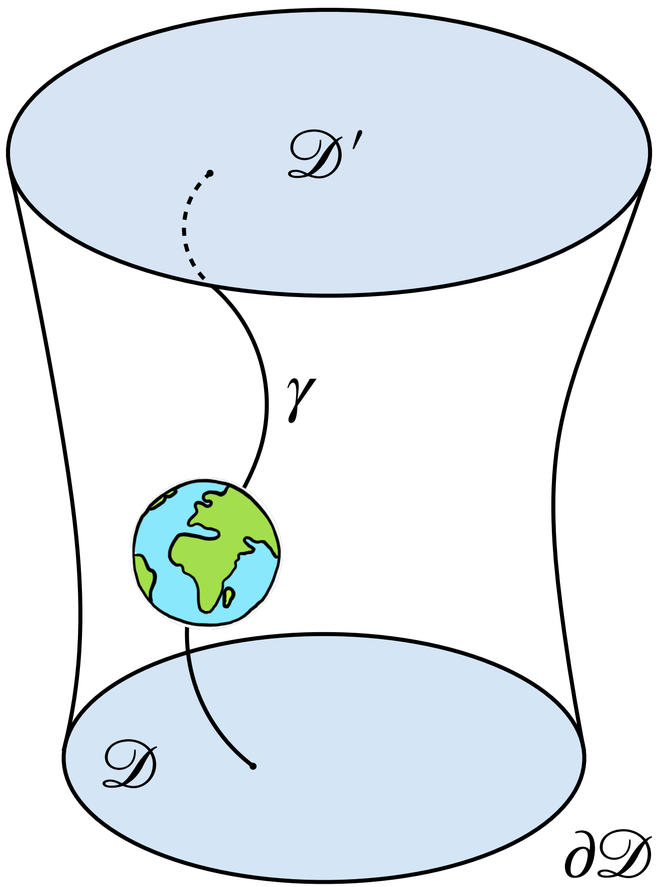
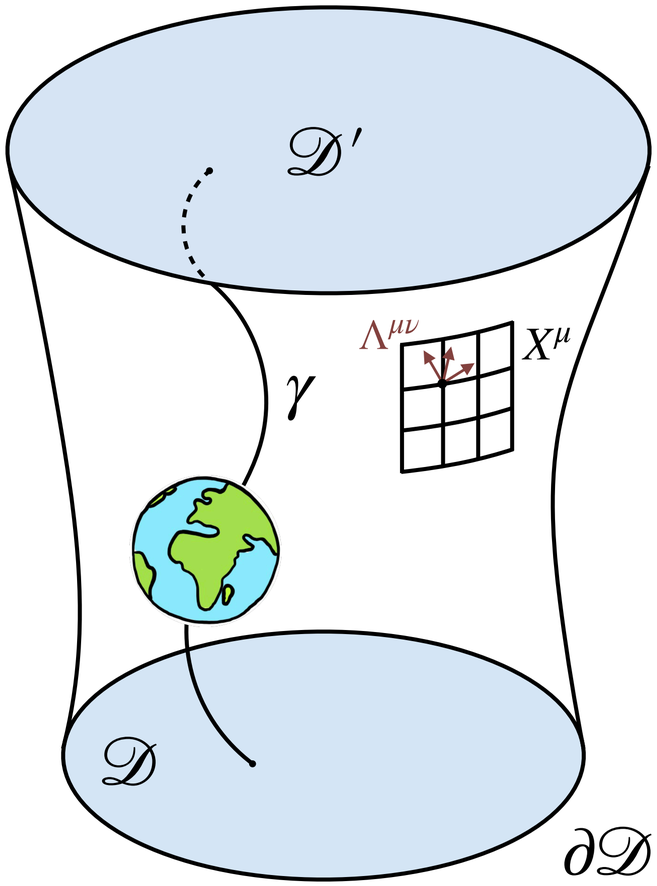
Gravity + Quantum Reference Frames



Gravity + Quantum Reference Frames

QRFs at the Boundary of Spacetime



Question: Can we derive quantum reference frames from within a general relativistic setup?

- QRFs formulated in the field-theoretic language of general relativity.
- Generalised QRF transformations due to larger symmetry group.

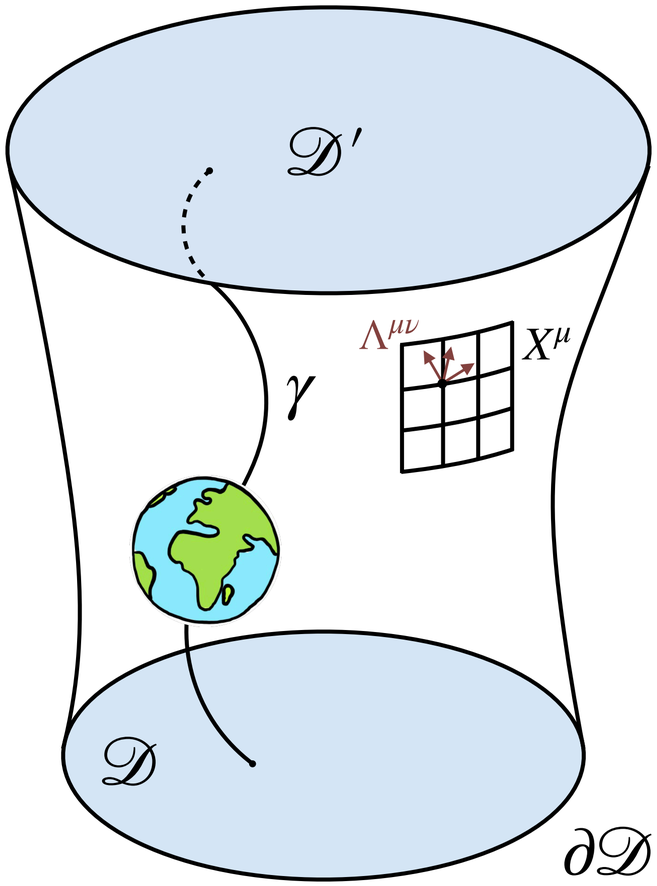
First step: Linearised general relativity.

→ QRFs as edge modes at the boundary.

arXiv:2302.11629

Gravity + Quantum Reference Frames

QRFs at the Boundary of Spacetime



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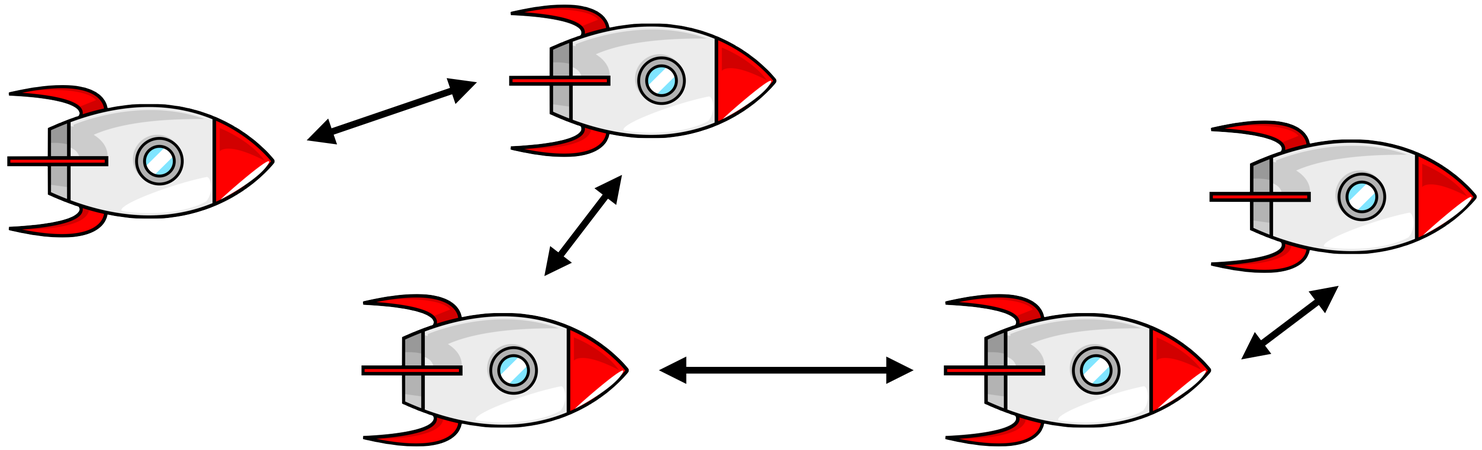
Cf. Carrozza, Eccles, Höhn (2022)

Edge Modes

in an N-Particle System



Rovelli, “Why Gauge?” (2014)

Edge Modes

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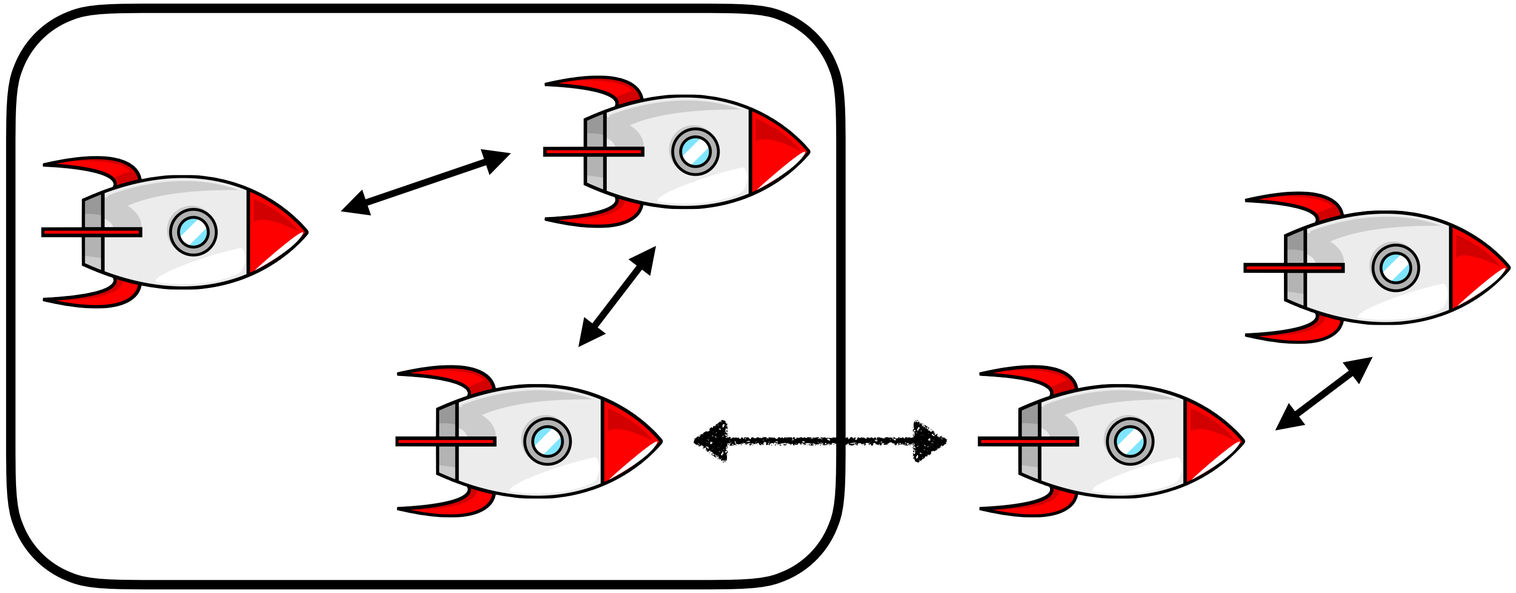

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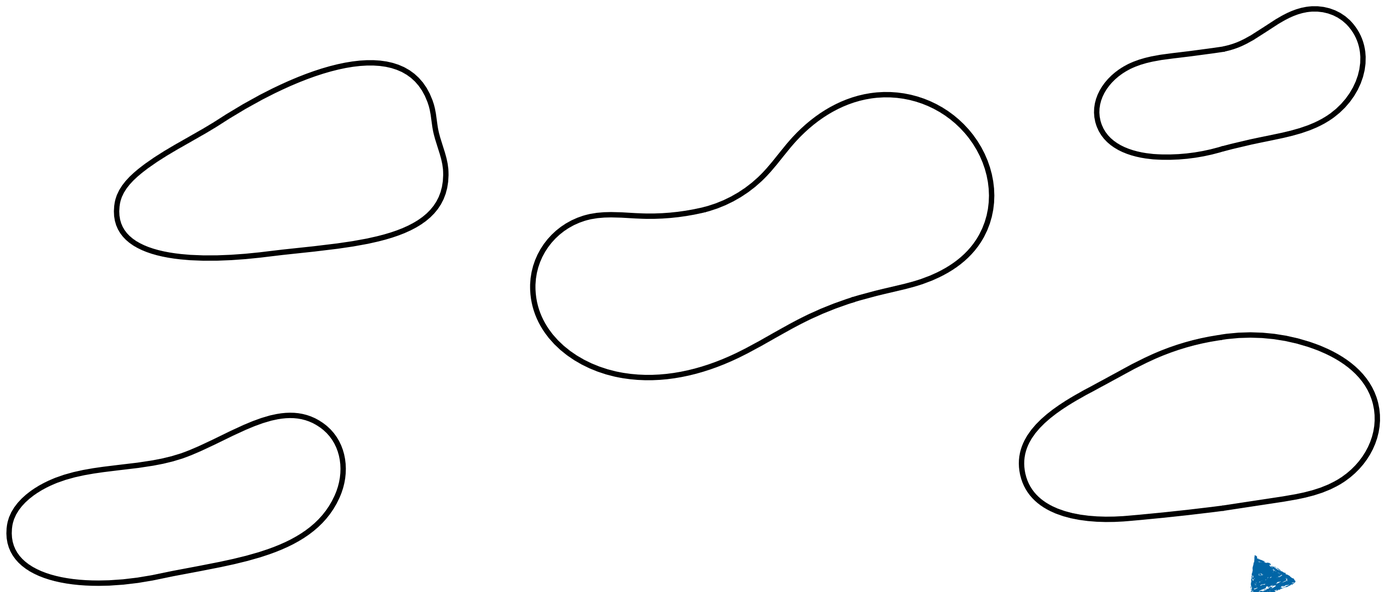
Edge Modes
in Electrodynamics



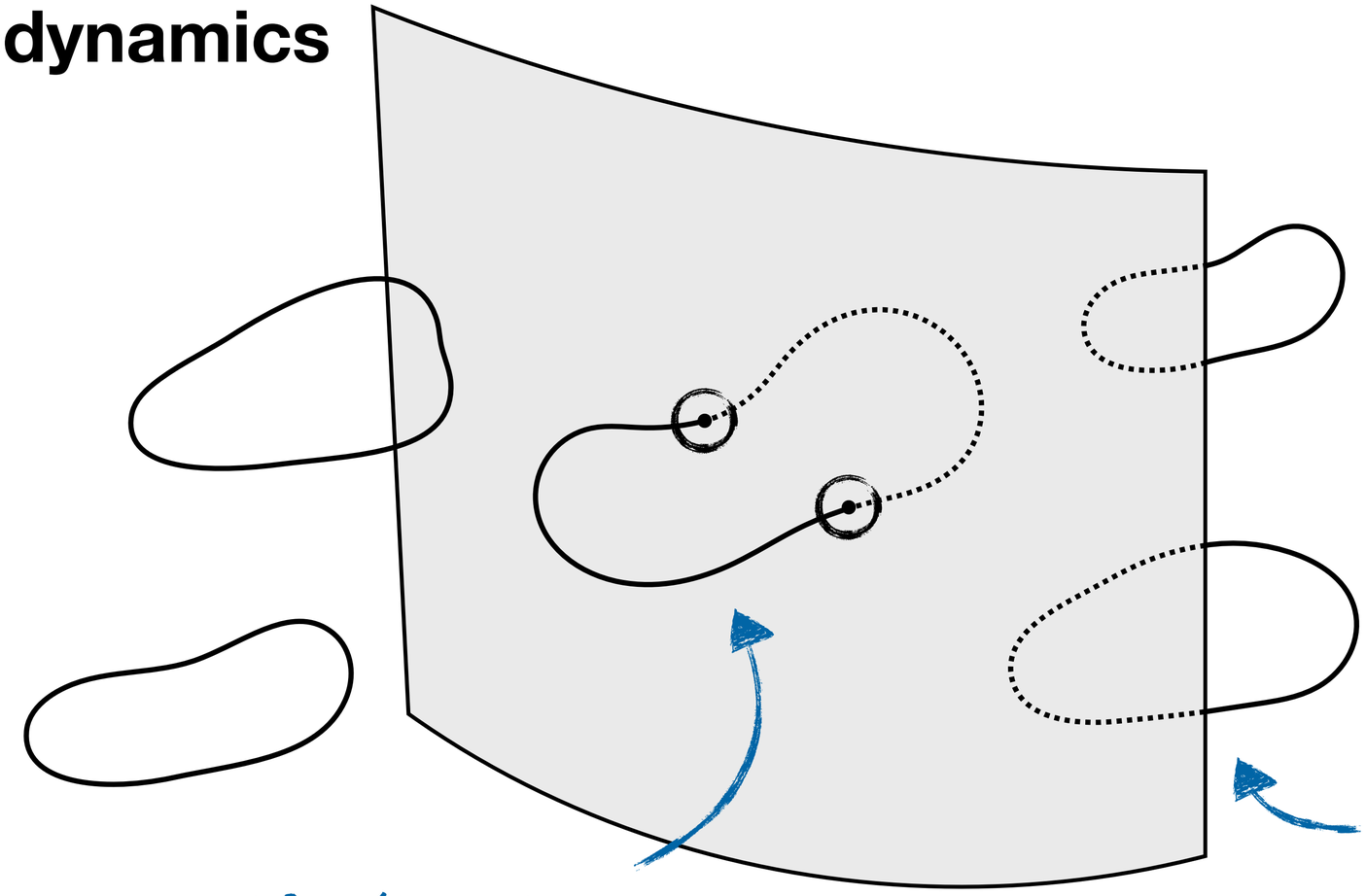
Wilson loop

$$h_\alpha[A] = e^{-iq\oint_\alpha A}$$

e.g. Donnelly and Freidel (2016)

Edge Modes

in Electrodynamics



half-Wilson loop:

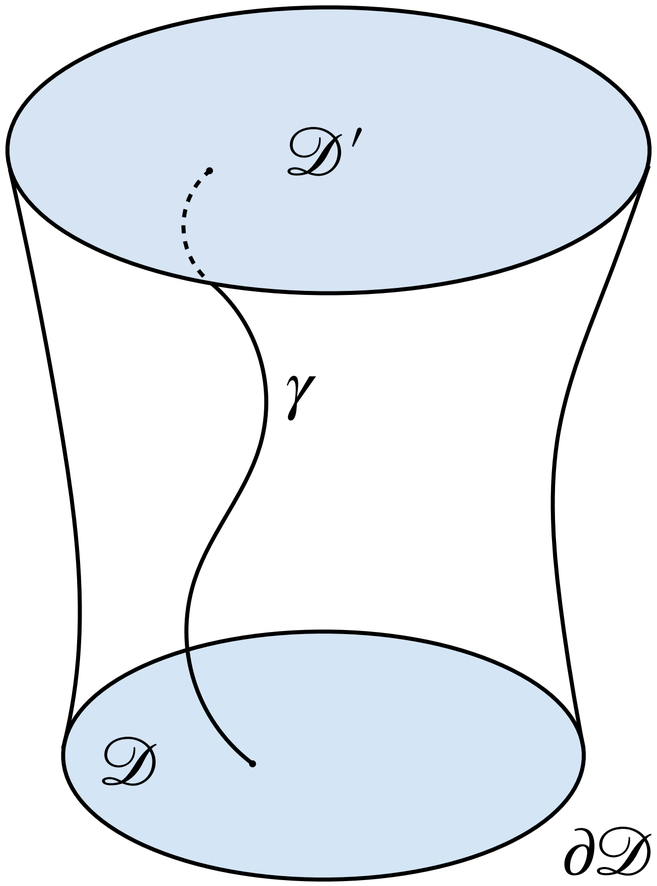
$$h_\gamma[A] \to h_\gamma[A] e^{-iq \sum_p \epsilon_p \lambda(p)}$$

Wilson loop

$$h_\alpha[A] = e^{-iq \oint_\alpha A}$$

e.g. Donnelly and Freidel (2016)

Edge Modes in Linearised Gravity



Classical Setup: Linearised gravity in subregion of spacetime with N point particles.

$$e^\alpha = \Lambda^\alpha{}_\mu \left(dX^\mu + f^\mu\right) \qquad A^\alpha{}_\beta = \Lambda^\alpha{}_\mu d\Lambda_\beta{}^\mu + \Lambda^\alpha{}_\mu \Delta^\mu{}_\nu \Lambda_\beta{}^\mu$$

where

$$X^\mu : \mathscr{M} \to \mathbb{R}^4 \qquad \Lambda^\mu{}_\nu : \mathscr{M} \to SO(1,3)$$

Governed by **Hilbert-Palatini action** (+ matter):

$$\mathcal{S}\left[A, e, \{\gamma_i, N_i, p_i\}_{i=1}^N\right] = \frac{1}{16\pi G}\int_M *(e_\alpha \wedge e_\beta) \wedge F^{\alpha\beta}[A] + \mathcal{S}_{\text{matter}}$$

Interlude: Covariant Phase Space

Provides manifestly covariant description of phase space:

$$\delta\mathscr{L} = (EOM)_I \delta\Phi^I + d\theta(\delta)$$

$$\boxed{\Omega_\Sigma = \int_\Sigma \mathbf{d}\theta}$$

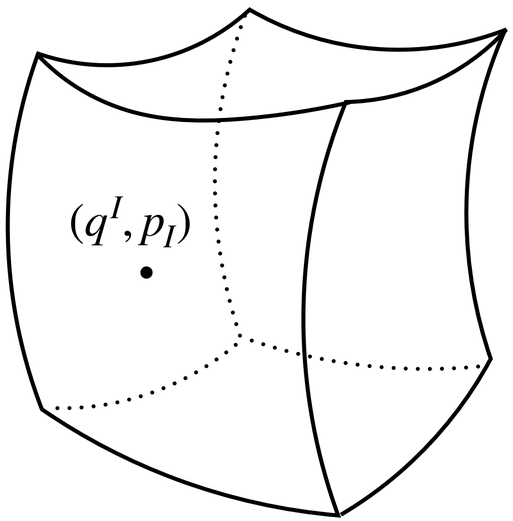
N Point Particles

$$df = \Omega(X_f, \cdot\,)$$

$$\Omega = \sum_{I=1}^{N} dp_I \wedge dq^I \quad \longrightarrow \quad \{f, g\} \equiv \Omega(X_f, X_g)$$

symplectic form

Poisson bracket



e.g. Harlow and Wu (2020)

Interlude: Covariant Phase Space

Provides manifestly covariant description of phase space:

$$\delta\mathscr{L} = (EOM)_I \delta\Phi^I + d\theta(\delta)$$

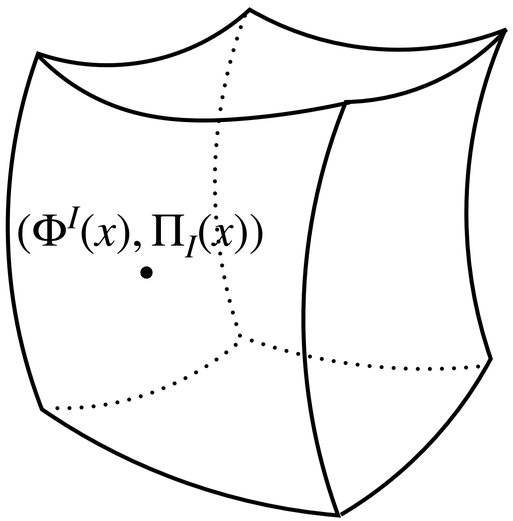
$$\boxed{\Omega_\Sigma = \int_\Sigma \mathbf{d}\theta}$$

Field-Space

$$\Omega_\Sigma = \int_\Sigma \mathbf{d}\Pi_I \mathbf{d}\Phi^I$$

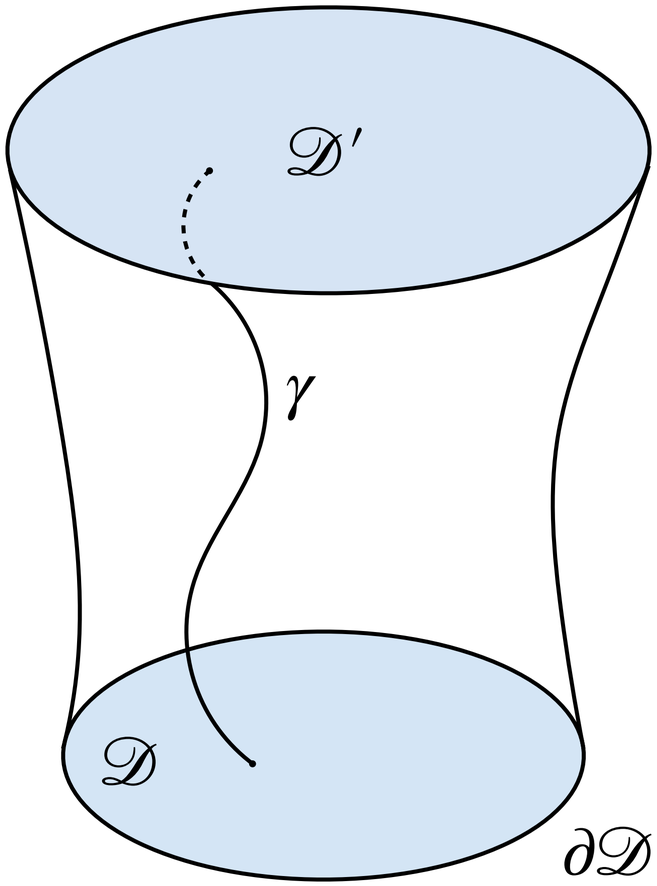
field-space
exterior derivative

$$\mathbf{d}\Phi^I(\delta) = \delta\Phi^I$$



e.g. Harlow and Wu (2020)

Edge Modes in Linearised Gravity



Variation of action → **symplectic form**:

$$\boxed{\Omega_{\mathscr{D}} = \Omega_{\mathscr{D}}^{matter} + \Omega_{\mathscr{D}}^{rad} + \Omega_{\partial\mathscr{D}}}$$

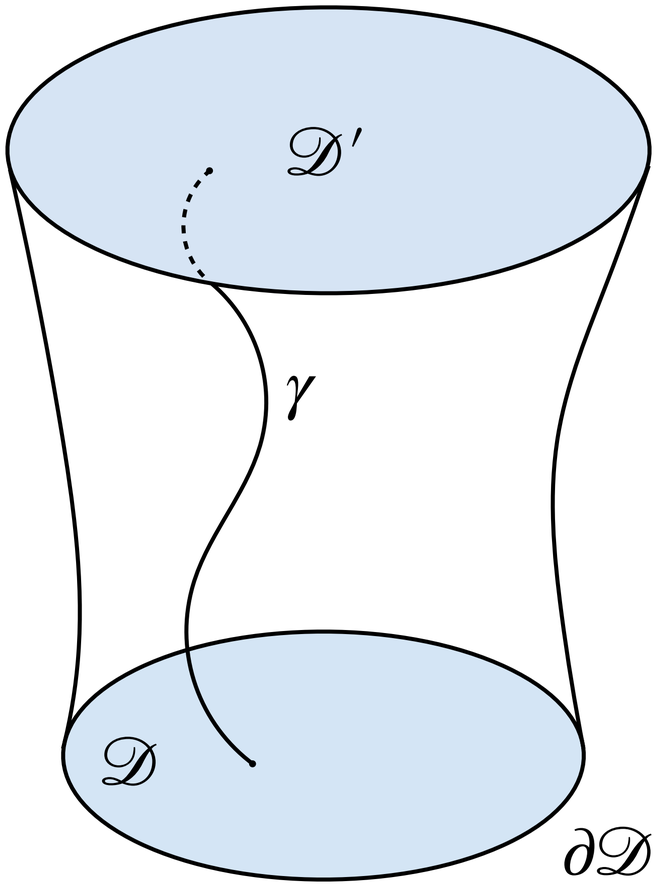
$$\mathbf{D}q_i^\mu = \mathbf{d}q_i^\mu + \mathbf{d}X^\mu\,|_{\gamma_i}$$
$$= \mathbf{d}(X^\mu \circ \gamma_i)$$

where

$$\Omega_{\mathscr{D}}^{matter} = \sum_{i=1}^{N} \mathbf{d}p_\mu^i \boxed{\left(\mathbf{d}q_i^\mu + \mathbf{d}X^\mu\Big|_{\gamma_i \cap \mathscr{D}}\right)} \equiv \sum_{i=1}^{N} \mathbf{d}p_\mu^i \mathbf{D}q_i^\mu$$

relative or "dressed" position

Edge Modes in Linearised Gravity



Variation of action → **symplectic form**:

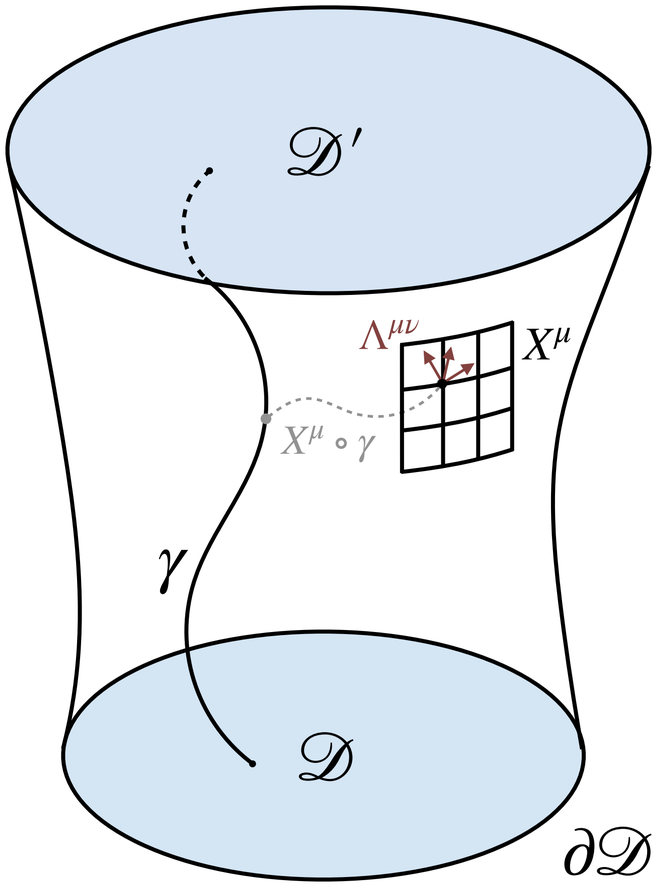
$$\boxed{\Omega_{\mathscr{D}} = \Omega_{\mathscr{D}}^{matter} + \Omega_{\mathscr{D}}^{rad} + \Omega_{\partial\mathscr{D}}}$$

$$\mathbf{D}^{(n)} f^{\mu} = \mathbf{d}^{(n)} f^{\mu} - \mathscr{L}_{\mathbf{X}}\, {}^{(n)} f^{\mu}$$

where

$$\Omega_{\mathscr{D}}^{rad} = \int_{\mathscr{D}} * \left(dX_{[\mu} \wedge \boxed{\mathbf{D}^{(1)} f_{\nu]}} \right) \wedge \boxed{\mathbf{D}^{(1)} \Delta^{\mu\nu}}$$

Edge Modes in Linearised Gravity



Variation of action → **symplectic form**:

$$\boxed{\Omega_{\mathscr{D}} = \Omega_{\mathscr{D}}^{matter} + \Omega_{\mathscr{D}}^{rad} + \Omega_{\partial\mathscr{D}}}$$

where

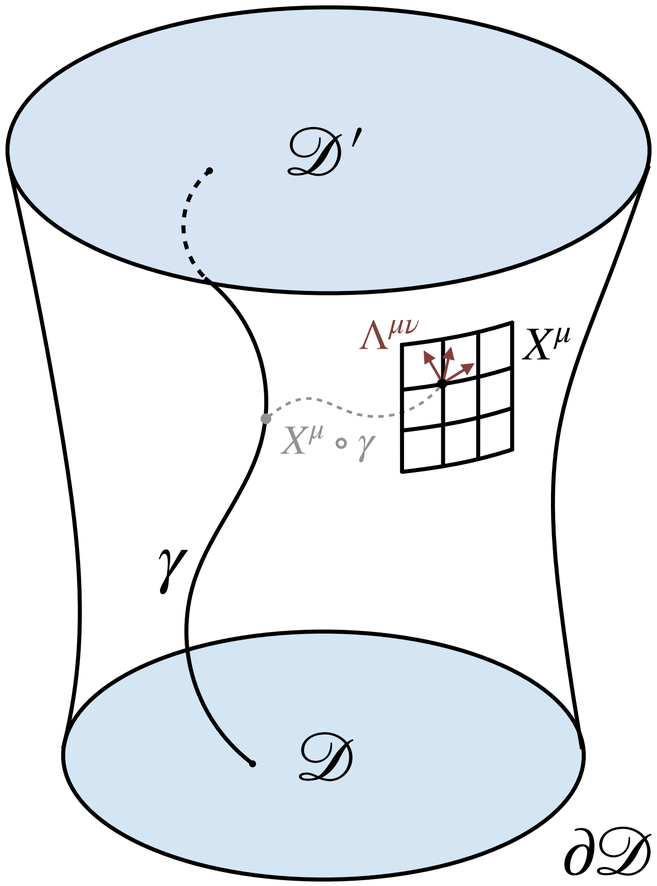
$$\Omega_{\partial\mathscr{D}} = \oint_{\partial\mathscr{D}} \mathbf{d}P_\mu \boxed{\mathbf{d}X^\mu} - \frac{1}{2}\oint_{\partial\mathscr{D}} \left[\mathbf{d}S^\mu{}_\nu \boxed{\mathbf{m}^\nu{}_\mu} + \frac{1}{2}S^\mu{}_\nu\,[\mathbf{m},\mathbf{m}]^\nu{}_\mu\right]$$

coordinate fields

Lorentz frames

$\mathbf{m} = (d\Lambda^{-1})\Lambda$

Edge Modes in Linearised Gravity



$$\Omega_{\partial\mathscr{D}} = \oint_{\partial\mathscr{D}} \mathbf{d}P_\mu \mathbf{d}X^\mu - \frac{1}{2}\oint_{\partial\mathscr{D}} \left[\mathbf{d}S^\mu{}_\nu \mathbf{m}^\nu{}_\mu + \frac{1}{2}S^\mu{}_\nu\,[\mathbf{m},\mathbf{m}]^\nu{}_\mu\right]$$

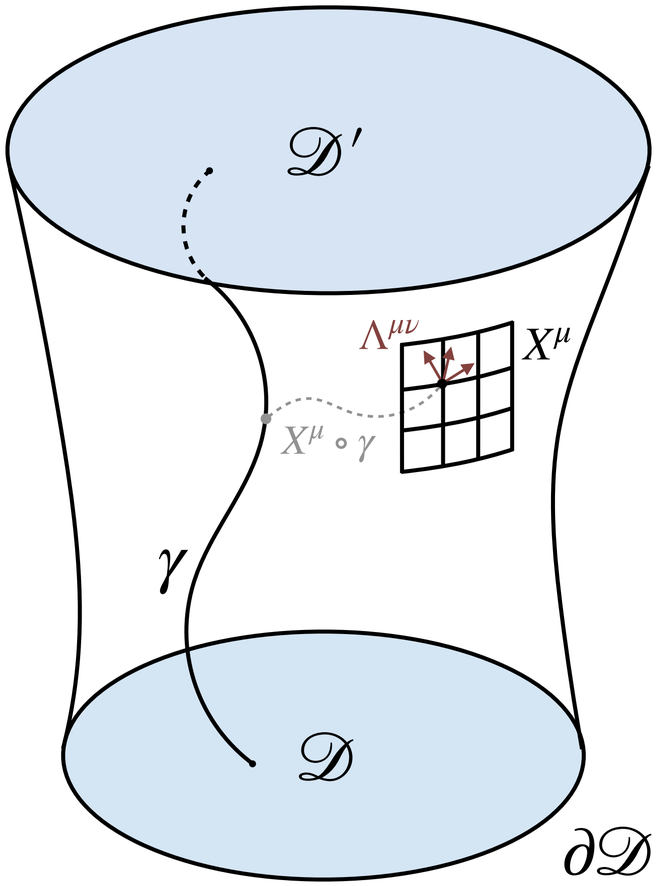
coordinate fields

Lorentz frames

$$\mathbf{m} = (d\Lambda^{-1})\Lambda$$

- Render $\Omega_{\mathscr{D}}$ **invariant** under diffeomorphisms.
- Provide reference for point particles' path through the **dressing** $X^\mu \circ \gamma$.
- Define the **location of the boundary**.

Edge Modes in Linearised Gravity

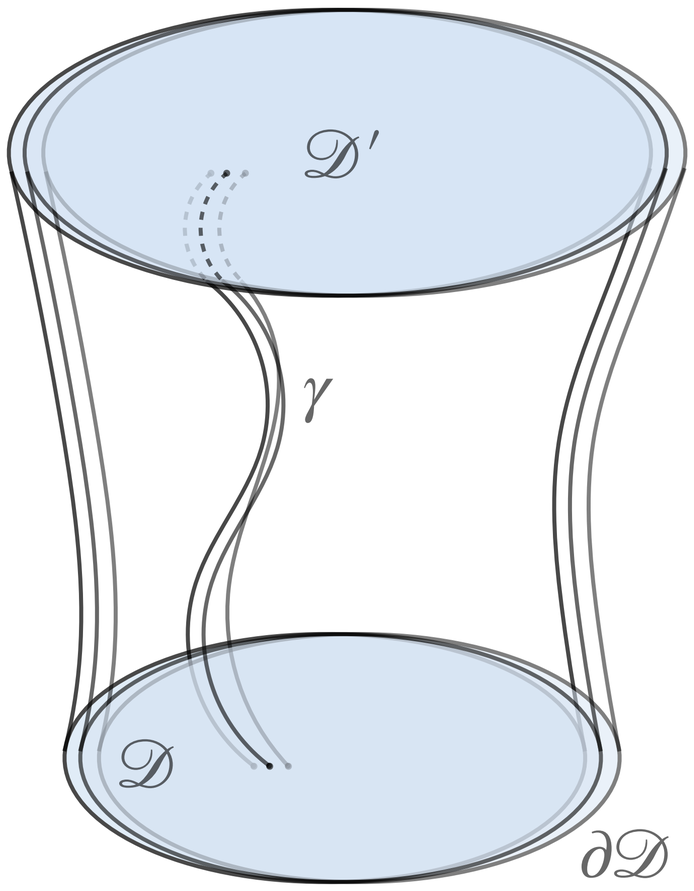


$$\Omega_{\partial\mathscr{D}} = \oint_{\partial\mathscr{D}} \mathbf{d}P_\mu \mathbf{d}X^\mu - \frac{1}{2}\oint_{\partial\mathscr{D}} \left[\mathbf{d}S^\mu{}_\nu \mathbf{m}^\nu{}_\mu + \frac{1}{2}S^\mu{}_\nu\,[\mathbf{m},\mathbf{m}]^\nu{}_\mu\right]$$

conjugate momentum: $P_\mu = \varphi^*_{\mathscr{C}}\left((*^{(2)}\Delta_{\mu\nu}) \wedge dX^\nu\right)$

- **Conserved quantity** whose integral agrees with ADM momentum at infinity.
- Definition → **constraint** relating bulk and boundary.

Implications for the Quantum Theory



- Separation of classical phase space → **partition of kinematical Hilbert space** as

$$\underbrace{\mathscr{H}^{matter}_{\mathscr{D}} \otimes \mathscr{H}^{rad}_{\mathscr{D}}}_{\mathscr{H}_{bulk}} \otimes \mathscr{H}^{boundary}_{\partial\mathscr{D}}$$

- Constraints relating bulk and boundary imply relational Schrödinger evolution at the boundary

$$\hat{P}_\mu \Psi_{phys}[X^\mu, \Lambda^\alpha_\mu] = -i\hbar \frac{\delta}{\delta X^\mu} \Psi_{phys}[X^\mu, \Lambda^\alpha_\mu] = \hat{H}_\mu \Psi_{phys}[X^\mu, \Lambda^\alpha_\mu]$$

- Obtain **quantum reference frames transformations** for asymptotic symmetries.

Quantum Reference Frames

for Asymptotic Symmetries

✦ Split coordinates into finite and divergent part: $X^\mu = \Omega^\mu{}_\nu \left(X_o^\nu + Q^\nu \right) + \mathcal{O}(\rho^{-1})$

fiducial coordinates

$$\rho \equiv \sqrt{X^\mu X_\mu} \to \infty$$

Quantum Reference Frames

for Asymptotic Symmetries

finite, angle-dependent translation

✦ Split coordinates into finite and divergent part: $X^\mu = \boxed{\Omega^\mu{}_\nu}\left(X_o^\nu + \boxed{Q^\nu}\right) + \mathcal{O}(\rho^{-1})$

global Lorentz rotation

asymptotic symmetry

✦ Generic state in the kinematical Hilbert space (formally):

$$|\Psi\rangle = \int_{[\mathbb{R}^4]^{S_2}} \mathscr{D}[Q^\mu] \int_{SO(1,3)} d\mu_\Omega \Psi_{bulk}[Q^\mu, \Omega^\mu{}_\nu) \otimes \boxed{|Q^\mu, \Omega^\mu{}_\nu\rangle}$$

$$\rho \equiv \sqrt{X^\mu X_\mu} \to \infty$$

Quantum Reference Frames

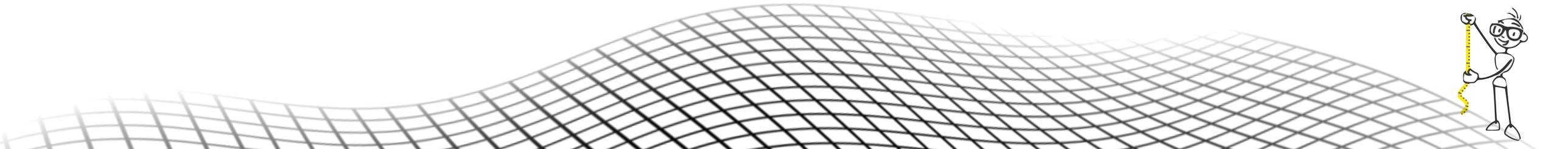
for Asymptotic Symmetries

✦ To obtain QRF transformations, consider two kinematical states of the form

$$\Psi = \Psi[Q_0, \Omega_0] \otimes \boxed{|\, Q_0, \Omega_0 \rangle} \qquad \Phi = \sum_i \Phi_i[Q_i, \Omega_0] \otimes \boxed{|\, Q_i, \Omega_0 \rangle}$$

QRF peaked on Q_0, Ω_0

QRF in superposition of configurations Q_i, Ω_0



Quantum Reference Frames

for Asymptotic Symmetries

✦ To obtain QRF transformations, consider two kinematical states of the form

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✦ Physical states obey **momentum constraint** → projector

$$\boldsymbol{P} = \int_{[\mathbb{R}^4]^{S_2}} \mathscr{D}[N] \exp\left(\frac{i}{\hbar} \oint_{S_2} N^\mu \boxed{(P_\mu - H_\mu)}\right)$$

✦ Assuming **orthogonality** of peaked states

$$\langle \Psi | \Phi \rangle_{phys} = \sum_i \langle \Psi[Q_0, \Omega_0] | \boxed{\langle Q_0, \Omega_0 | \boldsymbol{P} | Q_i, \Omega_i \rangle} \Phi[Q_i, \Omega_0] \rangle$$

$$\boxed{U_{Q_i \to Q_0}}$$

Quantum Reference Frames

for Asymptotic Symmetries

✦ Unitary **QRF transformation** operator:

$$\boxed{\sum_i U_{Q_i \to Q_0} \Phi_i[Q_i, \Omega_0] = \sum_i e^{\frac{i}{\hbar} \oint (Q_i - Q_0)^\mu H_\mu} \Phi_i[Q_i, \Omega_0]}$$

quantum-controlled,
point-wise translation

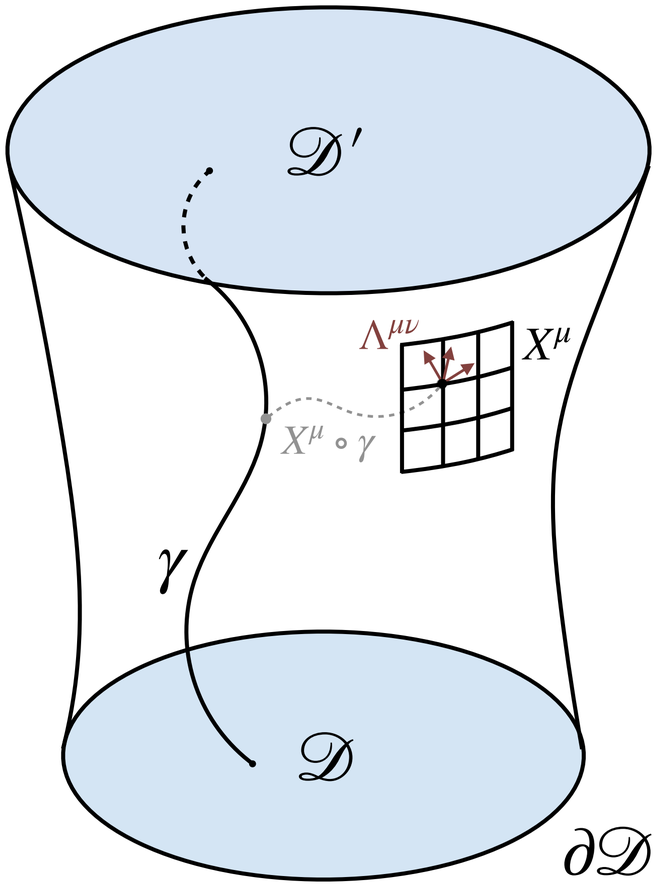
✦ **Generalises** quantum controlled translations:

$$\mathscr{P}_{CA} e^{\frac{i}{\hbar} \hat{x}_A \hat{p}_B} = \mathscr{P}_{CA} \int dx_A e^{\frac{i}{\hbar} x_A \hat{p}_B} |x_A\rangle\langle x_A|$$

↳ Strategy to derive/justify QRF transformations from GR description.

QRFs at the Boundary of Spacetime

Summary



- In linearised gravity, **coordinate fields** emerge naturally as **edge modes** at the boundary of spacetime.
- These ought to be **included in the quantum description**.
- Momentum constraint leads to a **relational Schrödinger evolution** with respect to these reference fields.
- At asymptotic boundaries, we obtain **QRF transformations for point-wise translations and rotations**.

arXiv:2302.11629

Summary

Quantum Reference Frames…

- Generalise the idea of a classical reference frame.
- Changes are implemented by **quantum-controlled symmetry transformations**.
- **Superposition and entanglement** become frame-dependent features.

… in Spacetime

- Allow us to study problems at the **interface between quantum physics and gravity** from a new perspective.
- QRFs for asymptotic symmetries arise naturally as **edge modes** in a bounded region of spacetime in linearised GR.

Outlook & Connections

material reference frames in QG

edge modes

algebras of observables

QRFs

Noether charges

Super-selection sectors

Thank you for your attention!