Effective Field Theory for Quantum Gravity from Shape Dynamics

International Loop Quantum Gravity Seminar

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Outline

- Introduction: motivation, goal of talk
- Symmetry Trading and Symmetry Doubling: symmetry trading, Shape Dynamics and General Relativity, symmetry doubling and Doubly General Relativity
- Doubly General Relativity: effective field theory, revised gravity theory space
- Consequences: new construction principle $\Rightarrow$ possible observational consequences, bulk/bulk duality, renormalization
- Summary

Motivation

General Relativity is not renormalizable:

perturbation expansion of Einstein-Hilbert action, unitarity problem with higher derivative gravity
$\Rightarrow$ problem is finding a different universality class

Importance of Symmetries:

- RG flow stays on (evolving) symm. surface
- encoded as BRST-invariance of path integral
  $\Rightarrow$ Slavnov-Taylor identities for eff. action
- universality classes are often explained by symmetries

Possibility: Hidden Symmetry

- FP may not be detected without adapting search to symmetry
  $\Rightarrow$ important heuristic for finding new universality class

Main Message

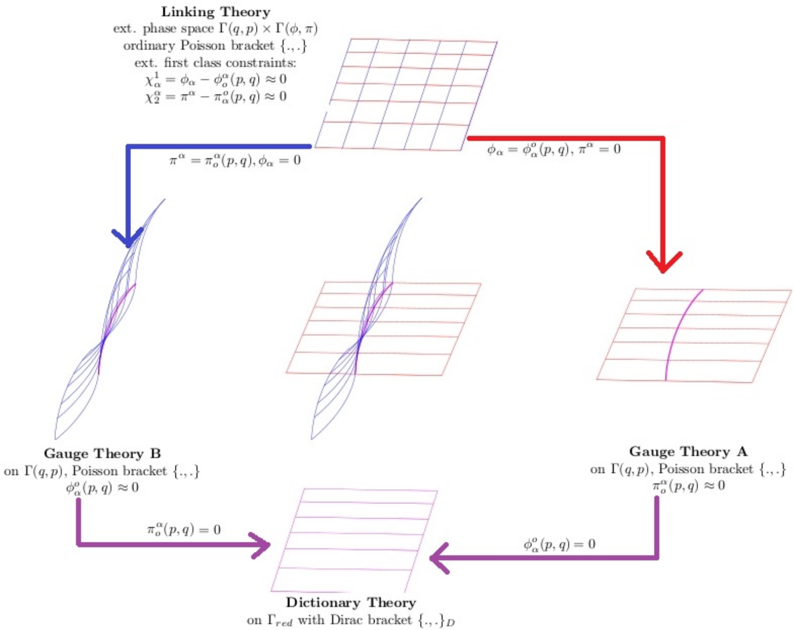
"Doubly General Relativity" in one line:

There is a hidden BRST-invariance in gravity due to Shape Dynamics.

BACKGROUND:

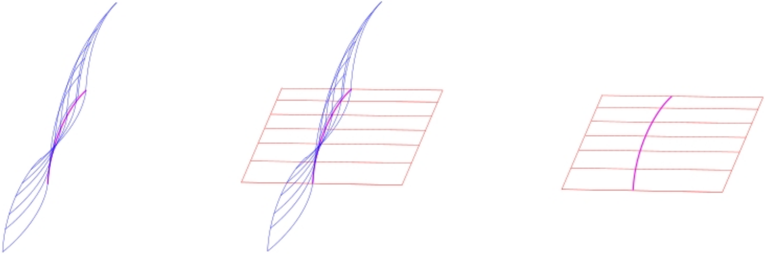
symmetry trading, Shape Dynamics and BRST-formalism

Symmetry Trading Mechanism



Construction of Shape Dynamics

Construction on ADM-phase space:



ADM Gravity

$$S(N) = \int N \left(\frac{G(\pi,\pi)}{\sqrt{|g|}} - (R - 2\Lambda)\sqrt{|g|} \right)$$
$$H(v) = \int \pi^{ab} \mathcal{L}_v g_{ab}$$

Shape Dynamics

$$H_{SD} = V - V_o$$
$$Q(\rho) = \int (\pi - \langle \pi \rangle \sqrt{|g|})$$
$$H(v) = \int \pi^{ab} \mathcal{L}_v g_{ab}$$

Relation of Dirac Observables

Intersecting constraint surfaces in local coordinates

- $\chi_\alpha = M_\alpha^\beta q_\beta \approx 0$ and $\sigma^\beta = N_\alpha^\beta (p^\alpha - p_o^\alpha)$ with M_α^β and N_α^β intertible
- equiv. doubly Abelian set: $\tilde{\chi}_\alpha = q_\alpha \approx 0$ and $\tilde{\sigma}^\beta = p^\beta - p_o^\beta$

$\Rightarrow$ vector fields $\{\tilde{\chi}_\alpha, .\}$ and $\{\tilde{\sigma}^\beta, .\}$ are Frobenius integrable
$\Rightarrow$ for every function f_{red} on intersection Γ_{red} there exists a local function f on $U \subset \Gamma$ s.t. $\{\tilde{\chi}_\alpha, f\} = 0 = \{\tilde{\sigma}^\beta, f\}$ (doubly strong observables)
$\Rightarrow$ for these preferred representatives the identification on full Γ is trivial

$$f_A \equiv f_B.$$

Abelianization generally spoils locality:

Nonabelian case: there is still a dictionary for observables through the two phase space reductions of the linking theory.

BRST-Formalism

Abelian constraints χ_α

- BRST-generator $\Omega = \eta^\alpha \chi_\alpha$ satisfies $\{\Omega, \Omega\} = 0$ (nontrivial: $gh(\Omega) = 1$)

$\Rightarrow$ defines graded differential $s : f \to \{\Omega, f\}$, i.e. $s^2 = 0$

- Observables as cohomology of s at $gh(.) = 0$:
  - gauge invariance: $\{\Omega, f(p, q)\} = 0 \Rightarrow f$ (strong observable)
  - equivalence: $\tilde{f} = f + \{\Omega, \Psi\} = f + \sigma^\alpha \chi_\alpha + \mathcal{O}(\eta)$ (weak observable) for gauge fixing $\Psi = \sigma^\alpha P_\alpha + \mathcal{O}(\eta)$ with $gh(\Psi) = -1$

$\Rightarrow$ always strong equations on extended phase space

- gauge fixed Hamiltonian $H_{BRS} = H_o + \{\Omega, \Psi\}$ when $\{H_o, \Omega\} = 0$.

Nonabelian constraints $\tilde{\chi}_\alpha = M_\alpha^\beta \chi_\beta$

apply canonical transform $\exp(\{\eta_\alpha c^\alpha_\beta P^\beta, .\})$ to Abelian case

$\tilde{\Omega} = \eta^\alpha M_\alpha^\beta \chi_\beta + \mathcal{O}(\eta^2)$ defines $\tilde{s}$, cohomology same as of s at $gh(.) = 0$.

DOUBLY GENERAL RELATIVITY

symmetry doubling, Extended Shape Dynamics, Doubly General Relativity

From Symmetry Trading to Symmetry Doubling

Symmetry Trading requires

Two first class surfaces (original and equivalent gauge symmetry) that **gauge fix** one another

BRST-gauge-fixing

- Ω is nilpotent because orig. system is first class
- Ψ can be chosen nilpotent because equiv. system is first class
- if H_o (on shell) Poisson commutes with Ω and Ψ then gauge fixed

$$H_{BRS} = H_o + \{\Omega, \Psi\}$$

is annihilated by both s_Ω and s_Ψ

Symmetry Doubling (**NOT anti-BRST!**)

Canonical action $S = \int dt(p_i \dot{q}^i + P_\alpha \dot{\eta}^\alpha - H_{BRST})$ is invariant under two BRST-transformations (up to a boundary term).

Construction of Doubly General Relativity (I)

Extending Shape Dynamics

- fixed CMC condition $Q(x) = \pi(x) + \lambda\sqrt{|g|}$
- conformal spatial harmonic gauge
  $F^k(x) = (g^{ab}\delta^k_c + \frac{1}{3}g^{ak}\delta^b_c)e^c_\alpha(\nabla_a - \hat{\nabla}_a)e^\alpha_b$
- First class system: $\{Q(x), Q(y)\} = 0 = \{F^i(x), F^j(y)\}$
  as well as $\{Q(x), F^i(y)\} = F^i(y)\delta(x, y)$

Interpretation as "local conformal system"

Q generates spatial dilatations and Poisson brackets resemble $C(3)$ at each point

Gauge fixing ADM

- gauge fixing operator is elliptic and invertible in a region R
- out side R: meager set with finite dimensional kernel

Construction of Doubly General Relativity (II)

BRST-charges

$$\Omega_{ADM} = \int d^3x \left(\eta S + \eta^a g_{ac}\pi^{cd}_{;d} + \eta^b \eta^a_{,b} P_a + \frac{1}{2}\eta^a \eta_{,a} P + \eta \eta_{,c} P_b g^{bc}\right)$$
$$\Omega_{ESD} = \int d^3x \left(P \frac{\pi}{\sqrt{g}} + P_a F^a + \frac{1}{2}\frac{P}{\sqrt{g}} P_a \eta^a\right)$$

Gauge-fixed gravity action

$S_{gf} = \int dt\,(symp.pot. + \{\Omega_{ADM}, \Omega_{ESD}\})$ is invariant under usual ADM-BRST transformations **and**

a hidden BRST-invariance of S_{gf} under

$$\begin{aligned} s_{ESD} g_{ab} &= \frac{P}{\sqrt{g}} g_{ab} & s_{ESD}\pi^{ab} &= \text{“}long\text{''} \\ s_{ESD}\eta &= -\frac{1}{\sqrt{g}}(\pi + \frac{1}{2}P_c\eta^c) & s_{ESD}P &= 0 \\ s_{ESD}\eta^a &= -F^a + \frac{P}{2\sqrt{g}}\eta^a & s_{ESD}P_a &= \frac{P}{2\sqrt{g}}P_a \end{aligned}$$

due to extended Shape Dynamics.

Construction of Doubly General Relativity (III)

Interpretation

The Hamiltonian of Doubly General Relativity is

$$H_{DGR} = S(\frac{\pi}{\sqrt{|g|}} + \lambda) + H(F^a) + \mathcal{O}(\eta)$$

The ghost-free part is neither the "frozen Hamiltonain" $H = 0$
nor the CMC-Hamiltonian $H = S(N_{CMC}[g, \pi])$,
but a generator of dynamics for $\lambda + \frac{\pi}{\sqrt{|g|}} > 0$.

EFFECTIVE FIELD THEORY

standard reasoning, symmetry doubling, definition of a gravity theory

Renormalization Group and Effective Field Theory

- Renormalization group: Γ_k interpolates between local bare action S_Λ (at UV cut-off Λ) and effective action Γ (in IR)
- Γ_k: functional on theory space (field content, symmetries, approx. locality)
- Asymptotic safety progr.: Find UV-fixed point with few relevant directions $\Rightarrow$ predictive theory
- Approximations: Fixed points $\leftrightarrow$ broken BRST-symmetries, RG-relevance $\leftrightarrow$ dimensional analysis (weak IR coupling)
- Universality: IR attractive critical manifold (i.e. S_Λ unimportant for IR)

Two Uses for RG:

- Fundamental theory: hard to verify w/o heuristic
- Theory Space constr. ppl. for local effective actions

(field content, BRST-symmetries, dimensionality, locality)

Theory Space of Gravity

Slavnov-Taylor Identities:

- assuming an invariant path integral measure $\Rightarrow$ BRST-variations yield:

$$\langle s_{ADM}\phi_A\rangle \frac{\delta_L \Gamma}{\delta \phi_A} = 0 \text{ and } \langle s_{ESD}\phi_A\rangle \frac{\delta_L \Gamma}{\delta \phi_A} = 0$$

BRST-variations are nonlinear $\Rightarrow$ difficult Legendre transform

- Nonlinearity obstructs use of two Zinn-Justin equations

$(\Gamma,\Gamma)_1|_{\hat\phi_2=0} = 0 = (\Gamma,\Gamma)_2|_{\hat\phi_1=0}$

- in semiclassical approximation $(\langle F[\phi]\rangle_{sc} = F[\phi_{sc}] + \mathcal{O}(\hbar))$:

$$s_{ADM}\Gamma = \mathcal{O}(\hbar) \text{ and } s_{ESD}\Gamma = \mathcal{O}(\hbar)$$

Refined definition of a gravity theory (from semiclassical reasoning):

Gravity = local action for $g_{ab}, \pi^{ab}, \eta, P, \eta^a, P_a$ at gh. number 0 that is invariant under ADM- and ESD- BRST-transformations s_{ADM}, s_{ESD}

FURTHER DIRECTIONS

dualities, experimental implications, renormalization

(first baby steps)

Classical Theory and Observations

Construction of classical gravity:

effective field theory:

- revised theory space
- dimensional analysis

$\Rightarrow$ construction ppl. for classical Doubly General Relativity

Possible Observable Consequences

- Effective field theory for GR: all higher derivative curvature invariants are allowed (just suppressed at low energies)
- these are generally not compatible with Extended Shape Dynamics
  $\Rightarrow$ DGR can be experimentally distinguished from usual GR
  (but only beyond Einstein-Hilbert)

This theory space has **not** been explored!

Bulk/bulk duality

Usual Shape Dynamics: bulk/boundary duality

- Hamiltonian at large volume $H_{SD} = \langle \pi \rangle^2 - 12\Lambda$
- gauge group: diffeomorphisms, vol. pres. conf. trfs. $\pi - \langle \pi \rangle \sqrt{|g|} \approx 0$
  $\Rightarrow$ dynamical large CMC-volume CFT-correspondence of gravity

Duobly General Relativity:

- True evolution generated by $H_{BRST} \Rightarrow$ "bulk"
- compatible with two equivalent symmetry principles describe gravity

Remark:

The symmetry doubling mechanism is very generic:
possible explanation for dualities like (A)dS/CFT

Renormalization

Immediate Question:

Are there implications of symmetry doubling for counter terms?
Possibly yes, but seems unfeasible in metric formulation.

Current Directions:

- Find a formulation of DGR where enough transofrmations are linearly realized:
  This makes prediction about counter terms very feasible
- Find a gauge fixing with improved power counting:
  Idea: part. fkt. Z is independent of gaug.fix. (quant. mast equ.)
  view action as ESD action and gauge fix with $\Omega' - \Omega_{ADM}$ (in BV)
  $\Rightarrow$ gives arbitrary gauge fixing of ESD

Conclusions

1. Symmetry trading is generic and gives equivalent gauge theories
2. Symmetry trading implies symmetry doubling in BRST formalism
3. Equivalence of Shape Dynamics and GR $\Rightarrow$ Doubly General Relativity
4. DGR implies a new theory space for gravity. To explore:
   - are there semiclassical predictions (beyond E-H-action)?
   - universality classes on this revised theory space (FRGE methods)?
   - new view on dualities?
5. Just started homogeneous quantum cosmology

"Doubly General Relativity" in one line:

There is a hidden BRST-invariance in gravity due to Shape Dynamics.

THANK YOU

and

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