New LQC modifications from symplectic structures

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Introduction

- Loop Quantum Cosmology presents a quantisation of isotropic spacetime based on techniques from the full LQG
- In recent years a possible window for relating LQC and LQG has opened: *effective dynamics from coherent states*
- LQC dynamics follows an effective Hamiltonian [Ashtekar, Pawlowski, Singh '06] agreeing with the expectation value of LQC Gaussian states [Taveras '08]
- Further insights beyond LQC, QRLG [Alesci&Cianfrani '13], LQG [Dapor&KL '17],...
- Still many unanswered questions in this procedure (e.g. continuum limit, validity of the effective Hamiltonian,...)
- Here: issue of $\mathrm{SU}(2)$-gauge-fixing the coordinate system
- We present that this can be avoided (e.g.) by using gauge covariant fluxes and discuss the resulting modifications

Gauge transformation

General relativity is equivalent to the phase space of a $\mathrm{SU}(2)$ gauge theory coordinatised by the Ashtekar-Barbero variables

$$\{E^a_J(x), A^K_b(y)\} = 8\pi G\gamma\delta^a_b\delta^J_K\delta^{(3)}(x,y) \tag{1}$$

A gauge transformation $g(x) \in \mathrm{SU}(2)$ acts thereon as:

$$A_a(x) \mapsto (gA_ag^\dagger - [\partial_a g]g^\dagger)(x), \qquad E^a(x) \mapsto (gE^ag^\dagger)(x) \tag{2}$$

where e.g. scalar constraint C is invariant under (2).

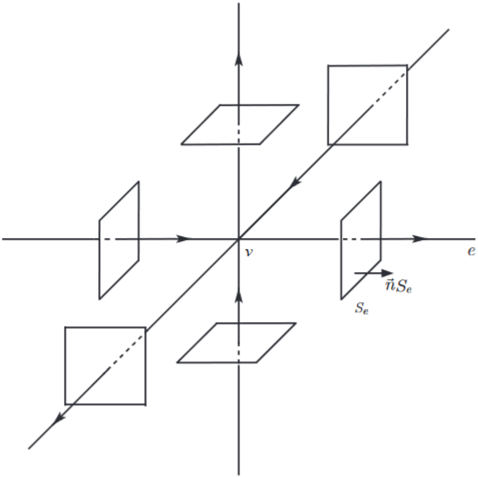
However for a curve $e : [0,1] \to \sigma$, and a face S:

$$h(e) \mapsto g(e[0])h(e)g^\dagger(e[1]), \qquad E(S) := \int_S (\star E) \mapsto ??? \tag{3}$$

$\Rightarrow$ conventional flux transforms not feasibly under gauge transf!

Example: discretised GR

Let us consider discretised classical GR on a lattice $\Gamma \subset \sigma$ of edge length ϵ. Let S_e be the face corresponding to $e \in \Gamma$ of the associated dual-cell complex.



Example: discretised GR

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Under any gauge transformation

$$Q(v) := \det(E)(v) \mapsto \det(E)(v) \tag{4}$$

i.e. Q is a $\mathrm{SU}(2)$ gauge-invariant function.

$$Q^{\epsilon}(v) := \frac{1}{48} \sum_{e_a \cap e_b \cap e_c = v} \mathrm{sgn}(\det(\dot{e}_a, \dot{e}_b, \dot{e}_c)) \times \epsilon^{IJK} E_I(S_a) E_J(S_b) E_K(S_c) \tag{5}$$

is an approximation in the sense that $\lim_{\epsilon \to 0} Q^{\epsilon}(v) = Q(v)$. However, it is not invariant under gauge transformations.

Example: discretised GR

Concrete example:
Fix $\epsilon > 0$. Then there exists a certain transformation $g^\epsilon(x)$ such that it acts non-trivially only on the the faces $S_{\pm 3}$. It can be chosen such, that for the *degenerate metric*

$$\tilde{E}^a(x) = p(\delta^1_a + \delta^3_a)\,\tau_1 + p\delta^2_a\,\tau_2 \tag{6}$$

and the *isotropic metric*

$$\bar{E}^a(x) = p\delta^I_a\,\tau_I \tag{7}$$

the result of Q^ϵ interchanges:

$$Q^\epsilon[\bar{E}](v) = p^3\epsilon^6 \mapsto 0 \tag{8}$$

$$Q^\epsilon[\tilde{E}](v) = 0 \quad \mapsto p^3\epsilon^6 \tag{9}$$

$\Rightarrow$ Singularity resolution should not be deduced from studying non-invariant quantities!

Possible Resolutions

There are several ways to resolve this conundrum:

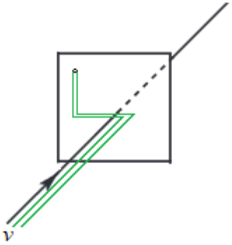
- **Continuum limit**: Upon $\epsilon \to 0$ the function $Q^\epsilon(x)$ becomes gauge-invariant as well and everything works as normal
- If one wants to keep finite regulator (e.g. μ_0 or $\bar{\mu}$) it suggests itself to work not with conventional fluxes $E(S)$, but with some **covariant fluxes** $P(S)$

Having comparison with LQC in mind, we will follow in this talk the second strategy!

Instead of conventional fluxes $E(S)$, we want to consider a quantity that transforms covariantly under $\mathrm{SU}(2)$-gauge transformations.
In the literature, plenty of work has been done regarding covariant fluxes [Thiemann, Bonzom, Dittrich, Dupuis, Freidel, Geiller, Girelli, Ziprick,...]
The first - to our knowledge - construction of a gauge-covariant flux is from Thiemann in 2001:

$$P(e) := h(e_{1/2}) \int_S h(l_x)(\star E)(x) h^\dagger(l_x) h^\dagger(e_{1/2}) \qquad (10)$$

In the remainder of this talk, we want to investigate the influence of the $P(e)$ for LQC.



Main Idea

- Computations can be done by choosing any coordinate system, since the result will be independent of this choice - given the considered observables are gauge invariant.
- Construction of gauge invariant quantities is possible using the Thiemann fluxes $P(e)$, e.g. in the definition of Q^ϵ instead of $E(S_e)$.
- This eases moreover the interpretation of semi-classical objects, as the Poisson algebra

$$\{h(e), h(e')\} = 0, \quad \{P^I(e), h(e')\} = 8\pi G\gamma\delta_{ee'} h(e)\tau^I, \\ \{P^I(e), P^J(e')\} = -8\pi G\epsilon_{IJK} P^K(e)\delta_{ee'} \tag{11}$$

is upon quantisation exactly mirrored in the commutator algebra of right-invariant vector fields.

Main Idea

How to obtain LQC modifications from the full theory?

- Coherent state expectation value procedure [Taveras, Bojowald, Corichi, Ashtekar, Gupt, Montoya, Alesci, Cianfrani, Dapor, KL,...]
- Consider a regularisation of the scalar constraint C in terms of the new fluxes, which can be quantised on the Ashtekar-Lewandowski Hilbert space $\rightarrow \hat{C}$
- Construct semiclassical gauge coherent states $\Psi_{c,p}$ [Thiemann& Winkler '01], peaked on the new fluxes P for flat cosmology (parametrised by c, p)
- Compute the expectation value $\langle \Psi_{c,p}, \hat{C}\Psi_{c,p}\rangle =: C'(c,p)$
- **Conjecture**: $C'(c,p)$ will be used as an effective scalar constraint, capturing modifications in LQC due to presence of the gauge covariant fluxes

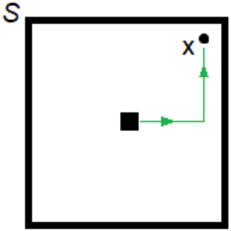
Gauge covariant fluxes in flat cosmology

We are allowed to choose a coordinate system knowing, that the values of physical quantities will not depend thereon:

$$A_a(x) = \tau_a c, \qquad E^a(x) = \tau_a p, \qquad h(e_a) = e^{\epsilon c \tau_a} \tag{12}$$

Computing the gauge covariant fluxes from Thiemann [QSD VII, '01]:

$$P(e_a) = e^{\epsilon c \tau_a/2} \int_S h(l_x) \star E(x) h^\dagger(l_x) e^{-\epsilon c \tau_a/2} \tag{13}$$



Choose a set of paths in S.

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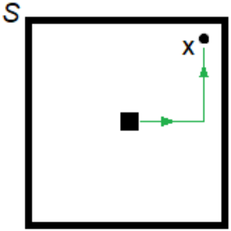

Choose a set of paths in S.
In some parametrisation:

$$\ldots \int_{-\epsilon/2}^{\epsilon/2} du\; e^{\epsilon u \tau_1} (p\tau_3) e^{-\epsilon u \tau_1} \ldots$$

Gauge covariant fluxes in flat cosmology

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$$\int_{-\epsilon/2}^{\epsilon/2} du\ e^{\epsilon u \tau_1}(p\tau_3)e^{-\epsilon u \tau_1}$$

$$= p \int_{-\epsilon/2}^{\epsilon/2} du\ [cos(\frac{cu}{2})\mathbb{1} + 2\sin(\frac{cu}{2})\tau_1]\tau_3[\cos(\frac{cu}{2})\mathbb{1} - 2\sin(\frac{cu}{2})\tau_1] =$$

$$= p \int_{-\epsilon/2}^{\epsilon/2} du[\cos(cu/2)^2\tau_3 - \sin(cu/2)^2 4\tau_1\tau_3\tau_1] = \tau_3 p \int_{-\epsilon/2}^{\epsilon/2} du \cos(cu)$$

$$= p\tau_3\epsilon\ \mathrm{sinc}(c\epsilon/2)$$

Gauge covariant fluxes in flat cosmology

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Computing the gauge covariant fluxes from Thiemann [QSD VII, '01]:

$$P(e_a) = e^{\epsilon c \tau_a/2} \int_S h(l_x) \star E(x) h^\dagger(l_x) e^{-\epsilon c \tau_a/2} \tag{17}$$

After short computation:

$$P(e_a) = p\, \tau_a\, \epsilon^2\, \mathrm{sinc}(c\epsilon/2)^2 \tag{18}$$

The difference with conventional fluxes $E(S_a) = p\tau_a\epsilon^2$ is merely in the additional $\mathrm{sinc}(\epsilon c/2)^2$ terms!

New modifications for the effective dynamics of LQC

We present a treatment mirroring (conjectured) effective dynamics as in LQC.
Instead of the usual LQC Hamiltonian minimal coupled to an isotropic scalar field ϕ, we have now for the conventional regularisation in the literature

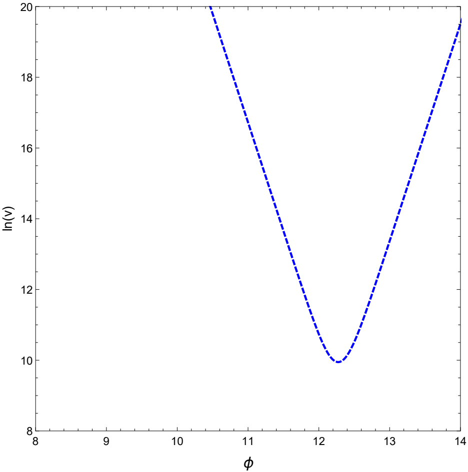
$$C^{\epsilon} = -\frac{6\sqrt{|p|}}{\kappa\gamma^2}\frac{\sin(c\epsilon)^2}{\epsilon^2}\mathrm{sinc}(c\epsilon/2) + \frac{\pi_\phi^2}{2\sqrt{|p^3|}}\mathrm{sinc}(c\epsilon/2)^{-3} \tag{19}$$

and in terms of the newly rediscovered Thiemann regularisation:

$$C^{\epsilon}_{\mathrm{TR}} = \frac{6\sqrt{|p|}}{\kappa\gamma^2}\left(\frac{\sin(c\epsilon)^2}{\epsilon^2} - \frac{1+\gamma^2}{4\gamma^2\epsilon^2}\sin(2c\epsilon)^2\right)\mathrm{sinc}(c\epsilon/2) + \frac{\pi_\phi^2}{2\sqrt{|p^3|}}\mathrm{sinc}(c\epsilon/2)^{-3} \tag{20}$$

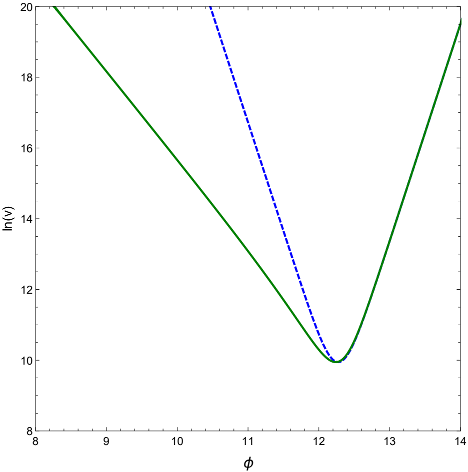
Note that our proposal works intrinsically with the regularisation scheme $\epsilon \equiv \mu_0 = 3\sqrt{3}$. (Later on, we present also the case $\epsilon \equiv \bar{\mu}$)

$$C^{\mu_0} = -\frac{6\sqrt{|p|}}{\kappa\gamma^2}\frac{\sin(c\mu_0)^2}{\mu_0^2} \qquad + \frac{\pi_\phi^2}{2\sqrt{|p^3|}}$$



Effective dynamics with μ_0 with gauge-covariant flux

$$C^{\mu_0} = -\frac{6\sqrt{|p|}}{\kappa\gamma^2}\frac{\sin(c\mu_0)^2}{\mu_0^2}\mathrm{sinc}(c\mu_0/2) + \frac{\pi_\phi^2}{2\sqrt{|p^3|}}\mathrm{sinc}(c\mu_0/2)^{-3}$$



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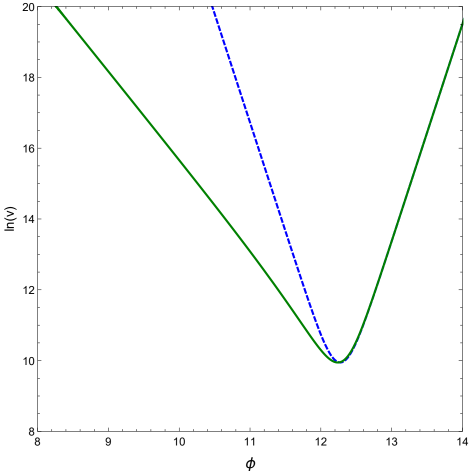


Conventionally:

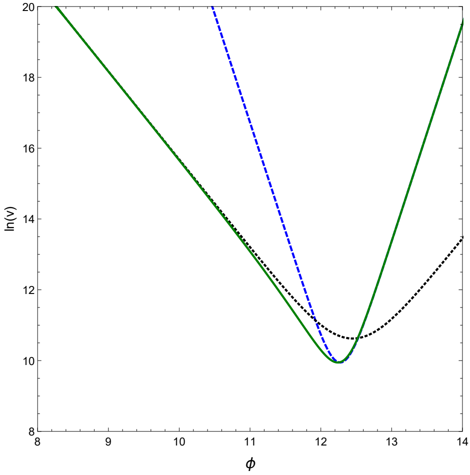
$v = p^{3/2}$

For gauge cov. fluxes:

$v_{gc} := p^{3/2}\mathrm{sinc}(c\mu_0/2)^3$

Effective dynamics with μ_0 & rescaled asymptotics

$$C^{\mu_0} = -\frac{6\sqrt{|p|}}{\kappa\gamma^2}\frac{\sin(c\mu_0)^2}{\mu_0^2}\mathrm{sinc}(c\mu_0/2) + \frac{\pi_\phi^2}{2\sqrt{|p^3|}}\mathrm{sinc}(c\mu_0/2)^{-3}$$



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Asymptotically:
$G \to \bar{G} = G[2/\pi]^4$,
$\pi_\phi \to \bar{\pi}_\phi = \pi_\phi[\pi/2]^3$

Problems of the μ_0-scheme

In conventional LQC the μ_0-scheme had several issues. These do reappear also in context of the gauge covariant flux modifications:

- No invariance under residual diffeomorphisms, i.e. rescaling of the fiducial cell
- Energy density at the bounce can be made arbitrarily small (by choice of π_ϕ)
- Inclusion of positive cosmological constant causes again recollapse and periodic behaviour (seen in numerical study)

$\Rightarrow$ a transition to the $\bar{\mu}$-scheme seems necessary

The $\bar{\mu}$-scheme

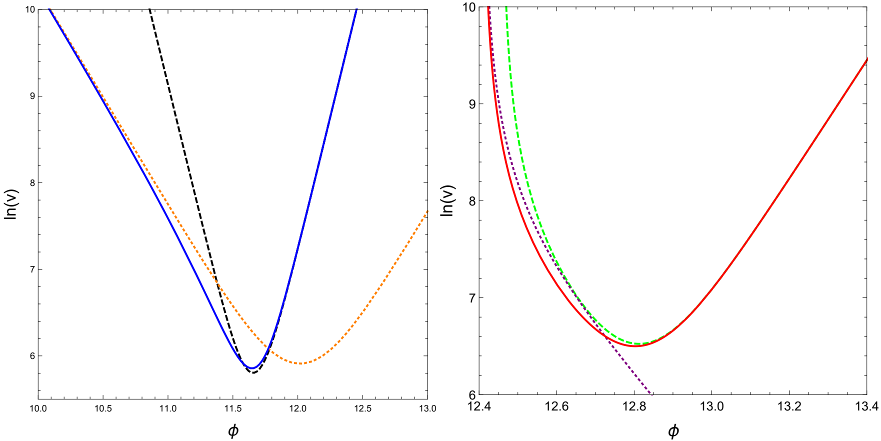
We will now turn towards the topic of switching to the $\bar{\mu}$-scheme.

- No clear derivation yet for the $\bar{\mu}$-scheme from a full theory context $\Rightarrow$ here: *ad hoc* replacement of $\epsilon \to \bar{\mu} = \sqrt{\Delta/p}$, where $\Delta = 4\sqrt{3}\pi\gamma\ell_P^2$
- For scale invariance under residual diffeos, we have to change to $\bar{\mu}$ after the gauge covariant flux corrections $\to \mathrm{sinc}(\bar{\mu}c)$
- Analysis can be repeated as before: again asymmetric asymptotic behaviour,

$$G \to \bar{G} = G[2/\pi]^4, \pi_\phi \to \bar{\pi}_\phi = \pi_\phi[\pi/2]^3 \tag{21}$$

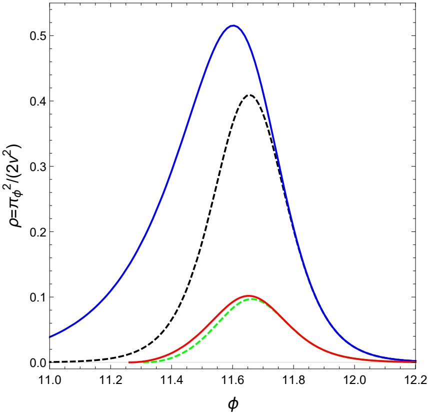
but unique bounce energy density (in Planck units)

$$\rho_{\mathrm{Bounce}} := \frac{\pi_\phi^2}{2v_{gc}^2} \,|_{\mathrm{Bounce}} \approx 0.515 \tag{22}$$



Blue: new $C^{\bar{\mu}}$, Black: conventional LQC, Orange: rescaled $\bar{G}, \bar{\pi}_\phi$
Red: new $C^{\bar{\mu}}_{\mathrm{TR}}$, Green: conventional TR-LQC, Purple: rescaled G', π'_ϕ, Λ'.

Energy density in $\bar{\mu}$ scheme



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Concrete quantization on LQC Hilbert space

Can a regularisation including gauge covariant fluxes be promoted to an operator on the LQC Hilbert space $\mathcal{H}_{LQC}$?

- Challenge lies in the $\mathrm{sinc}(\bar{\mu}c/2) = \frac{2}{b}\sin(b/2)$, due to the $1/b$
- However sinc is still a bounded function!
- Classical observation: rescaling forces us to be in the principal branches, i.e. $c \in [-\pi, \pi]$
- We restrict c classically to this region $\Rightarrow$ on a bounded interval sinc can be approximated by its *Fourier series*

$$\mathrm{sinc}(b)^2 \approx TF_N(b) := a_0 + \sum_{n=1}^{N} a_n \cos(nb/2) + R_N \qquad (23)$$

  where $a_n \in \mathbb{R}$ and $R_N \to 0$ for $N \to \infty$.
- Quantisation $\widehat{TF}_N$ is well defined bounded operator on $\mathcal{H}_{LQC}$

Conclusion

- On a discrete lattice with finite UV cut-off, gauge transformation may act non-trivially on the fluxes
- We followed Thiemanns construction to built a symplectic structure on lattice using gauge covariant fluxes
- We studied the influence on those new fluxes to the scalar constraint (regularised on said lattice)
- Asymmetric bounce in the conventional regularisation as well as the Thiemann regularisation
- Quantisation on $\mathcal{H}_{LQC}$ is possible as infinite order difference equation
- Non-trivial changes from LQC and TR-LQC in Planck regime physics (works in progress)

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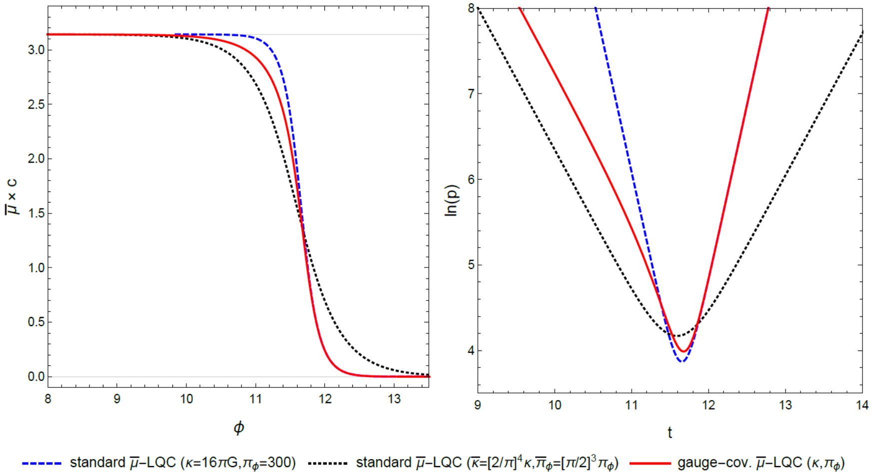
THANK YOU!

App*: (c, p) in $\bar{\mu}$-scheme and conventional reg.



App*: Ricci & Hubble in $\bar{\mu}$-scheme and conventional reg.