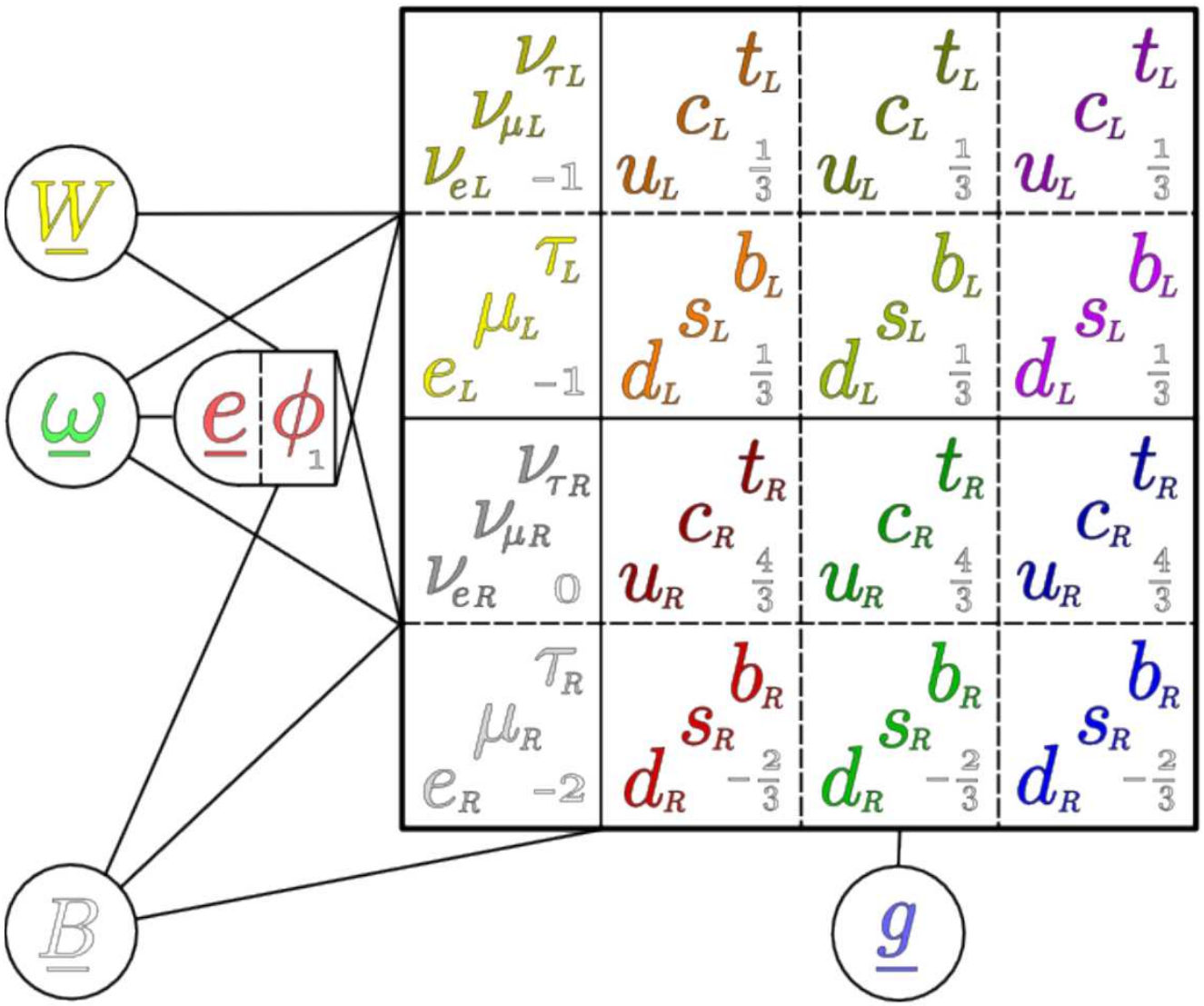
Periodic table of the standard model



An Exceptionally Simple Theory of Everything

http://arxiv.org/abs/0711.0770

Garrett Lisi
FQXi

A connection with everything

$$\underline{\omega} = d\underline{x}^k \tfrac{1}{2}\omega_k{}^{\mu\nu}\gamma_{\mu\nu} \in \underline{Cl}^2(3,1) \qquad \underline{e} = d\underline{x}^k (e_k)^\mu \gamma_\mu \in \underline{Cl}^1(3,1)$$

$$\underline{W} = d\underline{x}^k W_k{}^\pi \tfrac{i}{2}\sigma_\pi \in \underline{su}(2) \qquad \begin{bmatrix} \phi_+ \\ \phi_0 \end{bmatrix} \quad \begin{bmatrix} \nu_{eL} \\ e_L \end{bmatrix}$$

$$\underline{B} = d\underline{x}^k B_k i \in \underline{u}(1) \qquad Y$$

$$\underline{g} = d\underline{x}^k g_k{}^A \tfrac{i}{2}\lambda_A \in \underline{su}(3) \qquad [u^r, u^g, u^b]$$

$$\begin{bmatrix} e^\wedge_L \\ e^\vee_L \\ e^\wedge_R \\ e^\vee_R \end{bmatrix}$$

$$\updownarrow$$

$$\underline{A}_\cdot = \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi + \underline{W} + \underline{B} + \underline{g} + (\underset{\cdot}{\nu}_e + \underset{\cdot}{e} + \underset{\cdot}{u} + \underset{\cdot}{d})$$
$$+ (\underset{\cdot}{\nu}_\mu + \underset{\cdot}{\mu} + \underset{\cdot}{c} + \underset{\cdot}{s}) + (\underset{\cdot}{\nu}_\tau + \underset{\cdot}{\tau} + \underset{\cdot}{t} + \underset{\cdot}{b})$$

$$\underline{\underline{F}}_\cdot = \underline{d}\,\underline{A}_\cdot + \tfrac{1}{2}[\underline{A}_\cdot, \underline{A}_\cdot]$$

Review of some representation theory

Cartan subalgebra: $C = C^a T_a \quad \subset \ \mathrm{Lie}(G)$

Built from a maximal commuting set of R generators,

$$\left[T_a, T_b\right] = T_a T_b - T_b T_a = 0 \qquad \forall \quad 1 \leq a, b \leq R$$

Root vectors, V_β, are eigenvectors of C in the Lie bracket,

$$[C, V_\beta] = \alpha_\beta V_\beta = \sum_a i C^a \alpha_{a\beta} V_\beta$$

Roots, $\alpha_{a\beta}$, are the eigenvalue coefficients. The pattern of roots in R dimensions corresponds to the Lie algebra,

$$[V_\beta, V_\gamma] = V_\delta \quad \Leftrightarrow \quad \alpha_\beta + \alpha_\gamma = \alpha_\delta$$

Weight vectors and **weights** are eigenvectors and eigenvalue coefficients of C acting on some representation space,

$$C\, V_\beta = \alpha_\beta V_\beta$$

Weight vectors are particles, weights are their quantum numbers.

Gluon and quark weights

$$g = g^A T_A = g^A \frac{i}{2}\lambda_A = \frac{i}{2}\begin{bmatrix} g^3+\frac{1}{\sqrt{3}}g^8 & g^1-ig^2 & g^4-ig^5 \\ g^1+ig^2 & -g^3+\frac{1}{\sqrt{3}}g^8 & g^6-ig^7 \\ g^4+ig^5 & g^6+ig^7 & -\frac{2}{\sqrt{3}}g^8 \end{bmatrix}$$

Cartan subalgebra: $C = g^3 T_3 + g^8 T_8$ (the diagonal)
Roots and root vectors:

$$\left[C, V_{g^{g\bar{b}}}\right] = i\left(\left(-\frac{1}{2}\right)g^3 + \left(\frac{\sqrt{3}}{2}\right)g^8\right) V_{g^{g\bar{b}}} \qquad V_{g^{g\bar{b}}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

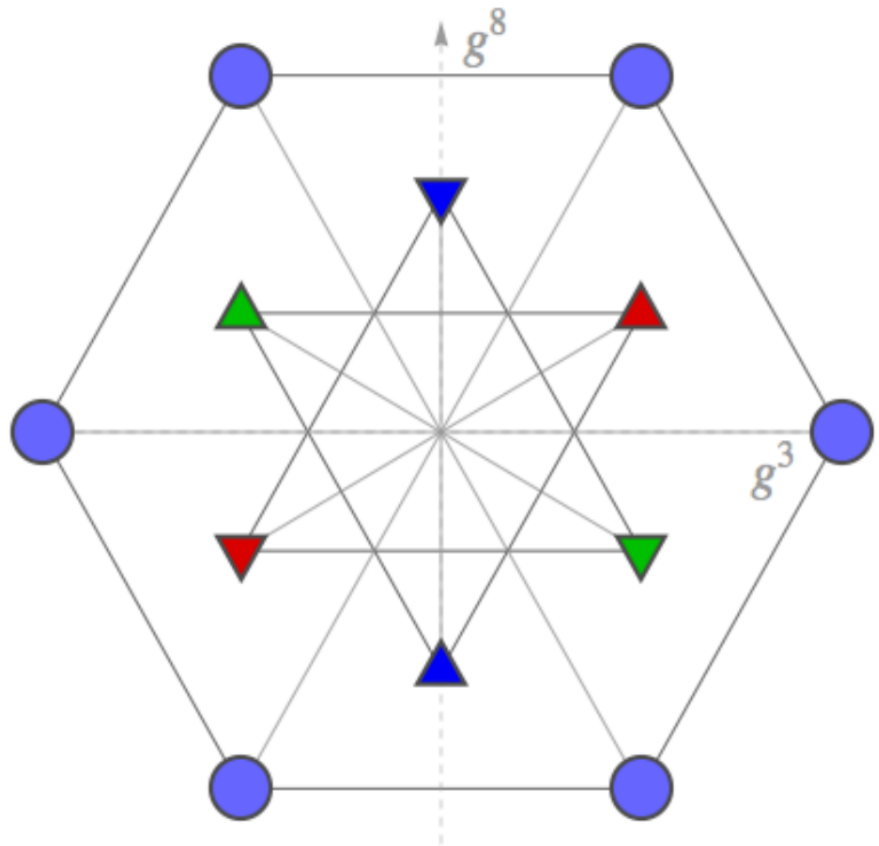
for the $g^{g\bar{b}}$ gluon. Weights and weight vectors:

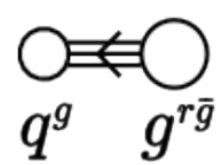
$$C\, V_{q^r} = i\left(\left(\frac{1}{2}\right)g^3 + \left(\frac{1}{2\sqrt{3}}\right)g^8\right) V_{q^r} \qquad V_{q^r} = [1, 0, 0]$$

for a red quark, q^r, and for their duals acted on by $-C^T$, the anti-quarks.

Strong G2

$G2$		V_β	g^3	g^8
●	$g^{r\bar{g}}$	$(T_2 - iT_1)$	1	0
●	$g^{\bar{r}g}$	$(-T_2 - iT_1)$	-1	0
●	$g^{r\bar{b}}$	$(T_5 - iT_4)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
●	$g^{\bar{r}b}$	$(-T_5 - iT_4)$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
●	$g^{\bar{g}b}$	$(-T_7 - iT_6)$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
●	$g^{g\bar{b}}$	$(T_7 - iT_6)$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
▲	q^r	$[1,0,0]$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$
▲	q^g	$[0,1,0]$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$
▲	q^b	$[0,0,1]$	0	$-\frac{1}{\sqrt{3}}$
▼	$\bar{q}^r$	$[1,0,0]$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$
▼	$\bar{q}^g$	$[0,1,0]$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$
▼	$\bar{q}^b$	$[0,0,1]$	0	$\frac{1}{\sqrt{3}}$





Exceptional Lie brackets

The 14 Lie algebra elements of the smallest exceptional Lie group, $G2$:

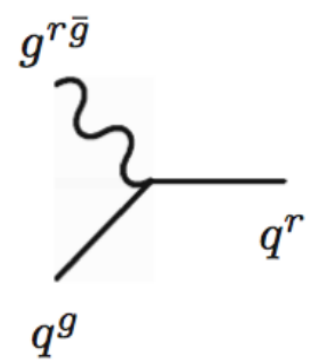
$$\begin{aligned} g2 = su(3) &+ 3 + \bar{3} \\ \underline{g} \quad &+ \underset{\cdot}{q} + \underset{\cdot}{\bar{q}} \in \underline{g}\underset{\cdot}{2} \end{aligned}$$

Structure of $G2$ implies Lie bracket equivalent to fundamental action,

$$[g, q] = \left[g^A T_A, q^B T_B\right] = g\, q = \begin{bmatrix} \frac{i}{2} g^3 + \frac{i}{2\sqrt{3}} g^8 & g^{r\bar{g}} & g^{r\bar{b}} \\ g^{\bar{r}g} & -\frac{i}{2} g^3 + \frac{i}{2\sqrt{3}} g^8 & g^{g\bar{b}} \\ g^{\bar{r}b} & g^{\bar{g}b} & -\frac{i}{\sqrt{3}} g^8 \end{bmatrix} \begin{bmatrix} q^r \\ q^g \\ q^b \end{bmatrix}$$

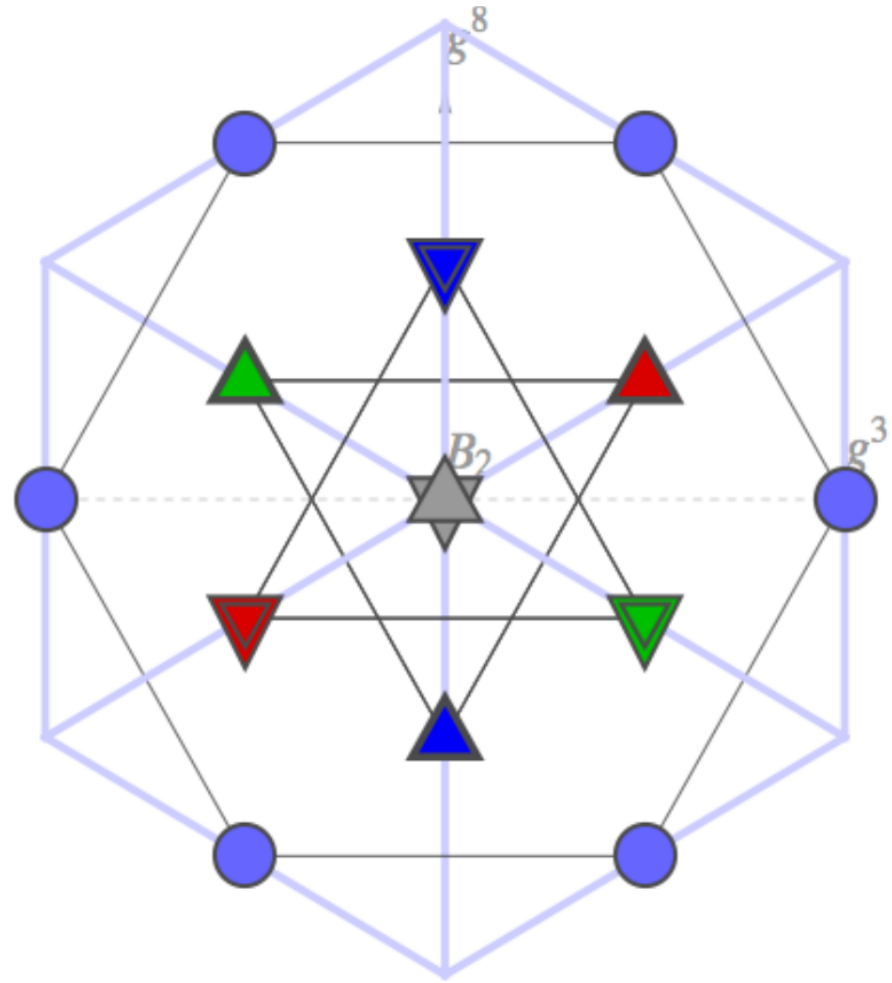
corresponding to the strong interactions, such as

$$\left[g^{r\bar{g}}, q^g\right] = q^r$$



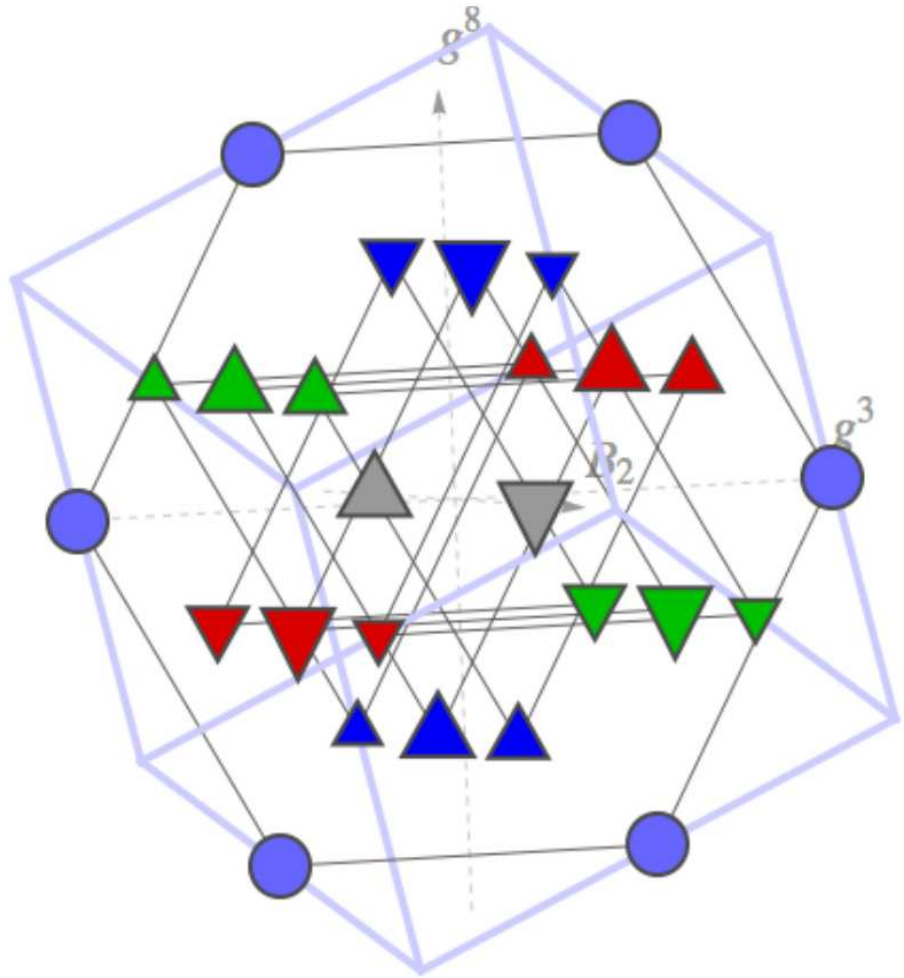
G2 in SO(7) .

$G2+$		x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
●	$g^{r\bar{g}}$	-1	1	0	1	0	0
●	$g^{\bar{r}g}$	1	-1	0	-1	0	0
●	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
▲	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
▼	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
✡	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
✡	q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
✡	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



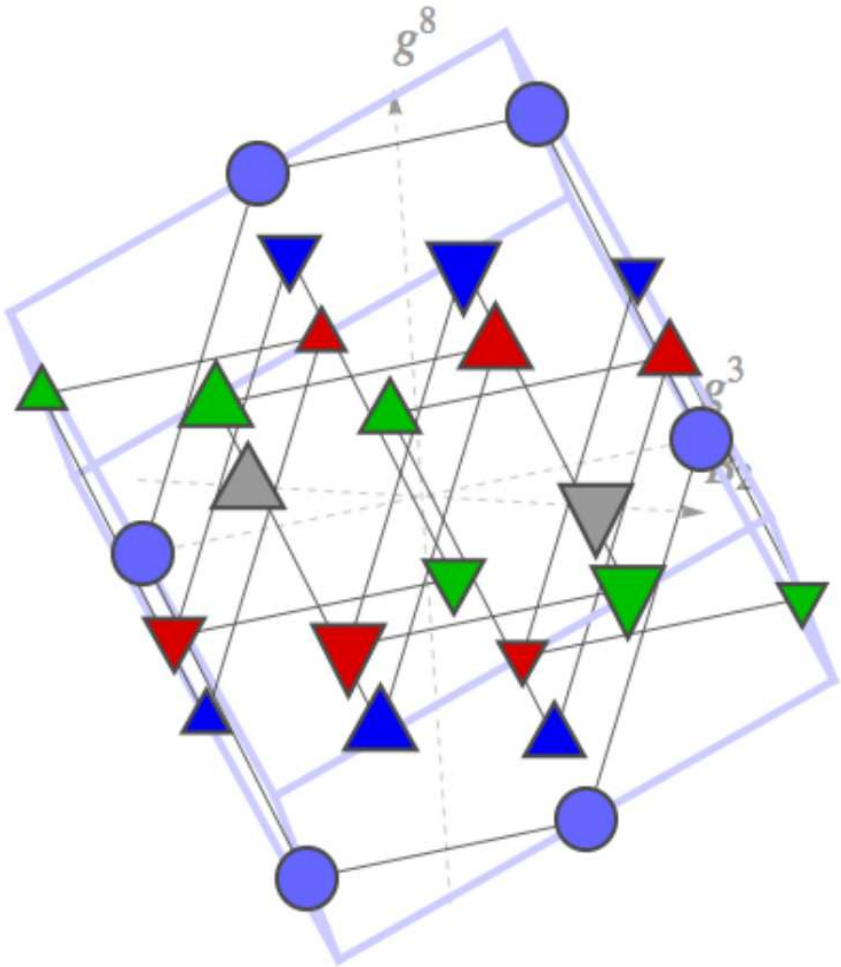
G2 in SO(7) ..

$G2+$		x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
●	$g^{r\bar{g}}$	-1	1	0	1	0	0
●	$g^{\bar{r}g}$	1	-1	0	-1	0	0
●	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
▲	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
▼	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
✶	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
✶	q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
✶	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



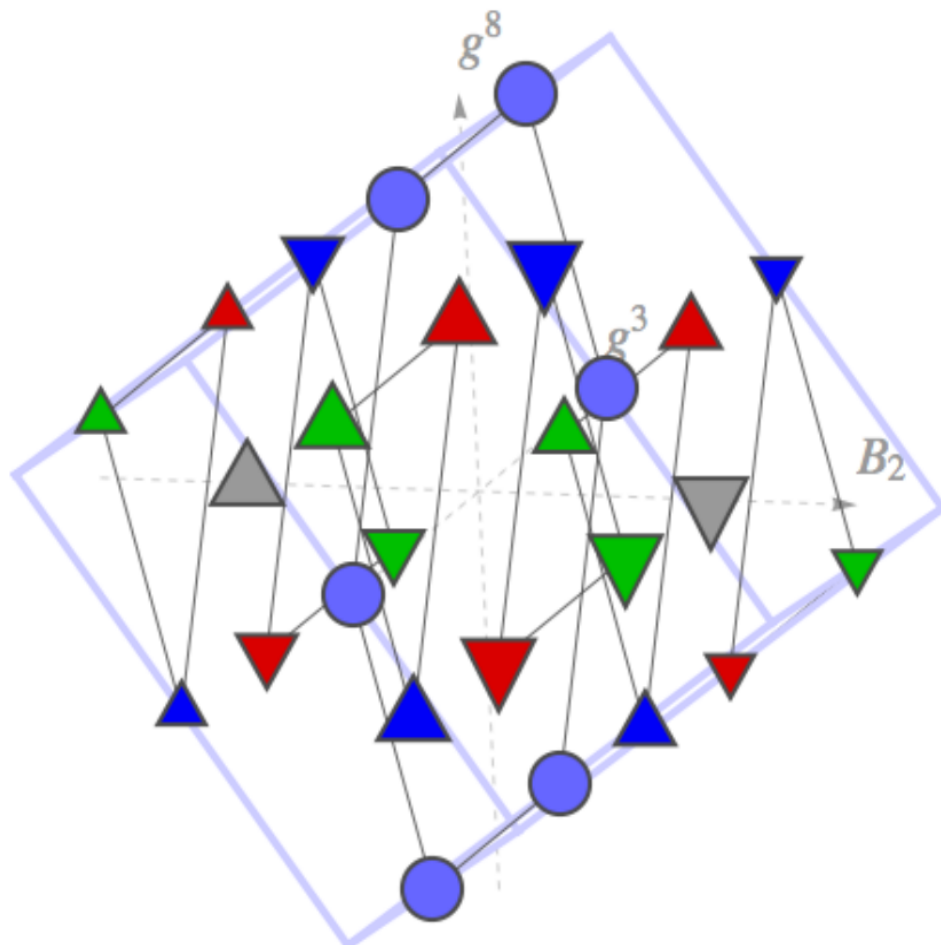
G2 in SO(7) ...

$G2+$		x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
●	$g^{r\bar{g}}$	-1	1	0	1	0	0
●	$g^{\bar{r}g}$	1	-1	0	-1	0	0
●	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
▲	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
▼	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
✡	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
✡	q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
✡	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



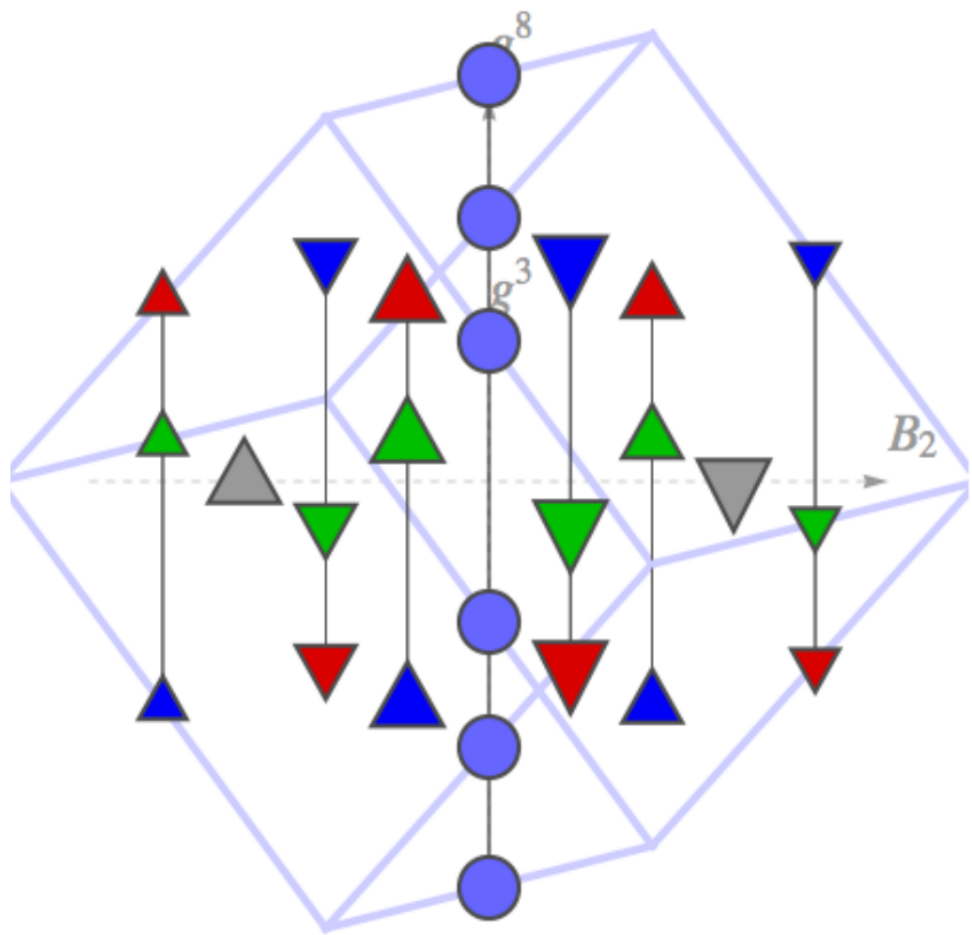
G2 in SO(7) ....

$G2+$		x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
●	$g^{r\bar{g}}$	-1	1	0	1	0	0
●	$g^{\bar{r}g}$	1	-1	0	-1	0	0
●	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
▲	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
▼	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
✡	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
✡	q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
✡	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1

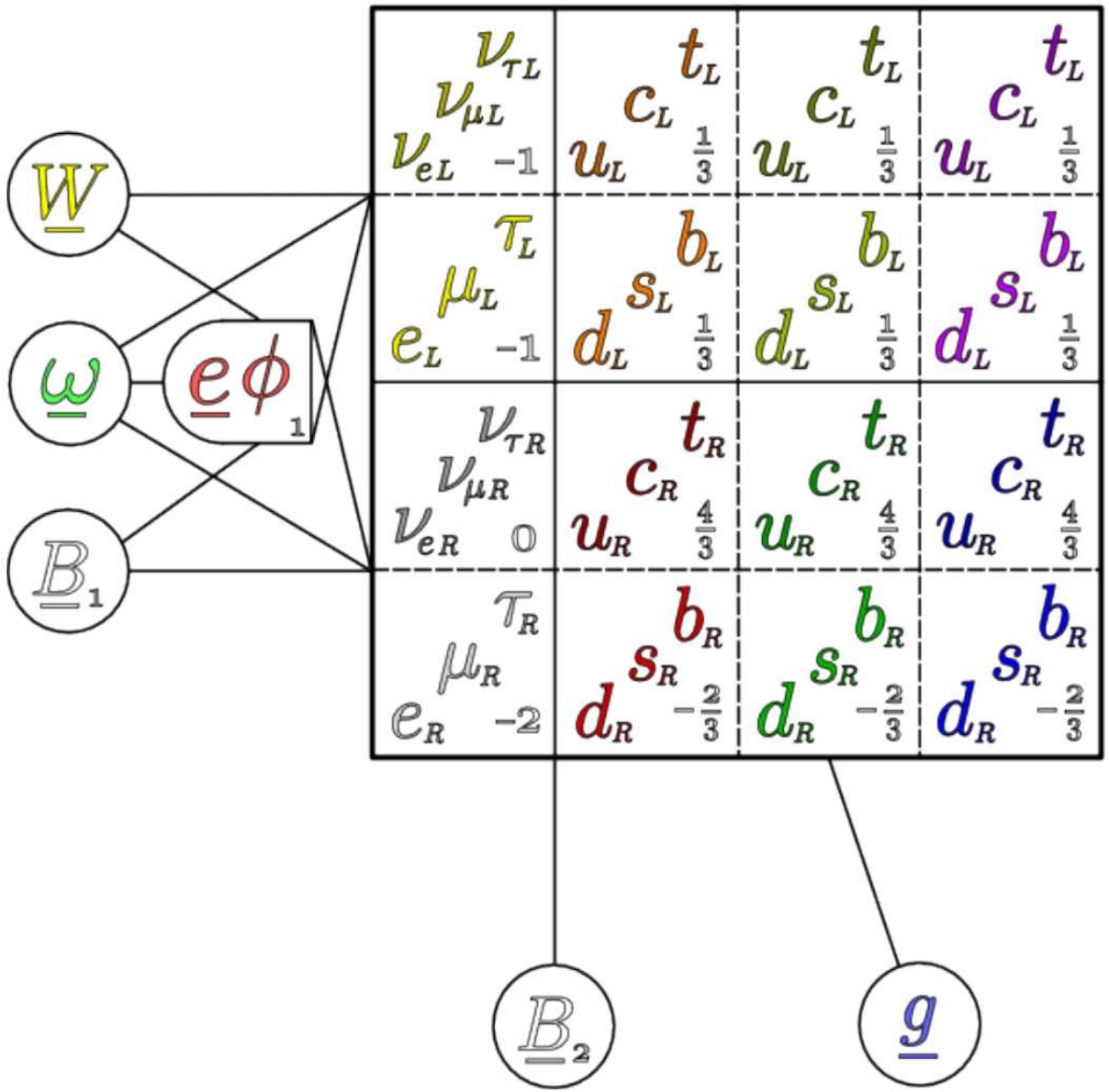


G2 in SO(7) x

$G2+$		x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
●	$g^{r\bar{g}}$	-1	1	0	1	0	0
●	$g^{\bar{r}g}$	1	-1	0	-1	0	0
●	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
●	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
▲	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
▲	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
▼	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
▼	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
✡	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
✡	q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
✡	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



Pati-Salam model plus gravity



$$\left(SO(3,1) + 4 \times 4 + SU(2)_L + SU(2)_R\right) + \left(U(1) + SU(3)\right)$$

Gravitational SO(3,1)

$$\omega = \tfrac{1}{2}\omega^{\mu\nu}\gamma_{\mu\nu} = \begin{bmatrix} \omega_L & \\ & \omega_R \end{bmatrix}$$

$$\omega_{L/R} = \begin{bmatrix} i\omega^3_{L/R} & \omega^\wedge_{L/R} \\ \omega^\vee_{L/R} & -i\omega^3_{L/R} \end{bmatrix}$$

$$e = e^\mu\gamma_\mu = \begin{bmatrix} & e_R \\ e_L & \end{bmatrix}$$

$$e_{L/R} = \begin{bmatrix} e^{\wedge/\vee}_T & \mp e^\wedge_S \\ \mp e^\vee_S & e^{\vee/\wedge}_T \end{bmatrix}$$

$$f = \begin{bmatrix} f_L \\ f_R \end{bmatrix} \qquad f_{L/R} = \begin{bmatrix} f^\wedge_{L/R} \\ f^\vee_{L/R} \end{bmatrix}$$

	$SO(3,1)$	$\frac{1}{2}\omega^3_L$	$\frac{1}{2}\omega^3_R$
●	$\omega^\wedge_L$	1	0
●	$\omega^\vee_L$	-1	0
●	$\omega^\wedge_R$	0	1
●	$\omega^\vee_R$	0	-1
■	$e^\wedge_S$	$\frac{1}{2}$	$\frac{1}{2}$
■	$e^\vee_S$	$-\frac{1}{2}$	$-\frac{1}{2}$
■	$e^\wedge_T$	$-\frac{1}{2}$	$\frac{1}{2}$
■	$e^\vee_T$	$\frac{1}{2}$	$-\frac{1}{2}$
△	$f^\wedge_L$	$\frac{1}{2}$	0
△	$f^\vee_L$	$-\frac{1}{2}$	0
▲	$f^\wedge_R$	0	$\frac{1}{2}$
▲	$f^\vee_R$	0	$-\frac{1}{2}$

Electroweak SU(2) and U(1)

$$W = \begin{bmatrix} \frac{i}{2}W^3 & W^+ \\ W^- & -\frac{i}{2}W^3 \end{bmatrix} \quad B_1 = \begin{bmatrix} \frac{i}{2}B_1^3 & B_1^+ \\ B_1^- & -\frac{i}{2}B_1^3 \end{bmatrix}$$

$$\left[\begin{bmatrix} W & \\ & B_1 \end{bmatrix}, \begin{bmatrix} & \phi_B \\ \phi_W & \end{bmatrix} \right]$$

$$\phi_{W/B} = \begin{bmatrix} -\phi_{0/1} & \phi_+ \\ \phi_- & \phi_{1/0} \end{bmatrix}$$

$$\begin{bmatrix} W & \\ & B_1 \end{bmatrix} \quad \begin{bmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{bmatrix} \quad \begin{bmatrix} u_L \\ d_L \\ u_R \\ d_R \end{bmatrix}$$

$$\left(\frac{\sqrt{3}}{\sqrt{5}}B_1^3 - \frac{\sqrt{2}}{\sqrt{5}}B_2\right) = \left(\frac{\sqrt{3}}{\sqrt{5}}\right)\frac{1}{2}Y \;\rightarrow\; g_1 = \frac{\sqrt{3}}{\sqrt{5}}$$

$SO(4)$		W^3	B_1^3	$\frac{\sqrt{2}}{\sqrt{3}}B_2$	$\frac{1}{2}Y$	Q
●	W^+	1	0	0	0	1
●	W^-	-1	0	0	0	-1
●	B_1^+	0	1	0	1	1
●	B_1^-	0	-1	0	-1	-1
■	ϕ_+	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	1
◆	ϕ_-	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
■	ϕ_0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0
◆	ϕ_1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
▲	ν_{eL}	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
▲	e_L	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	-1
▲	ν_{eR}	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
▲	e_R	0	$-\frac{1}{2}$	$\frac{1}{2}$	-1	-1
▲▲	u_L	$\frac{1}{2}$	0	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
▲▲	d_L	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
▲▲	u_R	0	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
▲▲	d_R	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$

Graviweak SO(7,1)

$$H_1 = (\tfrac{1}{2}\omega + \tfrac{1}{4}e\phi + W + B_1)$$

$$\in so(3,1) + 4\times 4 + \big(su(2) + su(2)\big)$$

$$= Cl^2(7,1) = so(7,1) = d4$$

$$8_{S+} \quad \rightarrow \quad H_1\,(\nu_e + e)$$

$$=$$

$$\begin{bmatrix} \frac{1}{2}\omega_L + \frac{i}{2}W^3 & W^+ & -\frac{1}{4}e_R\phi_1 & \frac{1}{4}e_R\phi_+ \\ W^- & \frac{1}{2}\omega_L - \frac{i}{2}W^3 & \frac{1}{4}e_R\phi_- & \frac{1}{4}e_R\phi_0 \\ -\frac{1}{4}e_L\phi_0 & \frac{1}{4}e_L\phi_+ & \frac{1}{2}\omega_R + \frac{i}{2}B_1^3 & B_1^+ \\ \frac{1}{4}e_L\phi_- & \frac{1}{4}e_L\phi_1 & B_1^- & \frac{1}{2}\omega_R - \frac{i}{2}B_1^3 \end{bmatrix} \begin{bmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{bmatrix}$$

$D4$		$\frac{1}{2}\omega_L^3$	$\frac{1}{2}\omega_R^3$	W^3	B_1^3
●	$\omega_L^{\wedge/\vee}$	± 1	0	0	0
●	$\omega_R^{\wedge/\vee}$	0	± 1	0	0
●	$W^{\pm}$	0	0	± 1	0
●	$B_1^{\pm}$	0	0	0	± 1
■	$e_T^{\wedge/\vee}\phi_+$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
■	$e_S^{\wedge/\vee}\phi_+$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
◆	$e_T^{\wedge/\vee}\phi_-$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
◆	$e_S^{\wedge/\vee}\phi_-$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
■	$e_T^{\wedge/\vee}\phi_0$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
■	$e_S^{\wedge/\vee}\phi_0$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
◆	$e_T^{\wedge/\vee}\phi_1$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
◆	$e_S^{\wedge/\vee}\phi_1$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
▲	$\nu_{eL}^{\wedge/\vee}$	$\pm\frac{1}{2}$	0	$\frac{1}{2}$	0
▲	$e_L^{\wedge/\vee}$	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$	0
▲	$\nu_{eR}^{\wedge/\vee}$	0	$\pm\frac{1}{2}$	0	$\frac{1}{2}$
▲	$e_R^{\wedge/\vee}$	0	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$

Graviweak F4

A **triality** rotation, T, of $D4$:

$$\begin{bmatrix} \frac{1}{2}\omega'^3_L \\ \frac{1}{2}\omega'^3_R \\ W'^3 \\ B'^3_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\omega^3_L \\ \frac{1}{2}\omega^3_R \\ W^3 \\ B^3_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\omega^3_R \\ B^3_1 \\ W^3 \\ \frac{1}{2}\omega^3_L \end{bmatrix}$$

$$T\,T\,T\,\omega^\wedge_R = T\,T\,\omega^\wedge_L = T\,B^+_1 = \omega^\wedge_R$$

8_V		$\frac{1}{2}\omega^3_L$	$\frac{1}{2}\omega^3_R$	W^3	B^3_1
tri		$\frac{1}{2}\omega^3_R$	B^3_1	W^3	$\frac{1}{2}\omega^3_L$
▲	$\nu^{\wedge/\vee}_{\mu L}$	0	0	$\frac{1}{2}$	$\pm\frac{1}{2}$
▲	$\mu^{\wedge/\vee}_{L}$	0	0	$-\frac{1}{2}$	$\pm\frac{1}{2}$
▲	$\nu^{\wedge/\vee}_{\mu R}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	0	0
▲	$\mu^{\wedge/\vee}_{R}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	0	0

Roots invariant under this T:

$$\{W^+,\, W^-,\, e^\wedge_S\phi_+,\, e^\wedge_S\phi_0,\, e^\vee_S\phi_-,\, e^\vee_S\phi_1\}$$

Rotations to triality-equivalent vector and negative chiral spinor representation spaces:

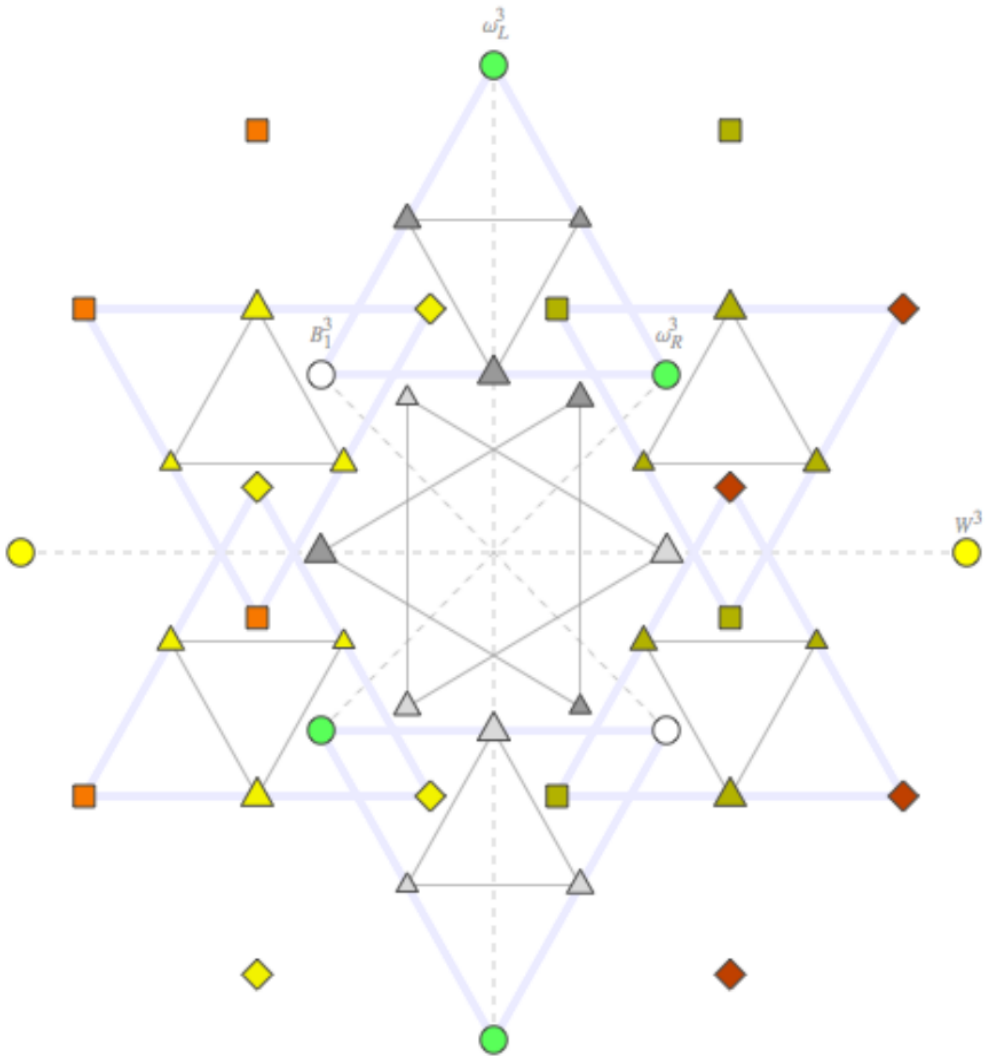
$$T\,8_{S+} = 8_V \qquad T\,8_V = 8_{S-} \qquad T\,8_{S-} = 8_{S+}$$

8_{S-}		$\frac{1}{2}\omega^3_L$	$\frac{1}{2}\omega^3_R$	W^3	B^3_1
tri		B^3_1	$\frac{1}{2}\omega^3_L$	W^3	$\frac{1}{2}\omega^3_R$
▲	$\nu^{\wedge/\vee}_{\tau L}$	0	$\pm\frac{1}{2}$	$\frac{1}{2}$	0
▲	$\tau^{\wedge/\vee}_{L}$	0	$\pm\frac{1}{2}$	$-\frac{1}{2}$	0
▲	$\nu^{\wedge/\vee}_{\tau R}$	$\frac{1}{2}$	0	0	$\pm\frac{1}{2}$
▲	$\tau^{\wedge/\vee}_{R}$	$-\frac{1}{2}$	0	0	$\pm\frac{1}{2}$

Three generations, related by triality:

$$T\,e^\wedge_L = \mu^\wedge_L \qquad T\,\mu^\wedge_L = \tau^\wedge_L \qquad T\,\tau^\wedge_L = e^\wedge_L$$

F4 root system



F4 and G2 together

$$F4 \quad : \quad (\tfrac{1}{2}\omega_L^3, \tfrac{1}{2}\omega_R^3, W^3, B_1^3) \qquad \left\{ \begin{array}{l} \text{graviweak interactions} \\ \text{three generations} \end{array} \right.$$

$$G2 \quad : \quad (B_2, g^3, g^8) \qquad \left\{ \begin{array}{l} \text{strong interactions} \\ \text{anti-particles} \end{array} \right.$$

$$E8 \quad : \quad (\tfrac{1}{2}\omega_L^3, \tfrac{1}{2}\omega_R^3, W^3, B_1^3, w, B_2, g^3, g^8) \quad \{ \text{ everything}$$

Breakdown of E8 to the standard model and gravity:

$$e8 = f4 + g2 + 26 \times 7$$

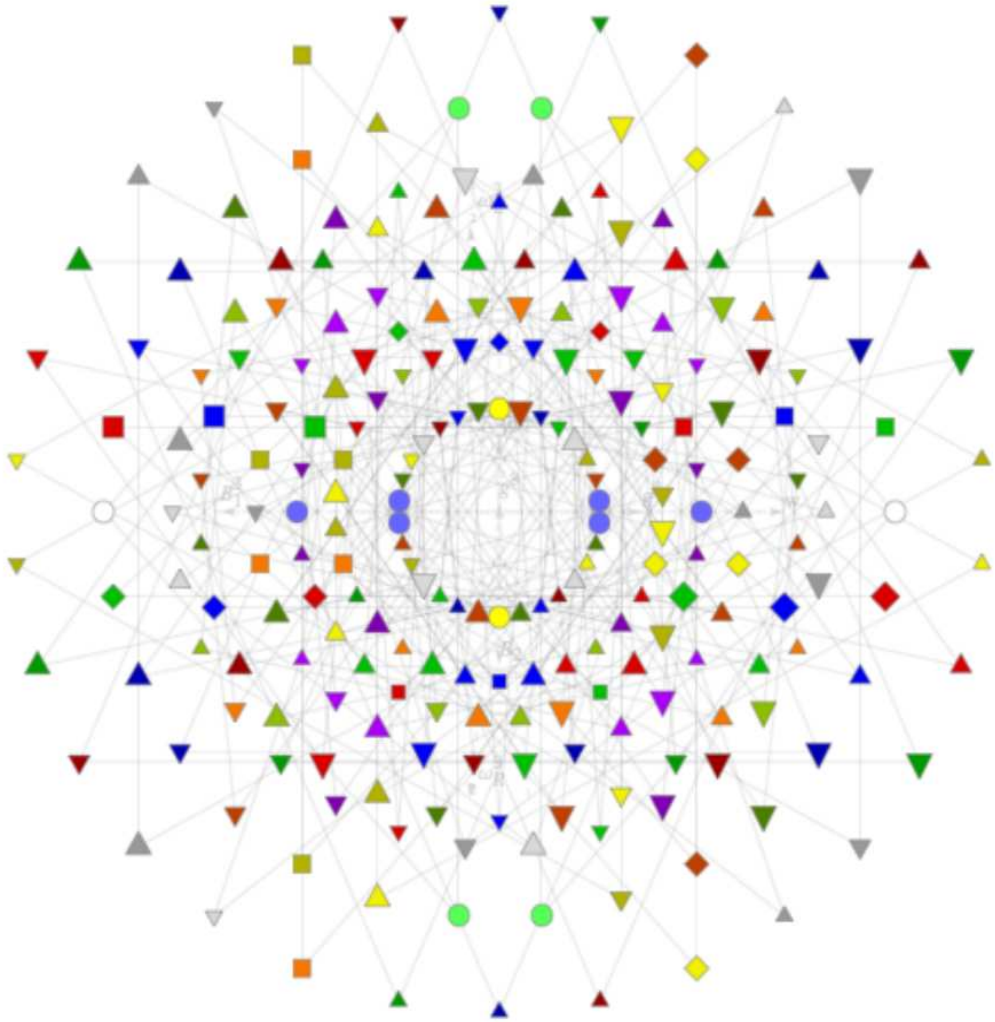
$$= so(7,1) + su(3) + (8_{S+} + 8_V + 8_{S-}) \times (1+1+3+\bar{3}) + 3 \times (3+\bar{3}) + 2$$

$$A = \left(\tfrac{1}{2}\omega + \tfrac{1}{4}e\phi + W + B_1\right) + g + 3 \times \Psi + x\Phi + B_2 + w$$

Two new quantum numbers and some non-standard particles:

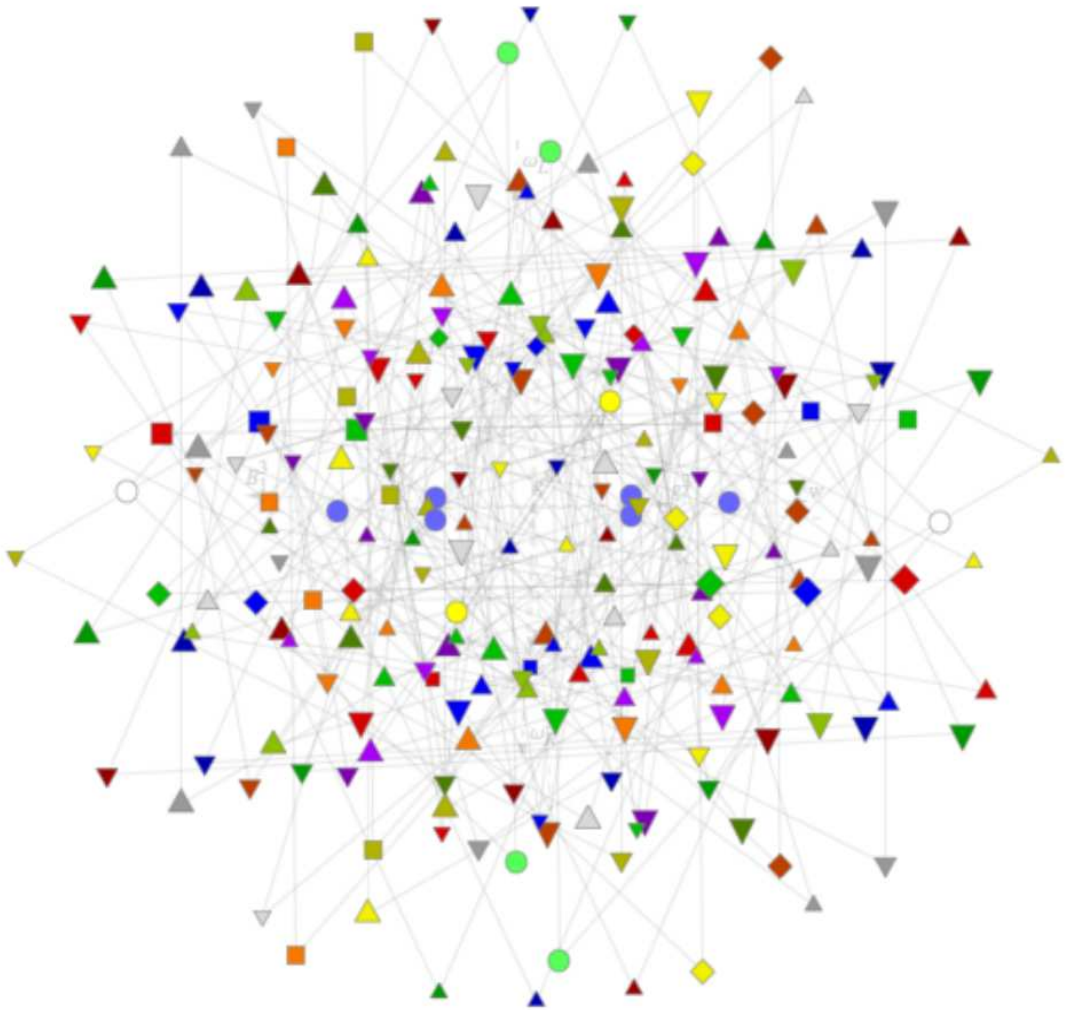
$$\{ \; w \quad (B_1^3 + B_2) \quad B_1^{\pm} \quad x_{1/2/3}\Phi^{r/g/b} \quad x_{1/2/3}\Phi^{\bar{r}/\bar{g}/\bar{b}} \; \}$$

E8 root system .



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ..



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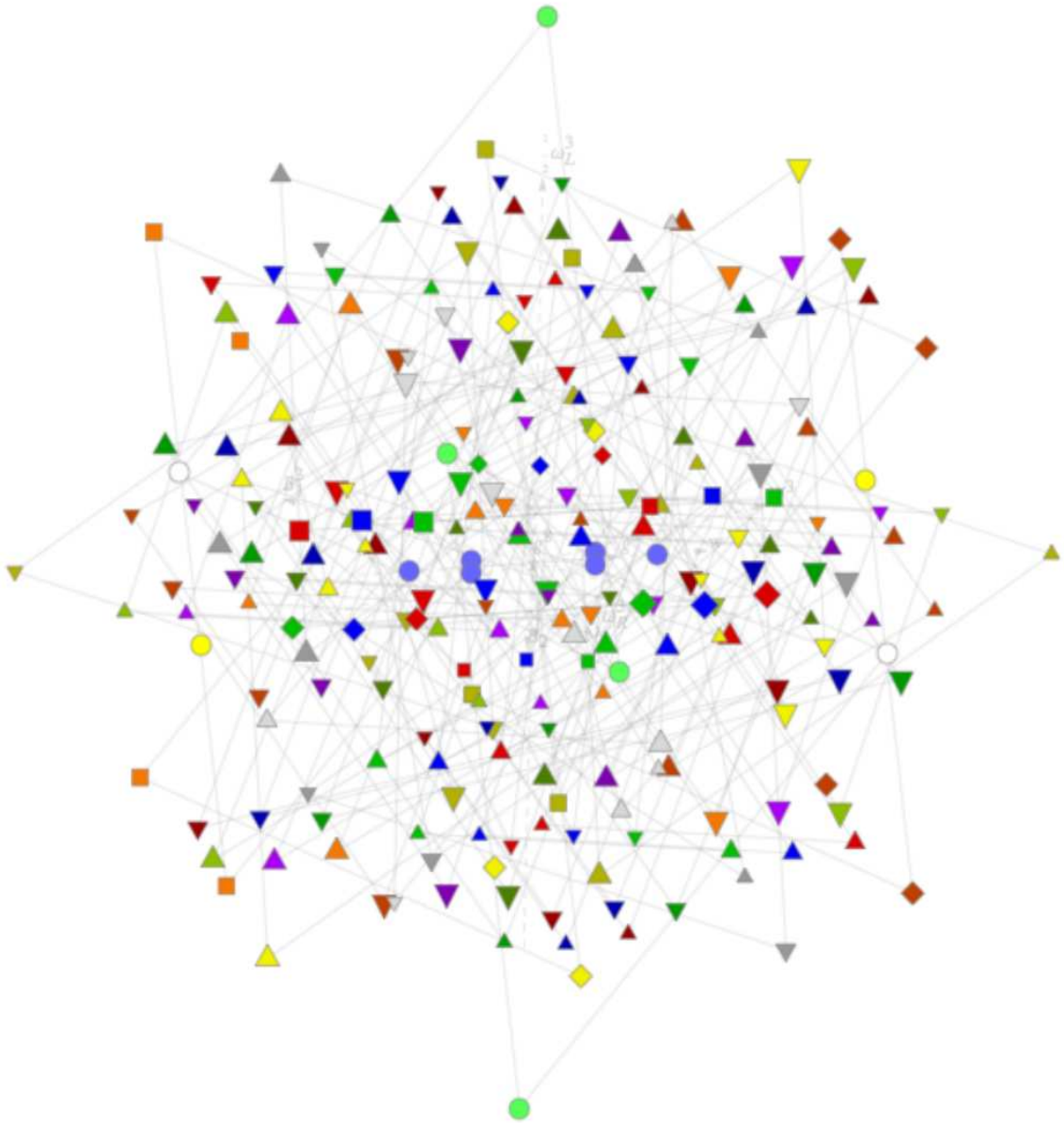
E8 root system ...



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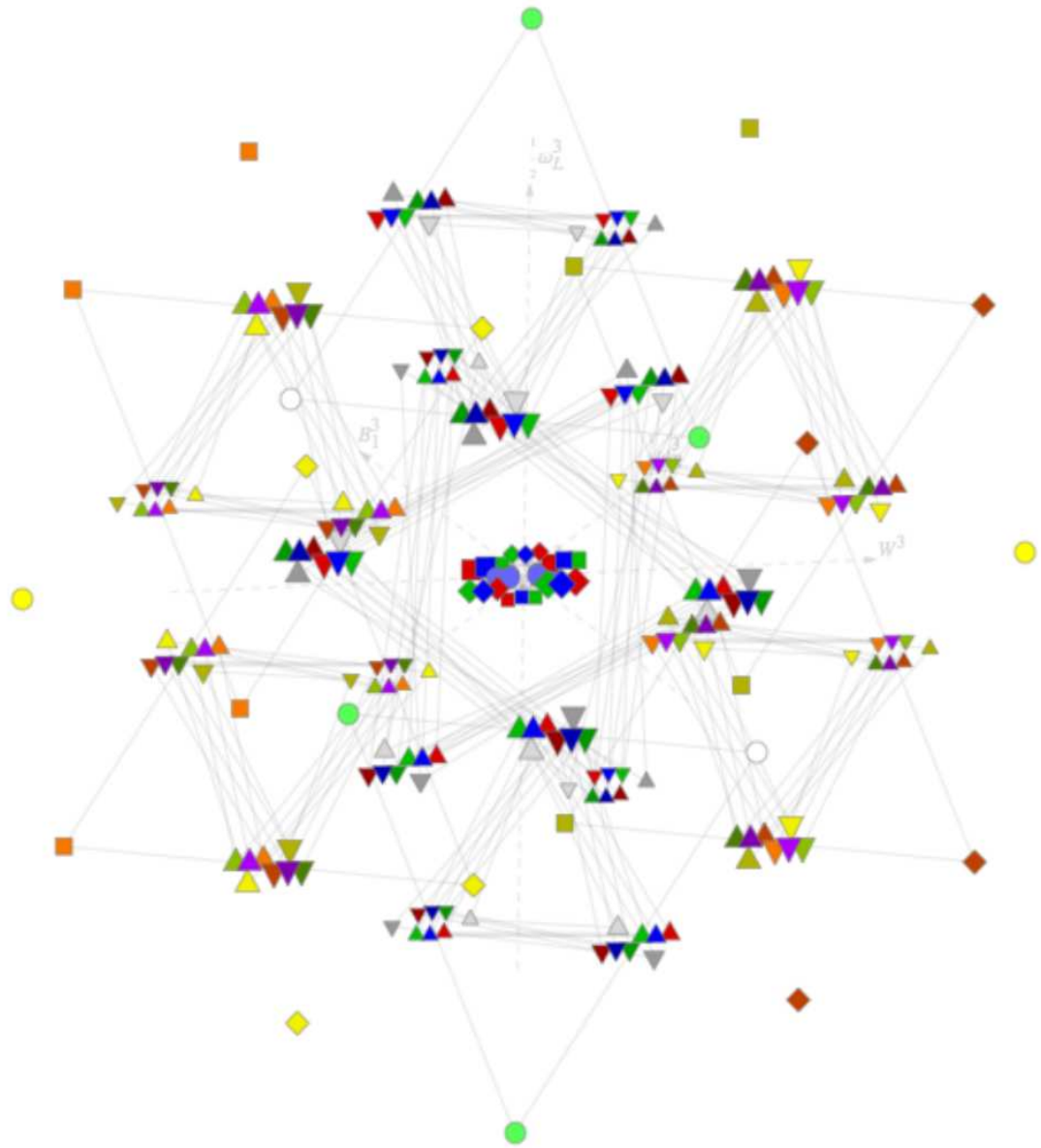
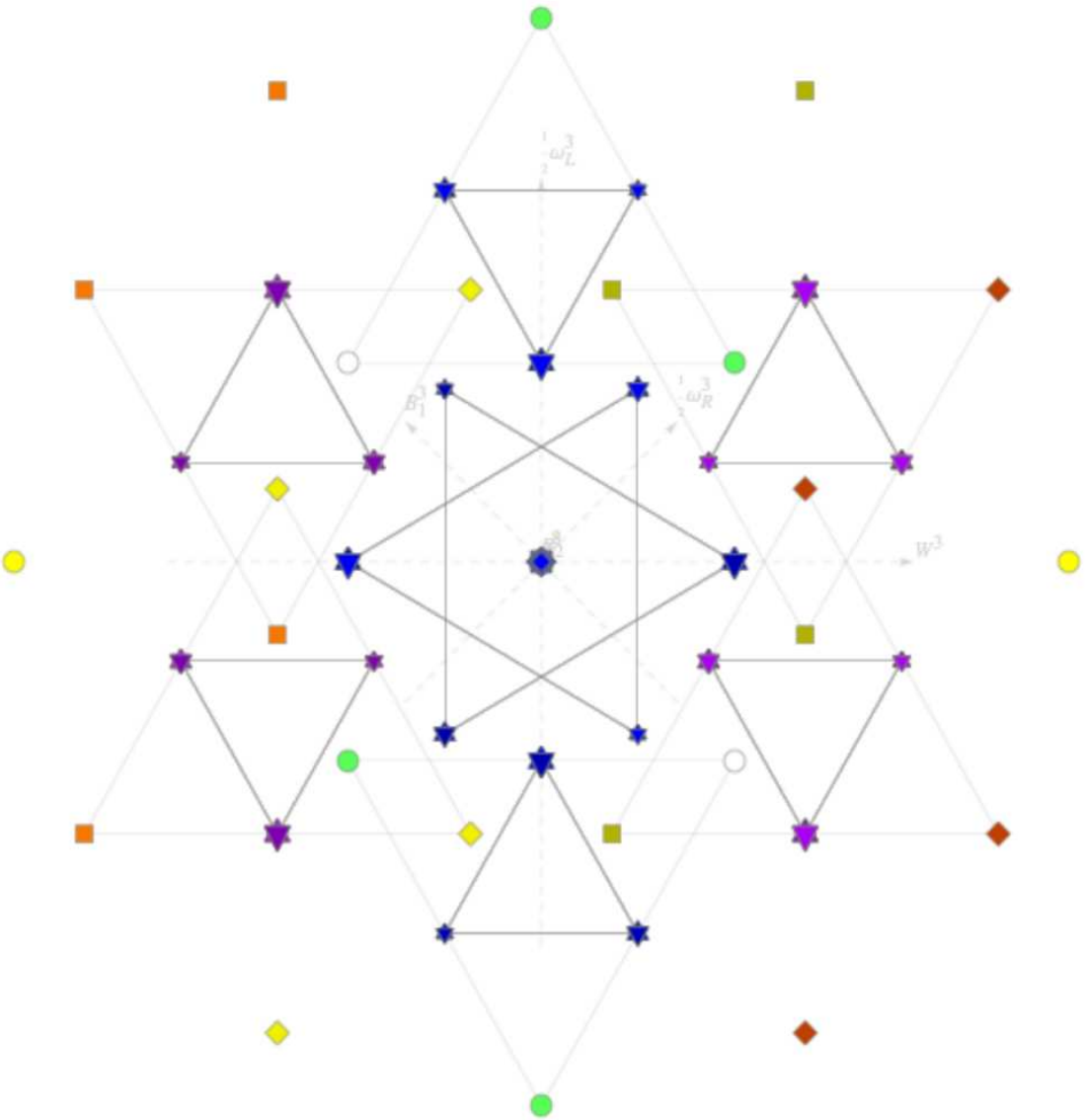
E8 root system ....



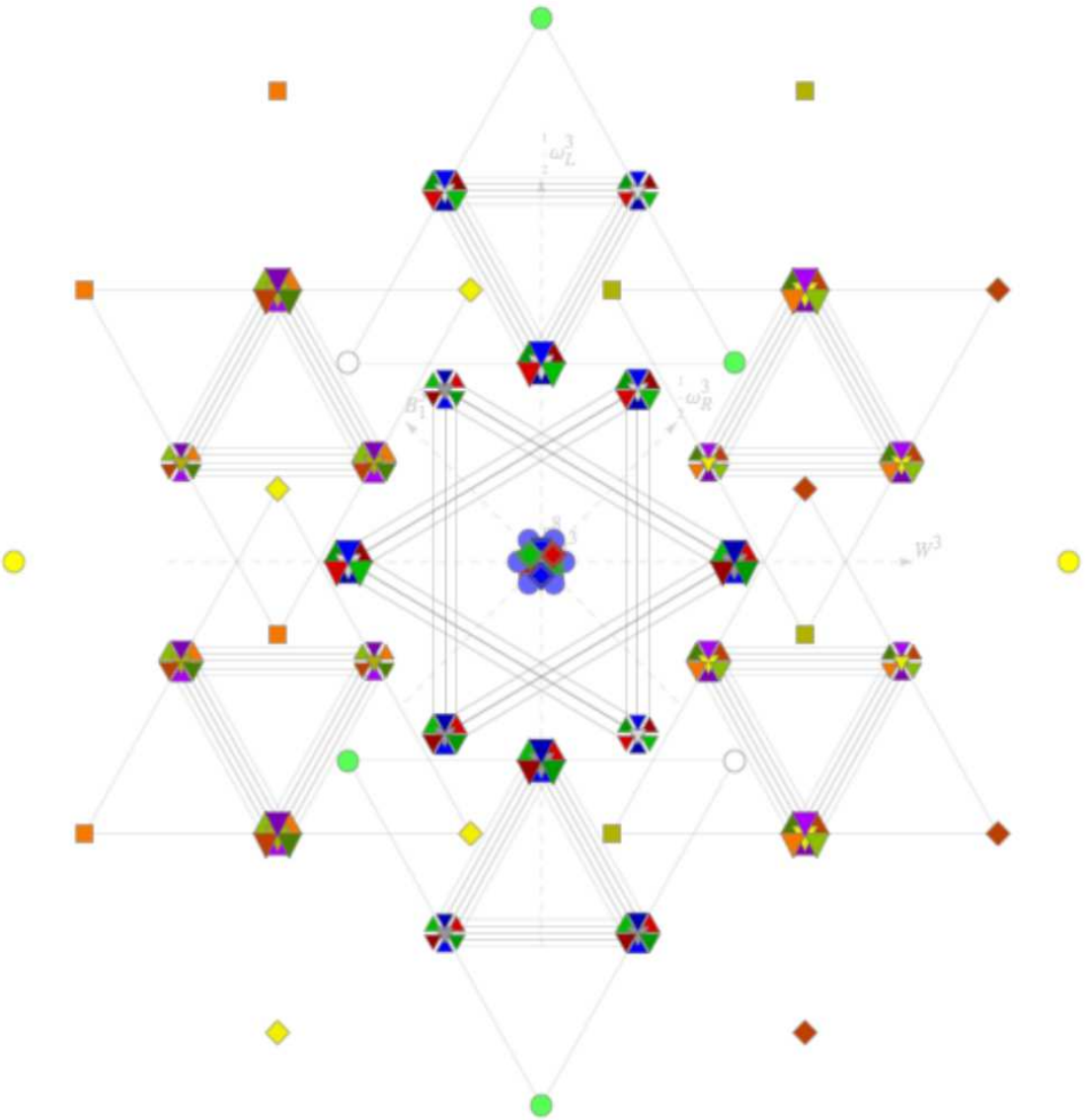
E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system x



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.



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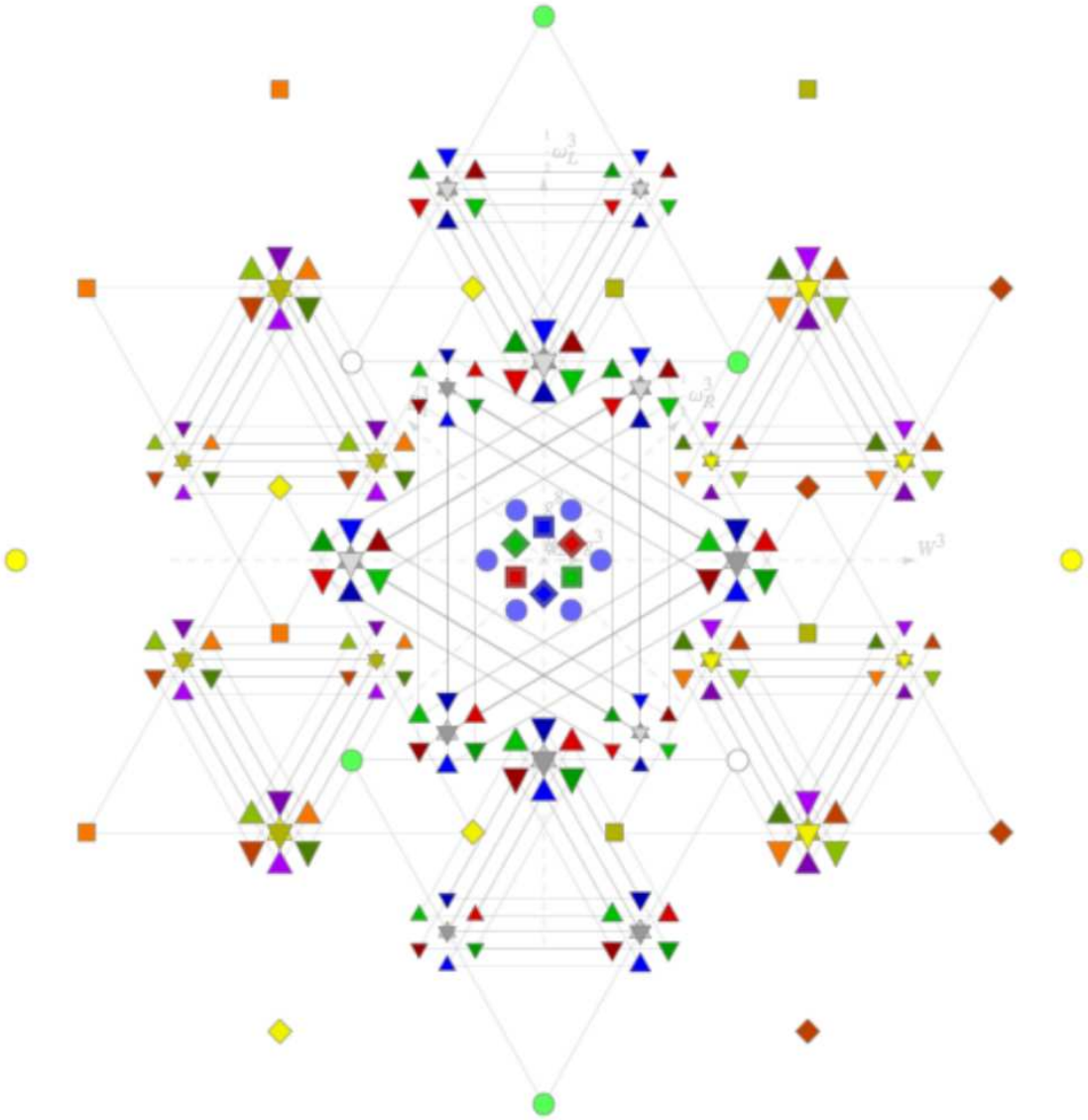
E8 root system ....x.x



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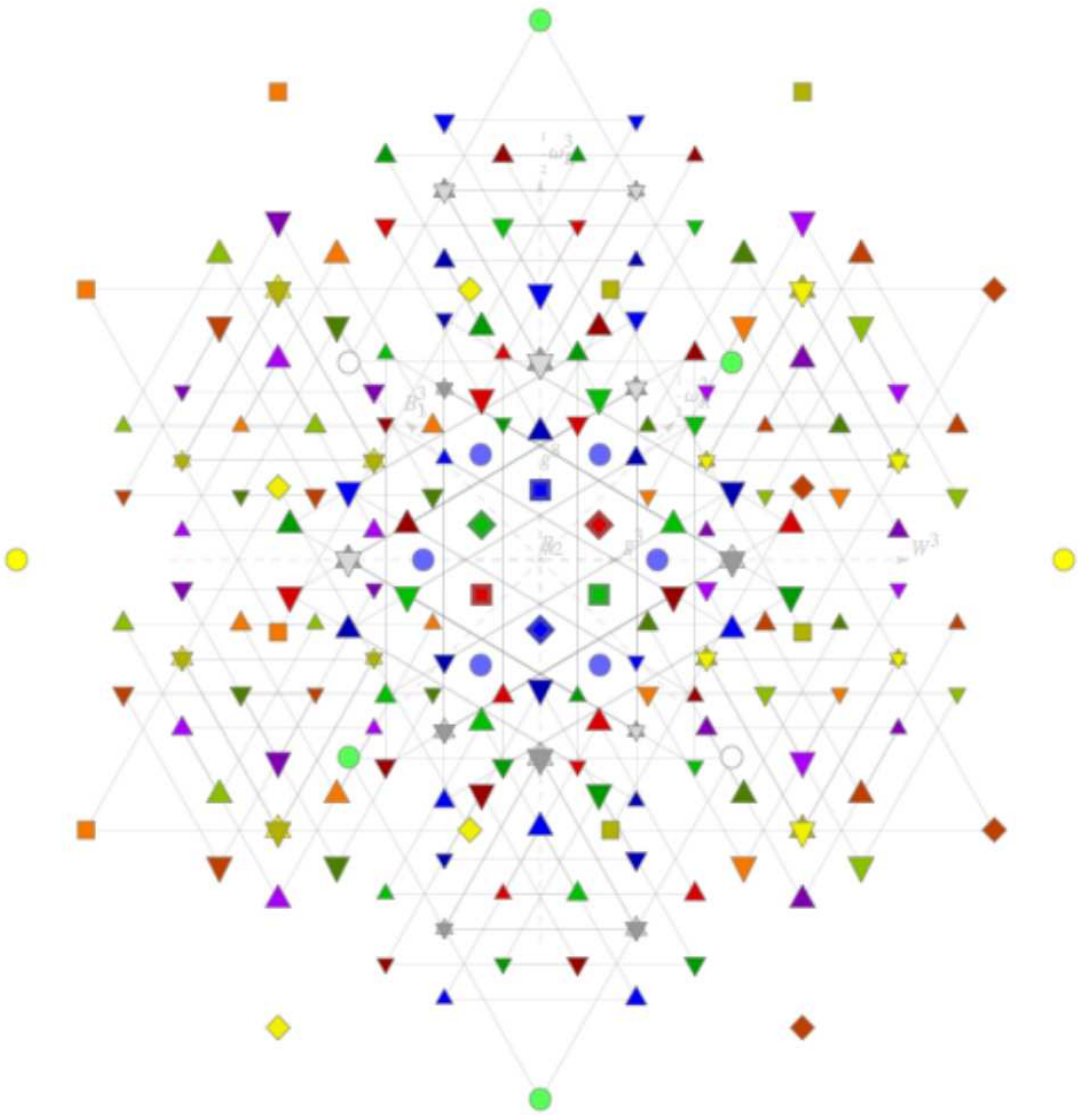
E8 root system ....x.x.



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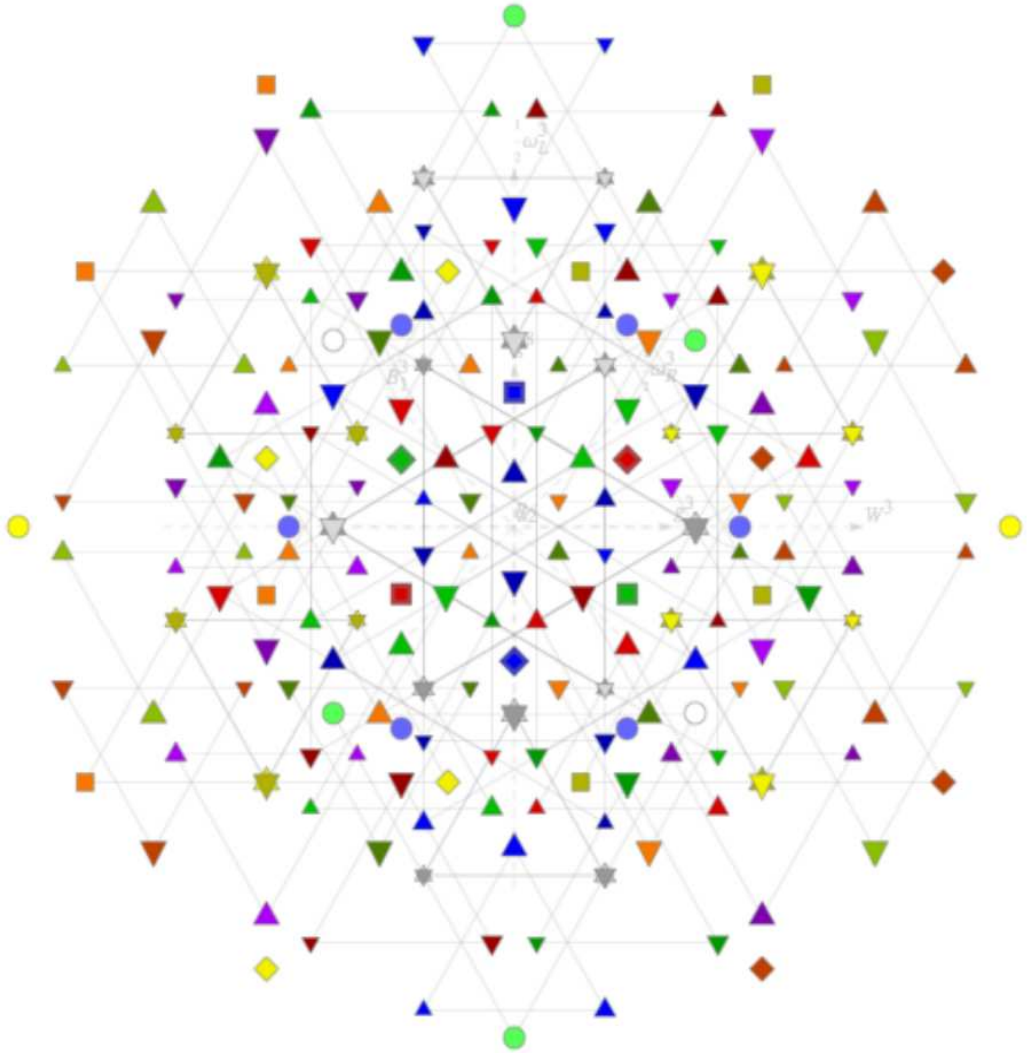
E8 root system ....x.x..



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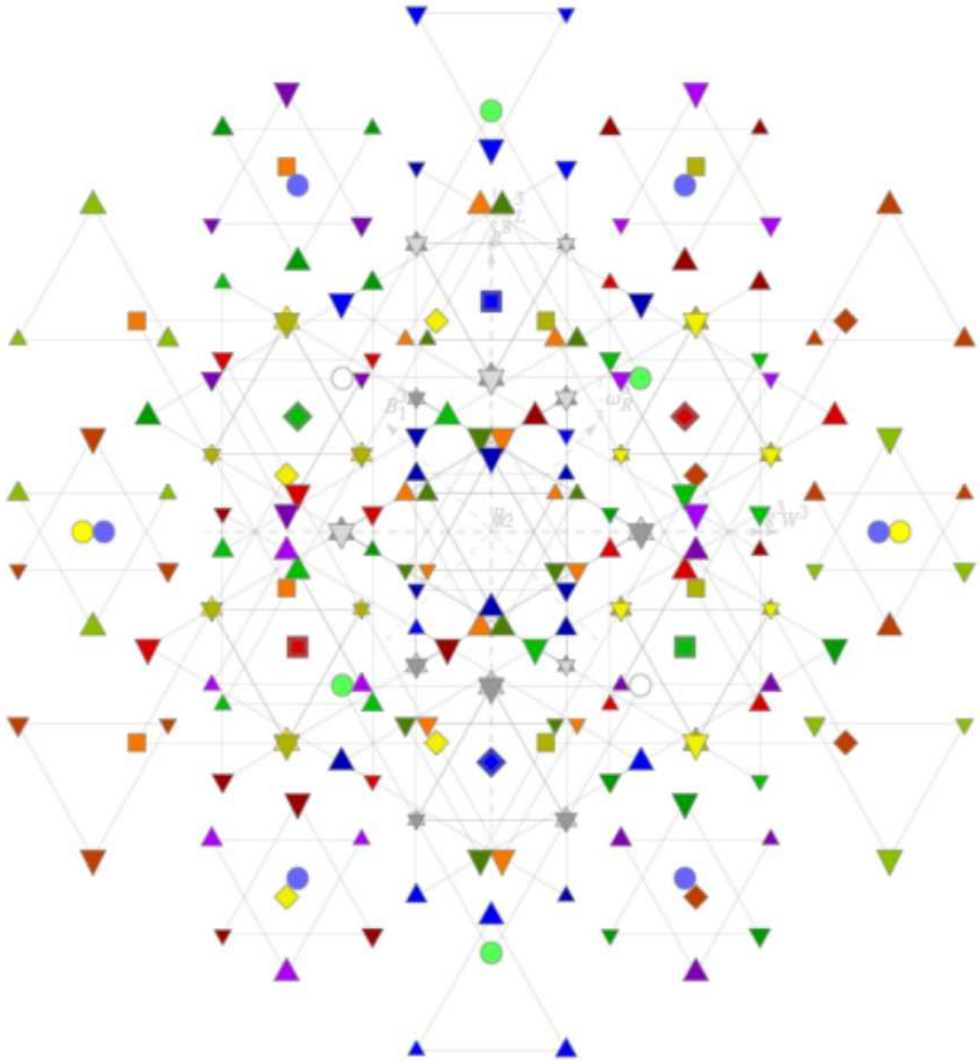
E8 root system ....x.x...



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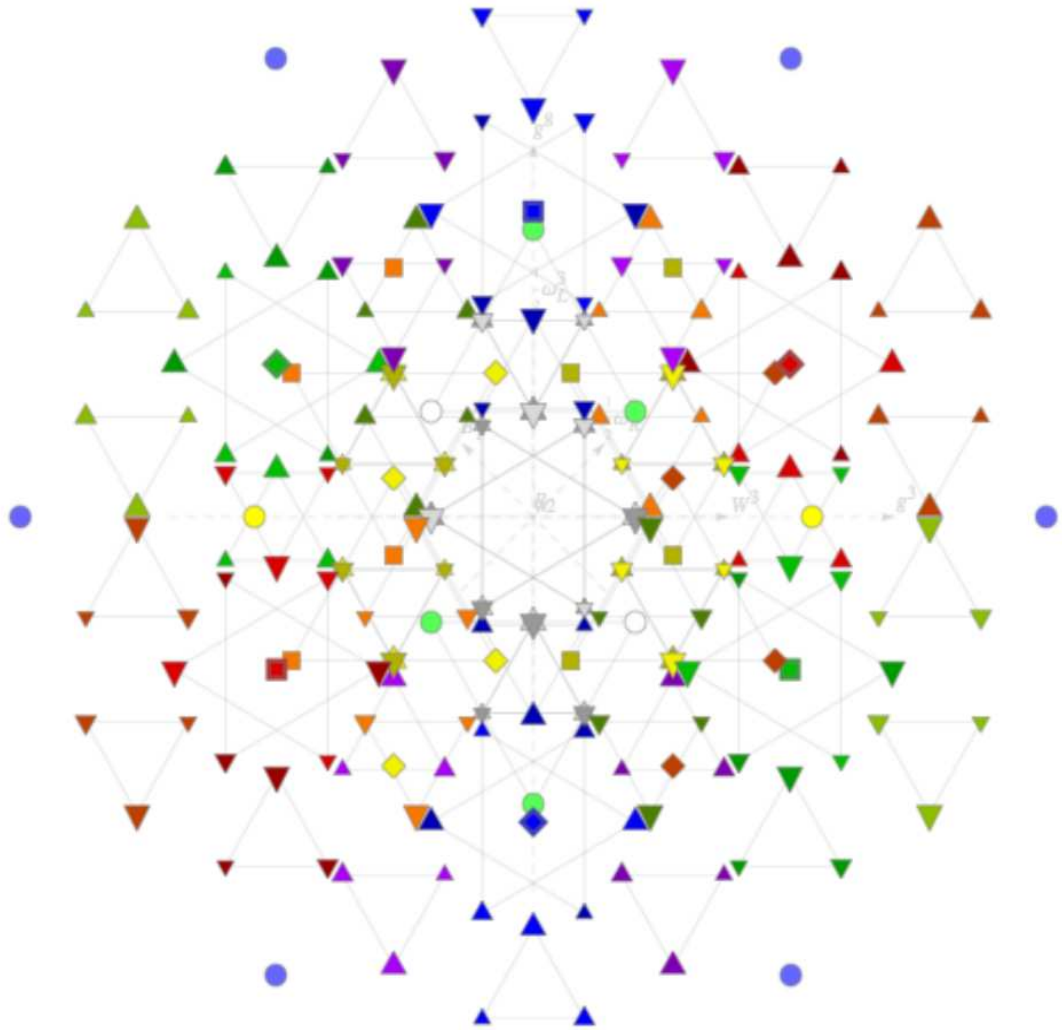
E8 root system ....x.x....



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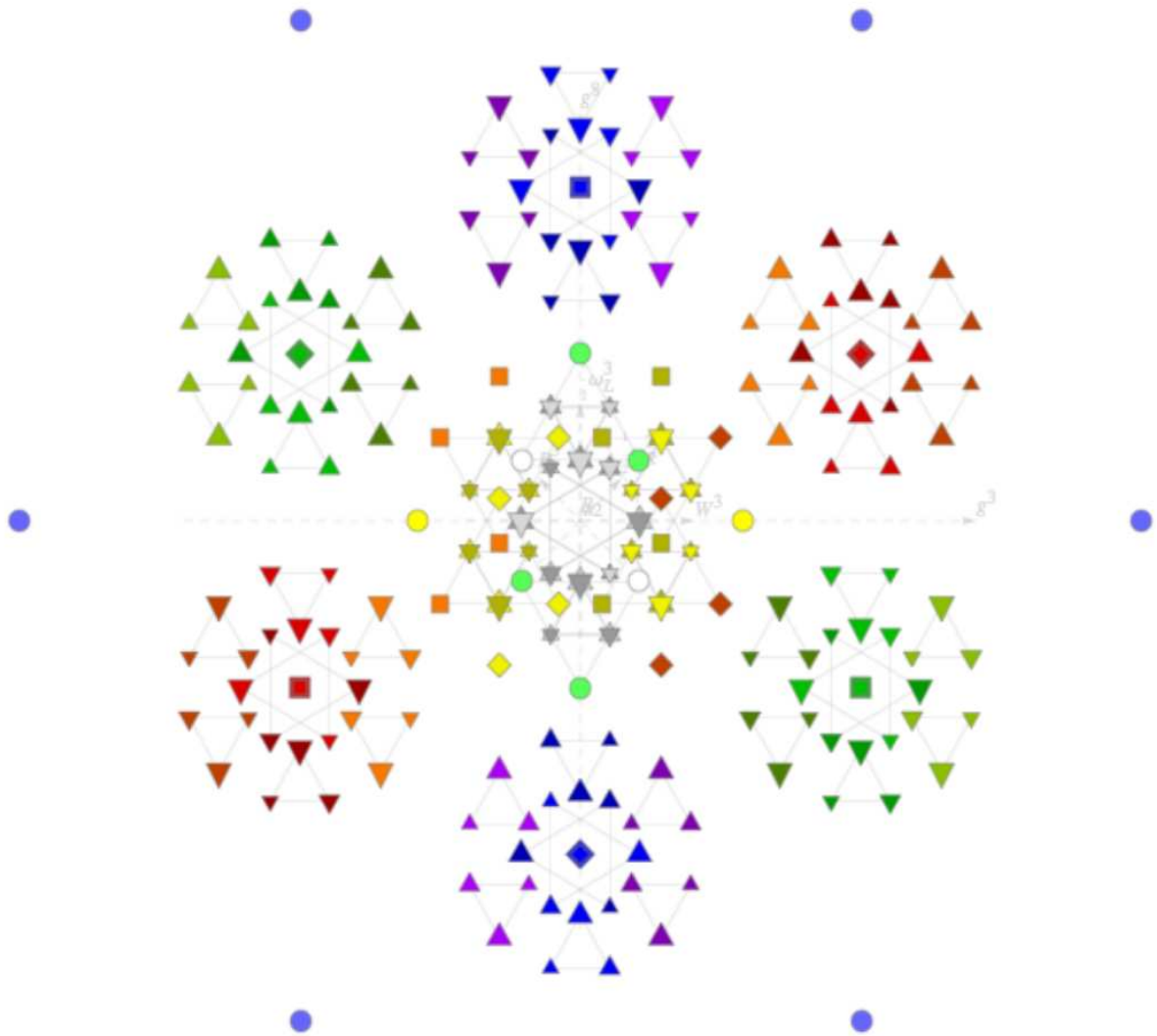
E8 root system ....x.x....x



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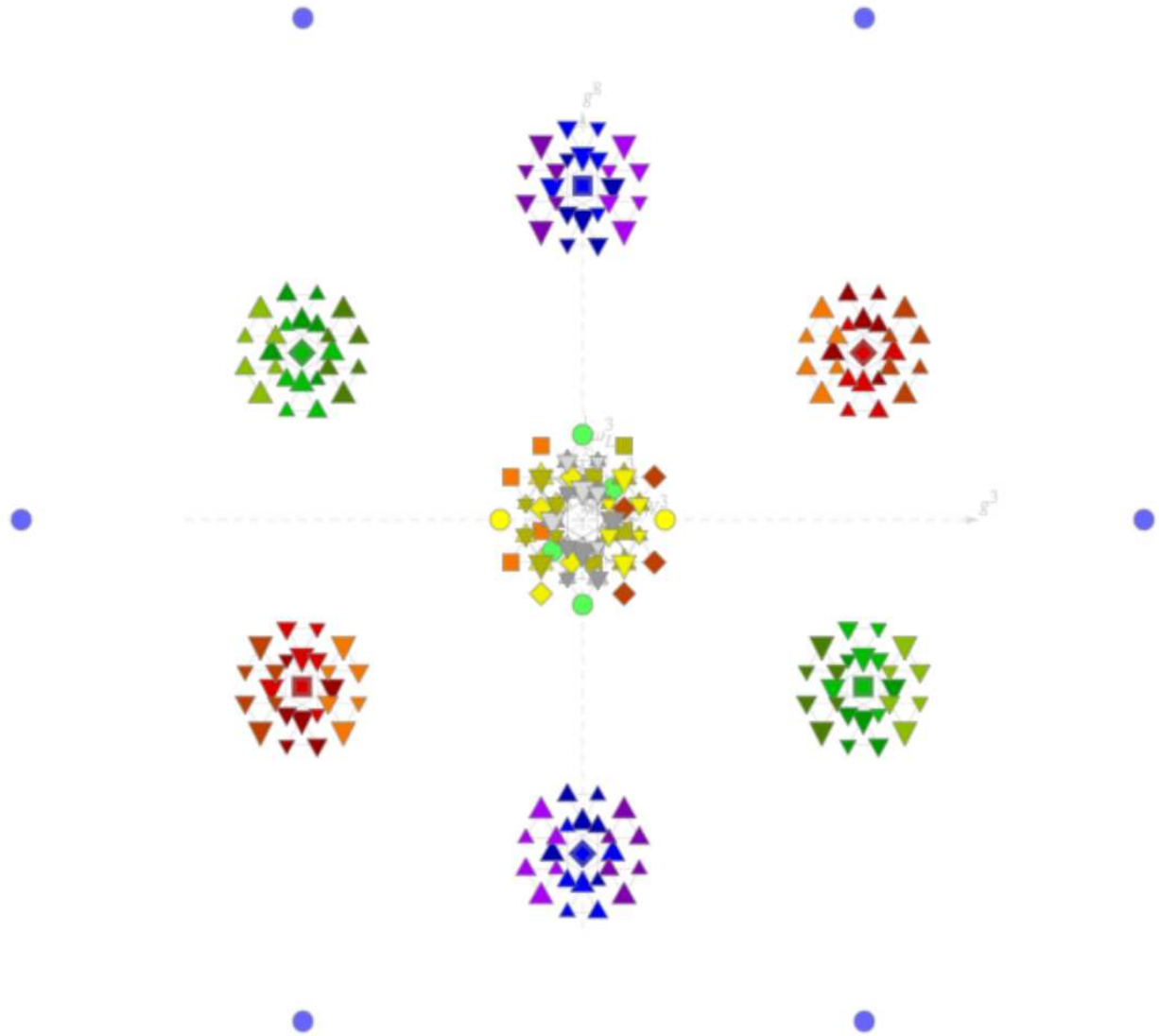
E8 root system ....x.x....x.



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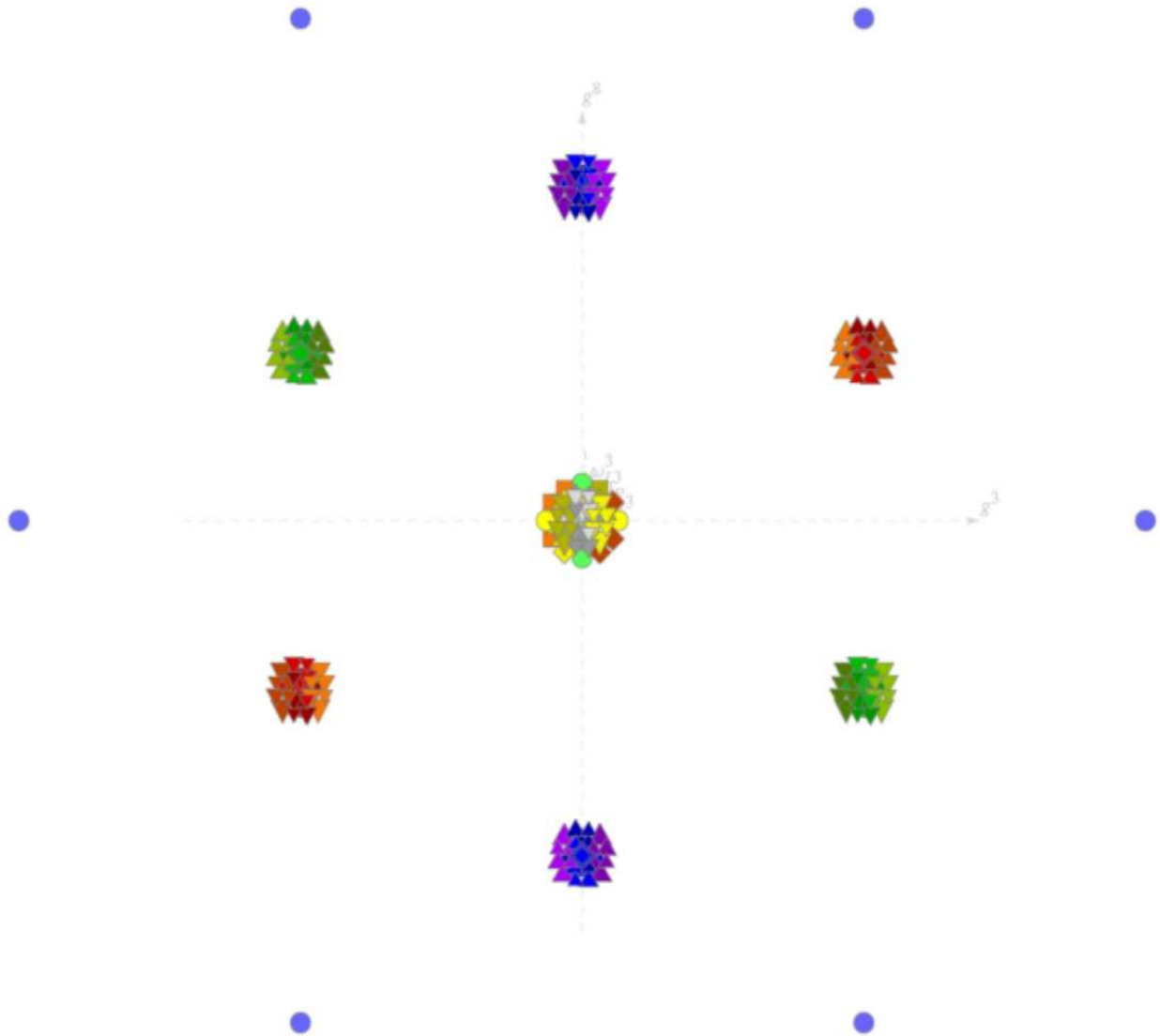
E8 root system ....x.x....x..



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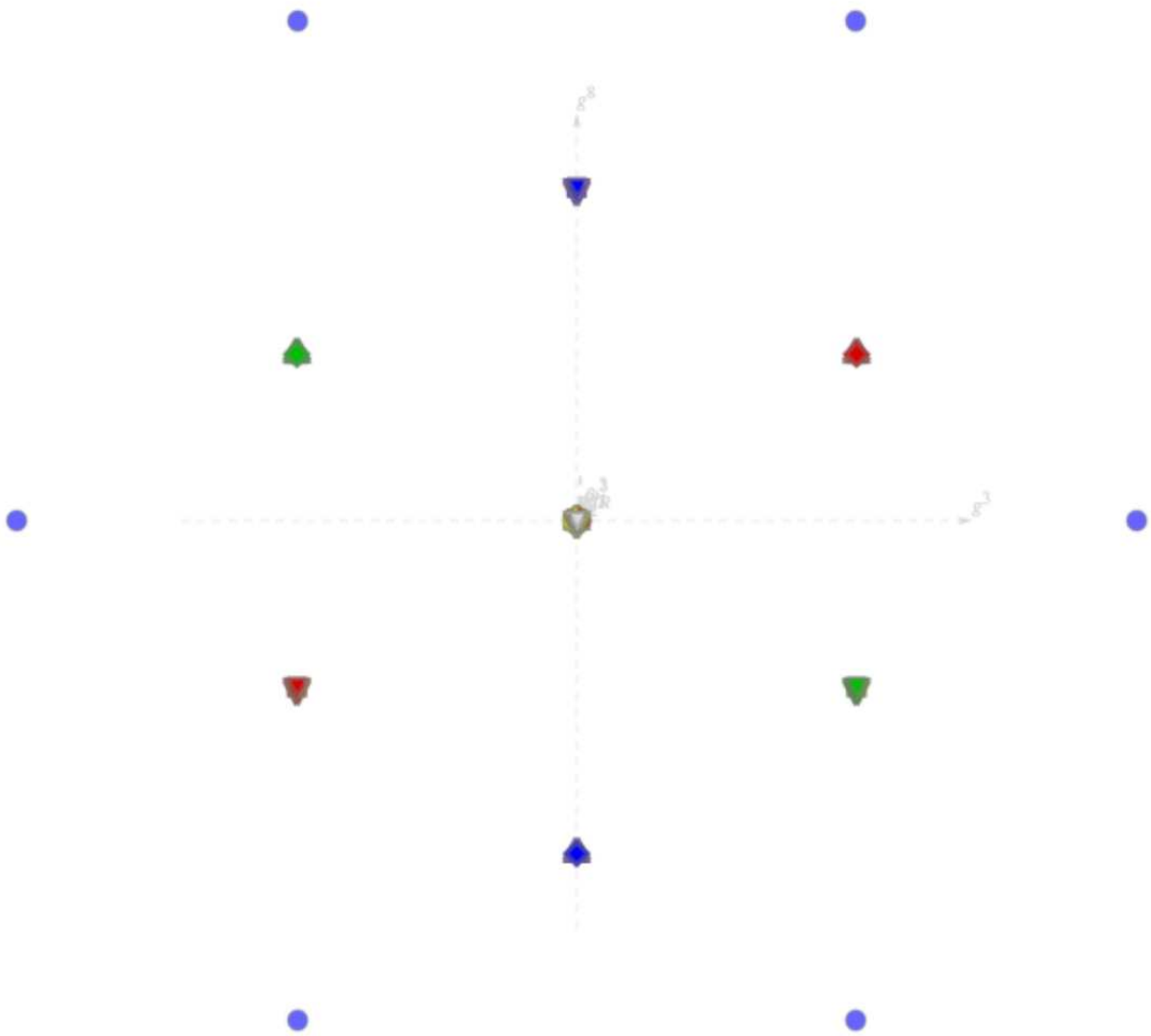
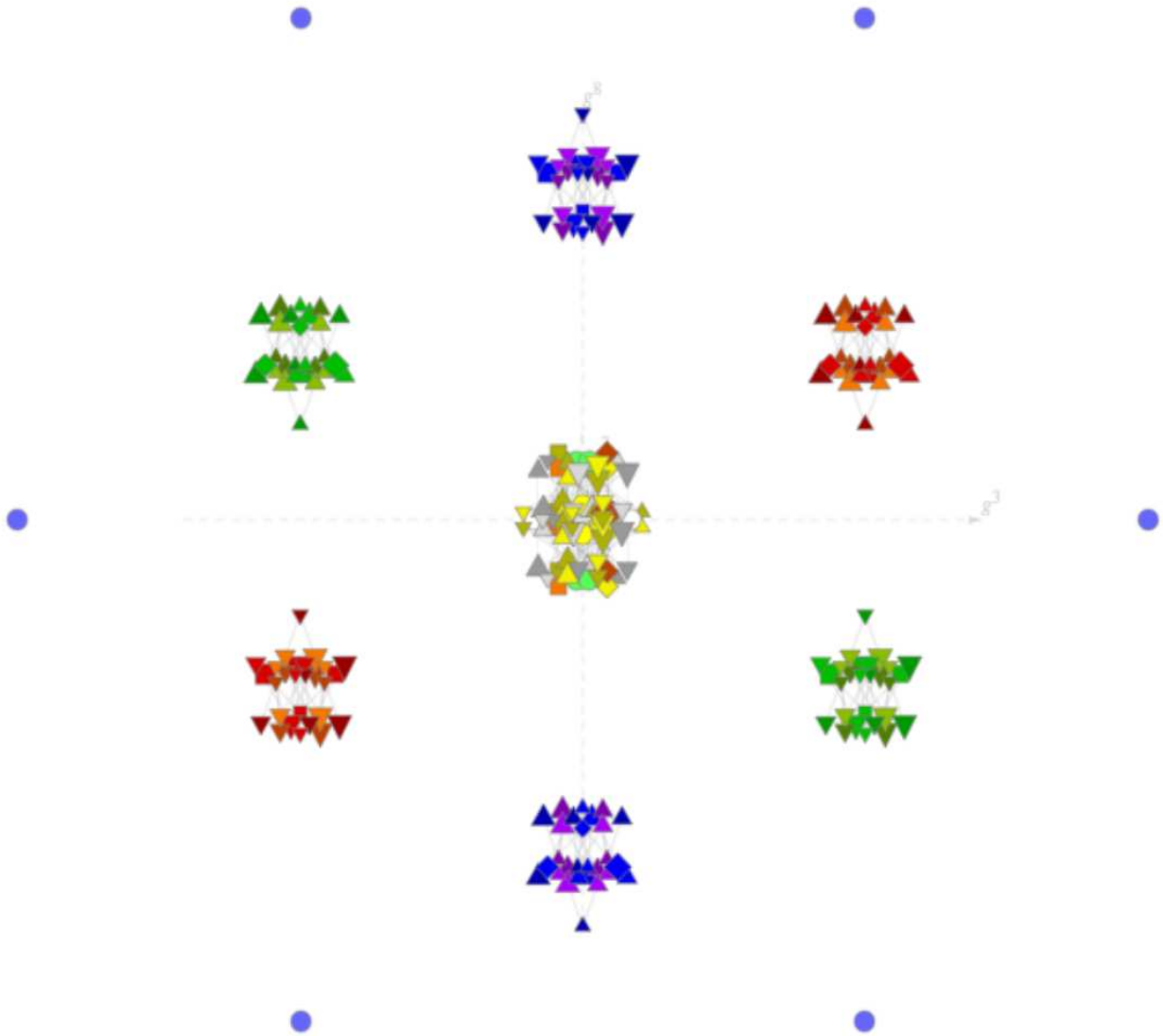
E8 root system ....x.x....x..x



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x....x..x.



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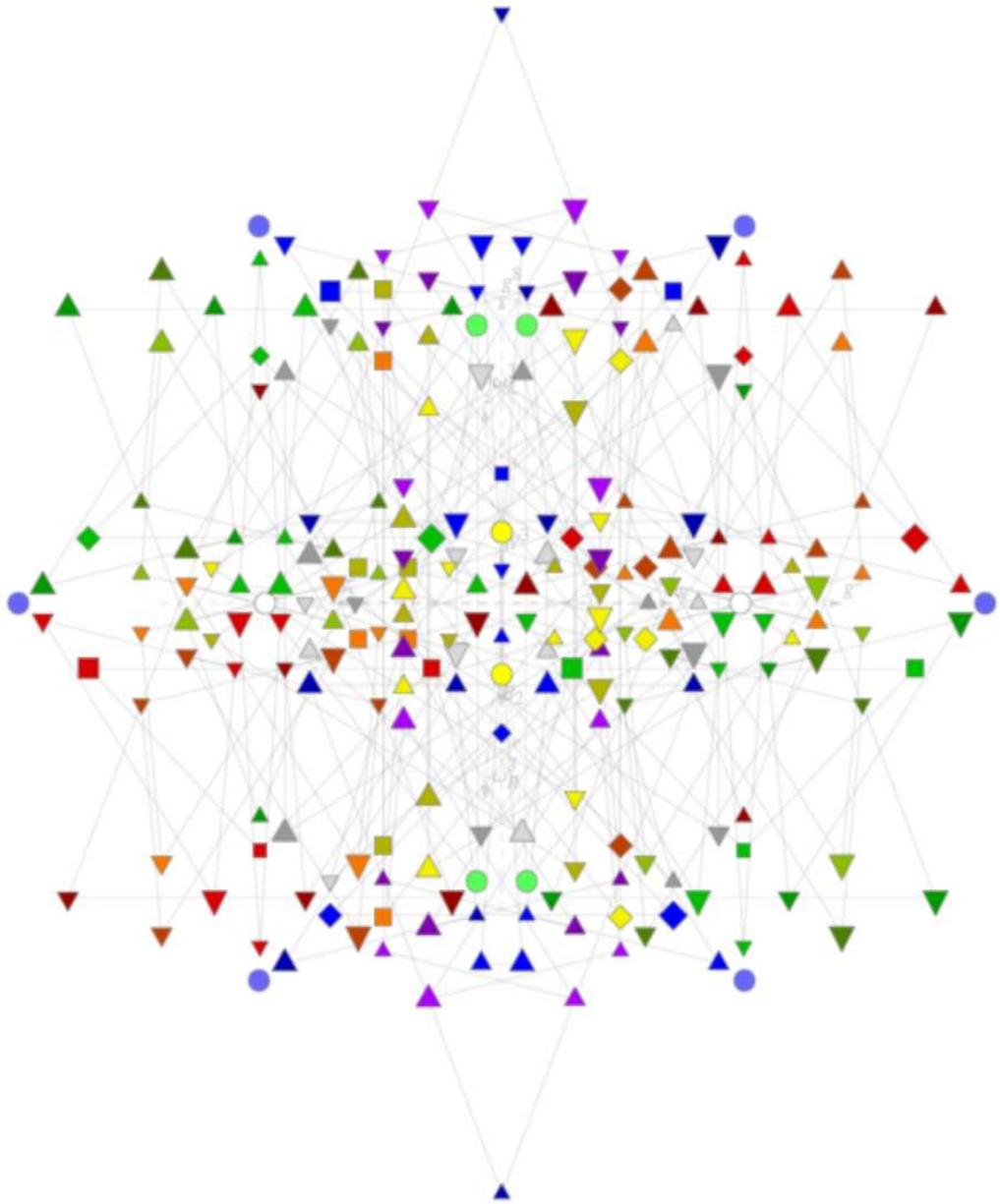
E8 root system ....x.x....x..x..



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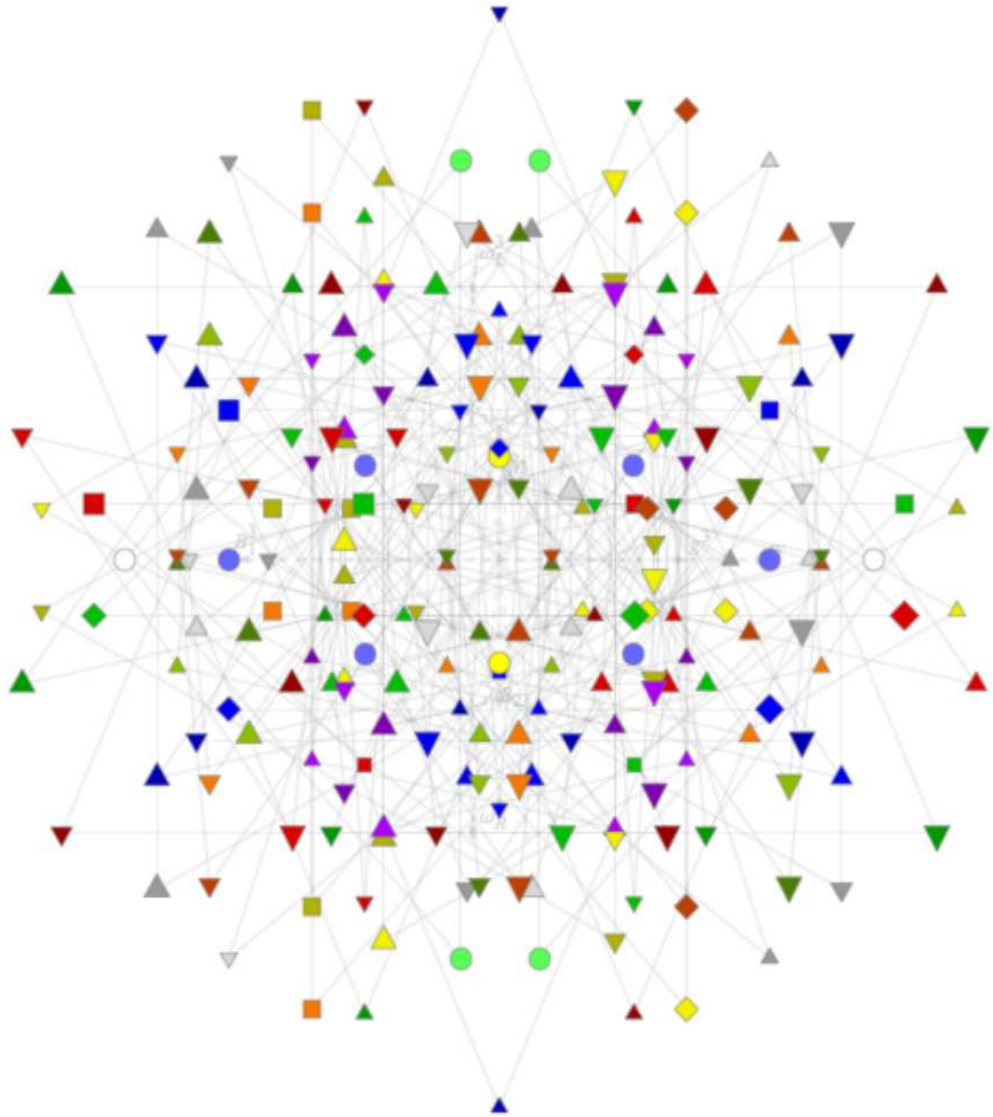
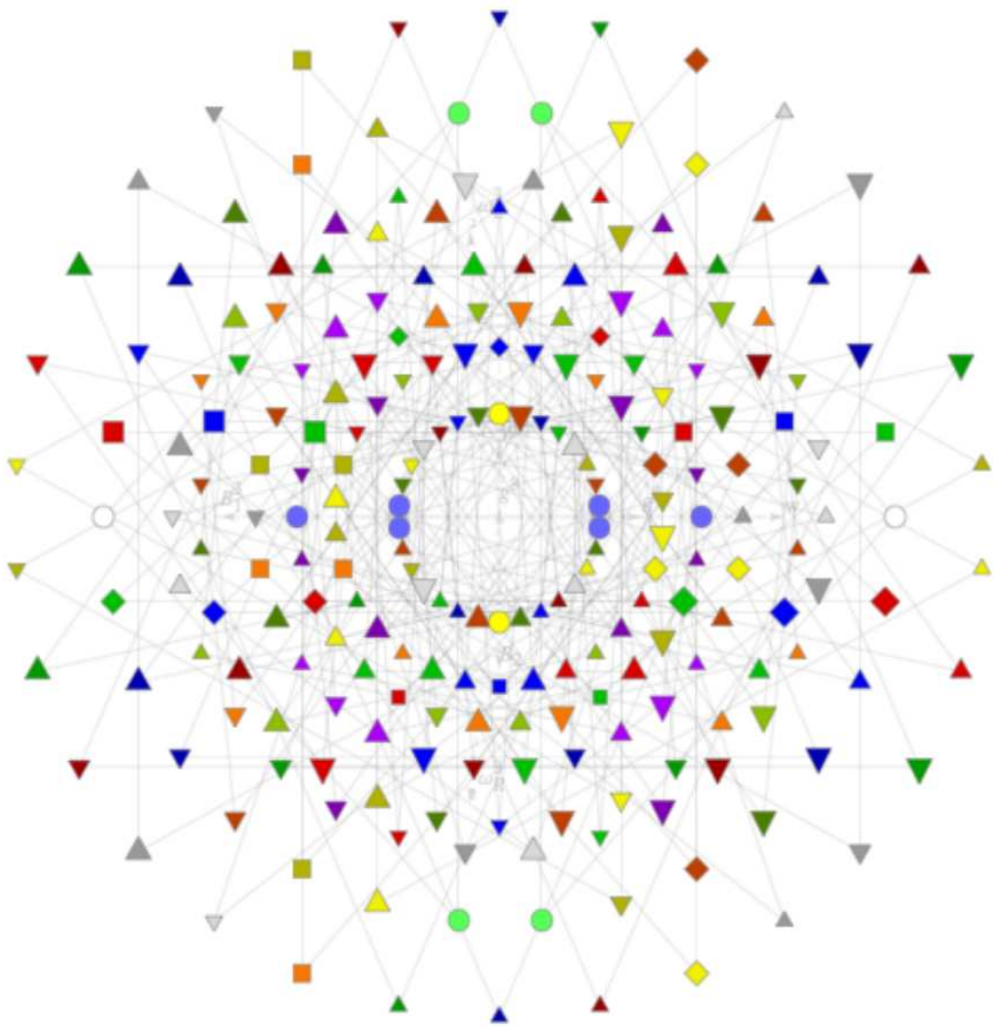
E8 root system ....x.x....x..x...



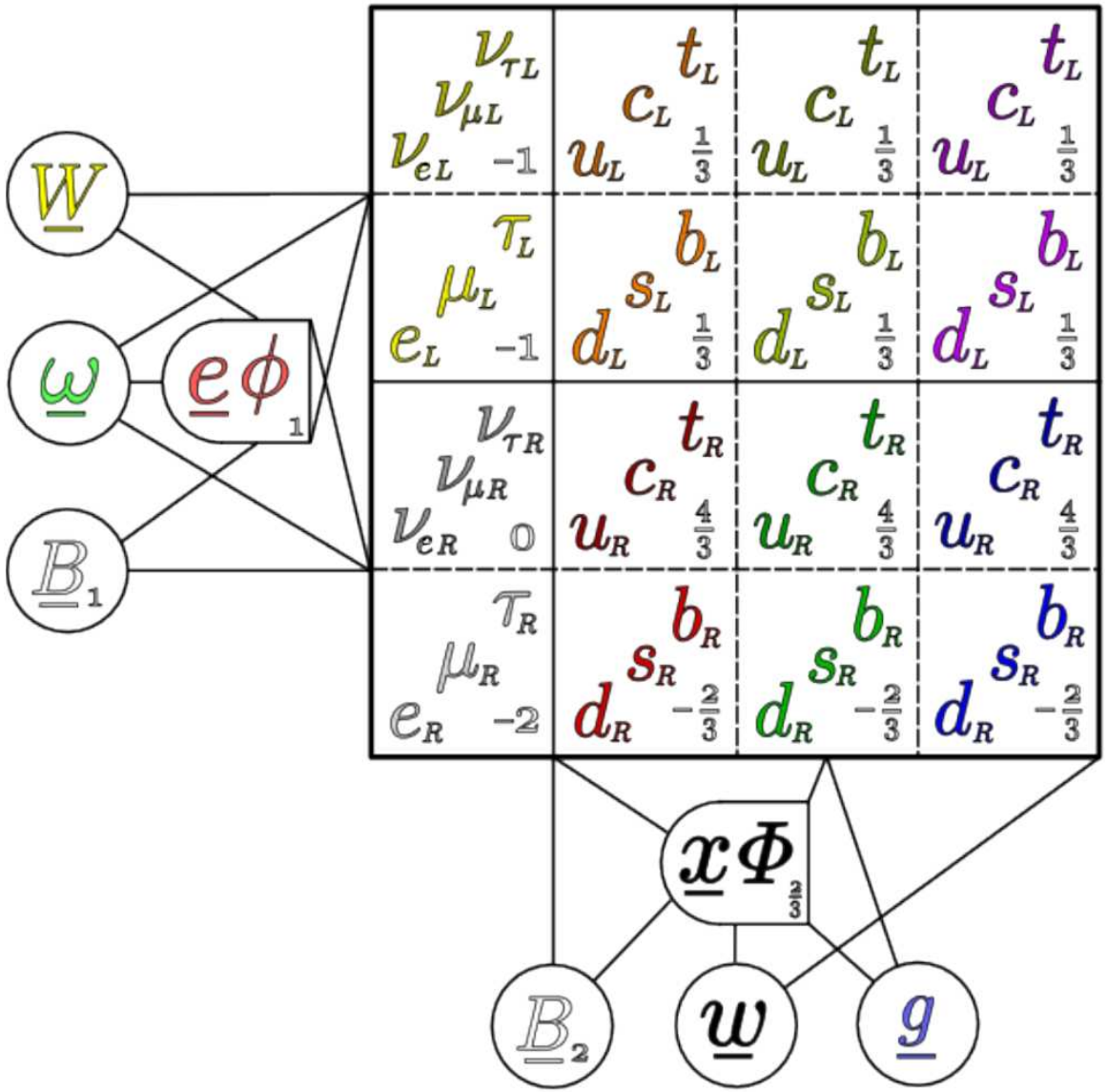
E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x....x..x...x



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 periodic table



"E8 is perhaps the most beautiful structure in all of mathematics, but it's very complex." — Hermann Nicolai

E8 connection

$$\underline{A}_{\cdot} = \underline{H}_1 + \underline{H}_2 + \underset{\cdot}{\Psi}_I + \underset{\cdot}{\Psi}_{II} + \underset{\cdot}{\Psi}_{III} \quad \in \quad \underline{e8}_{\cdot}$$

$$\begin{aligned}
\underline{H}_1 = \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi + \underline{W} + \underline{B}_1 & \quad \in \quad \underline{so}(7,1) \\
\underline{\omega} & \quad \in \quad \underline{so}(3,1) \\
\underline{e}\phi = (\underline{e}_1 + \underline{e}_2 + \underline{e}_3 + \underline{e}_4) \times (\phi_{+/0} + \phi_{-/1}) & \quad \in \quad \underline{4} \times (2 + \bar{2}) \\
\underline{W} + \underline{B}_1 & \quad \in \quad \underline{su}(2) + \underline{su}(2) \\
\underline{H}_2 = \underline{w} + \underline{B}_2 + \underline{x}\Phi + \underline{g} & \quad \in \quad \underline{so}(8) \\
\underline{w} + \underline{B}_2 & \quad \in \quad \underline{u}(1) + \underline{u}(1) \\
\underline{x}\Phi = (\underline{x}_1 + \underline{x}_2 + \underline{x}_3) \times (\Phi^{r/g/b} + \Phi^{\bar{r}/\bar{g}/\bar{b}}) & \quad \in \quad \underline{3} \times (3 + \bar{3}) \\
\underline{g} & \quad \in \quad \underline{su}(3) \\
\underset{\cdot}{\Psi}_I = \underset{\cdot}{\nu}_e + \underset{\cdot}{e} + \underset{\cdot}{u} + \underset{\cdot}{d} & \quad \in \quad 8_{S+} \times 8_{S+} \\
\underset{\cdot}{\Psi}_{II} = \underset{\cdot}{\nu}_\mu + \underset{\cdot}{\mu} + \underset{\cdot}{c} + \underset{\cdot}{s} & \quad \in \quad 8_V \times 8_V \\
\underset{\cdot}{\Psi}_{III} = \underset{\cdot}{\nu}_\tau + \underset{\cdot}{\tau} + \underset{\cdot}{t} + \underset{\cdot}{b} & \quad \in \quad 8_{S-} \times 8_{S-}
\end{aligned}$$

E8 curvature

$$\underline{\underline{F}}_{\cdot} = \underline{d}\,\underline{A}_{\cdot} + \underline{A}_{\cdot}\underline{A}_{\cdot} = \underline{\underline{F}}_1 + \underline{\underline{F}}_2 + \underline{D}\big(\underset{\cdot}{\Psi}_I + \underset{\cdot}{\Psi}_{II} + \underset{\cdot}{\Psi}_{III}\big) \quad \in \quad \underline{\underline{e8}}_{\cdot}$$

$$\begin{aligned}
\underline{\underline{F}}_1 &= \tfrac{1}{2}\big(\underline{\underline{R}} - \tfrac{1}{8}\underline{e}\,\underline{e}\,\phi^2\big) + \tfrac{1}{4}\big(\underline{\underline{T}}\phi - \underline{e}\,\underline{D}\phi\big) + \big(\underline{\underline{F}}_{B_1} + \underline{\underline{F}}_W\big) && \in \quad \underline{so}(7,1) \\
\underline{\underline{R}} &= \underline{d}\,\underline{\omega} + \tfrac{1}{2}\underline{\omega}\,\underline{\omega} && \in \quad \underline{so}(3,1) \\
\underline{\underline{T}}\phi - \underline{e}\,\underline{D}\phi &= \big(\underline{d}\,\underline{e} + \tfrac{1}{2}[\underline{\omega},\underline{e}]\big)\phi - \underline{e}\big(\underline{d}\phi + [\underline{B}_1 + \underline{W},\phi]\big) && \in \quad \underline{4} \times (2 + \bar{2}) \\
\underline{\underline{F}}_{B_1} + \underline{\underline{F}}_W &= (\underline{d}\,\underline{B}_1 + \underline{B}_1\underline{B}_1) + (\underline{d}\,\underline{W} + \underline{W}\,\underline{W}) && \in \quad \underline{su}(2) + \underline{su}(2)
\end{aligned}$$

$$\begin{aligned}
\underline{\underline{F}}_2 &= \big(\underline{\underline{F}}_w + \underline{\underline{F}}_{B_2} + \underline{x}\Phi\underline{x}\Phi\big) + \big((\underline{D}\underline{x})\Phi - \underline{x}\,\underline{D}\Phi\big) + \underline{\underline{F}}_g && \in \quad \underline{so}(8) \\
\underline{\underline{F}}_w + \underline{\underline{F}}_{B_2} &= \underline{d}\,\underline{w} + \underline{d}\,\underline{B}_2 && \in \quad \underline{u}(1) + \underline{u}(1) \\
(\underline{D}\underline{x})\Phi - \underline{x}\,\underline{D}\Phi &= \big(\underline{d}\,\underline{x} + [\underline{w} + \underline{B}_2, \underline{x}]\big)\Phi - \underline{x}\big(\underline{d}\Phi + [\underline{g},\Phi]\big) && \in \quad \underline{3} \times (3 + \bar{3}) \\
\underline{\underline{F}}_g &= \underline{d}\,\underline{g} + \underline{g}\,\underline{g} && \in \quad \underline{su}(3)
\end{aligned}$$

$$\underline{D}\underset{\cdot}{\Psi} = \big(\underline{d} + \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi\big)\underset{\cdot}{\Psi} + \underline{W}\,\underset{\cdot}{\Psi}_L + \underline{B}_1\underset{\cdot}{\Psi}_R - \underset{\cdot}{\Psi}\big(\underline{w} + \underline{B}_2 + \underline{x}\Phi\big) - \underset{\cdot}{\Psi}_q\,\underline{g}$$

Action for everything

Modified BF action, using $\dot{\underline{\underline{B}}} = \underline{\underline{B}} + \dot{\underline{\underline{\underline{B}}}}$:

$$
\begin{aligned}
S &= \int \left\langle \dot{\underline{\underline{B}}}\,\underline{\underline{F}}_{\cdot} + \underset{\sim}{\Phi}(\underline{H}_1, \underline{H}_2, \underline{\underline{B}}) \right\rangle \\
&= \int \left\langle \dot{\underline{\underline{\underline{B}}}}\,\underline{D}\underset{\cdot}{\Psi} + \underline{\underline{B}}\,\underline{\underline{F}} + \tfrac{\pi G}{4}\underline{\underline{B}}_G \underline{\underline{B}}_G \gamma + \underline{\underline{B}}' * \underline{\underline{B}}' \right\rangle \\
&= \int \left\langle \dot{\underline{\underline{\underline{B}}}}\,\underline{D}\underset{\cdot}{\Psi} + \underset{\sim}{e}\,\frac{1}{16\pi G}\phi^2\big(R - \frac{3}{2}\phi^2\big) + \frac{1}{4}\underline{\underline{F}}' * \underline{\underline{F}}' \right\rangle
\end{aligned}
$$

Cosmological constant from the Higgs VEV: $\Lambda = \frac{3}{4}\phi^2$

Implies frame VEV is de Sitter: $\underline{\underline{R}} = \frac{\Lambda}{6}\underline{e}\,\underline{e}$ $\qquad R = 4\Lambda$

Vacuum expectation value of the curvature vanishes: $\underline{\underline{F}}_{\cdot} = 0$

Gravitational part of the action

$$S_G = \int \left\langle \underline{\underline{B}}_G \underline{\underline{F}}_G + \frac{\pi G}{4} \underline{\underline{B}}_G \underline{\underline{B}}_G \gamma \right\rangle \qquad \underline{\underline{F}}_G = \frac{1}{2}\left(\underline{\underline{R}} - \frac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right) \in \underline{\underline{so}}(3,1)$$

$$\delta \underline{\underline{B}}_G \to \underline{\underline{B}}_G = \frac{1}{\pi G}\left(\underline{\underline{R}} - \frac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right)\gamma \qquad \gamma = \gamma_1\gamma_2\gamma_3\gamma_4$$

$$S_G = \frac{1}{\pi G}\int \left\langle \underline{\underline{F}}_G \underline{\underline{F}}_G \gamma \right\rangle = \frac{1}{4\pi G}\int \left\langle \left(\underline{\underline{R}} - \frac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right)\left(\underline{\underline{R}} - \frac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right)\gamma \right\rangle$$

$$\left\langle \underline{\underline{R}}\,\underline{\underline{R}}\gamma \right\rangle = \underline{d}\left\langle \left(\underline{\omega}\,\underline{d}\,\underline{\omega} + \frac{1}{3}\underline{\omega}\,\underline{\omega}\,\underline{\omega}\right)\gamma \right\rangle \qquad \leftarrow \text{Chern-Simons}$$

$$\frac{1}{4!}\left\langle \underline{e}\,\underline{e}\,\underline{e}\,\underline{e}\,\gamma \right\rangle = -\underset{\sim}{e} \qquad \leftarrow \text{volume element}$$

$$\left\langle \underline{e}\,\underline{e}\,\underline{\underline{R}}\,\gamma \right\rangle = -\underset{\sim}{e}R \qquad \leftarrow \text{curvature scalar}$$

$$S_G = \frac{1}{16\pi G}\int \underset{\sim}{e}\,\phi^2\left(R - \frac{3}{2}\phi^2\right) \qquad \text{cosmological constant:} \quad \Lambda = \frac{3}{4}\phi^2$$

Fermionic part of the action

Choosing the anti-Grassmann 3-form to be $\underset{\equiv}{\dot{B}} = \underset{\sim}{e}\dot{\Psi}\vec{e}$ gives the massive Dirac action in curved spacetime:

$$
\begin{aligned}
S_f &= \int \langle \underset{\equiv}{\dot{B}} \underset{\equiv}{F}_{\cdot} \rangle = \int \langle \underset{\equiv}{\dot{B}} \underline{D} \underset{\cdot}{\Psi} \rangle \\
&= \int \langle \underset{\sim}{e}\dot{\Psi}\vec{e}\big(\underline{d}\underset{\cdot}{\Psi} + \underline{H}_1\underset{\cdot}{\Psi} - \underset{\cdot}{\Psi}\underline{H}_2\big)\rangle \\
&= \int \langle \underset{\sim}{e}\dot{\Psi}\vec{e}\big((\underline{d} + \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi + \underline{W} + \underline{B}_1)\underset{\cdot}{\Psi} - \underset{\cdot}{\Psi}(\underline{w} + \underline{B}_2 + \underline{x}\Phi + \underline{g})\big)\rangle \\
&= \int d^4_{\sim}x\, |e| \,\langle \dot{\Psi}\gamma^\mu (e_\mu)^i \big(\partial_i \underset{\cdot}{\Psi} + \tfrac{1}{4}\omega_i{}^{\mu\nu}\gamma_{\mu\nu}\underset{\cdot}{\Psi} + W_i\underset{\cdot}{\Psi} + B_{1i}\underset{\cdot}{\Psi} \\
&\qquad\qquad + \underset{\cdot}{\Psi} w_i + \underset{\cdot}{\Psi} B_{2i} + \underset{\cdot}{\Psi} x_i \Phi + \underset{\cdot}{\Psi} g_i\big) + \dot{\Psi}\,\phi\,\underset{\cdot}{\Psi}\rangle
\end{aligned}
$$

The $\dot{\Psi}\,\phi\,\underset{\cdot}{\Psi}$ is the standard Higgs mass term.

The $\dot{\Psi}\gamma^\mu\underset{\cdot}{\Psi}x_\mu\Phi$ term... I don't understand yet — promising for CKM.

E8 Theory summary

Everything in an $E8$ principal bundle connection,

$$\underline{A}_{\cdot} \in \underline{e8}_{\cdot}$$

Periodic table of interactions (Feynman vertices) from curvature,

$$\underline{\underline{F}}_{\cdot} = \underline{d}\,\underline{A}_{\cdot} + \tfrac{1}{2}\left[\underline{A}_{\cdot}, \underline{A}_{\cdot}\right]$$

described by the $E8$ root polytope. Three generations through triality,

$$T\,e = \mu \qquad T\,\mu = \tau \qquad T\,\tau = e$$

Pati-Salam $SU(2)_L \times SU(2)_R \times SU(4)$GUT and MM gravity together,

$$S = \int \left\langle \dot{\underline{\underline{B}}}\,\underline{\underline{F}}_{\cdot} + \tfrac{\pi}{4}\underline{\underline{B}}_G\underline{\underline{B}}_G\gamma + \underline{\underline{B}}' * \underline{\underline{B}}' \right\rangle$$

No free parameters — masses from Higgs VEV's,

$$g_1 = \sqrt{\frac{3}{5}} \qquad g_2 = 1 \qquad g_3 = 1 \qquad \Lambda = \frac{3}{4}\phi^2 \qquad \phi_0, \phi_1, \Phi \ldots$$

Everything is pure geometry, and it's very beautiful.

E8 Theory discussion

- Quantization
  - Coupling constants run.
    - Large Λ compatible with UV fixed point.
  - Just a connection — amenable to LQG, spin foams, etc.
- Understand triality-generation relationship better
  - Possible collapse or mixing to graviweak $SL(2,\mathbb{C})$.
  - The role of $\underline{w} + \underline{x}\Phi$ and symmetry breaking.
  - Getting the CKMPMNS matrix would be nice.
- Why is the action what it is?
  - Pulling $\underline{e}$ out and putting it into $\underline{\underline{F}} * \underline{\underline{F}}$ and $\dot{\underline{\underline{B}}}$ seems weird.
    - Why $\underline{e}\,\phi$ simple?
    - Four dimensional base manifold emergent?

What this theory will mean, if it all works:

- Combines standard model with gravity — with LQG, it's a T.o.E.
- Our universe is very pretty.

BRST extended connection

Start with $E8$ principal bundle connection and its curvature,

$$\underline{A} = \underline{H} + \underline{\Psi} \qquad \underline{\underline{F}} = (\underline{d}\,\underline{H} + \underline{H}\,\underline{H} + \underline{\Psi}\,\underline{\Psi}) + (\underline{d}\,\underline{\Psi} + \underline{H}\,\underline{\Psi} + \underline{\Psi}\,\underline{H})$$

Action such that $\underline{\Psi}$ part is pure gauge,

$$S = \int \langle \underline{\underline{B}}\,\underline{\underline{F}} + \tfrac{\pi G}{4}\underline{\underline{B}}_G\underline{\underline{B}}_G\gamma + \underline{\underline{B}}' * \underline{\underline{B}}' \rangle$$

BRST: Replace $\underline{\Psi}$ part with ghosts, $\underset{\cdot}{\Psi}$, in extended connection,

$$\underline{A}_{\cdot} = \underline{H} + \underset{\cdot}{\Psi} \qquad \underline{\underline{F}}_{\cdot} = \big(\underline{d}\,\underline{H} + \underline{H}\,\underline{H}\big) + \big(\underline{d}\,\underset{\cdot}{\Psi} + [\underline{H}, \underset{\cdot}{\Psi}]\big) = \underline{\underline{F}}_H + \underline{D}\,\underset{\cdot}{\Psi}$$

Effective action for gauge fields, ghosts, and anti-ghosts:

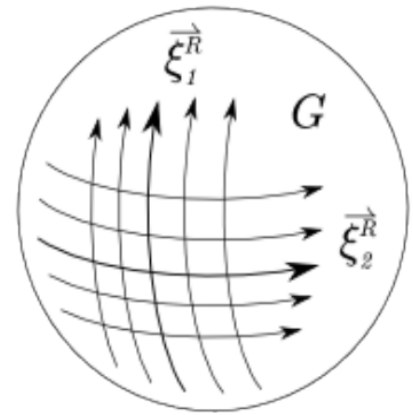
$$S = \int \langle \dot{\underline{\underline{B}}}\,\underline{\underline{F}}_{\cdot} + \tfrac{\pi G}{4}\underline{\underline{B}}_G\underline{\underline{B}}_G\gamma + \underline{\underline{B}}' * \underline{\underline{B}}' \rangle$$

$$= \int \langle \dot{\underline{\underline{B}}}\,\underline{D}\,\underset{\cdot}{\Psi} + \underset{\sim}{e}\,\frac{1}{16\pi G}\phi^2\big(R - \frac{3}{2}\phi^2\big) + \frac{1}{4}\underline{\underline{F}}' * \underline{\underline{F}}' \rangle$$

Geometry of Yang-Mills theory

Start with a **Lie group manifold** (*torsor*), G, coordinatized by y^p.
Two sets of invariant vector fields (*symmetries*, ***Killing vector*** *fields*):

$$\vec{\xi}^L_A(y)\,\underline{d}g = T_A\,g(y) \qquad \vec{\xi}^R_A(y)\,\underline{d}g = g(y)\,T_A$$



Lie derivative: $[\vec{\xi}^R_A, \vec{\xi}^R_B] = C_{AB}{}^C \vec{\xi}^R_C$

Lie bracket: $[T_A, T_B] = C_{AB}{}^C T_C$

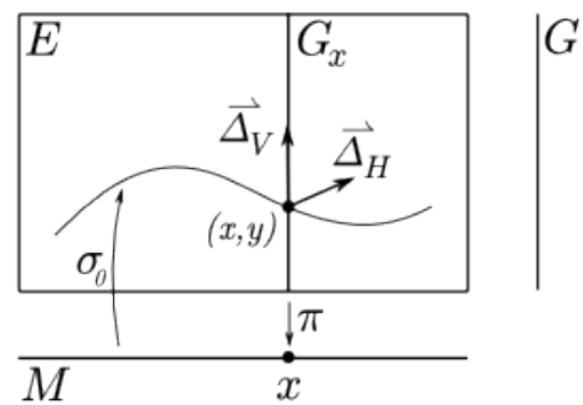
Killing form (***Minkowski metric***): $g_{AB} = C_{AC}{}^D C_{BD}{}^C$

Maurer-Cartan form (***frame***): $\underline{\mathcal{I}} = d\underline{y}^p (\xi^R_p)^A T_A$

Entire space of a **principal bundle**: $E \sim M \times G$

Ehresmann principal bundle connection over patches of E:

$$\underline{\vec{\mathcal{E}}}(x,y) = d\underline{x}^i A_i{}^B(x)\,\vec{\xi}^L_B(y) + d\underline{y}^p \vec{\partial}_p$$



Gauge field **connection** over M:

$$\underline{A}(x) = \sigma_0^* \underline{\vec{\mathcal{E}}}\,\underline{\mathcal{I}} = d\underline{x}^i A_i{}^B(x)\,T_B$$

The Coleman-Mandula theorem

> Let G be a connected symmetry group of the S matrix, and let the following five conditions hold: (1) G contains a subgroup locally isomorphic to the Poincaré group. (2)... Then, we show that G is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

E8 Theory does not allow a subgroup locally isomorphic to the Poincaré group. The S matrix only exists as an approximation, in which the theorem is satisfied.

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