

Developments in Group Field Theory Cosmology

A collective effort: D. Oriti, E. Wilson-Ewing, S. Gielen, M. Sakellariadou, A. Pithis, M. de Cesare, A. Polaczek, A. Jercher, A. Calcinari, R. Dekhil, X. Pang, L. Mickel, T. Ladstätter, P. Fischer, ...

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Overview

• Introduction to Group Field Theory

- Group Field Theory and spinfoam models
- Group Field Theory and Loop Quantum Gravity
- Including scalar matter

• Group Field Theory Cosmology

- Basic principles
- Homogeneous and isotropic sector
 - Volume
 - Matter
- Inhomogeneous sector
 - First steps and limitations
 - Super-horizon limit
 - Perturbations at all scales

Introduction to Group Field Theory

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or, in some cases, $G = \mathrm{SU}(2)$.

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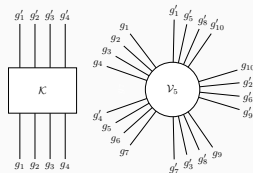
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



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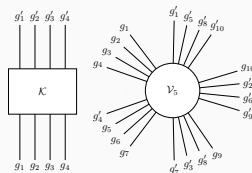
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Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

- Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.

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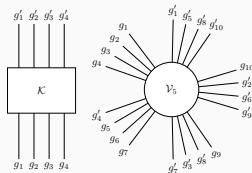
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Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{complete spinfoam model.}$$

- Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.
- \mathcal{K} and \mathcal{V}_{γ} chosen to match the desired spinfoam model.

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

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Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a simplicial interpretation of the fundamental quanta:

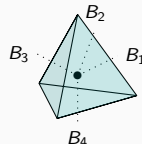
Closure

$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
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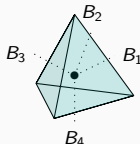
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Non-comm.

FT

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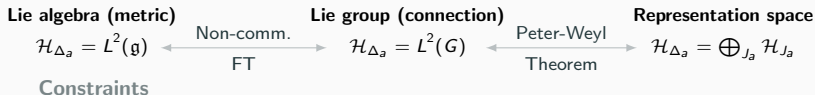
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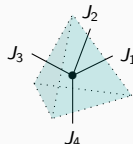
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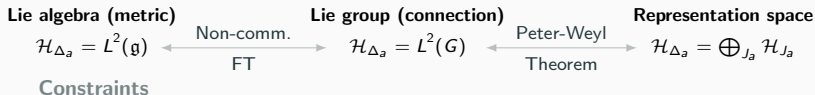
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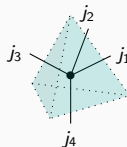
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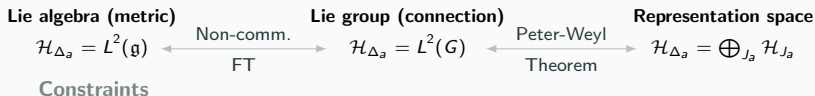


- ▶ Impose simplicity and reduce to $G = \text{SU}(2)$.
- ▶ Impose closure (gauge invariance).

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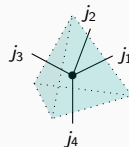
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$$\mathcal{H}_{\text{tetra}} = \bigoplus_{\vec{j}} \text{Inv} \left[\bigotimes_{a=1}^4 \mathcal{H}_{j_a} \right]$$

= open spin-network vertex space

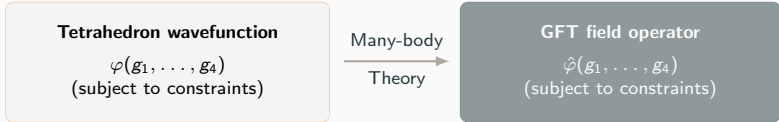
The Group Field Theory Fock space

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

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Many-body
Theory

GFT field operator

$\hat{\varphi}(g_1, \dots, g_4)$
(subject to constraints)

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
- ▶ Similar to \mathcal{H}_{LQG} but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

GFT Fock space

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Operators

Volume operator $\hat{V} = \int dg_a^{(1)} dg_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^\dagger(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{j_a, m_a, \ell} V_{j_a, \ell} \hat{\varphi}_{j_a, m_a, \ell}^\dagger \hat{\varphi}_{j_a, m_a, \ell}$

- ▶ Generic second quantization prescription to build a $m + n$ -body operator: sandwich matrix elements between spin-network states between m powers of $\hat{\varphi}^\dagger$ and n powers of $\hat{\varphi}$.

Group Field Theory and matter: scalar fields

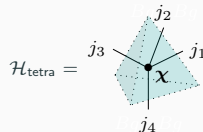
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Kinematics

Quanta are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ **Geometricity constraints** imposed analogously as before.



Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- ▶ Geometric data enter the action in a **non-local and combinatorial** fashion.

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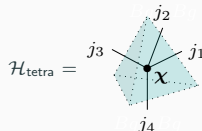
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- ▶ **Geometricity constraints** imposed analogously as before.
- ▶ Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^d$.



Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- ▶ Geometric data enter the action in a **non-local and combinatorial** fashion.
- ▶ Scalar field data are **local** in interactions.
- ▶ For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_a, g_b; \chi^\alpha, \chi^{\alpha'}) = \mathcal{K}(g_a, g_b; (\chi^\alpha - \chi^{\alpha'})^2)$$

$$\mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)}, \chi) = \mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)})$$

Group Field Theory Cosmology

The main ingredients

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d\mathbf{g}_a \sigma(\mathbf{g}_a, \chi^\alpha) \hat{\phi}^\dagger(\mathbf{g}_a, \chi^\alpha) \right] |0\rangle .$$

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Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:
$$\left\langle \frac{\delta S[\hat{\phi}, \hat{\phi}^\dagger]}{\delta \hat{\phi}(\mathbf{g}_l, \chi^\alpha)} \right\rangle_\sigma = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (\chi^\alpha - \chi^\alpha)^2) \sigma(\mathbf{h}_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, \chi^\alpha)} \Big|_{\varphi=\sigma} = 0.$$
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Relationality

Condensate Peaked States

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Condensate Peaked States

- ▶ Constructing relational observables on \mathcal{F}_{GFT} is difficult (QFT with no continuum intuition).
- ▶ Relational localization implemented at an **effective** level on observable **averages**.
- ▶ If χ^μ constitute a reference frame, this can be achieved by assuming
$$\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$$

Homogeneous sector

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

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Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

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Macroscopic cosmological variables and effective relationality

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Relationality

► Averaged evolution wrt χ^0 is physical:

$$\text{Intensive} \longleftarrow \langle \hat{X}^0 \rangle_{\sigma_{\chi^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{\chi^0}} / \langle \hat{N} \rangle_{\sigma_{\chi^0}} \simeq \chi^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{\chi^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{\chi^0}}$.

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Wavefunction
isotropy

$$\langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_v v_v |\tilde{\sigma}_v|^2(x^0)$$

$$\langle \hat{N} \rangle_{\sigma_{x^0}} = \sum_v |\tilde{\sigma}_v|^2(x^0)$$

► $v = j \in \mathbb{N}/2$ (EPRL);

► $v = \rho \in \mathbb{R}$ (ext. BC).

Relationality

► Averaged evolution wrt x^0 is physical:

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► Emergent effective relational description:

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- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$.

Mean-field approximation

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Derivative expansion of \mathcal{K} (due to peaking properties).
- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables.

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Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\mathcal{F}_v V_v \rho_v \text{sgn}(\rho_v') \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3\mathcal{F}_v V_v \rho_v^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2\mathcal{F}_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\mathcal{F}_v V_v \rho_v^2}$$

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Classical limit (large ρ_v s, late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

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Bounce

- ▶ A non-zero volume bounce happens for a large range of initial conditions (at least one $Q_v \neq 0$ or one $\mathcal{E}_v < 0$).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

(T)GFT interactions and matter

Scalar field with potential

Running couplings and effective potentials

- ▶ Adding a scalar field ϕ with potential U_ϕ requires (T)GFT interactions, as $\mathcal{V}_\gamma = \mathcal{V}_\gamma(\{g\}, U_\phi)$.
- ▶ Interactions studied perturbatively at late times (mesoscopic regime) and in single j approx.

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Modulus interactions

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\mathbf{g}_a$

$$\text{Tr}_{\mathcal{V}_{\gamma_l}}^{(m)}[\varphi, \bar{\varphi}] \sim (\mathcal{V}_{\gamma_l}^{(m)}, \bar{\varphi}^{(l+1)/2} \varphi^{(l+1)/2})$$

- ✓ GR matching possible only if $l = 5$, and if
- ▶ Macroscopic constants (including G) **run** with relational time!

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Phantom dark energy

Emergent matter components

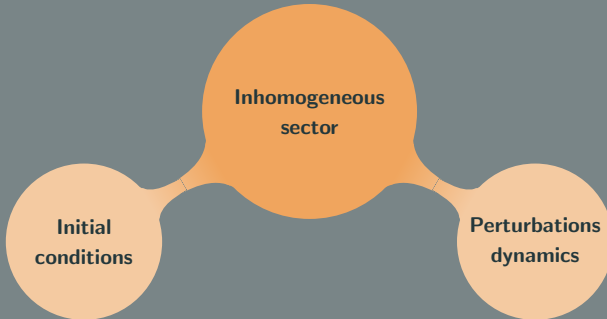
- ▶ Matter can also emerge as a result of pure QG effects!
- ▶ Consider modulus interactions at very late times, but include a subdominant spin j' :

$$w = 3 - 2(VV'')/(V')^2 \simeq -1 - b/V, \quad b > 0.$$

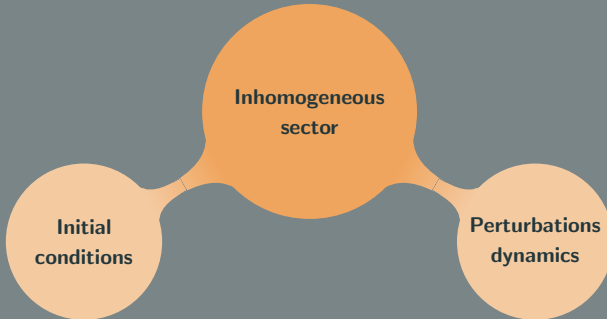
- ▶ Universe effectively dominated by (non-pathologic) **emergent** phantom dark energy.

Inhomogeneous sector

Two aspects of the inhomogeneity problem

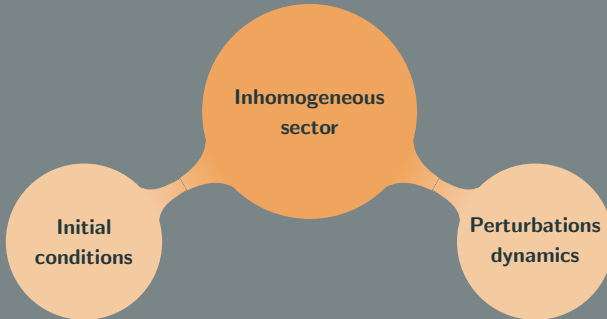


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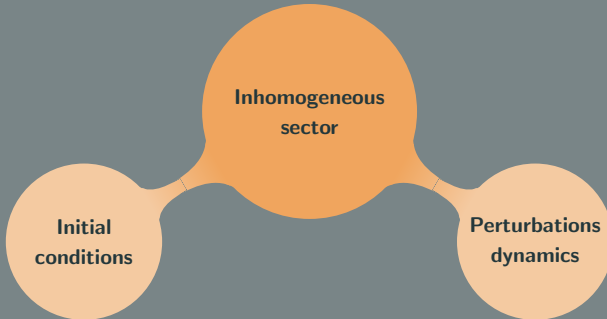
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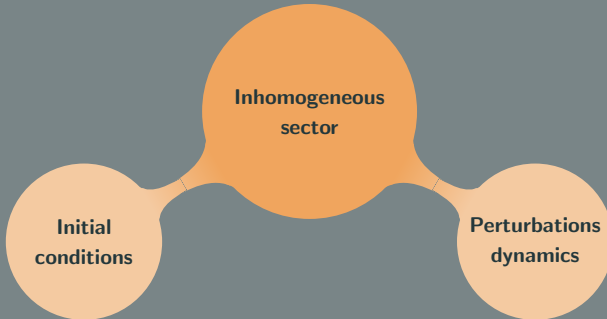
- ▶ Fock quantize the emergent perturbation dynamics.
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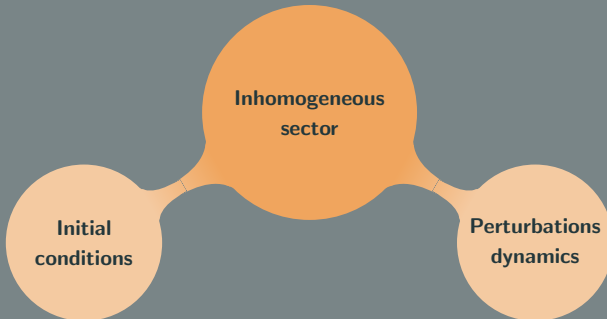
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 - What about higher moments? Is \hat{V} really relational? Only a background result?

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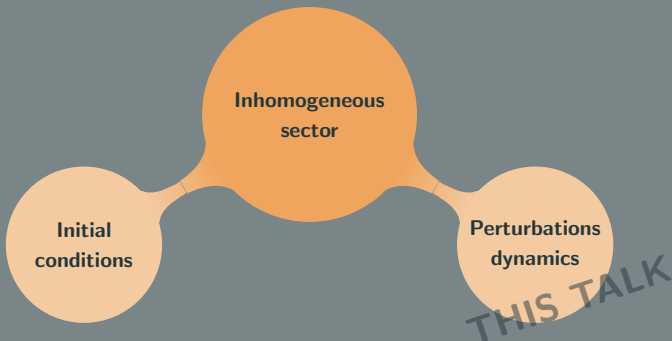
- ▶ Fock quantize the emergent perturbation dynamics. ▶ Goal: vector, tensor, scalar modes at all scales.
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Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

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Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) , with Lorentz/Euclidean invariant S_χ in field space.
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

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States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
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- Decoupling for a range of values of CPSs and large N (late times).

single
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- Dynamics of super-horizon scalar perturbations can be obtained generically for **any** MG theory.
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Perturbing background dynamics

- Study super-horizon scalar perturbations by perturbing background QG volume eq.
- **No matching** at early times with full effective GFT volume dynamics

Causal frame fields coupling

Causal properties of frame fields can be easily implemented in the complete extended BC model.

- ▶ $\varphi_\alpha \equiv \varphi(g_a, X_\alpha, \chi^\mu, \phi)$, $g \in \text{SL}(2, \mathbb{C})$, X_α tetrahedron normal defining its causal character, $\alpha = \pm$.

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Frame coupling

$$\mathcal{K}_+ = \mathcal{K}_+(\cdot, (\chi^0 - \chi'^0)^2),$$

$$\mathcal{K}_- = \mathcal{K}_+(\cdot, |\chi - \chi'|^2).$$

Scalar perturbations at all scales

Complete extended BC

Causal frame fields coupling

Causal properties of frame fields can be easily implemented in the complete extended BC model.

- ▶ $\varphi_\alpha \equiv \varphi(g_a, X_\alpha, \chi^\mu, \phi)$, $g \in \text{SL}(2, \mathbb{C})$, X_α tetrahedron normal defining its causal character, $\alpha = \pm$.

Two-sector Fock space

- ▶ Generic operators on $\mathcal{F} = \mathcal{F}_+ \otimes \mathcal{F}_-$ correlate spacelike and timelike tetrahedra.
- ▶ Volume operator is an exception: $\hat{V} = \hat{V}_+ \otimes \mathbb{I}_-$.

Frame coupling

$$\mathcal{K}_+ = \mathcal{K}_+(\cdot, (\chi^0 - \chi^{0'})^2),$$

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Collective states

Including two-body correlations

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \hat{\delta\Phi} \otimes \mathbb{I}_- + \hat{\delta\Psi} + \mathbb{I}_+ \otimes \hat{\delta\Xi}) |0\rangle$$

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Effective dynamics

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- ▶ Mean-field equations (negligible interactions):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ 2 coupled eqs. for 3 variables: $(\delta\Phi, \delta\Psi, \delta\Xi)$!
- ▶ Late times and single (spacelike) rep. label.

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- ▶ Late time GR matching fixes:
 - Parameters determining τ dynamics;
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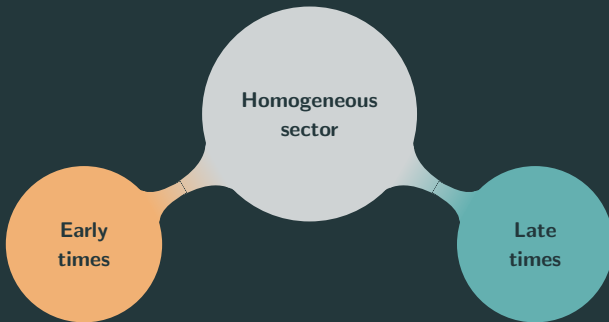
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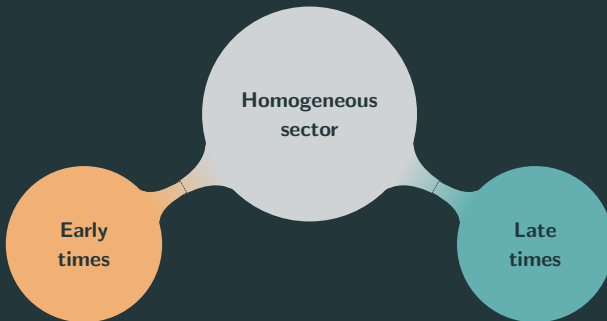
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Late times volume perturbations dynamics matches GR at all scales!

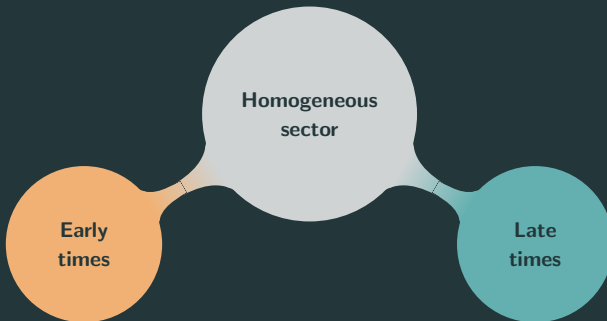




Results

- ✓ Singularity resolution into quantum bounce.
- ✓ Universal bounce (for MCMF scalar field).
- ✓ Impact of quantum effects on the bounce (and interplay with relationality).
- Acceleration produced by the bounce not long enough to sustain inflation.

LM, Oriti 2008.02774 - 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Oriti, Pithis 2112.00091; de Cesare, Pithis, Sakellariadou 1606.00352; Ladstätter, LM, Oriti (to appear); Oriti, Pang 2105.03751; Gielen, Polaczek 1912.06143 ; ...



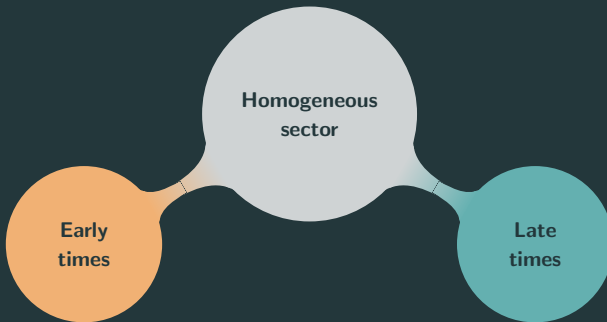
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Results

- ✓ Small interactions: classical regime identified (small quantum fluctuations and GR matching).
- ✓ Universal classical limit (for MCMF scalar field).
- ✓ Inclusion of scalar field with potential: emergent running couplings.
- Exotic matter can emerge from GFT interactions.

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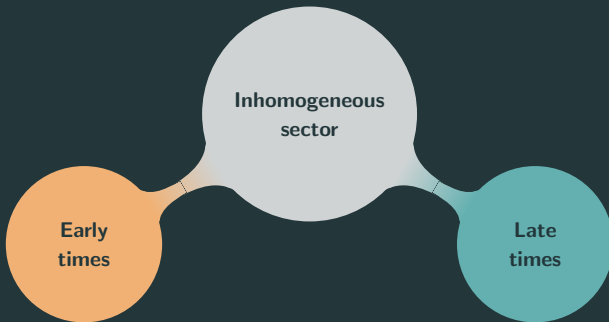


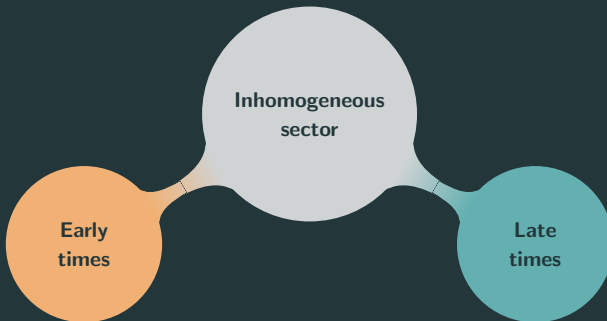
Perspectives

- ▶ Extend the analysis to more generic fluids.
- ▶ Universal bounce also for generic fluids?
- ⚠ In particular, would trigonometric modifications to a scalar field potential appear at early times?
- ⚠ What kind of inflationary physics is generated?

Perspectives

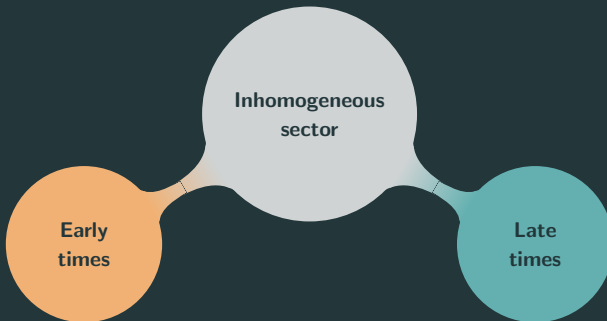
- ▶ Extend the analysis to more generic fluids.
- ▶ Universal classical limit also for generic fluids?
- ▶ Insights on the renormalization properties of GFTs from emergent running couplings?
- ▶ Can we rely on mean-field approx. at late times?





Results

- ✓ Super-horizon analysis in EPRL with MCMF scalar fields:
 - ✓ Scalar pert. dynamics differs from any MG model.
 - ✓ Full QG scalar pert. dynamics differs from perturbed background dynamics.

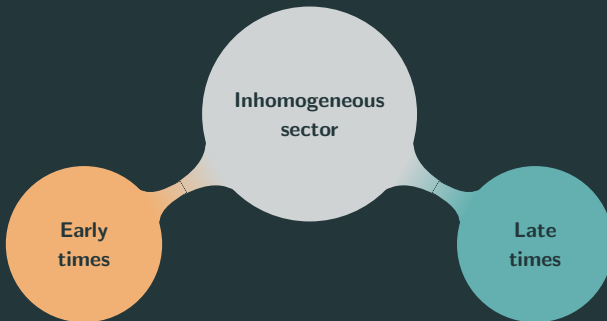


Results

- ✓ Super-horizon analysis in EPRL with MCMF scalar fields:
 - ✓ Scalar pert. dynamics differs from any MG model.
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Results

- ✓ All scales analysis for complete BC model with MCMF scalar fields:
 - ✓ Manifest causal properties of quanta allow for a careful coupling of the physical ref. frame.
 - ✓ Scalar pert. \longleftrightarrow quantum correlations!
 - ✓ Late-times scalar pert. dynamics matches GR!



Perspectives

- ⚠ Different fundamental d.o.f. \longrightarrow different perturbation dynamics?
- ⚠ Scalar field perturbations? EFT description?
 - ▶ Are the results universal? Analysis in BC!
 - ▶ Generalization to physically interesting fluids.
 - ▶ Extension to VT modes: more observables!
 - ▶ Initial conditions and power spectra?
 - Fock quantization of early-times dynamics.
 - Can we derive it from full QG?

Perspectives

- ⚠ Physical interpretation and consequences of matching conditions?
- ⚠ Scalar field perturbations? EFT description?
 - ▶ Are the results universal? Extension to EPRL!
 - ▶ Generalization to physically interesting fluids.
 - ▶ Extension to VT modes: more observables!
 - ▶ How do quantum perturbations classicalize?
 - ▶ How do GFT interactions change the picture?

Fischer, LM, Oriti (to appear); Jercher, LM, Pithis (to appear); Dekhil, Liberati, Oriti (to appear); Calcinari, Gielen 2210.03149.

Backup

Specifics of GFT models

EPRL model

$$S = \sum_{\{j_a\}, \{j'_a\}, \{m_a\}, \{m'_a\}, \ell, \ell'} \bar{\varphi}_{\{m_a\}}^{\{j_a\}\ell} \varphi_{\{m'_a\}}^{\{j'_a\}\ell'} \mathcal{K}_{\{m_a\}\{m'_a\}}^{\{j_a\}\{j'_a\}\ell\ell'} + V_5,$$

$$V_5 = \frac{1}{5} \sum_{\{j_a\}, \{m_a\}, \{\ell_b\}} \varphi_{m_1 m_2 m_3 m_4}^{j_1 j_2 j_3 j_4 \ell_1} \varphi_{-m_4 m_5 m_6 m_7}^{j_4 j_5 j_6 j_7 \ell_2} \varphi_{-m_7 - m_3 m_8 m_9}^{j_7 j_3 j_8 j_9 \ell_3} \varphi_{-m_9 - m_6 - m_2 m_{10}}^{j_9 j_6 j_2 j_{10} \ell_4} \varphi_{-m_{10} - m_8 - m_5 - m_1}^{j_{10} j_8 j_5 j_1 \ell_5}$$

$$\times \prod_{c=1}^{10} (-1)^{j_c - m_c} \mathcal{V}_5(j_1, \dots, j_{10}; \ell_1, \dots, \ell_5),$$

$$\begin{aligned} a &= 1, \dots, 4 \\ b &= 1, \dots, 5 \\ c &= 1, \dots, 10 \end{aligned}$$

$$\mathcal{V}_5(\{j_c\}, \{\ell_b\}) = \sum_{\{n_A\}} \int \left[\prod_A d\rho_A (n_A^2 + \rho_A^2) \right] \left[\bigotimes_b f_{\{n_A\}\{\rho_A\}}^{\ell_b}(\{j_a\}) \right] \{15j\}_{\text{SL}(2, \mathbb{C})},$$

where f maps $\text{SL}(2, \mathbb{C})$ data into $\text{SU}(2)$ ones by imposing the constraints $n = 2j$ and $\rho = 2j\gamma$.

Extended BC model

$$S = \left[\prod_i \int d\rho_i 4\rho_i^2 \sum_{j_i m_i} \right] \bar{\varphi}_{j_i m_i}^{\rho_i} \varphi_{j_i m_i}^{\rho_i} + \frac{\lambda}{5} \left[\prod_{a=1}^{10} \int d\rho_a 4\rho_a^2 \sum_{j_a m_a} \right] \left[\prod_{a=1}^{10} (-1)^{-j_a - m_a} \right] \{10\rho\}_{\text{BC}}$$

$$\times \varphi_{j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4}^{\rho_1 \rho_2 \rho_3 \rho_4} \varphi_{j_4 - m_4 j_5 m_5 j_6 m_6 j_7 m_7}^{\rho_4 \rho_5 \rho_6 \rho_7} \varphi_{j_7 - m_7 j_3 - m_3 j_8 m_8 j_9 m_9}^{\rho_7 \rho_3 \rho_8 \rho_9}$$

$$\times \varphi_{j_9 - m_9 j_6 - m_6 j_2 - m_2 j_{10} m_{10}}^{\rho_9 \rho_6 \rho_2 \rho_{10}} \varphi_{j_{10} - m_{10} j_8 - m_8 j_5 - m_5 j_1 - m_1}^{\rho_{10} \rho_8 \rho_5 \rho_1} + \text{c.c.}$$