

PERTURBATIONS IN LOOP QUANTUM COSMOLOGY

William Nelson

Pennsylvania State University

Work with: Abhay Astekar and Ivan Agullo

(see Ivan's ILQG talk, 29th March)

Institute for Gravitation and the Cosmos

- 1 INFLATION
- 2 HAMILTONIAN PERTURBATIONS IN COSMOLOGY
- 3 QFT IN QUANTUM (COSMOLOGICAL) SPACE-TIMES
- 4 PERTURBATIONS IN LQC
- 5 RESULTS AND CONCLUSIONS

INFLATION

See Ivan's ILQG talk 29th March 2011.

Quantum fluctuations of the scalar perturbations Q can be amplified by the background expansion.

The observable is the power-spectrum,

$$\langle 0 | \widehat{R}_{\mathbf{k}}(t) \widehat{R}_{\mathbf{k}'}(t) | 0 \rangle = \delta(\mathbf{k} + \mathbf{k}') |R_{\mathbf{k}}(t)|^2 := \delta(\mathbf{k} + \mathbf{k}') \left(4\pi k^3\right)^{-1} \Delta_R^2(k, t),$$

where $R_{\mathbf{k}} = \frac{\dot{\phi}}{H} Q_{\mathbf{k}}$.

(1)

For (quasi-) de Sitter (i.e. slow-roll)

$$\Delta_R^2(k, t \gg t_k) \approx \frac{H^2(t_k)}{\pi m_{\text{p}}^2 \epsilon(t_k)},$$
(2)

where t_k is given by $H(t_k)\lambda(t_k) \sim 1$.

INFLATION

This is *almost* constant w.r.t. k (scale invariant)

The deviation from scale invariance is parametrised by
(for $\mathcal{V}(\phi) = \frac{1}{2}m^2\phi^2$)

$$n_s := 1 + \frac{d}{d \ln k} \left(\ln \Delta_R^2(k) \right) \approx 1 - 4\epsilon \quad (3)$$

OBSERVATIONAL DATA: WMAP 7 ($k^* \approx 0.002 \text{ Mpc}^{-1}$)

$$\begin{aligned} \Delta_R^2(k^*) &= (2.43 \pm 0.091) \times 10^{-9} \\ n_s(k^*) - 1 &= 0.032 \pm 0.012 \end{aligned}$$

OPEN ISSUES WITH INFLATION

PARTICLE PHYSICS ISSUES

- What is the scalar field?
- Why is the potential flat?
- How does the inflaton couple to the standard model?

QUANTUM GRAVITY ISSUES

- Conditions to obtain inflation?
- Frequencies become transplanckian?
- Singularity?
- Perturbation theory?



INFLATION

For example what is $|0\rangle$?

Typically one takes the Bunch-Davis vacuum at some time t_{onset} , but what if we started in a different state?

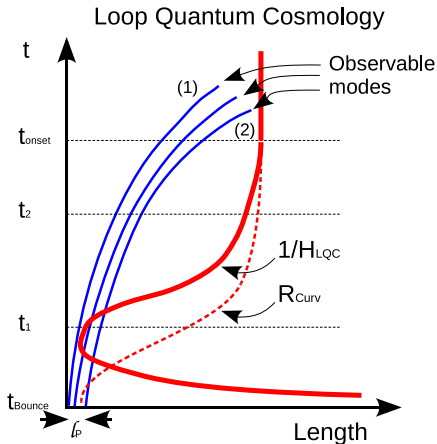
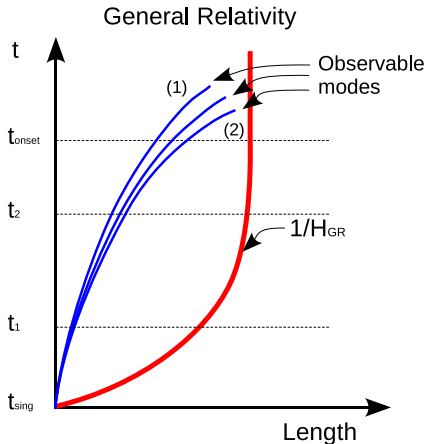
CONSIDER A STATE $|in\rangle$ WITH $N(k)$ PARTICLES

I. AGULLO, L. PARKER, PRD 2011

$$\begin{aligned}\Delta_R^2(k) &= {}^{(0)}\Delta_R^2(k) [1 + 2N(k)] \\ n_s - 1 &= {}^{(0)}n_s - 1 + \frac{d}{d \ln k} (1 + 2N(k))\end{aligned}$$

i.e. the initial state can have *observable* consequences.

INFLATION



$$\lambda_{(1)} < \lambda_{(2)}$$

Typical justification of Bunch-Davis vacuum: the modes were always inside the Hubble radius \Rightarrow requires putting the initial conditions at the singularity!

TAKE HOME POINT

Observations are (potentially) sensitive to the state at the onset of slow-roll.

The pre-inflationary dynamics will (typically) result in a state that contains particles (relative to $|0\rangle$) and hence we have a window on to the pre-inflationary era.

Note: Non-gaussianities will provide the really strong test/restriction on the form of this initial state.

INFLATION AND LQC: QUESTIONS AND ANSWERS?

HOMOGENEOUS LQC IS VERY SUCCESSFUL AT SOLVING THE PROBLEMS OF INFLATION

- Singularity \rightarrow Bounce
- Low densities \rightarrow Recover GR
- With a scalar field and potential \rightarrow Generically get inflation

How does it do for perturbations?

QUANTUM GRAVITY ISSUES

- Conditions to obtain inflation? (background) \checkmark (perturbations) ?
- Frequencies become transplanckian? (perturbations) ?
- Singularity? (background) \checkmark (perturbations) ?
- Perturbation theory? (background) \checkmark (perturbations) ?



THE PROGRAM

- Develop the framework using the most optimistic approximations (i.e. the standard matter constraints of QFT)
- Explore what this says for the issues of inflation.
- Within this framework, look for consistency with observations **and** freedom from the issues of GR **and** (ideally) deviations from the GR predictions.
- Go beyond the approximations and understand their consequences.

TRUNCATION

	Truncation
Isotropic LQC	Loop Quantise FRW Hamiltonian
Anisotropic LQC	Loop Quantise Bianchi Hamiltonians
Gowdy models	Loop Quantise Gowdy Hamiltonians

HAMILTONIAN PERTURBATIONS

Want to find a suitable truncation of full GR to cosmological perturbations:

[(e.g.) D. Langlois, L. Bethke . . .]

FULL GR WITH A MASSIVE SCALAR FIELD
(CONSIDER \mathbb{T}^3 OF SIZE ℓ^3)

$$\Gamma_{\text{Full}} = \underbrace{\Gamma_{\text{H}}}_{\substack{\uparrow \\ \text{Homogeneous}}} \times \underbrace{\Gamma_{\text{IH}}}_{\substack{\uparrow \\ \text{Purely inhomogeneous}}} \quad (4)$$

Symplectic structure and Poisson brackets factor:

- ⇒ Canonical pair for the Homogeneous modes
- ⇒ Canonical pairs for the Inhomogeneous modes

Plus constraints: $\mathcal{C} [N^i]$, $\mathcal{C} [\mathbf{N}]$ and $\mathcal{C} [M]$,

HAMILTONIAN PERTURBATIONS

We are interested in the inhomogeneous modes being *linear* perturbations:

Build up the constraints of 2nd order in the inhomogeneous variables

$$c = c^{(0)} + c^{(1)} + c^{(2)} . \quad (5)$$

Carry out the reduction to 1st order i.e.

Solve the 1st order constraint $\int N c^{(1)} \approx 0$ and find gauge invariant variables.

The dynamics are then given by

$$\int \underset{\substack{\uparrow \\ \text{Homogeneous}}}{N} (c^{(0)} + c^{(2)}) \approx 0$$

$$(6)$$

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HAMILTONIAN PERTURBATIONS

VARIABLES - BEFORE REDUCTION

$$\begin{aligned} A_a^i &= \mathbf{c}^{(0)} \omega_a^i + \alpha_a^i & E_i^a &= p \sqrt{q_0}^{(0)} e_i^a + \epsilon_i^a \\ \Phi &= \phi + \delta\phi & \Pi_\phi &= \pi_\phi + \delta\pi_\phi \end{aligned}$$

7- constraints, $(\alpha_a^i, \delta\phi) = 9 + 1$ inhomogeneous configuration variables

\Rightarrow 3 true, inhomogeneous degrees of freedom (1 scalar, 2 tensor)

VARIABLES - AFTER REDUCTION

Homogeneous degrees of freedom: $(V, \mathbf{b}, \phi, \pi_\phi)$
(rather than $(\mathbf{c}, \mathbf{p}, \phi, \pi_\phi)$)

Inhomogeneous degrees of freedom: $(Q_{\mathbf{k}}, P_{\mathbf{k}}), (h_{\mathbf{k}}^\pm, \pi_{h_{\mathbf{k}}}^\pm)$

HAMILTONIAN PERTURBATIONS

The scalar Hamiltonian constraint (in harmonic time) is

$$\underbrace{\frac{\pi_\phi^2}{2\ell^3} + \frac{m^2}{2\ell^3} V^2 \phi^2 - \frac{3}{8\pi G \ell^3} b^2 V^2}_{\substack{\uparrow \\ \text{Background part}}} + \int d^3k \left[\frac{1}{2} P_{\mathbf{k}}^2 + f(V, b, \phi, \pi_\phi, k) Q_{\mathbf{k}}^2 \right]$$

where $V = a^3 \ell^3$ is the physical volume of the torus.

Note: We use quantum theory from the bounce to $\rho_{\text{kinetic}} = \rho_{\text{potential}}$ ($\sim 10^4 t_p$), by which time the curvature is $\sim 10^{-10} m_p^2$.

After this, QFT in curved space-time is fine.

TAKE HOME POINT

Gauge invariant scalar perturbations satisfy

$$\frac{\pi_\phi^2}{2\ell^3} + \frac{m^2}{2\ell^3} V^2 \phi^2 - \frac{3}{8\pi G\ell^3} b^2 V^2 + \int d^3k \left[\frac{1}{2} P_{\mathbf{k}}^2 + f(V, b, \phi, \pi_\phi, k) Q_{\mathbf{k}}^2 \right] = 0 \quad (7)$$

Gauge invariant scalar perturbations look exactly like test scalar fields with a time dependent mass.

Note: tensor modes are automatically gauge invariant and their Hamiltonian has the same form.

(TEST) QFT IN (COSMOLOGICAL) QST

We know how to describe test Quantum Fields on Quantum Cosmological Space-Times. Ashtekar, Kaminski, Lewandowski, PRD 2009

- Write down the classical constraint and quantise this
(in \mathbb{T}^3 of volume $\ell^3 a^3(\tau)$)

Physical states obey,

$$\underbrace{-\hbar^2 \partial_\phi^2 \Psi}_{\substack{\text{Scalar field} \\ \text{(relational)time}}} = \underbrace{\left[\hbar^2 \Theta - 2\ell^2 \widehat{H}_{\tau, \mathbf{k}} \right]}_{\substack{\text{difference} \\ \text{operator of LQC}}} \Psi \quad (8)$$
$$\frac{1}{2} \left[\widehat{P}_{\mathbf{k}}^2 + g(a, k) \widehat{Q}_{\mathbf{k}}^2 \right]$$

τ is Harmonic time ($N = a^3$)

(TEST) QFT IN (COSMOLOGICAL) QST

Make three systematic approximations (test-field approx):

- Assume the state can be factored as: $\Psi = \Psi_0(V) \otimes \Psi_{\text{pert}}(Q_{\mathbf{k}})$
- Take expectation values w.r.t. geometry state.
- Approximate (e.g.) $\widehat{a^4} \approx \widehat{a}^4$

The procedure accounts for all the holonomy and inverse operator corrections.

(TEST) QFT IN (COSMOLOGICAL) QST

Now use the test field approximation i.e. $\sim \widehat{H}_0 \gg \widehat{H}_{\tau,\mathbf{k}}$ to get,

$$-i\hbar\partial_\phi\Psi = \left[\widehat{H}_0 - \underbrace{\left(\ell^{-3}\widehat{H}_0 \right)^{-1/2} \widehat{H}_{\tau,\mathbf{k}} \left(\ell^{-3}\widehat{H}_0 \right)^{-1/2}}_{\widehat{H}_{\phi,\mathbf{k}}} \right] \Psi \quad (9)$$

But $\widehat{H}_{\phi,\mathbf{k}}$ contains (e.g.) \widehat{a}^4 , which are *time independent*

Go to the interaction picture with $\Psi_{\text{int}} := e^{-\frac{i}{\hbar}\widehat{H}_0(\phi-\phi_0)}\Psi$.

$$\begin{cases} \widehat{H}_0 \rightarrow \text{Hamiltonian of 'Heavy' d.o.f.} \\ \widehat{H}_{\phi,\mathbf{k}} \rightarrow \text{Hamiltonian of 'light' d.o.f.} \end{cases}$$

$$i\hbar\partial_\phi\Psi_{\text{int}} = \widehat{H}_{\phi,\mathbf{k}}\Psi_{\text{int}}$$

(TEST) QFT IN (COSMOLOGICAL) QST

Finally, assume $\Psi_{\text{int}} = \Psi_0(\mathbf{a}) \otimes \Psi_{\text{pert}}(\mathbf{Q}_{\mathbf{k}})$

with $\Psi_0(\mathbf{a})$ sharply peaked around some classical back-ground trajectory.

Then taking the expectation value w.r.t. Ψ_0 we get,

$$i\hbar\partial_\phi\Psi_{\text{pert}} = \frac{1}{2} \left[\widehat{P}_{\mathbf{k}}^2 + g(\langle\hat{\mathbf{a}}\rangle, \mathbf{k}) \widehat{Q}_{\mathbf{k}}^2 \right] \Psi_{\text{pert}} . \quad (10)$$

Recall: $\Psi_0(\mathbf{a})$ is sharply peaked so $\langle\hat{\mathbf{a}}^2\rangle \approx \langle\hat{\mathbf{a}}\rangle^2$.

This precisely the QFT Hamiltonian for test fields on the *effective* background $\langle\hat{\mathbf{a}}\rangle$

PERTURBATIONS LQC

Recall that our classical Hamiltonian was:

$$H_{\text{GI}}^{\text{S}} = H^{(0)} + \int d^3k \left[\frac{1}{2} P^2 + \frac{1}{2} f(V, \mathbf{b}, \phi, \pi_\phi, k) Q^2 \right]. \quad (11)$$

But this is *exactly* the same as test scalar fields, with a 'time' dependent mass

$$f(V, \mathbf{b}, \phi, \pi_\phi, k) = 12\pi G \pi_\phi^2 - 16\pi^2 \gamma^2 G^2 \frac{\pi_\phi^4}{V^2 b^2} + 8\pi \gamma G \frac{V^2}{\ell^6} \frac{\pi_\phi}{V b} \frac{dV}{d\phi}. \quad (12)$$

TAKE HOME POINT

Up to fluctuations we get back to the standard QFT Hamiltonian on an *effective* LQC background.

But note:

- That is only true for the Hamiltonian in this form.
(difference between H^2 and ρ in LQC)
- Starting with the Hamiltonian formulation is crucial
(starting with the 4D perturbed Einstein's equations leads to difficulties interpreting gauge)
- This process is a systematic derivation with LQC.
 - ▶ 1 Truncate the classical theory (first order perturbations)
 - ▶ 2 Assume the state is a tensor product
 - ▶ 3 Take expectation values w.r.t. geometry state
 - ▶ 4 Assume the background part is sharply peaked

PERTURBATIONS LQC

TRANSPLANCKIAN FREQUENCIES

For QFT on a classical background, we do not trust transplanckian frequencies.

Here we have an Hamiltonian that includes the quantum geometry: transplanckian frequencies are fine, only total energy density in the modes is bounded.

Compare to background:

π_ϕ is *not* bounded, but kinetic energy density is: $< \rho_c$

This solves the transplanckian problem in the sense that *observable* modes can all be correctly treated in this set up. However, the conceptual question remains
(although the question is more refined than before)

PERTURBATIONS IN LQC

PERTURBATION THEORY

If we consider only those modes that are observable by WMAP, what is the total energy in the modes at the bounce? Are they really perturbations?

Subtle question! Ambiguities surround the definition of total energy . . .
Using adiabatic regularisation: the simplest vacuum state is a perturbation, even at the bounce.

For the modes we are interested in, this truncation is consistent.

RESULTS AND CONCLUSIONS

INFLATION IN GR

- Assume scalar field and potential
- Set 'natural' vacuum for perturbations at some time, $t = t_{\text{onset}}$.
- Evolve and calculate late time power-spectrum, $\Delta_R^2(|\mathbf{k}|)$.

First state of the program:

INFLATION IN LQC

- Assume scalar field and potential
- Set 'natural' vacuum for perturbations at bounce, $t = t_{\text{bounce}}$.
- Evolve and calculate late time power-spectrum, $\Delta_R^2(|\mathbf{k}|)$.



RESULTS AND CONCLUSIONS

LQC may not need inflation at all, but here we are looking at inflation within LQC. Can LQC solve the quantum gravity issues facing inflation?

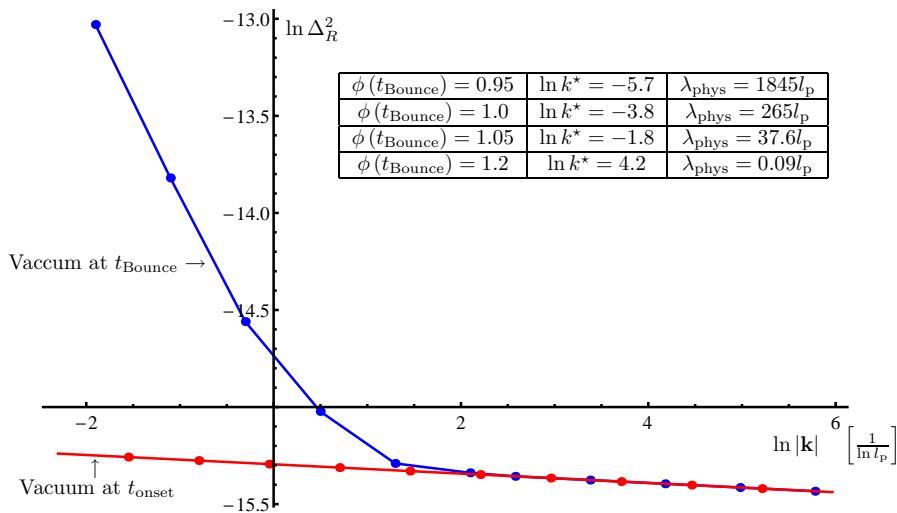
PARAMETERS

- (background) ϕ and mass m . (2 dofs)
- (perturbations) $Q_{\mathbf{k}}$ and $P_{\mathbf{k}}$ ($k \in (k_{\min}, k_{\max})$)

The initial state of the perturbations:

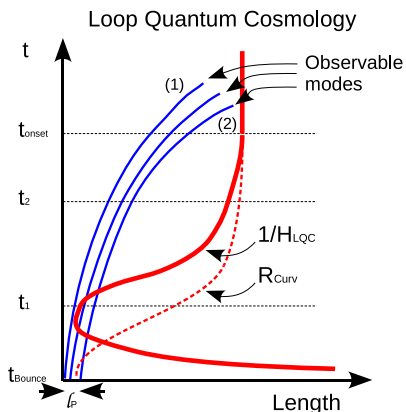
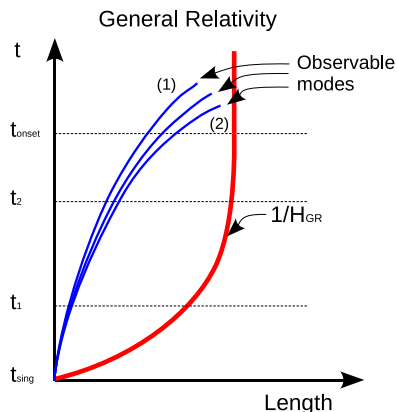
- Is there an initial state that is compatible with observations?
- If so, is this compatibility generic?
- Are there potential observables?

RESULTS AND CONCLUSIONS



Different $\phi(t_{\text{Bounce}}) \Leftrightarrow$ position of k^*

INFLATION



Wavelengths shorter than the curvature scale do not generate particles
 \Rightarrow for these modes the Bunch-Davis vacuum is a good approximation.

CONCLUSIONS

- There is a framework of inflation consistent with the quantum gravity era.
- The simplest initial state is consistent with observational data, for almost all values of $\phi(t_{\text{Bounce}})$ and m .
- The power-spectra are altered (in particular non-gaussianities): potential observables.

Next steps:

- Understand if this deviation is physical or due to our approximations.
- Remove the approximations: look for direct links from the LQG to derive the Hamiltonian from 1st principles.
- Understand what this says about the initial state: built in cut-off?
- Remove the restriction to a finite range of \mathbf{k} .