Vacuum state for Loop Quantum Cosmology

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- Cosmological perturbations in (L)QC: observational window;
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  - Minimization of renormalized stress-energy tensor[2], uncertainty relations[3],
  - Minimization of smeared quantities[4].

[1] Fahn, et. al, Universe **5**, 170 (2019), Elizaga Navascués, et. al, Class. Quant. Grav. **36**, 185010 (2019)

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Cosmological perturbations

- Inflaton field ϕ subject to potential $V(\phi)$,
- Scalar and tensor gauge invariant perturbations, minimally coupled: $\mathcal{Q}$, $\mathcal{T}^I$,
- Redefinition: $u = a\mathcal{Q}$, $\mu^I = a\mathcal{T}^I$,
- Fourier modes:

$$u(\eta, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \, u_{\vec{k}}(\eta) e^{i\vec{k}\vec{x}},$$

Cosmological perturbations

- Equations of motion

$$u''_{\vec{k}}(\eta) + \left(k^2 + s^{(s)}(\eta)\right) u_{\vec{k}}(\eta) = 0,$$
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- in hybrid LQC:

$$s^{(t)} = -\frac{4\pi G}{3} a^2 \left(\rho - 3P\right), \qquad s^{(s)} = s^{(t)} + \mathcal{U},$$

$$\mathcal{U} = \mathcal{U}\left[V(\phi), V_{,\phi}, V_{,\phi\phi}, \text{bckg}\right].$$

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- Power spectra ($z = a\dot{\phi}/H$):

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|u_k|^2}{z^2}\bigg|_{\eta=\eta_{\text{end}}}, \qquad \mathcal{P}_{\mathcal{T}}(k) = \frac{32k^3}{\pi} \frac{|\mu^I_k|^2}{a^2}\bigg|_{\eta=\eta_{\text{end}}}.$$

Cosmological perturbations

Adiabatic states

- ansatz $u_k(\eta) = \frac{1}{\sqrt{2W_k(\eta)}} e^{-i\int^\eta W_k(\bar{\eta})\mathrm{d}\bar{\eta}}$:

$$W_k^2 = k^2 + s(\eta) - \frac{1}{2}\frac{W_k''}{W_k} + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2.$$

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- $W_k^{(n)} \xrightarrow[k\to\infty]{} W_k$, at least as $\mathcal{O}\left(k^{n-\frac{1}{2}}\right)$:

$$\left(W_k^{(n+2)}\right)^2 = k^2 + s(\eta) - \frac{1}{2}\frac{W_k^{(n)\prime\prime}}{W_k^{(n)}} + \frac{3}{4}\left(\frac{W_k^{(n)\prime}}{W_k^{(n)}}\right)^2,$$

- $W_k^{(0)} = k$.

States of Low Energy [Olbermann, Class. Quant. Grav. **24**, 5011 (2007)]

- Fewster 2000 [Class. Quant. Grav. **17**, 1897–1911 (2000)]:
  - Renormalized energy density, smeared along time-like curve, is bounded from below as function of state.
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- Candidates for vacuum of perturbations in LQC:
  - Minimization of regularized energy density,
  - Exact Hadamard states.

States of Low Energy

A field T_k:

$$\ddot{T}_k + 3H\dot{T}_k + \omega_k^2(t)T_k = 0,$$

- scalar or tensor perturbations: $\omega_k^2(t) = \dfrac{k^2 + s(t)}{a^2} + H^2 + \dfrac{\ddot{a}}{a}$.

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Minimize mode contribution to smeared energy density:

$$E(T_k) = \frac{1}{2}\int dt\, f^2(t)\left(\frac{|\pi_{T_k}|^2}{a^6} + \omega_k^2|T_k|^2\right),$$

where $\pi_{T_k} = a^3\dot{T}_k$, $f(t)$ smearing function.

Procedure

- Arbitrary solution S_k,
- Bogoliubov transformation:

$$T_k = \lambda(k) S_k + \mu(k) \bar{S}_k,$$

with $|\lambda(k)|^2 - |\mu(k)|^2 = 1$, fixing $\mu(k) \in \mathbb{R}^+$,

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- Defining:

$$c_1(k) := \frac{1}{2} \int dt\, f^2(t) \left(|\dot{S}_k|^2 + \omega_k^2 |S_k|^2 \right) = E(S_k),$$
$$c_2(k) := \frac{1}{2} \int dt\, f^2(t) \left(\dot{S}_k^2 + \omega_k^2 S_k^2 \right).$$

Procedure

$$E(T_k) = \left(2\mu^2(k) + 1\right)\, c_1(k) + \underbrace{2\,\mu(k)\,\mathsf{Re}[\lambda(k)\,c_2(k)]}_{2\mu|\lambda||c_2|\cos[\mathsf{Arg}(\lambda)+\mathsf{Arg}(c_2)]} \quad .$$

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Minimizing $E(T_k)$ with respect to μ:

$$\mu(k) = \sqrt{\frac{c_1(k)}{2\sqrt{c_1^2(k) - |c_2(k)|^2}} - \frac{1}{2}},$$

$$\lambda(k) = -e^{-i\mathsf{Arg}[c_2(k)]}\sqrt{\frac{c_1(k)}{2\sqrt{c_1^2(k) - |c_2(k)|^2}} + \frac{1}{2}} \ .$$

Procedure

State of Low Energy associated with $f(t)$

$$T_k = \lambda(k) S_k + \mu(k) \bar{S}_k,$$

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State of Minimal Energy

The state T_k that is of low energy for *any* $f(t)$.
Only exists in ultrastatic spacetimes.

Properties

In maximally symmetric spacetimes:

- Minkowski: SLE trivially identical to Minkowski vacuum;
  - start with $S_k =$ Minkowski vacuum
  - $c_2 = 0$ trivially $\Rightarrow |\lambda| = 1, \mu = 0$
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- de Sitter: singles out Bunch-Davies
  - support of $f \to$ distant past
  - [massless case: our work]

Properties (Banerjee, Niedermaier, J. Math. Phys. **61**, 103511 (2020))

- Independence of fiducial solution S_k;
- T_k admits UV and IR expansions;
- UV aymptotic behavior of $|T_k|^2$ is independent of f;
- IR symptotic behavior is Minkowski-like for all f;
- As vacuum states of perturbations: agreement with observations for cosmological models with period of kinetic dominance prior to inflation.

SLEs in LQC

1. Obtain S_k numerically
   - give initial conditions (irrelevant)
     $\hookrightarrow$ 0th order adiabatic at bounce:
     $$u_k(0) = \frac{1}{\sqrt{2k}},\ u_k'(0) = -i\sqrt{\frac{k}{2}}$$
   - $V(\phi) = m^2\phi^2/2$
   - $m = 1.2 \times 10^{-6}\ m_{\mathsf{Pl}}$
   - $\phi_B = 1.225\ m_{\mathsf{Pl}}$

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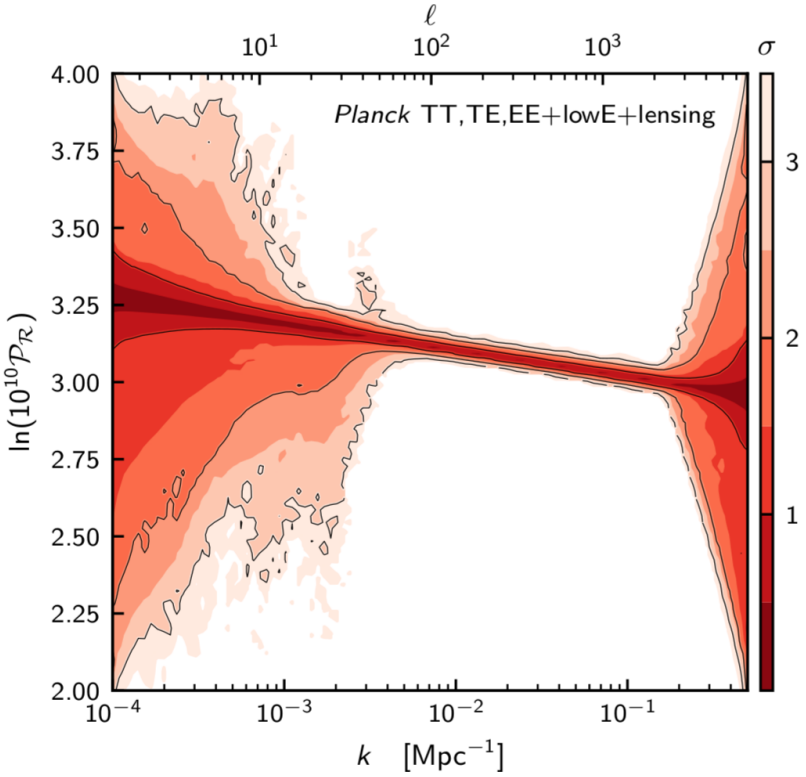

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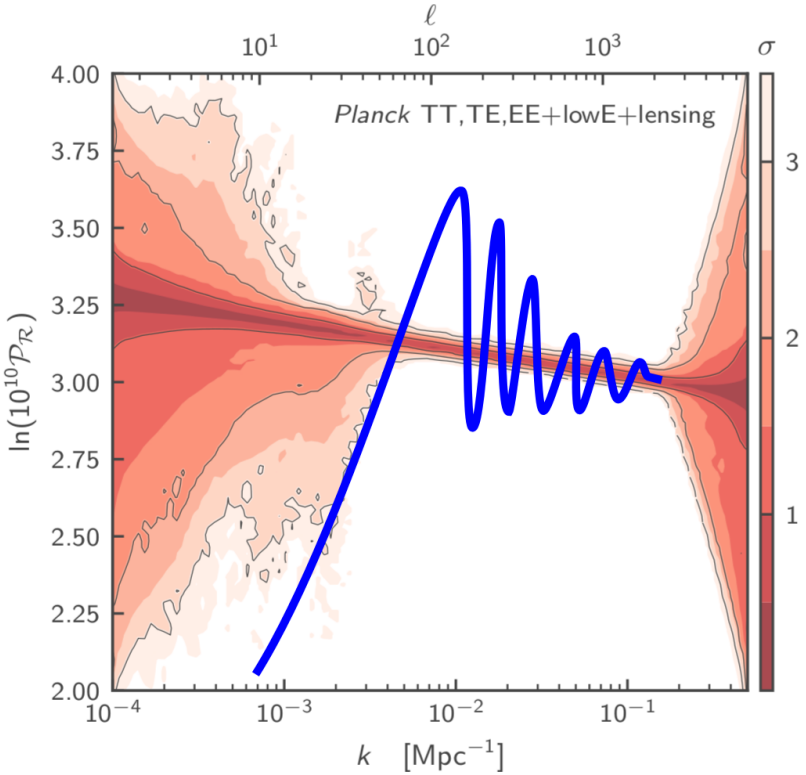

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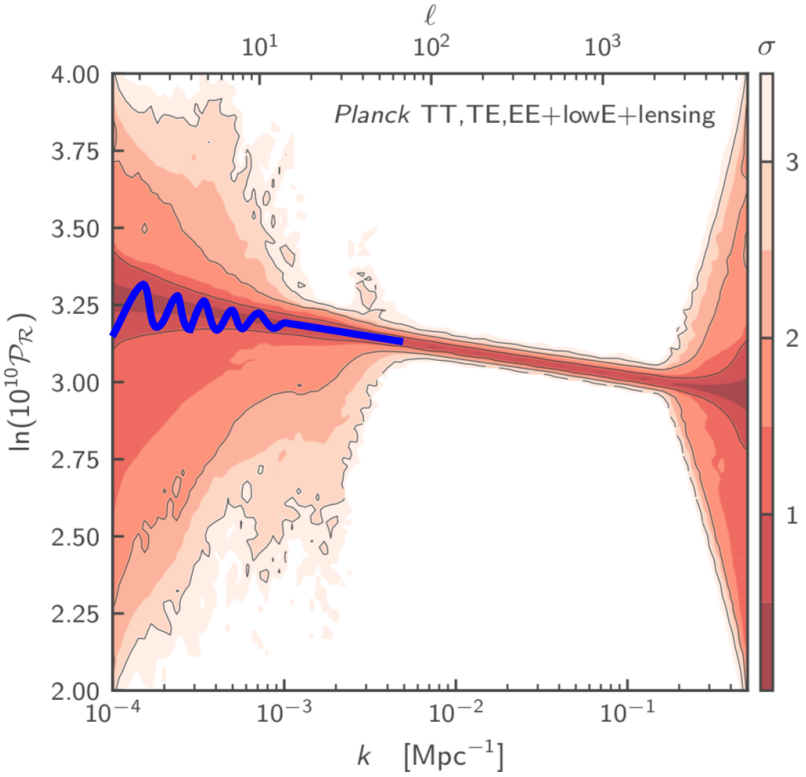

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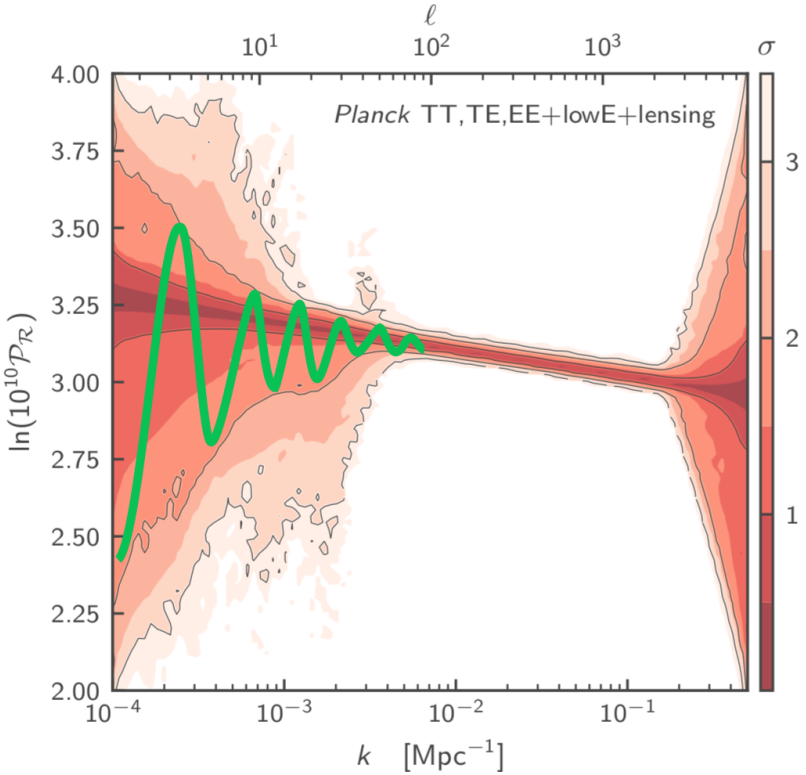

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   - usually in LQC, 2 strategies for choosing initial conditions of perturbations
     - at bounce
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   - usually in LQC, 2 strategies for choosing initial conditions of perturbations
     - at bounce
     - at asymptotic time well before bounce
   - here: no need for initial conditions, but need to choose f^2:
     - support on "whole" evolution (since well before to well after bounce):
       $\rightarrow$results insensitive to shape and support of f^2 as long as it is wide enough
     - support on expanding branch only (f^2 is sharp (but smooth) step function starting at bounce):
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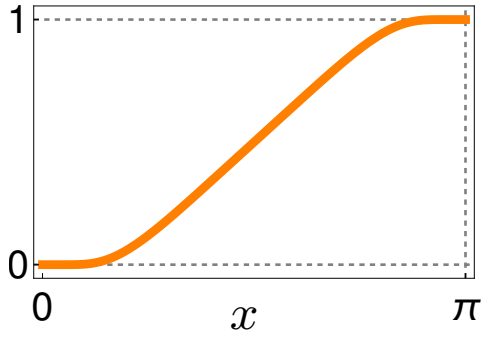

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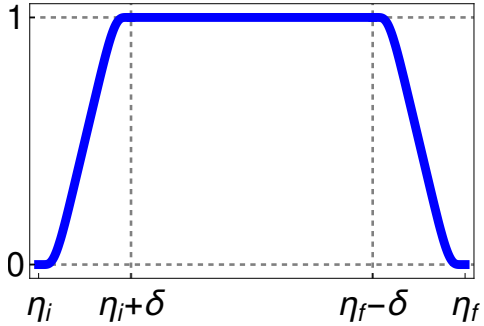

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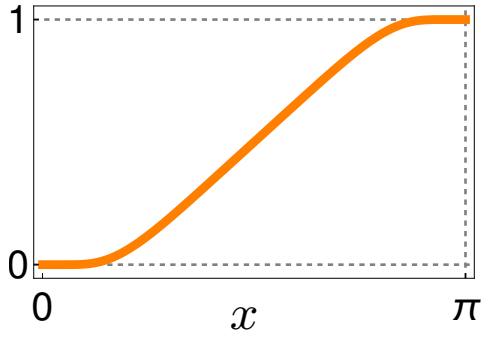

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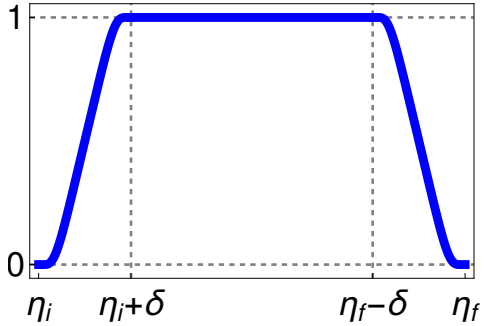

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3. compute $c_1, c_2 \longrightarrow \mu, \lambda \longrightarrow T_k$

SLEs in LQC - Results

- Initial conditions:

$$u_k(0) = \frac{1}{\sqrt{2D_k}},$$
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D_k: positive function,
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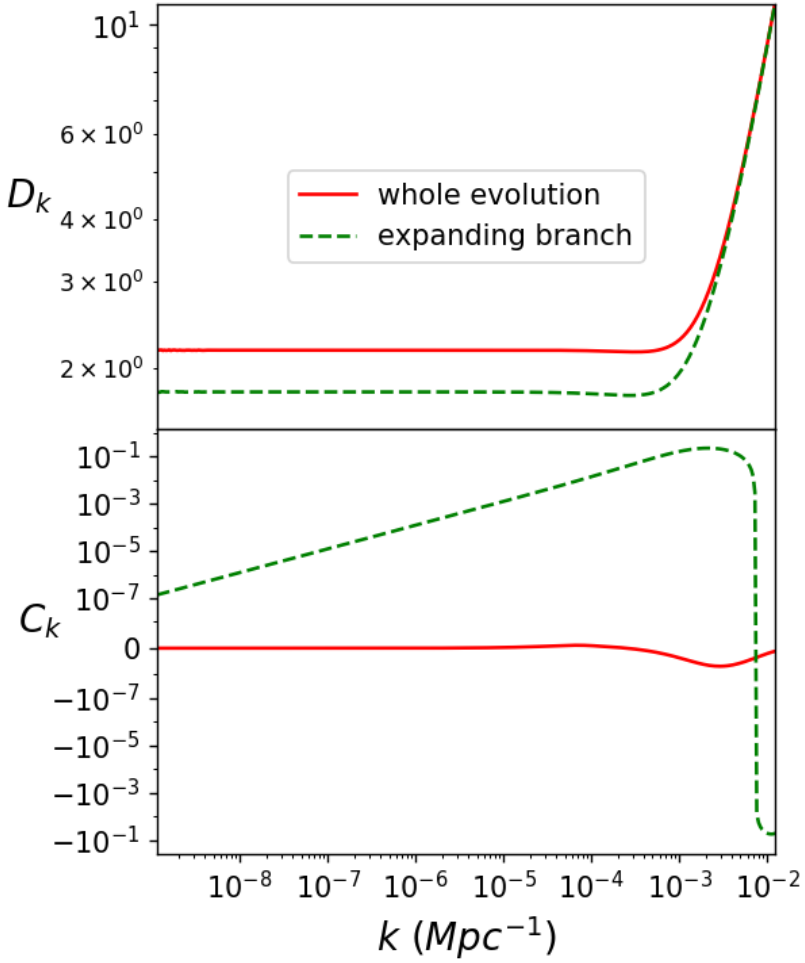

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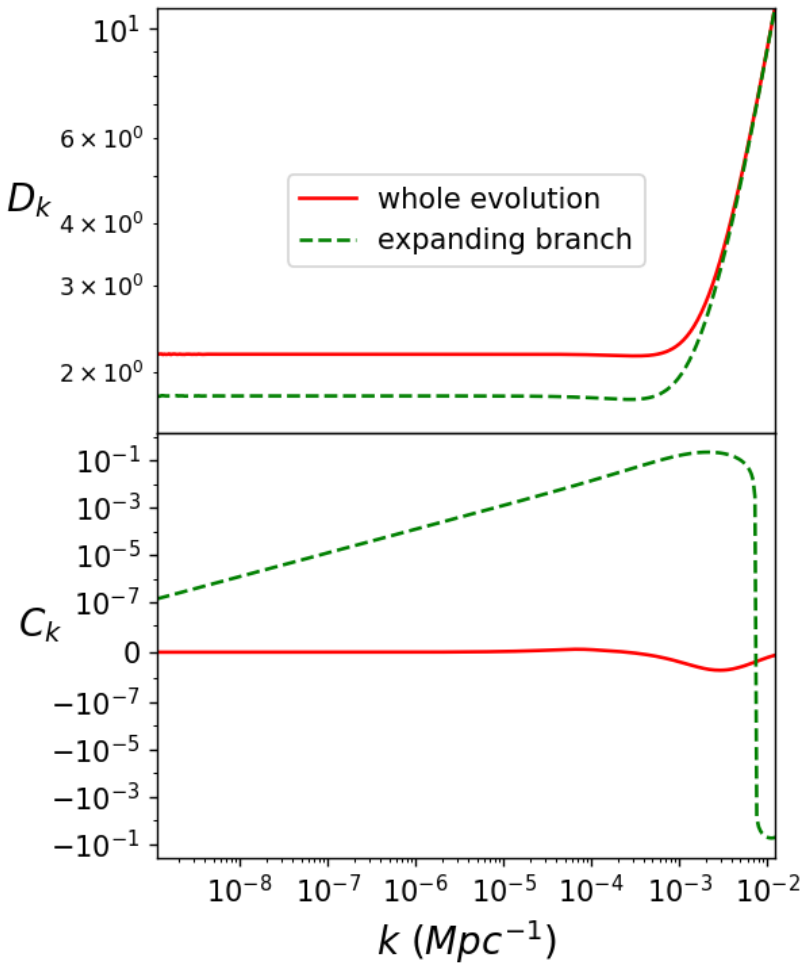
- scalar $\sim$ tensor *at bounce*
  - $s^{(s)} = s^{(t)} + \mathcal{U} \simeq s^{(t)}$
  - $S_k(0) \leftrightarrow$ 0th order adiabatic ($D_k = k, C_k = 0$)



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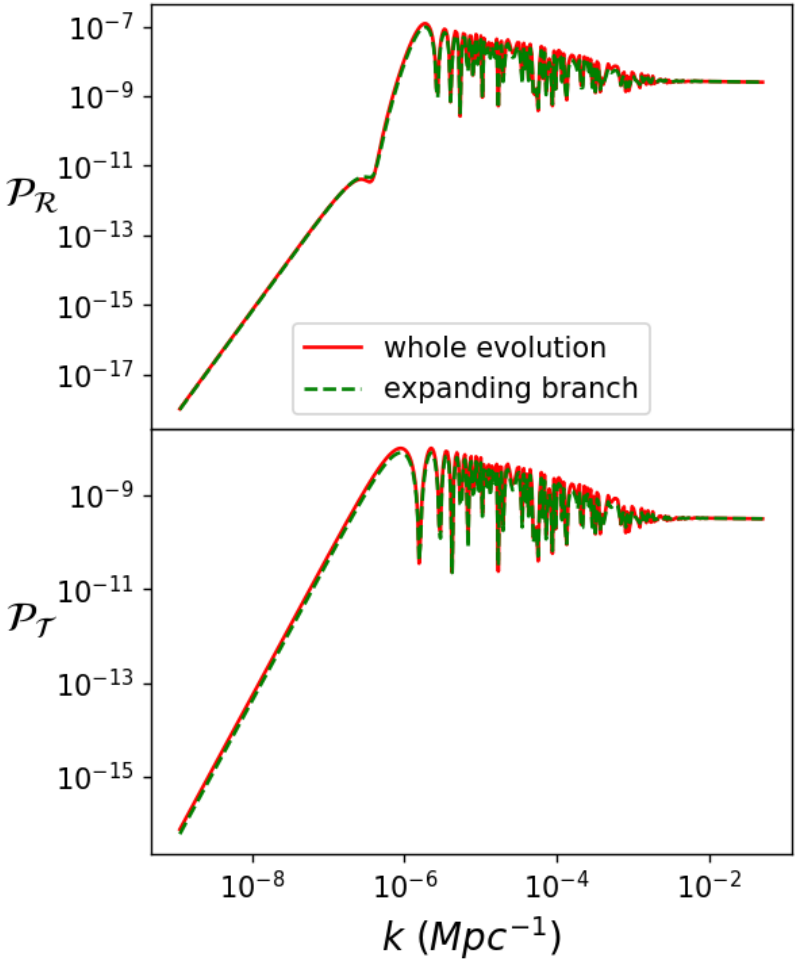

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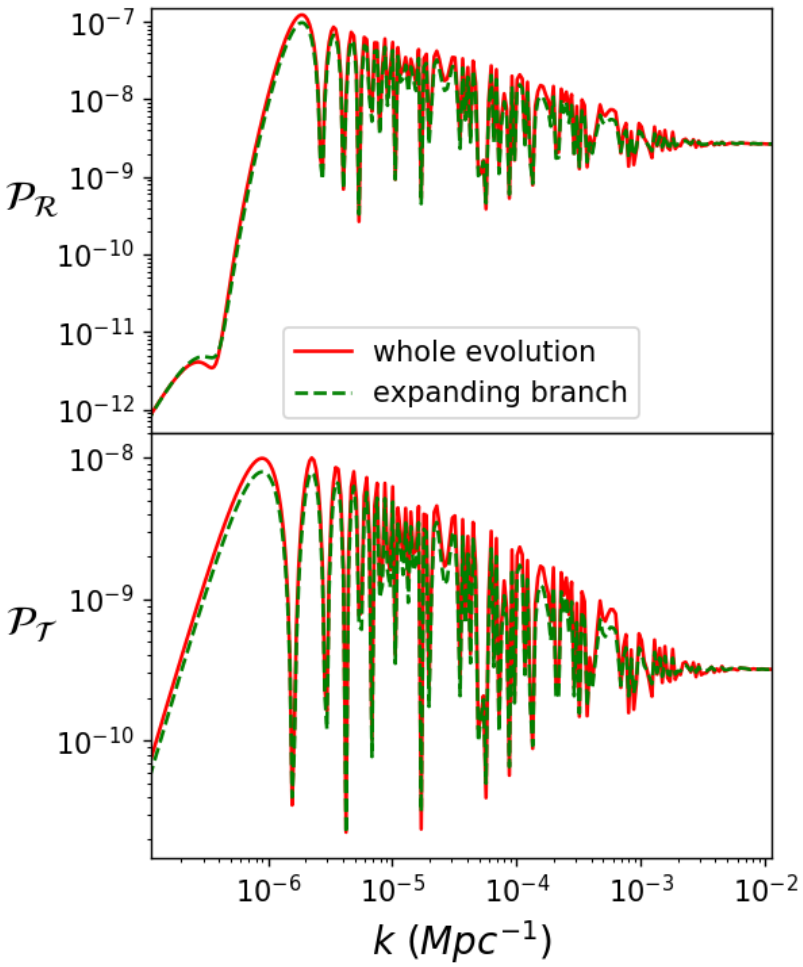

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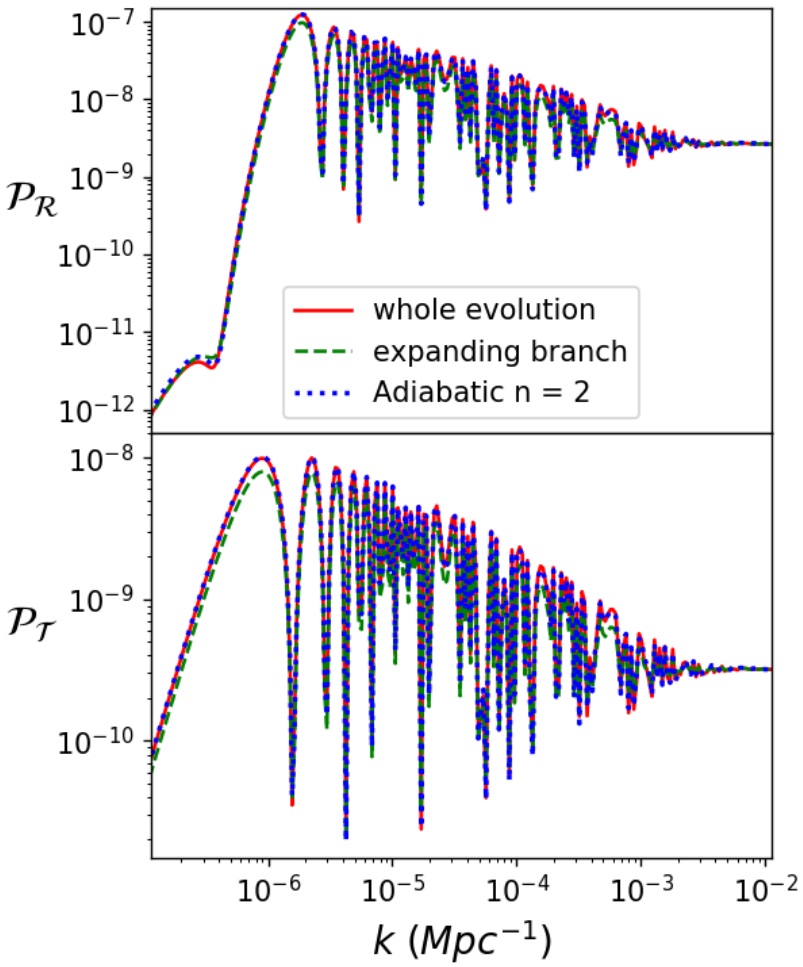

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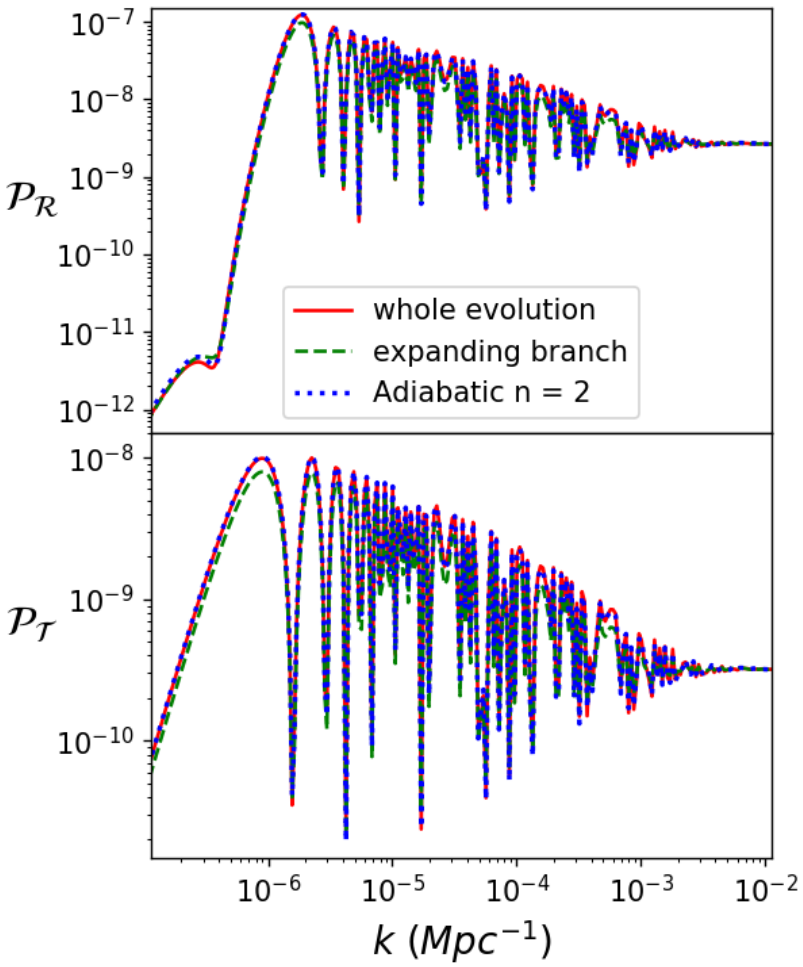
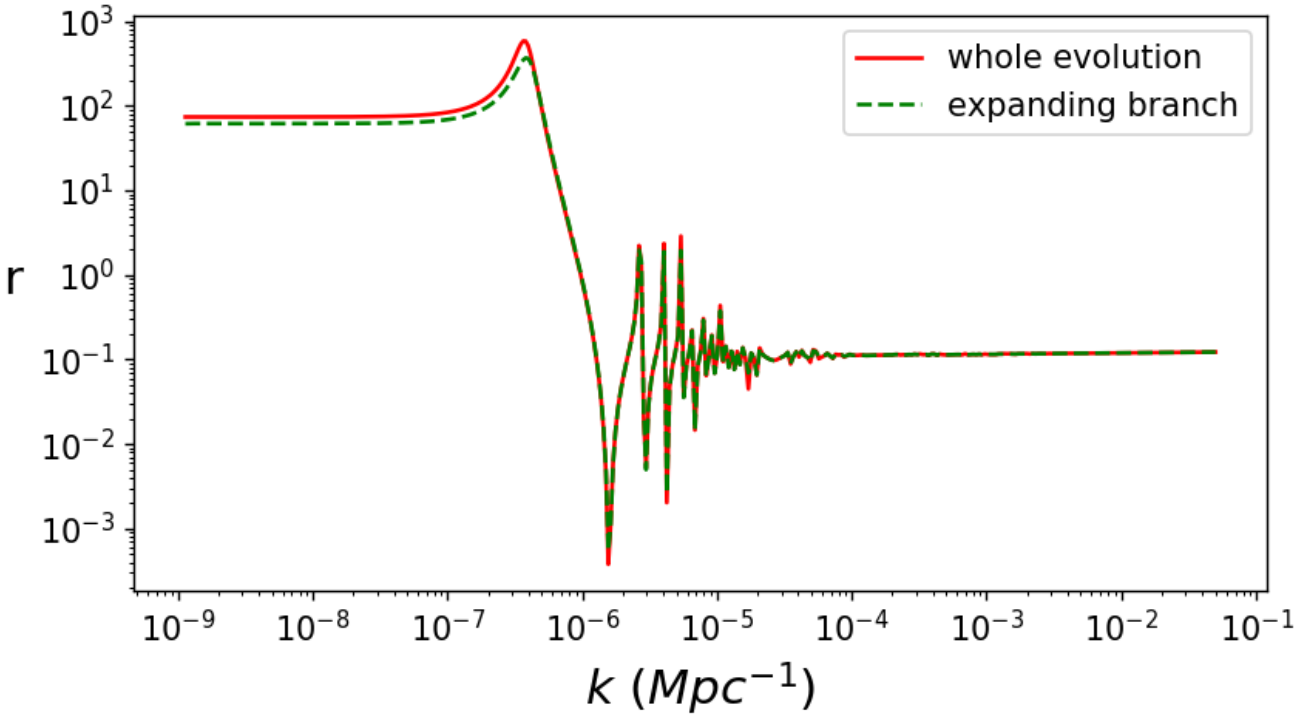
- Observationally: close to 2nd order Adiabatic;
- Fundamentally different, SLEs:
  - minimize smeared energy density,
  - are exact Hadamard states: computation of stress-energy tensor



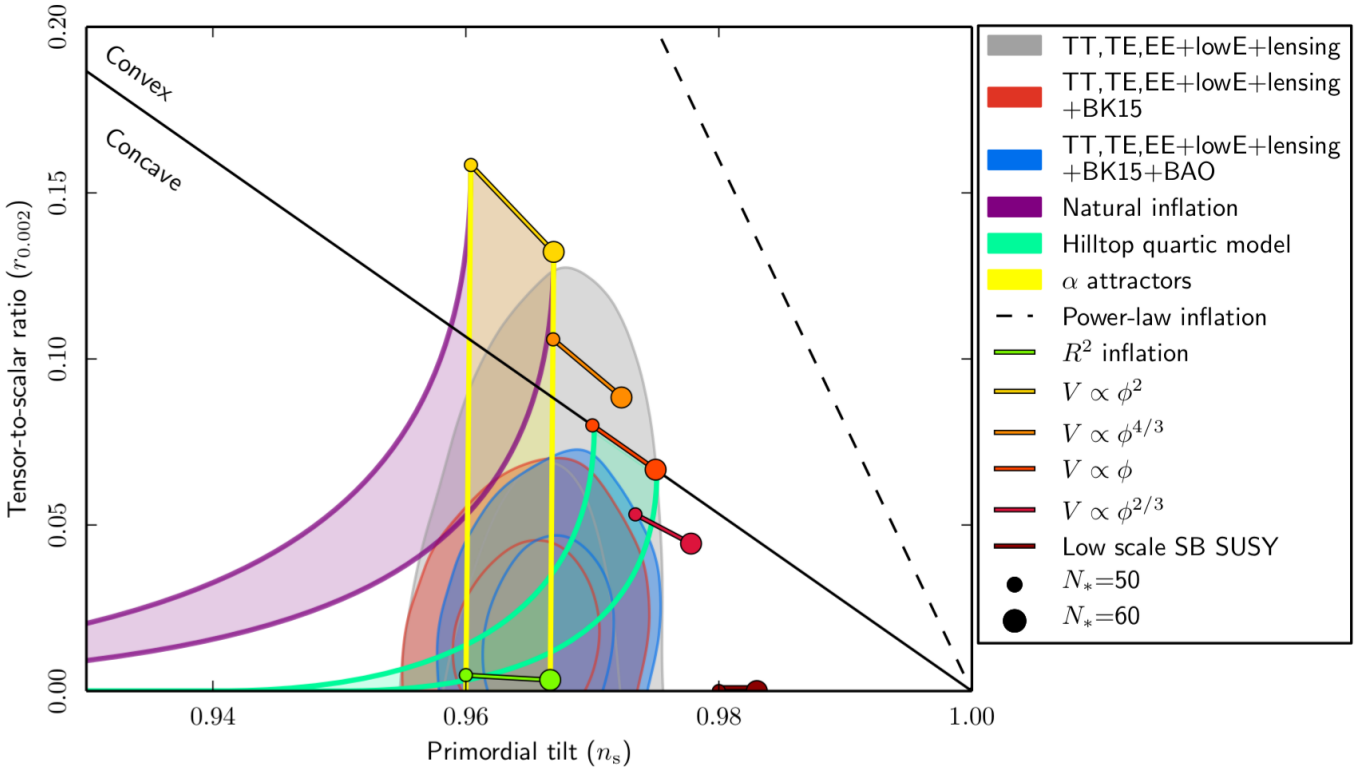
SLEs in LQC - Results

- Tensor-to-scalar ratio
  - scale invariance from smaller k.



SLEs in LQC - Results

- Tensor-to-scalar ratio:
  $r_{0.002} \simeq 0.117$
- Spectral index:
  $n_s \simeq 0.969$



Planck 2018: Y. Akrami et al.(Planck), Astron. Astrophys. **641**, A10 (2020)

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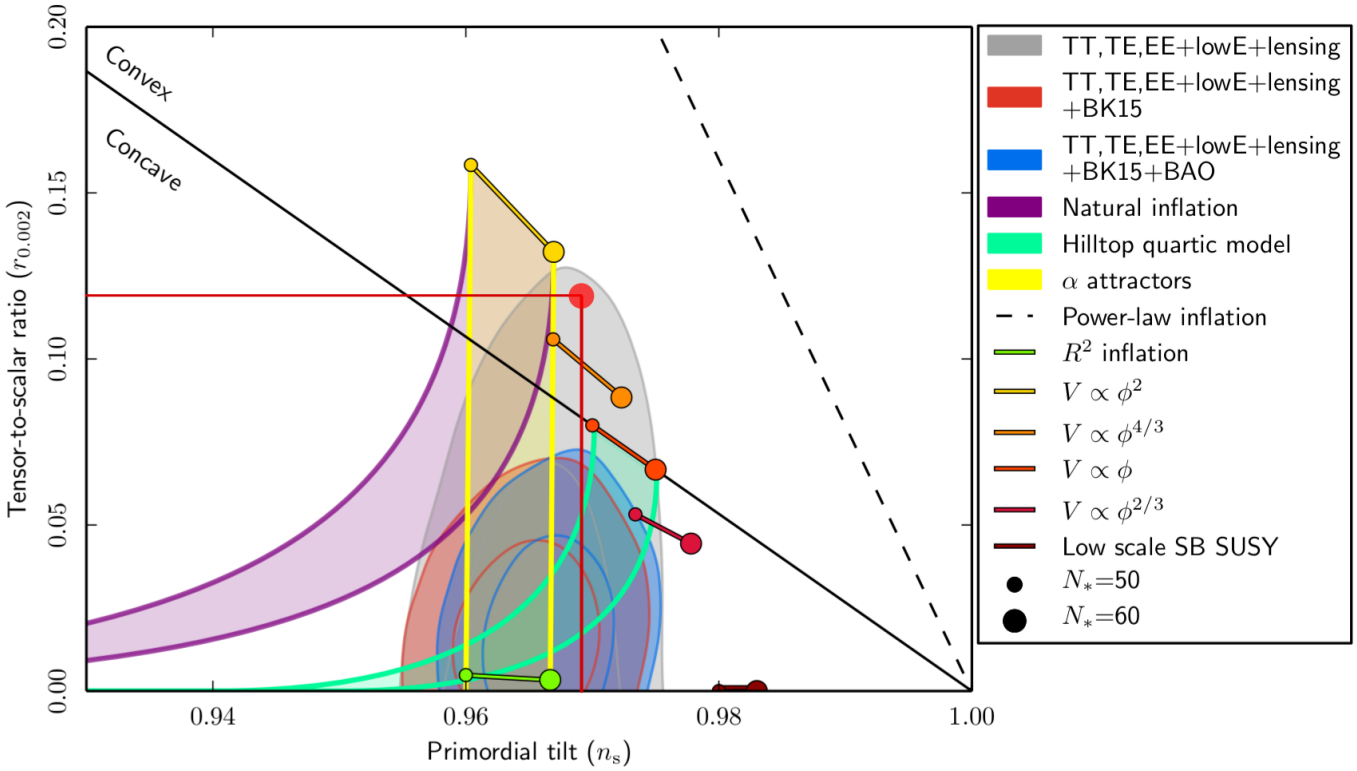

Planck 2018: Y. Akrami et al.(Planck), Astron. Astrophys. **641**, A10 (2020)

Summary and outlook

- SLEs are good candidates for vacua:
  - Hadamard states,
  - Minimize smeared energy density,
  - IR and UV asymptotic behaviors independent of test function;
- IR and UV agreement with observations when KD precedes inflation;
- In LQC:
  - qualitative agreement with observations,
  - insensitive to choice of test function;
- Rigorous statistical analysis:
  - parameters of the background,
  - quantization,
  - $V(\phi)$.

Thank you for your attention