Spherically symmetric quantum spacetimes coupled to a thin null-dust shell

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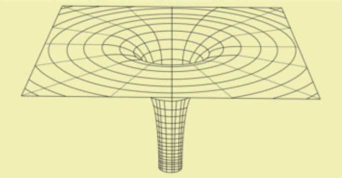
Plan of the talk

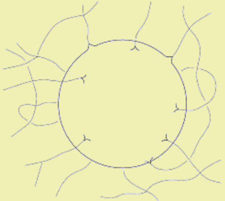
1) Introduction.
2) Classical description and Dirac observables.
3) Self-gravitating quantum shell.
   a) Consistent Dirac quantization (Abelian scalar constraint)
   b) Quantum Dirac (parametrized) observables.
   c) Immediate physical consequences (singularity resolution).
5) Conclusions.
6) Discussion and outlook.

Introduction

1) Spherically symmetric spacetimes:
   a) Gravitational collapse.

   b) Black hole physics: formation, singularity,



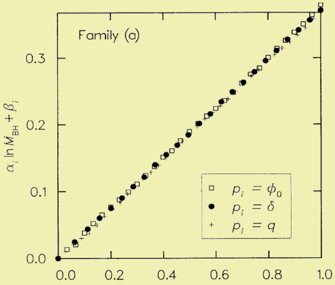


$$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ, \quad T_{\mathsf{H}} = \frac{\kappa}{2\pi}, \quad S_{\mathsf{BH}} = \frac{A}{4},$$

evaporation (Hawking radiation) and information loss paradox.

Introduction

2) Massless scalar field (Choptuik).



3) Thin null-dust shell (Louko, Whiting and Friedman).

Quantization: (Hájícek, Kiefer)

a) Embedding geometrical variables.

b) Partial quantization in the neighbor of the shell.

c) Selfadjointness of the true Hamiltonian prevents eternal black formation: bouncing shells.

Classical system: Ashtekar variables

1) Phase space $\Big(K_x(x), E^x(x)\Big)$, $\Big(K_\varphi(x), E^\varphi(x)\Big)$ and (r,p).

2) Spatial metric: $dh^2 = \frac{(E^\varphi)^2}{|E^x|}dx^2 + |E^x| d\Omega^2$

3) The Hamiltonian is a linear combination of constraints

$$H(N) := \int dx\, N\Bigg[\frac{((E^x)')^2}{8\sqrt{|E^x|}E^\varphi} - \frac{E^\varphi}{2\sqrt{|E^x|}} - 2K_\varphi\sqrt{|E^x|}K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{|E^x|}} - \frac{\sqrt{|E^x|}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} + \frac{\sqrt{|E^x|}(E^x)''}{2E^\varphi} + \frac{\sqrt{|E^x|}}{E^\varphi}\eta p\delta(x-r)\Bigg], \quad (1)$$

$$H_x(N^x) := \int dx\, N^x \left[E^\varphi K_\varphi' - (E^x)'K_x - p\,\delta\,(x-r)\right]\ . \quad (2)$$

fulfilling the algebra

$$\{H_x(N^x), H_x(\tilde{N}^x)\} = H_x\Big(N^x(\tilde{N}^x)' - (N^x)'\tilde{N}^x\Big),\ \{H(N), H_r(N^x)\} = H(N^xN'),$$

$$\{H(N), H(\tilde{N})\} = H_x\left(\frac{E^x}{(E^\varphi)^2}\left[N\tilde{N}' - N'\tilde{N}\right]\right). \quad (3)$$

Classical system: new constraint algebra

4) We Abelianize the scalar constraint (as in vacuum)

$$\tilde{H} = \frac{(E^x)'}{E^\varphi} H - 2K_\varphi \frac{\sqrt{|E^x|}}{E^\varphi} H_x, \quad N_{\rm new} = \frac{E^\varphi}{(E^x)'} N, \quad N^x_{\rm new} = N^x + 2K_\varphi \frac{\sqrt{|E^x|}}{(E^x)'} N. \qquad (4)$$

The total Hamiltonian with boundary terms now reads

$$H_T = \int dx \Bigg[- N'_{\rm new} \left(-\sqrt{|E^x|}\left(1 + K_\varphi^2\right) + \frac{\left((E^x)'\right)^2 \sqrt{|E^x|}}{4\left(E^\varphi\right)^2} + F(r) p\, \Theta(x - r) \right) + N^x_{\rm new} \left[-(E^x)' K_x + E^\varphi K'_\varphi - p\, \delta\left(x - r\right) \right] \Bigg] + N_+ \Big(F(r) p + 2M \Big) + N_- 2M, \qquad (5)$$

where

$$F(r) = \sqrt{E^x} \left(\eta \left(E^x\right)' \left(E^\varphi\right)^{-2} + 2K_\varphi \left(E^\varphi\right)^{-1} \right) |_{x=r}. \qquad (6)$$

The constraint algebra is $\{\tilde{H}(N_{\rm new}), \tilde{H}(\tilde{N}_{\rm new})\} = 0$ and the usual one with the diffeomorphism constraint.

Classical Dirac observables

2) We can identify two classical observables (in absence of a pre-existing black hole): the mass (total ADM mass)

$$m := F(r)p/2, \tag{7}$$

and its conjugate variable

$$V := \int_r^\infty dy \left(\frac{2}{F(y)} - [\eta\,(1 + 2m/y)] \right) + t - \eta\,[r + 2m\ln(r/(2m))]\,. \tag{8}$$

such that $\{m, V\} = 1$. It represents the Eddington–Finkelstein time of an ingoing/outgoing shell.

Parametrized classical observables

1) We can promote phase space variables to parametrized observables as functions of gauge parameters and Dirac observables.

2) Let us introduce the gauge fixing condition $K_\varphi = \frac{R_S[\Theta(x)-\Theta(-x)]}{\sqrt{|E^x|}\sqrt{1+\frac{R_S}{\sqrt{|E^x|}}}}$,

$$R_S = 2m\Big[\Theta(x)\,\Theta\left(\sqrt{|E^x|}+(t-V)\right)+\Theta(-x)\,\Theta\left(\sqrt{|E^x|}-(t-V)\right)\Big]. \qquad (9)$$

We obtain $N = \frac{1}{2}$ and $E^\varphi = \frac{(E^x)'}{2}\sqrt{1+\frac{R_S}{\sqrt{|E^x|}}}$.

3) At the classical level we can also consider $E^x(x) = x^2$, which yields $N^x = 0$.

Parametrized classical observables

4) We can now compute any metric component in terms of x, t, m and V.

$$g_{tt} = 1 - \frac{R_S}{\sqrt{|E^x|}}, \quad g_{tx} = -\frac{R_S}{2}\frac{(E^x)'}{|E^x|}\left[\Theta(x) - \Theta(-x)\right],$$
$$g_{xx} = \frac{\left[(E^x)'\right]^2}{4|E^x|}\left(1 + \frac{R_S}{\sqrt{|E^x|}}\right), \quad g_{\theta\theta} = E^x, \quad g_{\phi\phi} = E^x \sin^2\theta. \tag{10}$$

5) And solving the EOMs of the shell (see Louko et al. 1998) we get the parametrized observable for the position of the (ingoing) shell.

$$r = (V - t). \tag{11}$$

6) The resulting spacetime in Eddington–Finkelstein coordinates is explicitly time-dependent since $R_S = R_S(x,t)$ through Eq. (9).

Kinematical Hilbert space

1) Sectors:

a) Spin networks

$$\langle K_x, K_\varphi | g, \vec{k}, \vec{\mu}\rangle = \prod_{e_j \in g} \exp\left(i\frac{k_j}{2}\int_{e_j} dx\, K_x(x)\right) \prod_{v_j \in g} \exp\left(i\frac{\mu_j}{2} K_\varphi(x_j)\right), \tag{12}$$

$k_j \in \mathbb{Z}$ is the valence associated with the edge e_j, and $\mu_j \in \mathbb{R}$ the valence associated with the vertex x_j.

b) Matter $\psi(r) := \langle r|\psi\rangle$.

2) Kinematical Hilbert space:

$$\mathcal{H}^g_{\text{kin}} = \left[\bigotimes_j^n \ell^2_j \otimes \ell^2_{\delta_j}\right] \otimes L^2(\mathbb{R}, dr), \tag{13}$$

with $\mu_j = 2\rho_j(l_j + \delta_j)$ and $\delta_j \neq 0, 1, 2$. The inner product is

$$\langle g, \vec{k}, \vec{\mu}, r | g', \vec{k}', \vec{\mu}', r'\rangle = \delta(r - r')\delta_{\vec{k},\vec{k}'}\delta_{\vec{\mu},\vec{\mu}'}\delta_{g,g'}\ . \tag{14}$$

Kinematical operators

3) Operator representation: position of the shell and triads

$$\hat{r}|g,\vec{k},\vec{\mu},r\rangle = r|g,\vec{k},\vec{\mu},r\rangle, \quad \hat{p} = -i\partial_r,$$
$$\hat{E}^x(x)|g,\vec{k},\vec{\mu},r\rangle = \ell_{\rm Pl}^2 k_j|g,\vec{k},\vec{\mu},r\rangle,$$
$$\hat{E}^\varphi(x)|g,\vec{k},\vec{\mu},r\rangle = \ell_{\rm Pl}^2 \sum_{v_j\in g} \delta\big(x-x_j\big)\mu_j|g,\vec{k},\vec{\mu},r\rangle, \tag{15}$$

4) Holonomies (of K_φ) of length $\rho(x)$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j\rangle = |\mu_j \pm \rho_j\rangle, \quad x = x_j.$$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j,\mu_{j+1}\rangle = |\mu_j, \pm\rho_j, \mu_{j+1}\rangle, \quad x_j < x < x_{j+1}.$$

Representation of the scalar constraint

The scalar constraint will defined on the lattice

$$\hat{H}(x_j) := \mathbf{H}^g_j + \frac{1}{2}\sum_i \mathbf{F}_i(\boldsymbol{\theta}_j\mathbf{X}_i + \mathbf{X}_i\boldsymbol{\theta}_j), \tag{16}$$

such that the spacing of the vertices is $\epsilon_j = x_{j+1} - x_j$,

$$\mathbf{H}^g_j = \hat{b}_j\left(-1 - \widehat{K^2_\varphi}(x_j) + \hat{a}^2_j\widehat{[1/E^\varphi]^2}(x_j)\right), \quad \mathbf{F}_j = 2\,\epsilon_j^{-1}\hat{b}_j\left(\hat{a}_j\widehat{[1/E^\varphi]^2}(x_j) + \widehat{[K_\varphi/E^\varphi]}(x_j)\right). \tag{17}$$

The quantum algebra close if a) $[\mathbf{H}^g_i, \mathbf{F}_j] = i\hbar\mathbf{F}^2_i\delta_{i,j}$, which involves

$$[\widehat{K^2_\varphi}(x_i), \widehat{[1/E^\varphi]^2}(x_j)] = -2i\hbar\delta_{ij}\left(\widehat{[1/E^\varphi]^2}(x_i)\widehat{[K_\varphi/E^\varphi]}(x_i) + \widehat{[K_\varphi/E^\varphi]}(x_i)\widehat{[1/E^\varphi]^2}(x_i)\right),$$

$$[\widehat{K^2_\varphi}(x_i), \widehat{[K_\varphi/E^\varphi]}(x_j)] = -2i\hbar\delta_{ij}\left(\widehat{[K_\varphi/E^\varphi]}(x_i)\right)^2,$$

$$[\widehat{[1/E^\varphi]^2}(x_i), \widehat{[K_\varphi/E^\varphi]}(x_j)] = -2i\hbar\delta_{ij}\left(\widehat{[1/E^\varphi]^2}(x_i)\right)^2, \tag{18}$$

b) $[\boldsymbol{\theta}_i, \mathbf{X}_j] = -i\delta_{ij}\boldsymbol{\delta}_j$, $[\boldsymbol{\theta}_i, \boldsymbol{\theta}_j] = 0 = [\boldsymbol{\theta}_i, \boldsymbol{\delta}_j]$, $\boldsymbol{\delta}_i\boldsymbol{\delta}_j = \delta_{ij}\boldsymbol{\delta}_i$, $(\boldsymbol{\delta}_j\mathbf{X}_i + \mathbf{X}_i\boldsymbol{\delta}_j) = 2\delta_{ij}\mathbf{X}_i$.

Representation of the scalar constraint

They can all be written in terms of the elementary operators,

$$\widehat{K_\varphi^2}(x_j) = \frac{\sin\,(\widehat{\rho K_\varphi(x_j)})}{\rho}\hat{E}^\varphi(x_j)\frac{\sin\,(\widehat{\rho K_\varphi(x_j)})}{\rho}\hat{E}^\varphi(x_j)^{-1},$$
$$[\widehat{K_\varphi/E^\varphi}](x_j) = \frac{\sin\,(\widehat{\rho K_\varphi(x_j)})}{\rho}\cos\,(\widehat{\rho K_\varphi(x_j)})\hat{E}^\varphi(x_j)^{-1},$$
$$[\widehat{1/E^\varphi}]^2(x_j) = \cos\,(\widehat{\rho K_\varphi(x_j)})\hat{E}^\varphi(x_j)^{-1}\cos\,(\widehat{\rho K_\varphi(x_j)})\hat{E}^\varphi(x_j)^{-1},$$
$$\hat{a}_j = \frac{\eta}{2\epsilon_j}\left(\hat{E}^x(x_j) - \hat{E}^x(x_{j-1})\right), \quad \hat{b}_n = \sqrt{|\hat{E}^x(x_j)|}. \tag{19}$$

as well as $\boldsymbol{\theta}_j$ and $\mathbf{X}_j$ are operators on $\psi(r)$ defined as

$$\boldsymbol{\theta}_j\psi(r) := \int_0^{\epsilon_j} d\epsilon\Theta(x_j + \epsilon - r)\;\psi(r),$$
$$\mathbf{X}_j := \frac{1}{2}(\boldsymbol{\delta}_j\hat{p} + \hat{p}\boldsymbol{\delta}_j), \quad \boldsymbol{\delta}_j\psi(r) := \int_0^{\epsilon_j} d\epsilon\delta(x_j + \epsilon - r)\;\psi(r), \tag{20}$$

Quantum observables

1) Observables: the model is characterized by the mass of the shell $\hat{m} = \widehat{Fp}/2$ and its conjugated momentum $\hat{V}$. In the case of diffeo invariant states, there should also be the observable (with no analogue classical Dirac obs.)

$$\hat{O}(z)|\Psi_{\text{phys}}\rangle = \ell_{\text{Pl}}^2 k_{\text{Int}(zv)}|\Psi_{\text{phys}}\rangle, \quad z \in [-1,1], \quad n = 2v+1. \tag{21}$$

2) Parametrized observables: For the parameter functions $z(x) : x \to [-1,1]$ and $\mathcal{K}_\varphi(x,t)$

$$\hat{E}^x(x)|\Psi_{\text{phys}}\rangle = \ell_{\text{Pl}}^2 k_{\text{Int}\left(z(x)v\right)}|\Psi_{\text{phys}}\rangle, \tag{22}$$

$$\left(\hat{E}^x(x)\right)'|\Psi_{\text{phys}}\rangle = \ell_{\text{Pl}}^2 \left(k_{\text{Int}\left(z(x)v\right)} - k_{\text{Int}\left(z(x)v-1\right)}\right)|\Psi_{\text{phys}}\rangle. \tag{23}$$

Quantum observables

3) Metric components in Eddington–Finkelstein coordinates take the form

$$\hat{g}_{tt}(t,x) = 1 - \frac{\hat{R}_S(t,x)}{\sqrt{|\hat{E}^x(x)|}}, \quad g_{tx}(t,x) = -\frac{\hat{R}_S(t,x)}{2}\frac{\left(\hat{E}^x(x)\right)'}{|\hat{E}^x(x)|}\left[\Theta(x) - \Theta(-x)\right],$$

$$\hat{g}_{xx} = \frac{\left[\left(\hat{E}^x(x)\right)'\right]^2}{4|\hat{E}^x(x)|}\left(1 + \frac{\hat{R}_S(t,x)}{\sqrt{|\hat{E}^x(x)|}}\right), \quad \hat{g}_{\theta\theta}(t,x) = |\hat{E}^x(x)| = \frac{\hat{g}_{\phi\phi}(t,x)}{\sin^2\theta}. \tag{24}$$

with the time-dependent Schwarzschild radius

$$\hat{R}_S(t,x) =: 2\hat{m}\left[\Theta(x)\,\Theta\left(\sqrt{|\hat{E}^x(x)|} + (t - \hat{V})\right) + \Theta(-x)\,\Theta\left(\sqrt{|\hat{E}^x(x)|} - (t - \hat{V})\right)\right]:, \tag{25}$$

(symmetrization :: is required) and the position of the shell

$$\hat{r} = \hat{V} - t. \tag{26}$$

Singularity resolution

We lack a selfadjoint representation in the polymer space of $\hat{m}$ and $\hat{V}$. However, we still can adopt a standard representation for $[\hat{m}, \hat{V}] = i\hbar$ (semi reduced phase space quantization):

$$\langle \vec{k}, m | \vec{k}', m' \rangle = \delta_{\vec{k},\vec{k}'} \delta(m - m'). \quad (27)$$

1) Let us choose a state with radial positions $x_i \in [-L, L]$ where $L = \Delta(v + 1)$ such that $\Delta \geq \ell_{\rm Pl}^2 / x_r$. We choose $x_i = (i + 1)\Delta$ if $i \geq 0$. Then we have that $z(x_i) = x_i / L$ and $k_i = \text{Int}(x_i^2 / \ell_{\rm Pl}^2)$. Also that $(E^x(x_i))' \sim (2i + 1)\,\Delta$. At $(E^x(x_0))' \sim (k_0 - k_{-1}) = 2\Delta$. If $i < 0$ then $x_i = i\Delta$ and $k_i = -\text{Int}(x_i^2 / \ell_{\rm Pl}^2)$.

Singularity resolution

2) With these assumptions the result of the quantum construction is essentially a discretization of the above classical expressions of the metric on a lattice determined by a given spin network.

3) Away from the high quantum regime we would recover smooth geometries (even more if superpositions of m and $\vec{k}$ are considered). At the deep quantum regime the geometry would not be smooth but regular (the singularity can be resolved as in the vacuum case).

Conclusions

1) We have provided a quantum scalar constraint compatible with the Dirac quantization approach (formally).

2) We do not know yet the solutions to the constraints in closed form.

3) We are able to construct parametrized Dirac observables. Among them we provide the metric components.

4) We do not know yet a selfadjoint loop representation of some of the basic observables of the model.

5) But, assuming a standard one, we can complete the quantization and explain the way the singularity is resolved. We can also construct semiclassical geometries with a fundamental discretization where quantum gravity effects emerge at the high curvature regime.

Discussion and outlook

1) Since the topology does not change dynamically:
   a) If there are two asymptotic infinities, the shell can form a black hole and emerge into a different spacetime forming a white hole.
   b) If there is only one asymptotic infinity, the shell can form a black hole but its deep quantum dynamics (whether it bounces or not) has not been analyzed in detail yet.
2) We are looking for a selfadjoint representation of the observables in the polymer space.
3) We will study the solutions to the constraints.
4) Black hole evaporation process can be studied:
   a) Typical times of evaporation.
   b) The black hole information loss paradox.