The status of Group Field Theory

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Plan

I. GFT intro

II. GFT foundations and formal developments

III. GFT quantum consistency and continuum limit

IV. GFT and effective continuum physics

Part I:
GFT intro

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, Ryan, ……)

QFT of spacetime, not defined on spacetime

a QFT for the building blocks of (quantum) space

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Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

relevant classical phase space for "GFT quanta": $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$

can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"; for gravity models, G = local gauge group of gravity (e.g. Lorentz group)

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \rightarrow \mathbb{C}$

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can be defined for any (Lie) group and dimension d, any signature, ...

very general framework; interest rests on specific models/use
(most interesting QG models are for Lorentz group in 4d)

GFT “atom of space” (quantum geometric models)

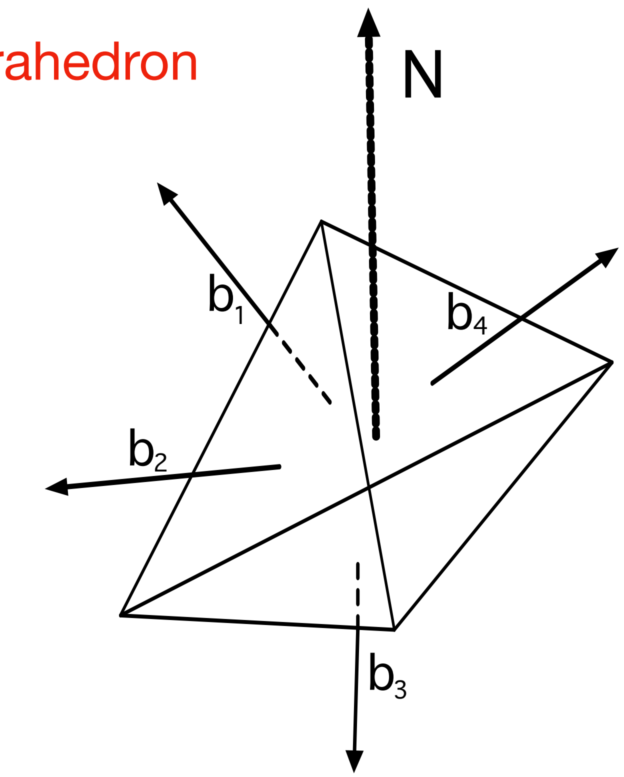
Barbieri ’97; Baez, Barrett, ’99; Rovelli, Speziale, ’06; Bianchi, Dona, Speziale, ’10; Baratin, DO, '11; ……

Elementary building block of 3d space: single polyhedron - simplest example: a tetrahedron

Classical geometry in group-theoretic variables

4 vectors normal to triangles that close (lying in hypersurface with normal N)

$$A_i\, n_i^I \;=\; b_i^I \in \mathbb{R}^{3,1} \qquad b_i \cdot N \;=\; 0 \qquad \sum_i b_i \;=\; 0$$



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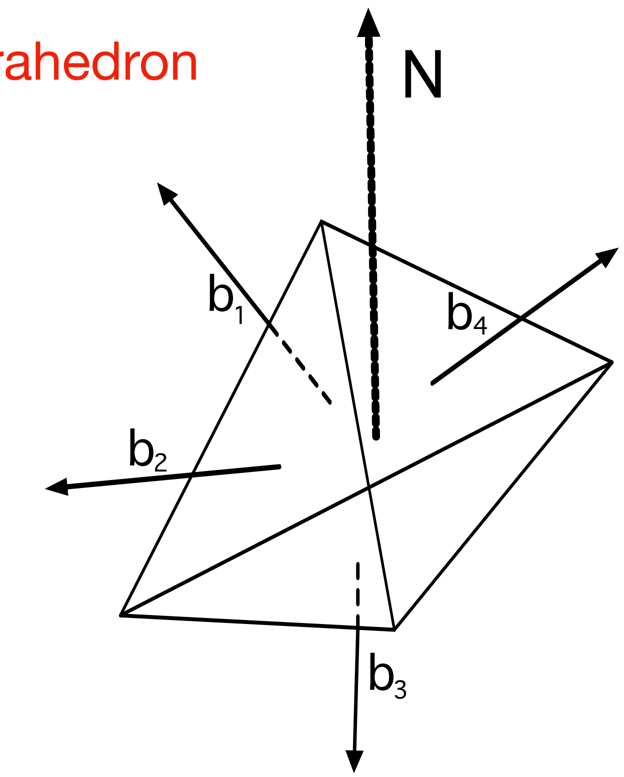

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equivalently: constrained 4d area 2-forms:

$$\boxed{(B_i^{IJ} \in \wedge^2 \mathbb{R}^{3,1} \simeq \mathfrak{so}(3,1)\,,\, N^I \in \mathcal{T}\mathbb{R}^{3,1}) \qquad N_I\,(*B_i^{IJ}) = 0 \qquad \sum_i B_i^{IJ} = 0}$$

$$B_i^{IJ} = N^I \wedge b_i^J \;(\sim \star e \wedge e)$$

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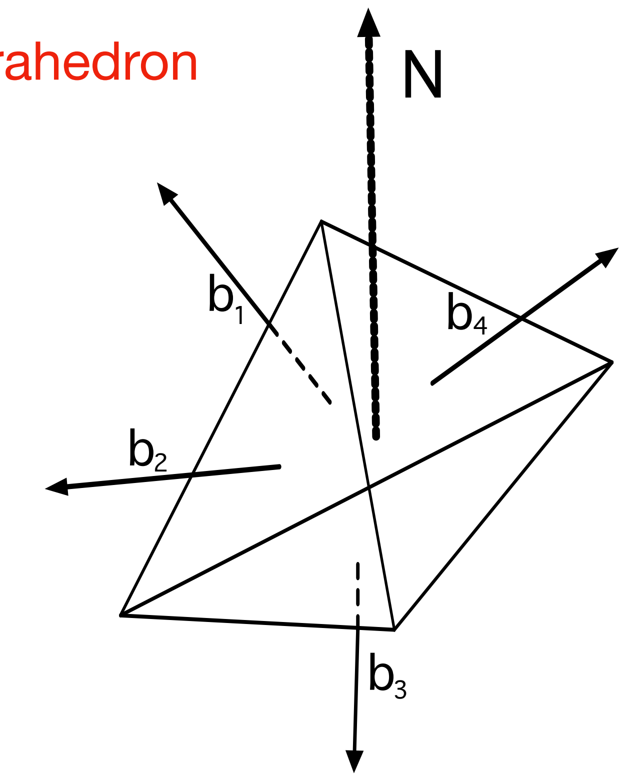

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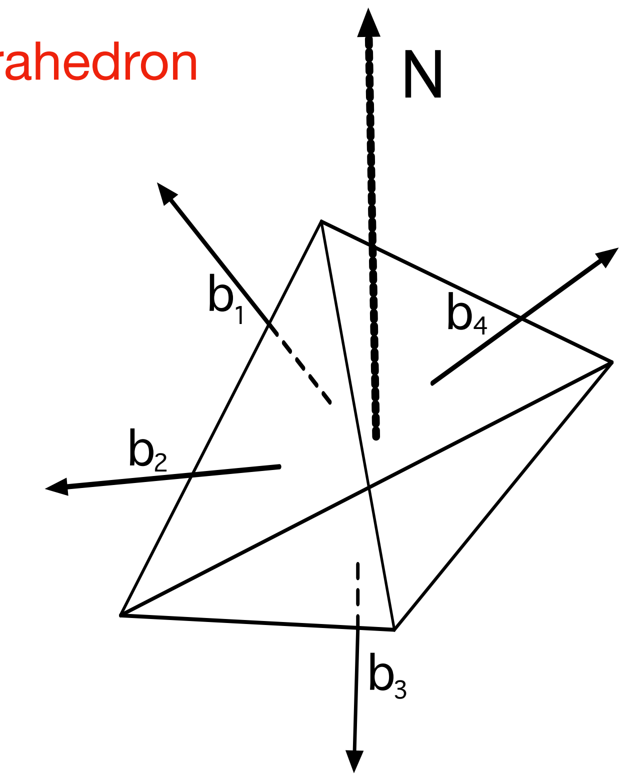

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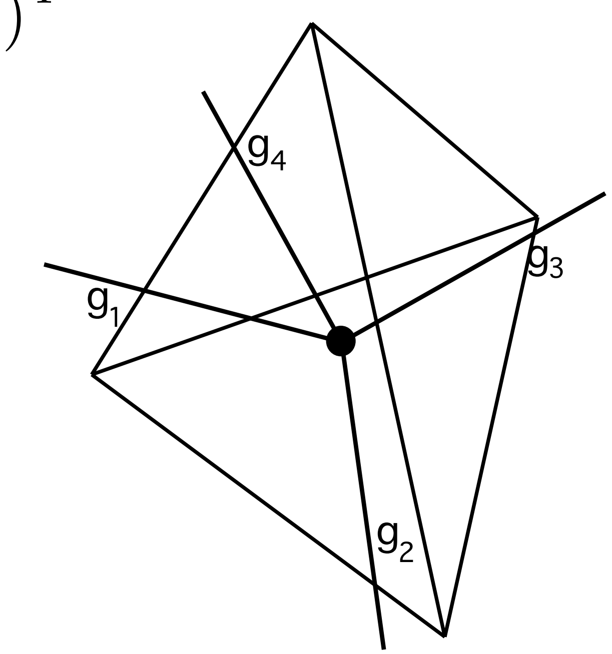
phase space: + constraints

$$(\mathcal{T}^* SO(3,1))^4 \simeq (\mathfrak{so}(3,1) \times SO(3,1))^4 \;\supset (\mathfrak{so}(3) \times SO(3))^4 \simeq (\mathcal{T}^* SO(3))^4$$

general: $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$



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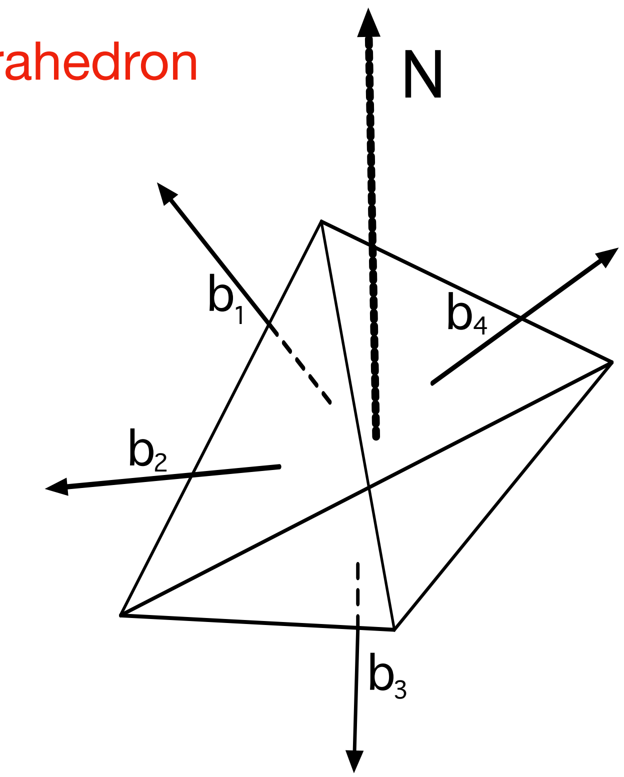

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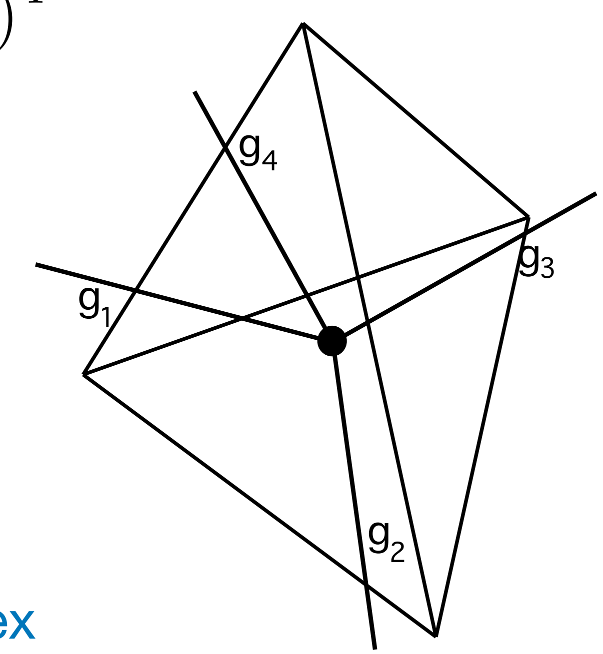
Quantum geometry in group-theoretic variables



Hilbert space

$$\mathcal{H}_v = L^2\left(G^d; d\mu_{Haar}\right)$$

+ constraints on states

$$B_i^{IJ} \rightarrow \hat{J}^{IJ} \in \mathfrak{so}(3,1) \qquad b_i^J \rightarrow \hat{J}_N^i \in \mathfrak{su}(2)$$

equivalently, in flux/Lie algebra variables:

$$\mathcal{H}_v = L^2_\star\left(\mathfrak{g}^d; d\mu_{Leb}\right)$$

Hilbert space of spin network vertex

GFT dynamics of quantum space

DO, '09; DO, '14

Dynamics governs gluing processes and formation of extended discrete structures

Interactions processes correspond to (simplicial) complexes in one dimension higher

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Interactions processes correspond to (simplicial) complexes in one dimension higher

details depend on (class of) models

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

"combinatorial non-locality"
in pairing of field arguments

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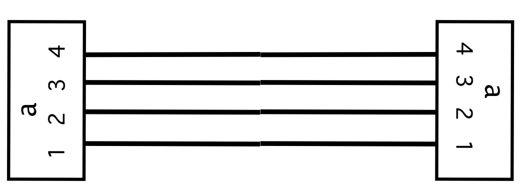
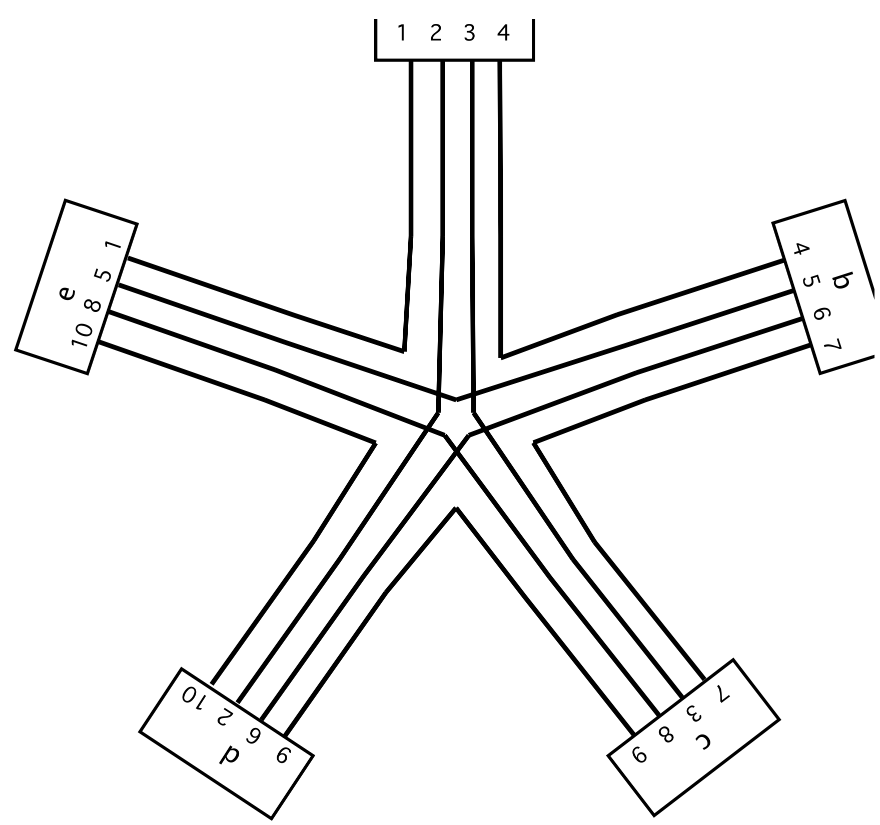
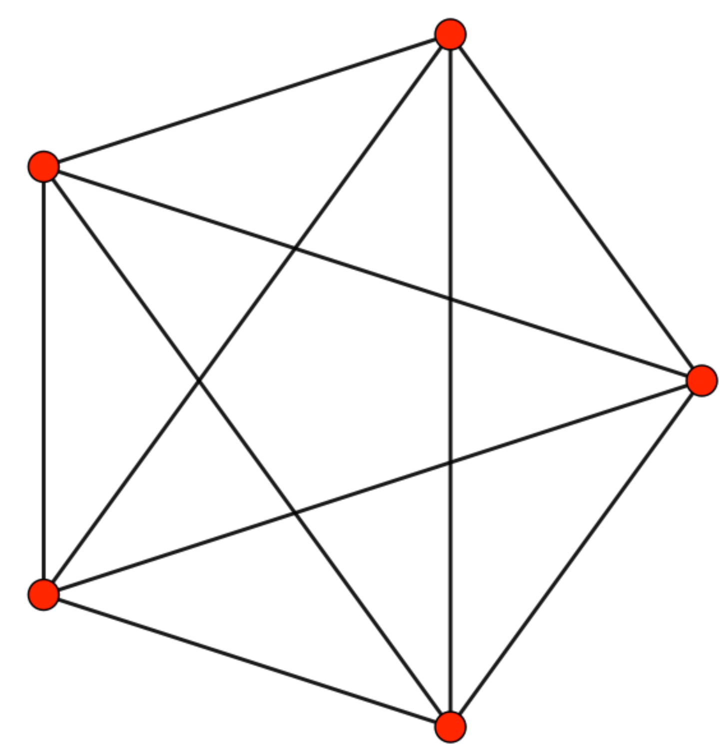
Example: simplicial interactions

GFT propagator = gluing rule for pairs of fundamental spacetime cells across faces



GFT interactions = fundamental polyhedra ("atoms of space") glue to form (boundary of) fundamental "spacetime" cell (i.e. building block of discrete spacetime)





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$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\,\mathcal{A}_\Gamma$$

Feynman diagrams = stranded diagrams dual
to cellular complexes of arbitrary topology

sum over triangulations/complexes

amplitude for each
triangulation/complex

GFT and random Tensor Models

(Ambjorn, Jonsson, Durhuus, Sasakura, …, Gurau, Rivasseau, Bonzom, Tanasa, Benedetti, Ryan, …..)

Tensor models

Lie group replaced by finite group or finite set; field remains tensor; combinatorics unchanged

example: d=3

$$\varphi(g_1, g_2, g_3) : G^{\times 3} \to \mathbb{C} \longrightarrow \begin{array}{l} T_{ijk} : \mathbb{Z}_N^{\times 3} \to \mathbb{C} \\ T_{ijk} : X^{\times 3} \to \mathbb{C} \end{array} \qquad X = 1, 2, ..., N$$

$$S(T) = \frac{1}{2}\sum_{i,j,k} T_{ijk}T_{kji} - \frac{\lambda}{4!\sqrt{N^3}} \sum_{ijklmn} T_{ijk}T_{klm}T_{mjn}T_{nli}$$

Quantum dynamics (purely combinatorial):

$$Z = \int \mathcal{D}T\, e^{-S(T,\lambda)} = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} Z_\Gamma = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} N^{F_\Gamma - \frac{3}{2}V_\Gamma}$$

can be recast in terms of Regge action for gravity discretised on equilateral triangulation

equivalent to (generating functional for) dynamical triangulations framework

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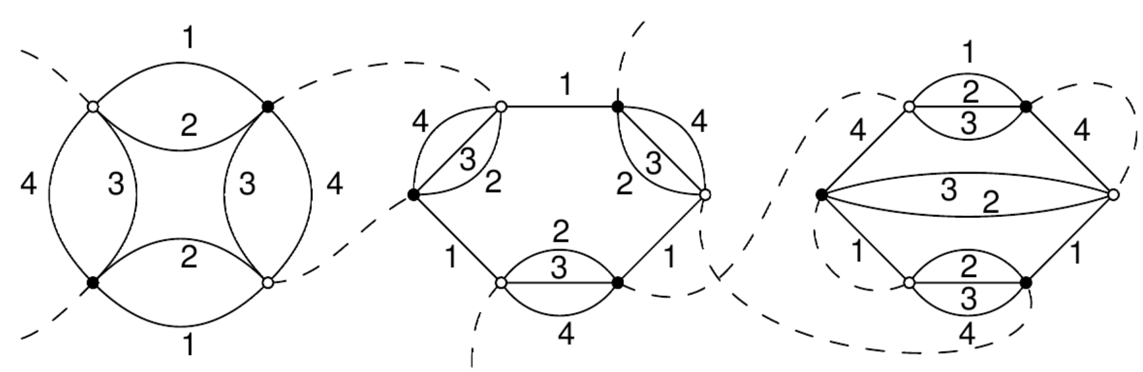
Tensorial group field theories

Built on unitary (or orthogonal) invariance of the action (dictates interaction terms to be “colored bubbles”)

U^d tranformations

$$S(\varphi, \overline{\varphi}) = \sum_{b \in \mathcal{B}} t_b I_b(\varphi, \overline{\varphi})$$



tensorial nature of field is given central role

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most (combinatorial) results of tensor models also apply to GFTs, e.g.:

- use of colors (colored tensors) to encode topology (2-complex not enough)
- large-N expansion
- double scaling
- universality of random tensors
- Virasoro algebra from SD equations

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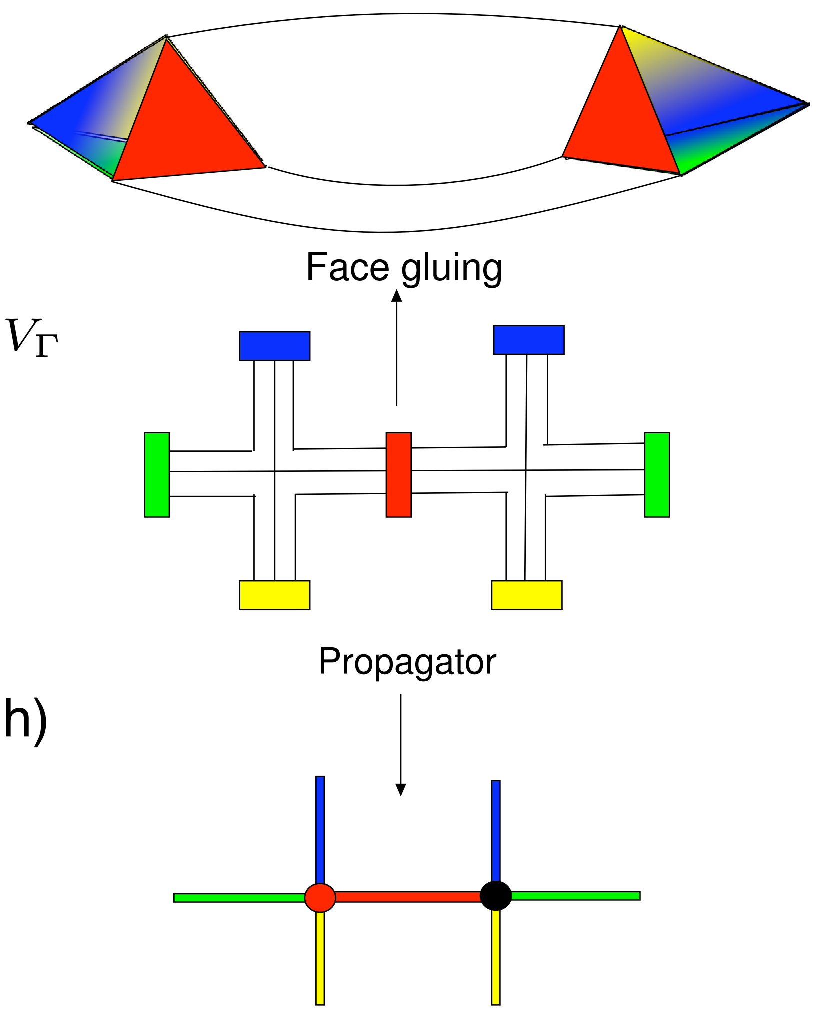

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(Every PL d-pseudomanifold M can be represented by a (d+1)-colored graph G)

arguably needed also in spin foam models

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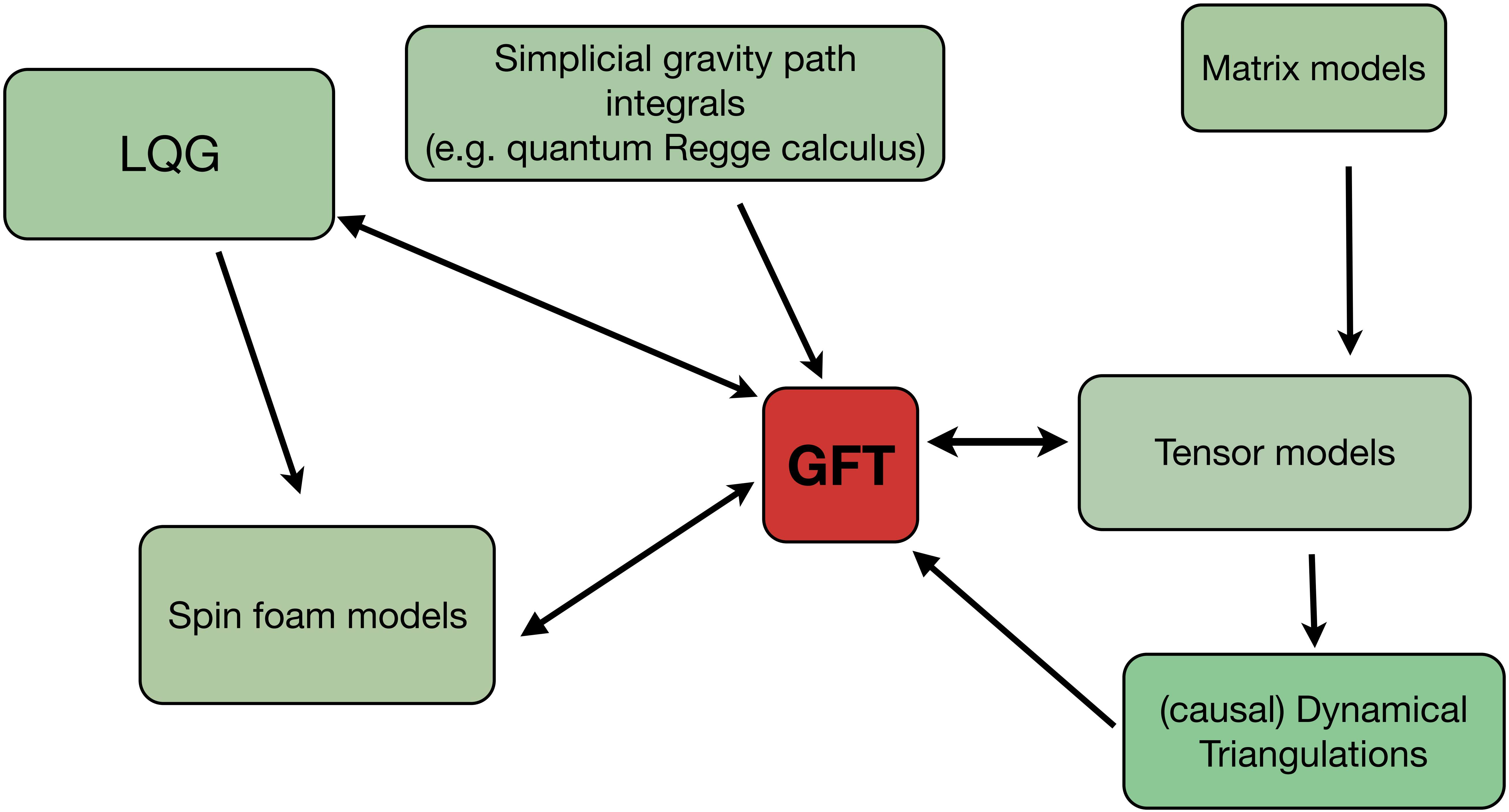
crucial for GFT renormalization

Group Field Theory: crossroad of QG formalisms



GFT and Loop Quantum Gravity

DO, '13

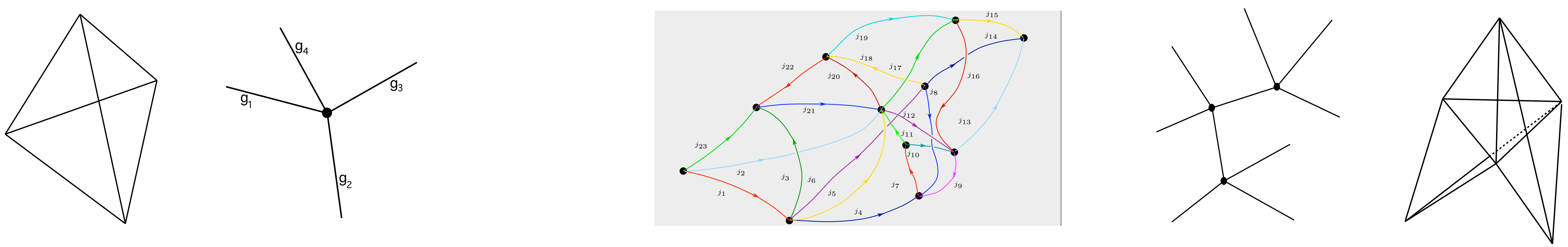
GFT Fock space: many-body Hilbert space for "quantum space"

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

Fock vacuum: "no-space" state $|0>$

Note: bosonic statistics is assumption

generic quantum state: collection of spin network vertices (incl. glued ones) or tetrahedra (incl. glued ones)



2nd quantised representation for (simplicial) geometric operators, incl. dynamical ones (Hamiltonian constraint)

$$\left[\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = \mathbb{I}_G(\vec{g},\vec{g}') \qquad \left[\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}(\vec{g}')\right] = \left[\hat{\varphi}^\dagger(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = 0$$

$$\rightarrow \quad \widehat{\mathcal{O}_{n,m}}\left(\hat{\varphi},\hat{\varphi}^\dagger\right) = \int [d\vec{g}_i][d\vec{g}'_j]\;\hat{\varphi}^\dagger(\vec{g}_1)..\hat{\varphi}^\dagger(\vec{g}_m)\mathcal{O}_{n,m}\left(\vec{g}_1,..,\vec{g}_m,\vec{g}'_1,..,\vec{g}'_n\right)\hat{\varphi}(\vec{g}'_1)..\hat{\varphi}(\vec{g}'_n)$$

e.g. total space volume (extensive quantity): $\hat{V}_{tot} = \int [dg_i][dg'_j]\hat{\varphi}^\dagger(g_i)\,V(g_i,g'_j)\,\hat{\varphi}(g'_j) = \sum_{J_i} \hat{\varphi}^\dagger(J_i)\,V(J_i)\,\hat{\varphi}(J_j)$

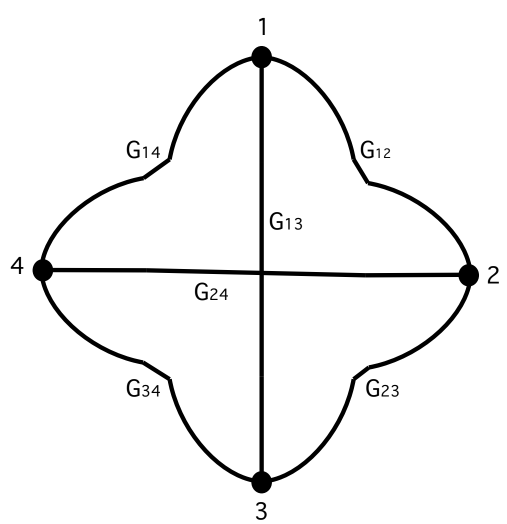
volume of single tetrahedron (from simplicial geometry)

GFT and Loop Quantum Gravity

LQG Hilbert space from canonical quantum GR:

$$\mathcal{H} = \lim_{\gamma} \frac{\bigcup_\gamma \mathcal{H}_\gamma}{\approx} = L^2\left(\bar{\mathcal{A}}\right)$$

$$\mathcal{H}_\gamma = L^2\left(G^E/G^V, d\mu = \prod_{e=1}^{E} d\mu_e^{Haar}\right)$$



links labelled by j=0 immaterial

equivalence classes of graphs

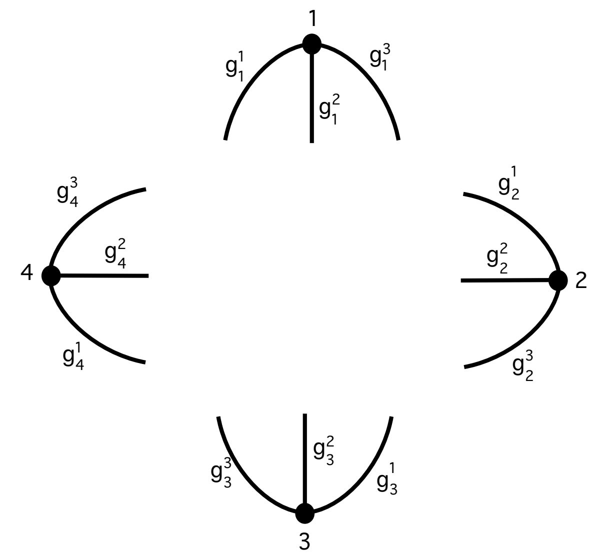
orthogonality of equivalence classes

space of continuum connection

GFT Hilbert space of discrete pre-geometric structures:

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

$$\mathcal{H}_v = L^2\left(G^{\times d}/G\right) \qquad \mathcal{H}_\gamma \subset \mathcal{H}_V$$



no cylindrical equivalence

orthogonality for different #vertices only

no immediate continuum interpretation

no knotting (or other post-embedding info)

GFT and Loop Quantum Gravity

LQG Hilbert space from canonical quantum GR:

$$\mathcal{H} = \lim_{\gamma} \frac{\bigcup_\gamma \mathcal{H}_\gamma}{\approx} = L^2\left(\bar{\mathcal{A}}\right)$$

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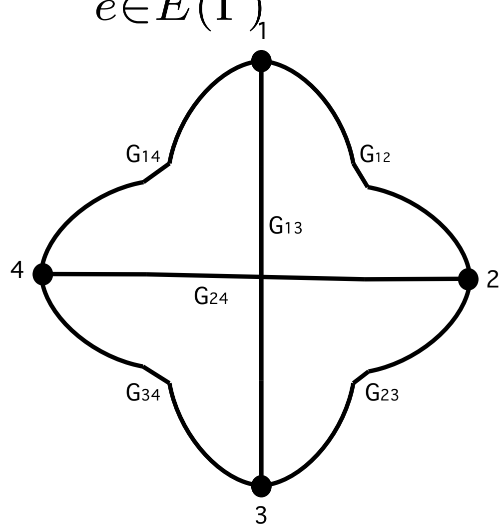
$$\mathcal{H}_v = L^2\left(G^{\times d}/G\right) \qquad \mathcal{H}_\gamma \subset \mathcal{H}_V$$

$$\Psi_\Gamma(\{G_{ij}^{ab}\}) = \prod_{e\in E(\Gamma)} \int_G \mathrm{d}\alpha_{ij}^{ab}\, \phi_\Gamma(\{g_i^a \alpha_{ij}^{ab}; g_j^b \alpha_{ij}^{ab}\}) = \Psi_\Gamma(\{g_i^a (g_j^b)^{-1}\})$$

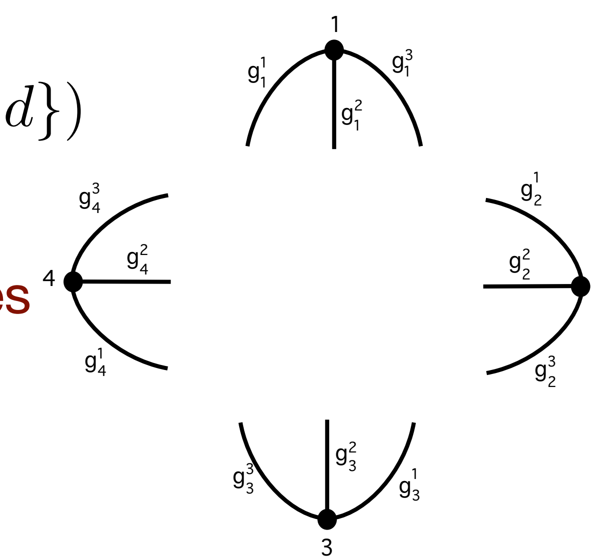
$$[(i\,a)\,(j\,b)] \in E(\Gamma)$$

$$E(\Gamma) \subset (\{1,\ldots,V\} \times \{1,\ldots,d\})$$

wave function for closed graph



wave function for many open spin net vertices



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GFT and spin foam models

complete definition of SF model: quantum amplitudes for all spin foam complexes + organization principle

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spin foam model with sum over complexes as perturbative expansion of GFT (valid for any SF model)

M. Reisenberger, C. Rovelli, '00

any combinatorics

DO, J. Ryan, J. Thürigen, '14

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$$Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I)$$

any combinatorics

DO, J. Ryan, J. Thürigen, '14

$$Z(\Gamma) \leftrightarrow \begin{cases} A_f(J) \\ A_e(J,I) \\ A_v(J,I) \end{cases} \qquad \longleftrightarrow \qquad \begin{cases} \mathcal{K}(J,I) \sim \mathcal{K}(g) \\ \mathcal{V}(J,I) \sim \mathcal{V}(g) \end{cases} \leftrightarrow \; S(\varphi,\bar{\varphi})$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

using local expression of SF amplitudes, needed for composition along (portions of) boundaries

M. Finocchiaro, DO, '18

$$\updownarrow$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma \qquad Z(\Gamma) \equiv \mathcal{A}_\Gamma$$

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advantages:

- prescription for combinatorial weights + way to parametrize SF ambiguities
- ways to go beyond spin foams (non-perturbative structure of theory)
- QFT tools for extracting physics

GFT and simplicial gravity path integrals

GFT Feynman amplitudes are simplicial gravity path integrals

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma \quad = \sum_\Delta w(\Delta) \int \mathcal{D}g_\Delta\, e^{i\, S_\Delta(g_\Delta)}$$

not just "heuristically", but, for ALL models based on (constrained) BF theory, literally true in flux representation, without any semi-classical approx. A. Baratin, DO, '10; A. Baratin, DO, '11; M. Finocchiaro, DO, '18

example: Boulatov (Ponzano-Regge) model (3d):

$$\mathcal{A}_\Gamma \quad = \quad \int \prod_l \mathrm{d}h_l \prod_f \delta\left(H_f(h_l)\right) = \int \prod_l \mathrm{d}h_l \prod_f \delta\left(\overrightarrow{\prod}_{l\in\partial f} h_l\right) =$$

$$= \sum_{\{j_e\}} \prod_e d_{j_e} \prod_\tau \begin{Bmatrix} j_1^\tau & j_2^\tau & j_3^\tau \\ j_4^\tau & j_5^\tau & j_6^\tau \end{Bmatrix} = \int \prod_l [\mathrm{d}h_l] \prod_e [\mathrm{d}^3 x_e]\, e^{i \sum_e \mathrm{Tr}\, x_e H_e}$$

example: constrained BF models (4d)

$$\mathcal{A}_\Gamma^\beta \quad = \int \left[\prod_{f\in\mathcal{F}} \frac{d^6 X_f}{(2\pi)^6}\right]\left[\prod_{e\in\mathcal{E}_{\mathrm{int}}} dk_e\right] \mathcal{D}_\beta^{H_{ve},\, k_e}(X_f) \star \prod_{f\in\mathcal{F}} E_{H_f}(X_f)$$

details of measure (incl. simplicity constraints) and action depend on chosen quantization map

$$E_{H_f}(X_f) = \ \eta(H_f) e^{i S_f^{\mathrm{BF}}[H_f, X_f]}$$

Part II: GFT foundations and formal developments

GFT symmetries

GFT symmetries

"structural" symmetries, affecting combinatorial features

GFT symmetries

"structural" symmetries, affecting combinatorial features

permutation symmetry R. De Pietri, C. Petronio, '00

$$\phi_{\alpha_{\tau(1)}\dots\alpha_{\tau(n)}} = \Re[\phi_{\alpha_1\dots\alpha_n}] + i\cdot\mathrm{sgn}(\tau)\cdot\Im[\phi_{\alpha_1\dots\alpha_n}] \qquad \tau \in \mathfrak{S}_n \text{ permutation group}$$

permutation of n strands used to define orientation of simplicial GFT Feynman (stranded) link

invariance under even permutations: orientable complexes

invariance under any permutations: un-orientable complexes

fixed in colored fields; colors fix order of GFT arguments and determine orientation of GFT diagrams: orientable complexes dual to bipartite (n+1)-colored Feynman diagrams

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"structural" symmetries, affecting combinatorial features

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tensorial unitary (or orthogonal) symmetry

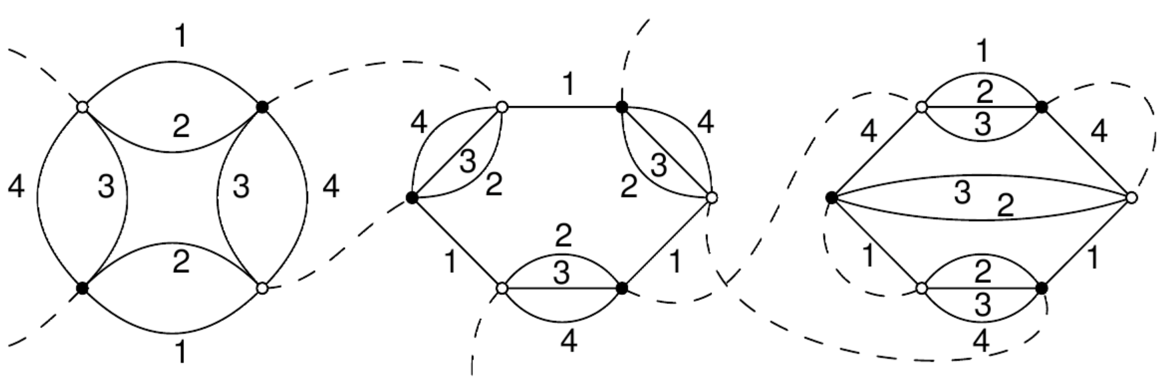
$$\varphi(g_1,..,g_d) \;\rightarrow\; \int [dg_i] U(g_1',g_1)\cdots U(g_d',g_d)\,\varphi(g_1,...,g_d)$$

unitary (or orthogonal) group $U^{\times d}$

requires coloring (thus fixed ordering) of GFT fields or arguments

characterizes infinite "tensorial" theory space, with allowed (invariant) interactions in 1-1 correspondence with "bubbles" (d-colored graphs)

$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$



in general does not include simplicial interactions

GFT symmetries

special symmetries, in models motivated by quantum geometry

GFT symmetries

special symmetries, in models motivated by quantum geometry

"local" Lorentz or SU(2) symmetry

e.g. d=3 $\varphi(g_1, g_2, g_3) = \varphi(hg_1, hg_2, hg_3) \qquad \forall h \in SU(2)$

equivalently imposed at the level of GFT acton, rather than GFT fields

L. Freidel, '06

or, better (to ensure covariance of simplicity constraints):

e.g. d=4 $(C \triangleright \phi)(g_1, \cdots, g_4; N) = \int dh\, \phi(hg_1, hg_2, hg_3, hg_4; h^{-1} \triangleright N)$ A. Baratin, DO, '11

produces corresponding gauge symmetry of spin network states and spin foam amplitudes

easily gauge-fixed A. Perez, C. Rovelli, '00; L. Freidel, E. Livine, '06; J. Engle, R. Pereira, '08

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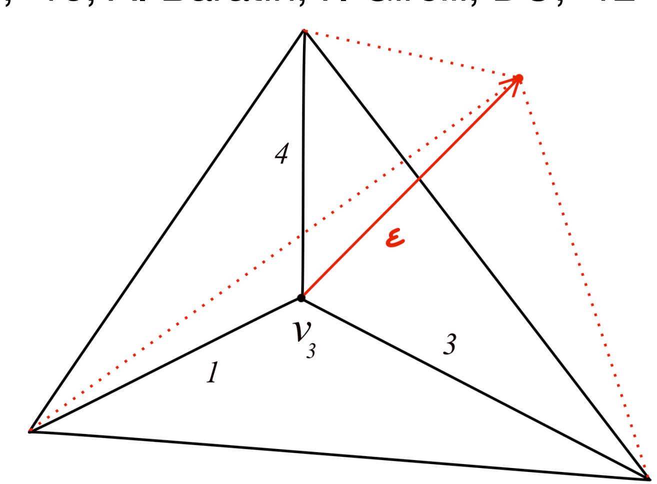
"simplicial diffeomorphism" (vertex translation) symmetry F. Girelli, E. Livine, '10; A. Baratin, F. Girelli, DO, '12

symmetry of BF models

D. Louapre, L. Fridel, '02; B. Dittrich, '08;
B. Bahr, R. Gambini, J. pullin, '11

transformation of GFT fields corresponding to vertex translations in triangulations dual to GFT Feynman diagrams



most easily seen in (non-commutative) flux (discrete triad) variables - quantum group symmetry

$$\begin{aligned}
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_1(x_1, x_2, x_3) &:= \bigstar_{\varepsilon_3}\, \widehat{\varphi}_1(x_1 - \varepsilon_3, x_2, x_3 + \varepsilon_3) \\
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_2(x_3, x_4, x_5) &:= \bigstar_{\varepsilon_3}\, \widehat{\varphi}_2(x_3 - \varepsilon_3, x_4 + \varepsilon_3, x_5) \\
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_4(x_6, x_4, x_1) &:= \bigstar_{\varepsilon_3}\, \widehat{\varphi}_4(x_6, x_4 - \varepsilon_3, x_1 + \varepsilon_3) \\
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_3(x_5, x_2, x_6) &:= \widehat{\varphi}_3(x_5, x_2, x_6)
\end{aligned}$$

requires coloring of GFT fields

not proper QFT symmetry (weird action)

Classical aspects of GFT

GFT symmetries and classical solutions

general treatment of classical GFT symmetries - generalization of Noether analysis to non-local QFTs

J. Ben Geloun, '11; A. Kegeles, DO, '15, '16

GFT are non-local field theories - action is sum of local terms on different vector bundles

$$S[\phi] = \underset{\int_{\mathbb{R}} \phi(x)\Delta\phi(x)}{S^0[\phi]} + \underset{\int_{\mathbb{R}\times\mathbb{R}} \phi(x)V(x-y)\phi(y)}{S^1[\phi]} + \cdots$$

transformations considered: point symmetries (diffeos) of associated vector bundles

generalised "conservation law": akin to continuity eqn with sources $\quad \mathrm{div}(J) = \Delta_{\mathrm{NL}}$

$$J(g) = J_{\mathrm{N}}(g) + J_{\mathrm{NL}}(g) \qquad \Delta_{\mathrm{NL}}(g) \sim \int \frac{\delta L_{NL}}{\delta\phi(g)}\, X^0$$

application to GFT: various models, various symmetries (incl. known ones)

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application to GFT: various models, various symmetries (incl. known ones)

classical solutions of various GFT models

GFT equations are, in group/Lie algebra variables, (complicated) integro-differential equations or, in representation variables, (complicated) algebraic equations

many scattered (often very interesting) results - no general scheme or computational tool

mostly for topological or simpler models; mostly simple (highly symmetric) solutions

W. Fairbairn, E. Livine, '07; F. Girelli, E. Livine, DO, '09; DO, L. Sindoni, '10;
E. Livine, DO, J. Ryan, '11; J. Ben Geloun, A. Kegeles, A. Pithis, '16

Inequivalent representations of quantum GFT algebra

Coherent state representations of GFT field algebra

question: what GFT representations, beside Fock one, are possible?

expect that:

A. Kegeles, DO, C. Tomlin, '17

- multitude of inequivalent representations exist (characterized by different symmetries) - different continuum phases
- Fock space representation does not exist for interacting theory (if non-local generalization of Haag's theorem applies)
- other relevant representations correspond to coherent state vacua

from GFT field algebra and smearing to Weyl algebra

$$[\phi(g), \phi^\dagger(\tilde{g})] = \delta(g\tilde{g}^{-1}) \qquad \phi(f) := \int \mathrm{d}\mu\ \phi(g)\bar{f}(g) \qquad \phi^\dagger(f) := \int \mathrm{d}\mu\ \phi^\dagger(g)f(g)$$

$$W(f)W(g) = W(f+g)\ e^{-\imath\Im(f,g)}$$
$$W^\star(f) = W(-f)$$
$$W(0) = 1$$

$$W(f) = e^{\imath\left(\phi(f)+\phi^\dagger(f)\right)}$$

GFTs with group/ Lie algebra variables only

start with Fock reprs at fixed particle number and finite (group) volume

construct coherent states $$|\alpha\rangle := W_F(\alpha)|\varnothing\rangle \qquad \langle N\rangle = \int_V |\alpha(g)|^2$$

in the thermodynamic limit N ---> oo, V --> oo (non-compact groups) at finite density:
inequivalent coherent state (condensate) representations with broken symmetry

(compact groups: dual construction (on Lie algebra) gives similar results)

Coupling to scalar fields
and
relational GFT dynamics

GFT models for QG coupled to scalar fields

early work (with different strategies/results): K. Krasnov, '05; L. Freidel, DO, J. Ryan, '05; DO, J. Ryan, '06; R. Dowdall, '09

basic idea: expand domain of GFT fields DO, L. Sindoni, E. Wilson-Ewing, '16

$$\hat{\varphi}(g_v) \to \hat{\varphi}(g_v, \phi)$$

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basic guideline for model-building (choosing GFT action): best done in flux variables

GFT Feynman amplitudes = simplicial path integrals for gravity coupled to scalar fields Y. Li, DO, M. Zhang, '17

$$Z^{\Delta}_{\phi\mathrm{G}} = \int \prod_{f\in\Delta} \mathcal{D}B_f \prod_{f\in\Delta*} \mathrm{d}g_\ell \prod_{v\in\Delta*} \mathrm{d}\phi_v \; \delta(S_\gamma(B_f)) \; \mathrm{e}^{\frac{i}{\hbar}(S^{\Delta}_{\phi}+S^{\Delta}_{G})}$$

$$S^{\Delta}_{\phi} \equiv \left(\sum_{l\in\Delta*} \tilde{V}_l \left(\frac{\delta_l\phi}{L_l}\right)^2 + \sum_{v\in\Delta*} V_v \mathbb{V}(\phi_v) \right)$$

geometric coupling potential

$$\tilde{S}^{\mathrm{GFT}}_{\phi\mathrm{G}} = \frac{1}{2}\int [\mathrm{d}x^4][\mathrm{d}x'^4] d\phi \psi(\vec{x};\phi) \star \tilde{P}^{-1}_{\phi\mathrm{G}}(\vec{x},\vec{x}';\partial^2_\phi) \star \psi(\vec{x}';\phi) +$$

$$\int \mathrm{d}\phi \left(\prod_{n=1}^{5} [\mathrm{d}x^4_n]\right) V_{\phi\mathrm{G}}(\vec{x}_1,\dots,\vec{x}_5;\phi) \star \left(\prod_{n=1}^{5} \psi(\vec{x}_n;\phi)\right) \quad + \quad \mathrm{c.c.}$$

$$\tilde{P}^{-1}_{\phi\mathrm{G}}(\vec{x},\vec{x}';\partial^2_\phi) = P^{-1}_{\mathrm{G}}(\vec{x},\vec{x}') \star \tilde{P}^{-1}_{\phi}(\vec{x}';\partial^2_\phi)$$

$$\tilde{P}^{-1}_{\phi} = \left(\frac{i\pi\hbar L^2}{\tilde{V}}\right)^{-\frac{1}{2}} e^{-i\frac{\hbar L^2}{4\tilde{V}}\partial^2_\phi}$$

$$V_{\phi\mathrm{G}}(\vec{x}_1,\dots,\vec{x}_5;\phi) = V_\phi(\vec{x}_1,\dots,\vec{x}_5;\phi) \star V_{\mathrm{G}}(\vec{x}_1,\dots,\vec{x}_5)$$

$$V_\phi = e^{\frac{i}{\hbar}V\mathbb{V}(\phi_v)},$$

various discretization & quantum ambiguities; important to capture continuum & classical limit

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geometric coupling — potential

$$\tilde{S}^{\rm GFT}_{\phi G} = \frac{1}{2}\int [\mathrm{d}x^4][\mathrm{d}x'^4] d\phi \psi(\vec{x};\phi) \star \tilde{P}^{-1}_{\phi G}(\vec{x},\vec{x}';\partial^2_\phi) \star \psi(\vec{x}';\phi) +$$

$$\int \mathrm{d}\phi \left(\prod_{n=1}^{5} [\mathrm{d}x^4_n]\right) V_{\phi G}(\vec{x}_1,\ldots,\vec{x}_5;\phi) \star \left(\prod_{n=1}^{5} \psi(\vec{x}_n;\phi)\right) \quad + \quad \mathrm{c.c.}$$

$$\tilde{P}^{-1}_{\phi G}(\vec{x},\vec{x}';\partial^2_\phi) = P^{-1}_{G}(\vec{x},\vec{x}') \star \tilde{P}^{-1}_{\phi}(\vec{x}';\partial^2_\phi)$$

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$$V_{\phi G}(\vec{x}_1,\ldots,\vec{x}_5;\phi) = V_\phi(\vec{x}_1,\ldots,\vec{x}_5;\phi) \star V_G(\vec{x}_1,\ldots,\vec{x}_5)$$

$$V_\phi = e^{\frac{i}{\hbar} V \mathbb{V}(\phi_v)},$$

various discretization & quantum ambiguities; important to capture continuum & classical limit

in fact, largely captured by symmetries
and simplified greatly, depending on context

e.g. GFT condensate cosmology

DO, L. Sindoni, E. Wilson-Ewing, '16

Relational ("deparametrised") GFT dynamics

E. Wilson-Ewing, '18

once massless scalar field is added, GFT action becomes:

$$S[\varphi] = \int \mathrm{d}\chi \, \mathcal{L}[\varphi]$$

$$\mathcal{L}[\varphi] = -\frac{1}{2} \sum_{j,m,\iota} \left(\partial_\chi \varphi^{\vec{\jmath},\iota}_{\vec{m}}(\chi)\right) \mathcal{K}^{(2)}_{\vec{\jmath},\vec{m},\iota} \left(\partial_\chi \varphi^{\vec{\jmath},\iota}_{\vec{m}}(\chi)\right) + \frac{1}{2} \sum_{j,m,\iota} \varphi^{\vec{\jmath},\iota}_{\vec{m}}(\chi) \mathcal{K}^{(0)}_{\vec{\jmath},\vec{m},\iota} \varphi^{\vec{\jmath},\iota}_{\vec{m}}(\chi) - U[\varphi],$$

e.g.

$$U[\varphi] = \frac{1}{5} \sum_{j,m,\iota} \mathcal{V}^{j_i,m_i,\iota_i} \prod_{a=1}^{5} \varphi^{\vec{\jmath}_a,\iota_a}_{\vec{m}_a}(\chi)$$

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e.g.

$$U[\varphi] = \frac{1}{5} \sum_{j,m,\iota} \mathcal{V}^{j_i,m_i,\iota_i} \prod_{a=1}^{5} \varphi^{\vec{\jmath}_a,\iota_a}_{\vec{m}_a}(\chi)$$

"local action in time variable" $\longrightarrow$ can deparametrise and canonically quantise wrt to scalar field clock

Relational ("deparametrised") GFT dynamics

E. Wilson-Ewing, '18

once massless scalar field is added, GFT action becomes:

$$S[\varphi] = \int \mathrm{d}\chi\, \mathcal{L}[\varphi]$$

$$\mathcal{L}[\varphi] = -\frac{1}{2}\sum_{j,m,\iota}\left(\partial_\chi \varphi^{\vec{j},\iota}_{\vec{m}}(\chi)\right)\mathcal{K}^{(2)}_{\vec{j},\vec{m},\iota}\left(\partial_\chi \varphi^{\vec{j},\iota}_{\vec{m}}(\chi)\right)$$
$$+\frac{1}{2}\sum_{j,m,\iota}\varphi^{\vec{j},\iota}_{\vec{m}}(\chi)\mathcal{K}^{(0)}_{\vec{j},\vec{m},\iota}\,\varphi^{\vec{j},\iota}_{\vec{m}}(\chi) - U[\varphi],$$

e.g.

$$U[\varphi] = \frac{1}{5}\sum_{j,m,\iota}\mathcal{V}^{j_i,m_i,\iota_i}\prod_{a=1}^{5}\varphi^{\vec{j}_a,\iota_a}_{\vec{m}_a}(\chi)$$

"local action in time variable" $\longrightarrow$ can deparametrise and canonically quantise wrt to scalar field clock

can define "equal (relational) time" commutation relations: $\left[\,\hat{a}_{\vec{j}_1,\vec{m}_1,\iota_1},\,\hat{a}^\dagger_{\vec{j}_2,\vec{m}_2,\iota_2}\,\right] = \delta_{\vec{j}_1,\vec{j}_2}\delta_{\vec{m}_1,\vec{m}_2}\delta_{\iota_1,\iota_2}$

get standard Hamiltonian evolution: $\hat{\mathcal{H}}\,\Psi = i\hbar\,\frac{d\Psi}{d\chi}$

$$\hat{\mathcal{H}} = \hbar\sum_{j,m,\iota}M_{\vec{j},\vec{m},\iota}\left(\hat{a}^\dagger_{\vec{j},\vec{m},\iota}\hat{a}_{\vec{j},\vec{m},\iota} + \frac{\omega^{\vec{j},\iota}_{\vec{m}}}{2}\right) + U[\hat{\varphi}]$$

or

$$\hat{\mathcal{H}} = \hbar\sum_{j,m,\iota}M_{\vec{j},\vec{m},\iota}\left((\hat{a}^\dagger_{\vec{j},\vec{m},\iota})^2 + \hat{a}^2_{\vec{j},\vec{m},\iota}\right) + U[\hat{\varphi}]$$

depending on relative sign of K^0 and K^2

$$M_{\vec{j},\vec{m},\iota} = \pm\sqrt{\left|\frac{\mathcal{K}^{(0)}_{\vec{j},\vec{m},\iota}}{\mathcal{K}^{(2)}_{\vec{j},\vec{m},\iota}}\right|}$$

GFT (quantum) statistical mechanics

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I. Kotecha, DO, '17; G. Chirco, I. Kotecha, DO, '18

General problem in background independent (classical and) quantum gravity: what is "equilibrium" in absence of preferred temporal direction?

C. Rovelli, '12; G. Chirco, T. Josset, C. Rovelli, '15; I. Kotecha, '19

two main strategies for identifying/constructing equilibrium states, then applied to GFT context:

applied to system of quantum tetrahedra (using Fock space)

$$Z_{\mu,\beta} = \mathrm{Tr}_{\mathcal{H}_F}\left[e^{-\beta(\hat{C}-\mu\hat{N})}\right] = \mathrm{Tr}_{\mathcal{H}_F}\left[e^{-\beta(\sum_a \frac{\beta_a}{\beta}\hat{C}_a - \frac{\tilde{\mu}}{\beta}\hat{N})}\right]$$

- Use a basis of field coherent states

$$|\psi\rangle = e^{-\frac{||\psi||^2}{2}} e^{\int_{SU(2)^4} d\vec{g}\, \psi(\vec{g})\hat{\varphi}^\dagger(\vec{g})}|0\rangle \quad , \quad \hat{\varphi}(\vec{g})|\psi\rangle = \psi(\vec{g})|\psi\rangle$$

- For dynamical constraint operator $\hat{C}$, & effective dynamics $\langle\hat{C}\rangle_\rho = 0$

choices for C: volume operator, gluing + simplicity constraints, LQG Hamiltonian constraint, LQG physical projector

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- Effective statistical field theory $Z \approx Z_{\mathrm{eff}} = \int [D\mu(\psi,\bar{\psi})]e^{-C_{\mathrm{eff}}(\psi,\bar{\psi})}$

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1- Jaynes' entropy maximization principle

M. Montesinos, C. Rovelli, '01; G. Chirco, I. Kotecha, DO, '18

Maximising $S[\rho] = -\langle \ln \rho \rangle_\rho$ under

a set of macrostate constraints $\{\langle \mathcal{O}_a \rangle_\rho = U_a\}$

gives $\quad \rho_{\{\beta_a\}} = \frac{1}{Z_{\{\beta_a\}}} e^{-\sum_a \beta_a \mathcal{O}_a}$

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GFT partition function (encodes LQG dynamics)

DO, '13

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GFT partition function (encodes LQG dynamics)

DO, '13

can then be used to define generalised QG thermodynamics

I. Kotecha, '19

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2- KMS conditions on algebra of observables ("thermal time")

C. Rovelli, '93; A. Connes, C. Rovelli, '94

· for $A, B \in \mathcal{B}$, and normal state ρ,

$$\langle A(\alpha_{\lambda+i\beta}B)\rangle_\rho = \langle(\alpha_\lambda B)A\rangle_\rho$$

$$\Rightarrow \rho \propto e^{-\beta\mathcal{G}}$$

· requires existence of a pre-defined (continuous) 1-parameter flow α_λ

· generalised (inverse) temperature β enters as periodicity in parameter λ

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applied to GFT system and (Weyl) algebra of field observables

- with respect to group (rotation) symmetries

I. Kotecha, DO, '17

- with respect to relational (scalar field) clock variable

corresponding equilibrium states constructed - effective GFT quantum statistical framework

GFT (quantum) statistical mechanics

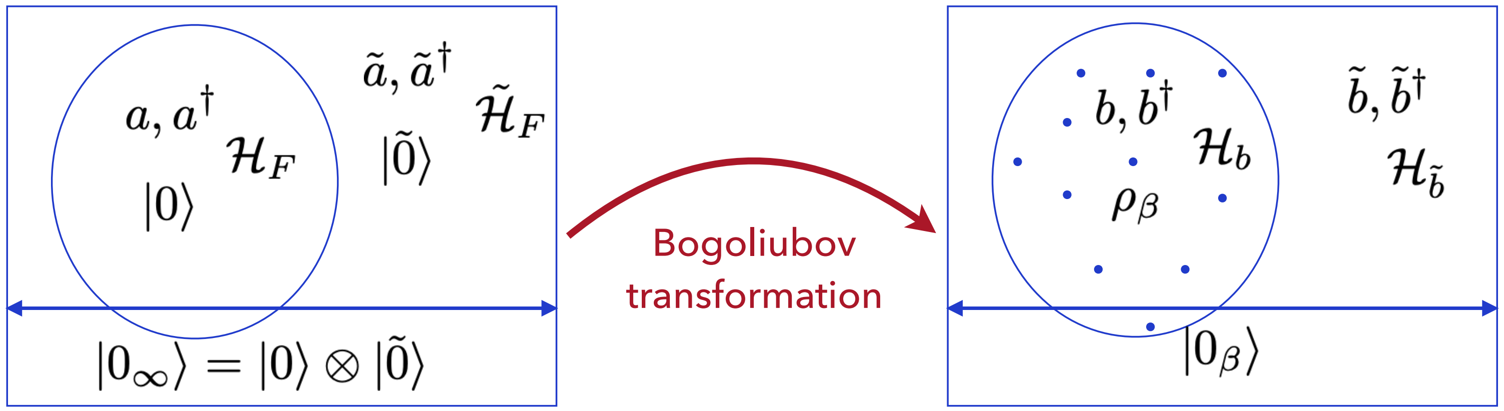
GFT thermal states and fluctuations using thermofield double formalism M. Assianoussi, I. Kotecha, '19, '20

GFT (quantum) statistical mechanics

GFT thermal states and fluctuations using thermofield double formalism

M. Assianoussi, I. Kotecha, '19, '20

Thermal vacuum for a given GGS $\hat{\rho}_\beta \Longleftrightarrow |0_\beta\rangle$ $\mathrm{Tr}(\rho_\beta \hat{A})_{\mathcal{H}_F} = \langle 0_\beta | \hat{A} | 0_\beta \rangle_{\mathcal{H}_\beta}$



$a_{J\alpha} |0_\infty\rangle = \tilde{a}_{J\alpha} |0_\infty\rangle = 0$

$\mathcal{H}_\infty = \mathcal{H}_F \otimes \tilde{\mathcal{H}}_F$

$b_{J\alpha} |0_\beta\rangle = \tilde{b}_{J\alpha} |0_\beta\rangle = 0$

$\mathcal{H}_\beta = \mathcal{H}_b \otimes \mathcal{H}_{\tilde{b}}$

defines inequivalent thermal representation

$$|0_\beta\rangle = e^{\sum_{J\alpha} \theta_{J\alpha}(a^\dagger_{J\alpha}\tilde{a}^\dagger_{J\alpha} - a_{J\alpha}\tilde{a}_{J\alpha})} |0_\infty\rangle$$

$$\boxed{\begin{aligned} b_{J\alpha} &= \cosh\theta_{J\alpha}(\beta)\, a_{J\alpha} - \sinh\theta_{J\alpha}(\beta)\, \tilde{a}^\dagger_{J\alpha} \\ \tilde{b}_{J\alpha} &= \cosh\theta_{J\alpha}(\beta)\, \tilde{a}_{J\alpha} - \sinh\theta_{J\alpha}(\beta)\, a^\dagger_{J\alpha} \end{aligned}}$$

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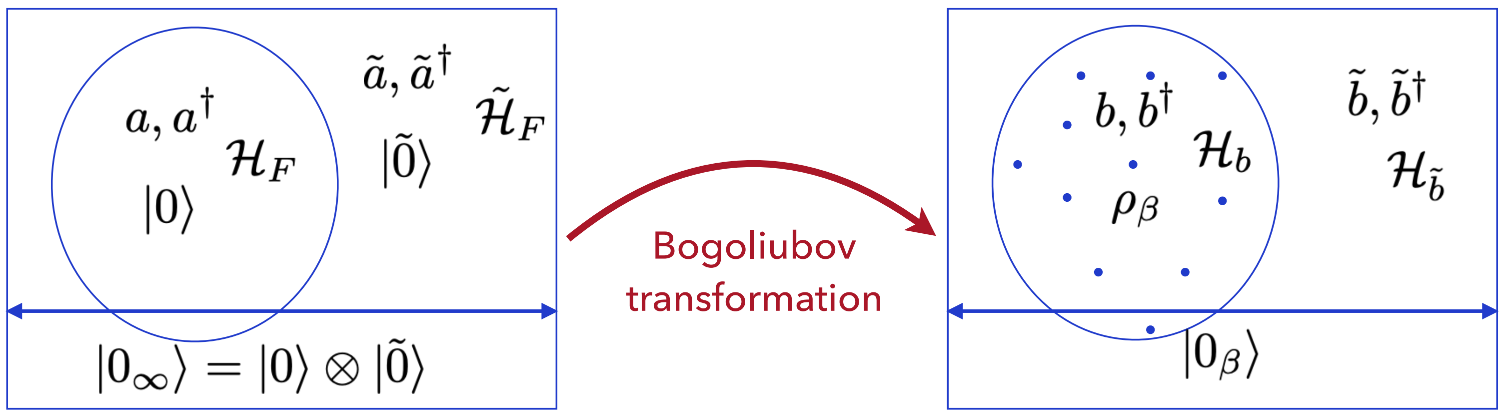

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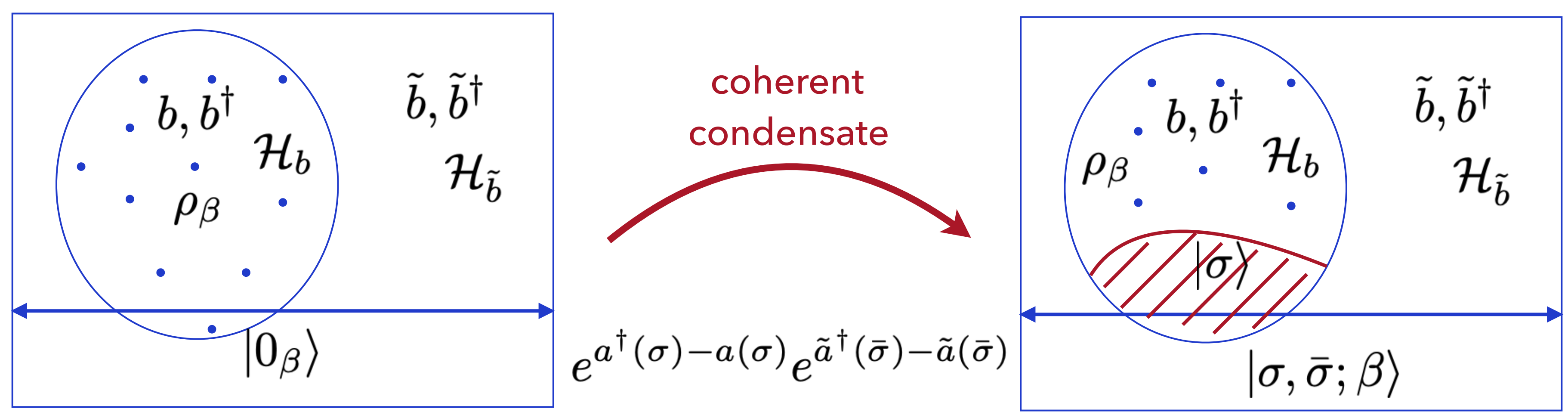
using thermal representation, can define further condensate (coherent) states (from one at zero temperature)



$$b_{J\alpha}|\sigma, \bar{\sigma}; \beta\rangle = (\cosh\theta_{J\alpha} - \sinh\theta_{J\alpha})\sigma_{J\alpha}|\sigma, \bar{\sigma}; \beta\rangle$$

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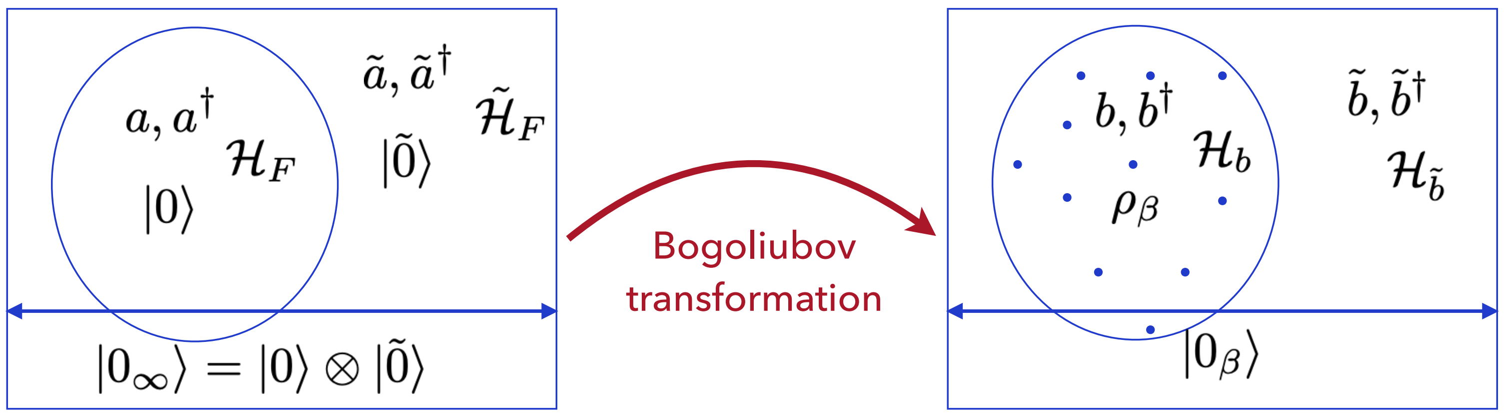

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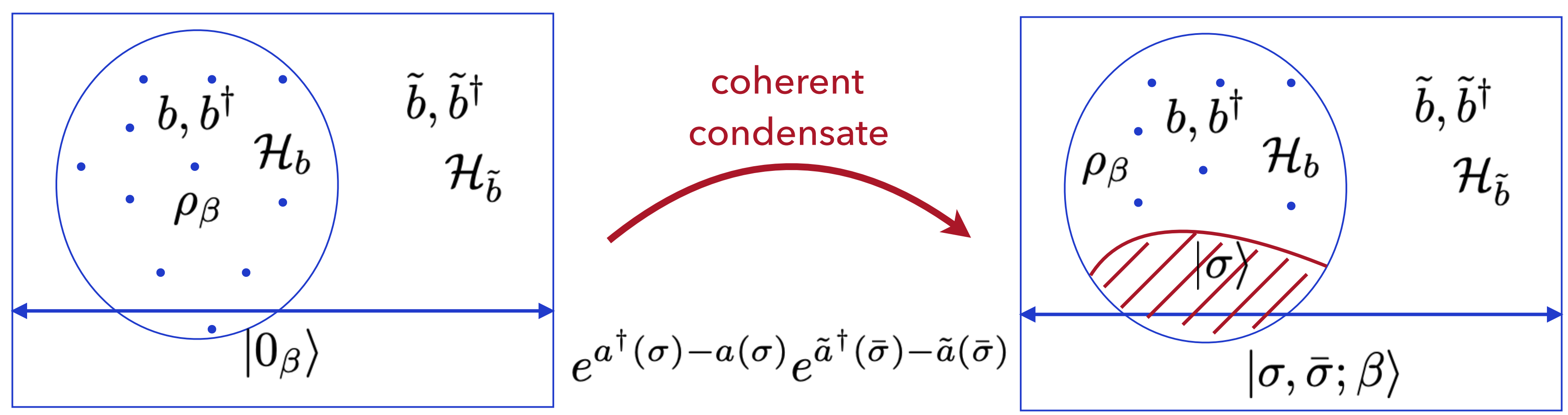

$$b_{J\alpha} |\sigma, \bar{\sigma}; \beta\rangle = (\cosh\theta_{J\alpha} - \sinh\theta_{J\alpha}) \sigma_{J\alpha} |\sigma, \bar{\sigma}; \beta\rangle$$

and compute thermal fluctuations of geometric observables in such condensate state

applications in GFT condensate cosmology

e.g. number operator: $\langle a^\dagger_{J\alpha} a_{J\alpha} \rangle_{\sigma, \bar{\sigma}; \beta} = |\sigma_{J\alpha}|^2 + \sinh^2\theta_{J\alpha}(\beta)$

Part III:
GFT quantum consistency and continuum limit

Quantum consistency of GFT models

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

quantum consistency

=

quantum dynamics is valid for all ranges of dynamical variables

↓

GFT perturbative renormalizability

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quantum consistency

=

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↓

GFT perturbative renormalizability

GFT model building mostly based on their Feynman amplitudes, i.e. lattice gravity path integrals and spin foam amplitudes

these present several ambiguities, e.g.:

- exact way of imposing simplicity constraints
- generalisations at combinatorial level (which spin foam complexes?)
- quantisation ambiguities (choice of quantisation map)
- quantum corrections and stability of spin foam amplitudes
- “measure” terms
- quantization/construction ambiguities in LQG Hamiltonian constraint

A. Perez, ‘07

quantum consistency implies taming such ambiguities

(not so much issue of divergences or existence of physical cut-offs)

Continuum limit of GFT models

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our QG d.o.f. are interacting → physics of few d.o.f. is very different from physics of many d.o.f. !

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our QG d.o.f. are interacting $\longrightarrow$ physics of few d.o.f. is very different from physics of many d.o.f. !

expect different phases

and phase transitions

as result of quantum dynamics

(what are the phases of LQG?)

which ones are “geometric”

in which one does spacetime emerge?

Koslowski, ’07; DO, ‘07

A. Ashtekar, J. Lewandowski, ’94 T. Koslowski, H. Sahlmann, ’10 B. Dittrich, M. Geiller, ’14; B. Bahr, B. Dittrich, M. Geiller, '16; S. Gielen, DO, L. Sindoni, ’13 A. Kegeles, DO, C. Tomlin, '16

Continuum limit of GFT models

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i] \overline{\varphi(g_i)} \mathcal{K}(g_i) \varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}] \varphi(g_{i1}) \ldots \varphi(\bar{g}_{iD}) \mathcal{V}(g_{ia}, \bar{g}_{iD}) \quad + \quad c.c.$$

our QG d.o.f. are interacting

physics of few d.o.f. is very different from physics of many d.o.f. !

expect different phases

and phase transitions

as result of quantum dynamics

(what are the phases of LQG?)

which ones are “geometric”

in which one does spacetime emerge?

Koslowski, ’07; DO, ‘07

controlling the continuum limit

=

evaluating full GFT partition function (in non-perturbative regime)

this means: evaluating full lattice gravity path integral, complete spin foam model (all complexes), whole LQG quantum dynamics

GFT non-perturbative renormalization group flow

A. Ashtekar, J. Lewandowski, ’94 T. Koslowski, H. Sahlmann, ’10 B. Dittrich, M. Geiller, ’14; B. Bahr, B. Dittrich, M. Geiller, '16; S. Gielen, DO, L. Sindoni, ’13 A. Kegeles, DO, C. Tomlin, '16

GFT renormalization

GFT renormalisation - general scheme

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\ e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

scales:

defined by propagator: e.g. spectrum of Laplacian on G = indexed by group representations

$$\sum_{\ell=1}^{d} j_\ell(j_\ell+1) \lesssim \Lambda^2$$

GFT renormalisation - general scheme

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scales:

defined by propagator: e.g. spectrum of Laplacian on G = indexed by group representations

$$\sum_{\ell=1}^{d} j_\ell (j_\ell + 1) \lesssim \Lambda^2$$

- need to have control over "theory space" (e.g. via symmetries)
- technical difficulties:
  controlling the combinatorics of GFT Feynman diagrams
  need to adapt/redefine many QFT notions: connectedness, subgraph contraction, Wick ordering, …..

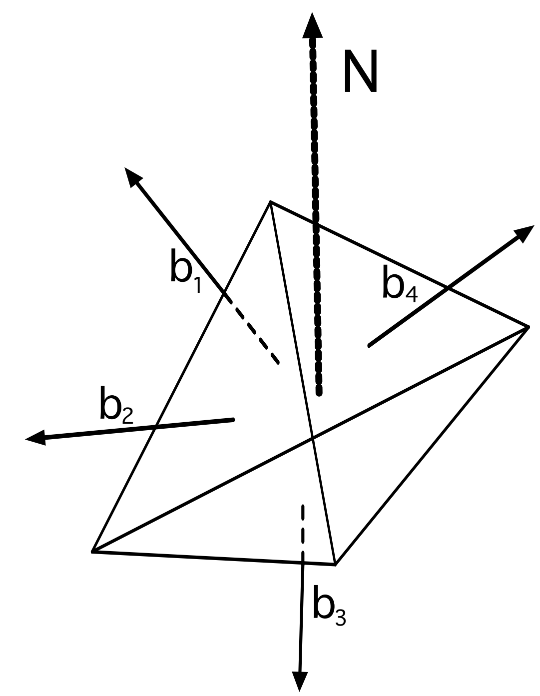
missing in simplicial GFT models

most work done for tensorial GFT models

GFT Renormalization: geometric interpretation?

arguments of GFT field: $b_i \in \mathfrak{su}(2)$ gravity case: d=4

I b I ~ J = irrep of SU(2) ~ “area of triangles”



“geometric” interpretation?

RG flow from large areas to small areas?

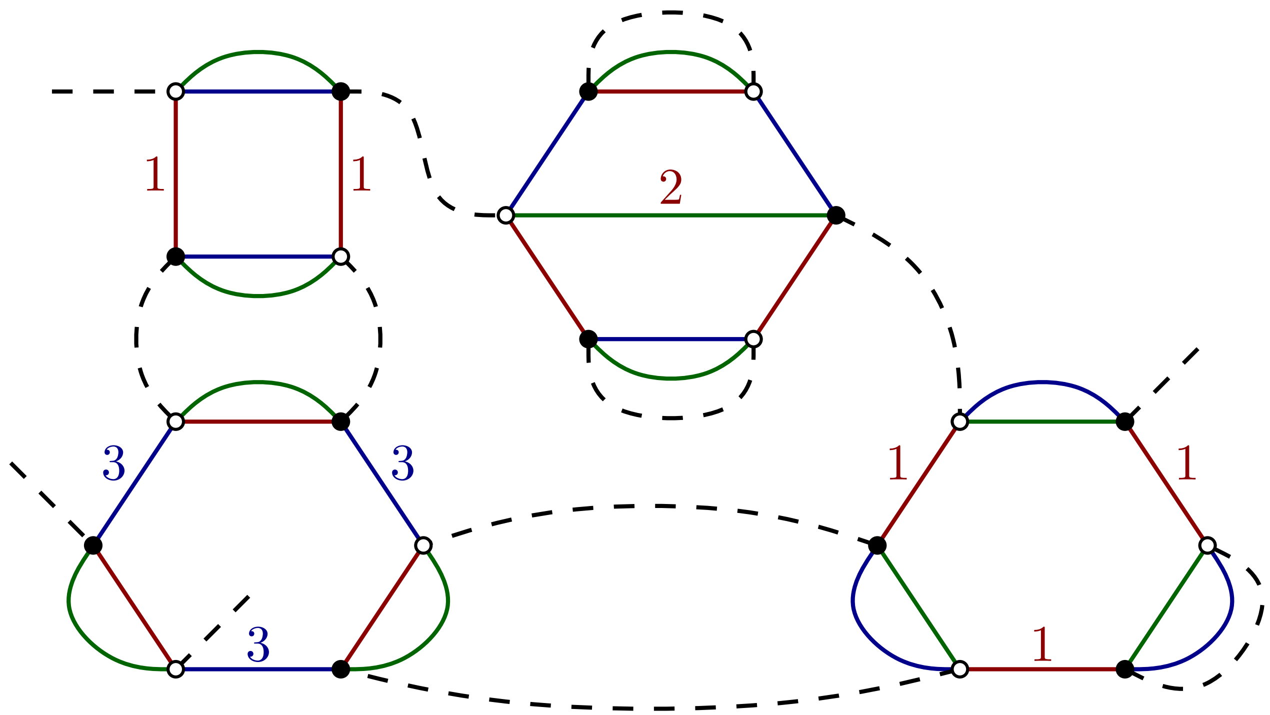
continuum geometric intepretation?

from LQG
from Regge calculus

- CAUTION in interpreting things geometrically outside continuum geometric approx.
- expect “physical” continuum areas A ~ < J > < n > , for < n > large
- expect proper continuum geometric interpretation (and effective metric field)

  for < J > small, < n > large, A finite (not too small)
- from continuum geometric perspective, large areas are result of coarse graining of microscopic dofs

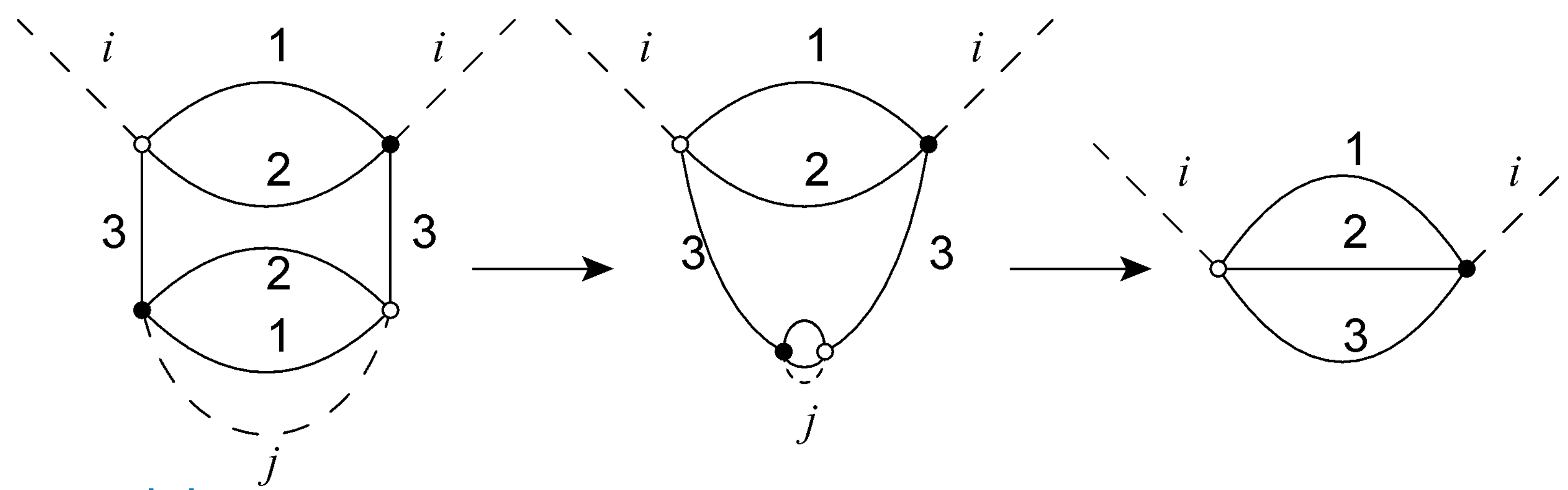
GFT Renormalization: combinatorics of FDs



example of Feynman diagram in 4d (interaction process of tetrahedra ~ 4d complex)

Example: when internal scales $j \gg$ external scales i

contraction of (divergent) subgraphs + absorption in effective vertices is coarse-graining of simplicial lattices



(perturbative) GFT renormalization = renormalization of lattice gravity path integral

spin foam amplitude consistency under coarse graining = RG consistency of GFT Feynman amplitudes

GFT perturbative renormalisation

S, Carrozza, '16

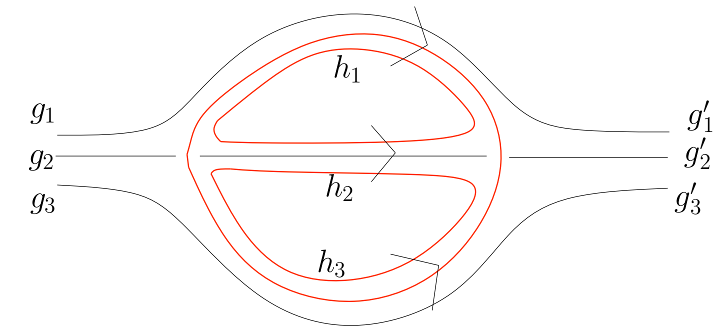
towards renormalizable 4d gravity simplicial GFT models:

- calculation of radiative corrections and basic divergences

  T. Krajewski et al., '10; A. Riello, '13; V. Bonzom, B. Dittrich, '15; P. Dona', '17; P. Dona et al, '19; M. Finocchiaro, DO, to appear

- finiteness results in 3d simplicial models (Boulatov with Laplacian kinetic term)

  Ben Geloun, Bonzom, '11; Ben Geloun, '13

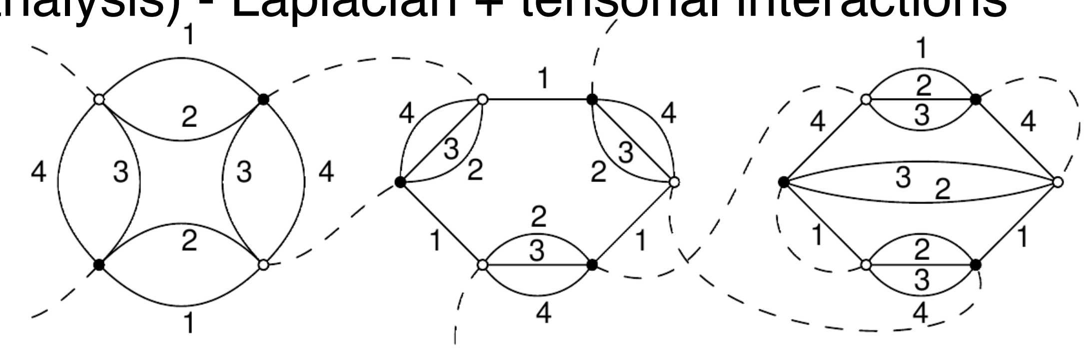


key goals: identify theory space, characterize divergent configurations

- renormalizable TGFT models (3d, 4d, and higher - multi scale analysis) - Laplacian + tensorial interactions

  Ben Geloun, Rivasseau, '11
  Carrozza, DO, Rivasseau, '12. '13

$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$



—> with gauge invariance

—> non-abelian (SU(2))

—> on homogeneous spaces (towards TGFTs for 4d QG): first steps

Lahoche, DO, '15

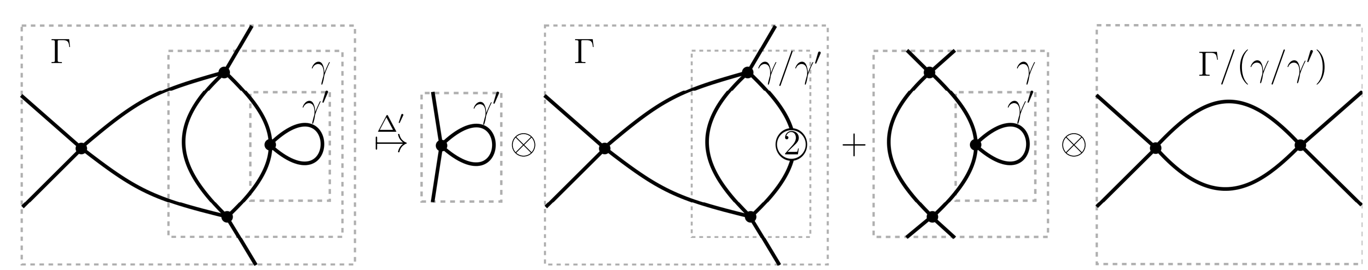
————> generic asymptotic freedom/safety

Ben Geloun, '12; Carrozza, '14; Carrozza, Lahoche, '16

key (most divergent, renormalizable) diagrams: melonic diagrams

- Hopf algebra methods in TGFT renormalization

  M. Raasakka, A. Tanasa, '13; R. Cochou, V. Rivasseau, A. Tanasa, '17



GFT non-perturbative renormalisation

S, Carrozza, '16

- GFT constructive analysis

Freidel, Louapre, Noui, Magnen, Smerlak, Gurau, Rivasseau, Tanasa, Dartois, Delpouve, …..

non-perturbative resummation of perturbative (spin foam) series

variety of techniques:

- intermediate field method (loop-vertex expansion)
- Borel summability
- proof of existence of phase transition

- FRG analysis of (discrete gravity) tensor models and SYK-like tensor models/QFTs

Eichhorn, Koslowski, Duarte Pereira, ….

Benedetti, Ben Geloun, Carrozza, Gurau, Rivasseau, Sfondrini, Tanasa, Wulkenhaar, ….

comparison with results from resummation of matrix models (FRG counterpart of double scaling limit)

- TGFT non-perturbative renormalization (e.g. FRG analysis ala Wetterich-Morris)

Benedetti, Ben Geloun, DO, Martini, Lahoche, Carrozza, Ousmane-Samary, Duarte Pereira, ….

GFT non-perturbative renormalisation

S, Carrozza, '16

recent results:

FRG for (tensorial) GFT models

(similar to matrix/tensor models but distinctively field-theoretic)

Eichhorn, Koslowski, '14

- Polchinski formulation based on SD equations   Krajewski, Toriumi, '14
- general set-up for Wetterich-Morris formulation based on effective action
  - RG flow and phase diagram for: Benedetti, Ben Geloun, DO, '14 ; Ben Geloun, Martini, DO, '15, '16, Benedetti, Lahoche, '15; Lahoche, Ousmane-Samary, '16; ……
    - TGFT on compact U(1)^d (with gauge invariance)
    - TGFT on non-compact R^d (with gauge invariance)
    - TGFT on SU(2)^3 (with gauge invariance)   Carrozza, Lahoche, '16
    - non-melonic TGFT on SU(2)^4 with gauge invariance   S. Carrozza, V. Lahoche, DO, '17
    - TGFT on U(1)^3 in full quartic truncation

      J. Ben Geloun, T. Koslowski, A. Duarte Pereira, DO, '18
- consequences of Ward identities on RG flow

  V. Lahoche, D. Ousmane-Samary, '17, '18,'19, '20
- epsilon-expansion   Carrozza, '14

challenges:

- scaling dimensions of couplings (depend on combinatorics of corresponding interactions)
- non-autonomous systems of flow equations
- more subtle thermodynamic limit
- combinatorics
- ....

GFT non-perturbative renormalisation

S, Carrozza, '16

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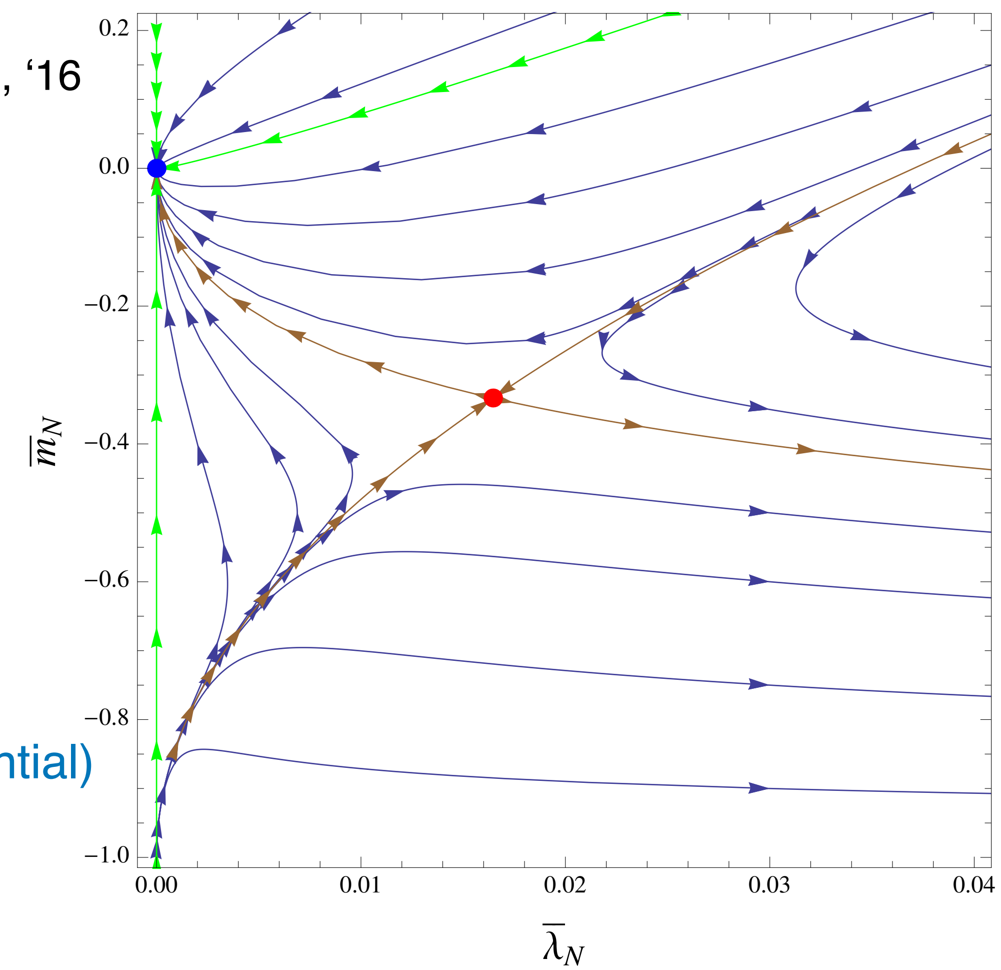

generically (so far):

- asymptotic freedom/safety
- hints of broken or condensate phase (non-trivial minimum of classical potential)
- TGFTs can flow naturally out of melonic sector
- non-melonic TGFTs can be renormalizable
- truncation (incl. non-melonic, disconnected diagrams) matters

Part IV:
effective continuum physics from GFT

GFT and (quantum) many-body systems

GFT, tensor models & (quantum) many-body systems

- recently, large body of work applying GFTs & tensor models to SYK systems
  - **Sachdev-Ye-Kitaev** models = disordered systems of N Majorana fermions

    [Sachdev, Ye, George, Parcollet '90s...; Kitaev '15, Maldacena, Stanford, Polchinski, Rosenhaus...]

$$H_{\rm int} \sim J_{i_1 i_2 i_3 i_4}\, \psi_{i_1}\psi_{i_2}\psi_{i_3}\psi_{i_4}\,, \qquad \left\langle J_{i_1 i_2 i_3 i_4}\right\rangle \sim 0\,, \quad \left\langle J^2_{i_1 i_2 i_3 i_4}\right\rangle \sim \frac{J^2}{N^3}$$

many related models have been constructed (bosonic, supersymmetric, different dimension, etc)

- Many interesting properties:
  - **solvable at large** N
  - emergent **conformal symmetry** at **strong coupling**
  - maximal **quantum chaos**
  - holography in low dimension: "$NAdS_2/NCFT_1$"

Witten, Gurau, Klebanov, Tarnopolsky, Benedetti, Bonzom, Carrozza, Tanasa, Ben Geloun, Rivasseau, Ferrari, ...

same large-N behaviour of simple GFT/tensor models - melonic diagrams

related TGFT/tensor models - abelian with extra real direction (time)

e.g. $$S[\bar\psi,\psi] = \int \mathrm{d}t\, \left(i\bar\psi_{abc}\partial_t\psi_{abc} - \frac{g}{2}\epsilon_{bg}\epsilon_{dh}\bar\psi_{abc}\bar\psi_{fge}\psi_{ade}\psi_{fhc}\right)$$

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- other applications to statistical systems on random lattices
  - Ising model
  - dimer models
  - ....

Bonzom, Gurau, Smerlak, Rivasseau, Lahoche, Ousmane-Samary, ...

GFT, entanglement, tensor networks and holography

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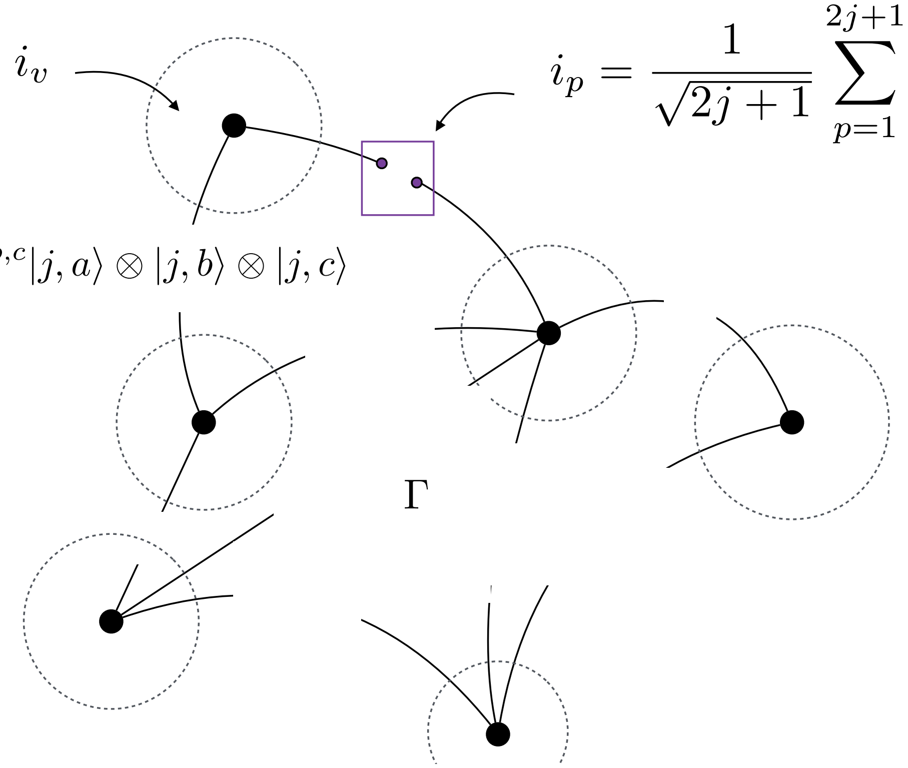
entanglement at the foundations of GFT/LQG description of quantum space

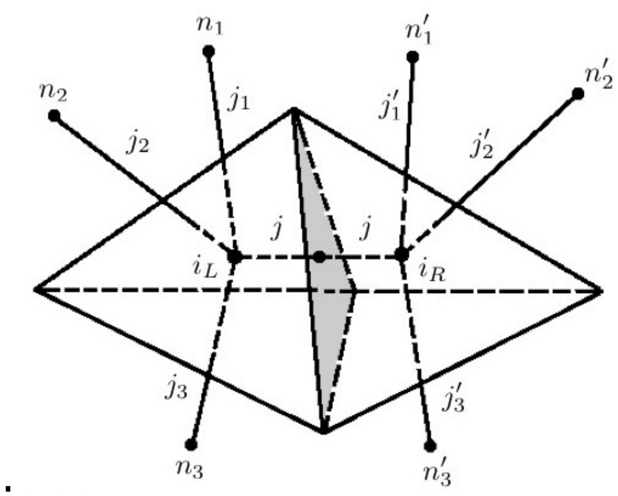
Donnelly, '12; Livine, Terno, '08; DO '13; Chirco, Mele, DO, Vitale, '17; .....

gluing = discrete space connectivity = entanglement between "atoms of space"

$$i_p = \frac{1}{\sqrt{2j+1}} \sum_{p=1}^{2j+1} |e_p\rangle\langle e_p|$$

$$|\mathcal{I}\rangle = \sum_{\{a,b,c\}} i^{a,b,c} |j,a\rangle \otimes |j,b\rangle \otimes |j,c\rangle$$





entanglement across link ~ j
~ area of dual surface

gauge invariance/closure at vertices ~
entanglement of link dofs ~ volume information

GFT, entanglement, tensor networks and holography

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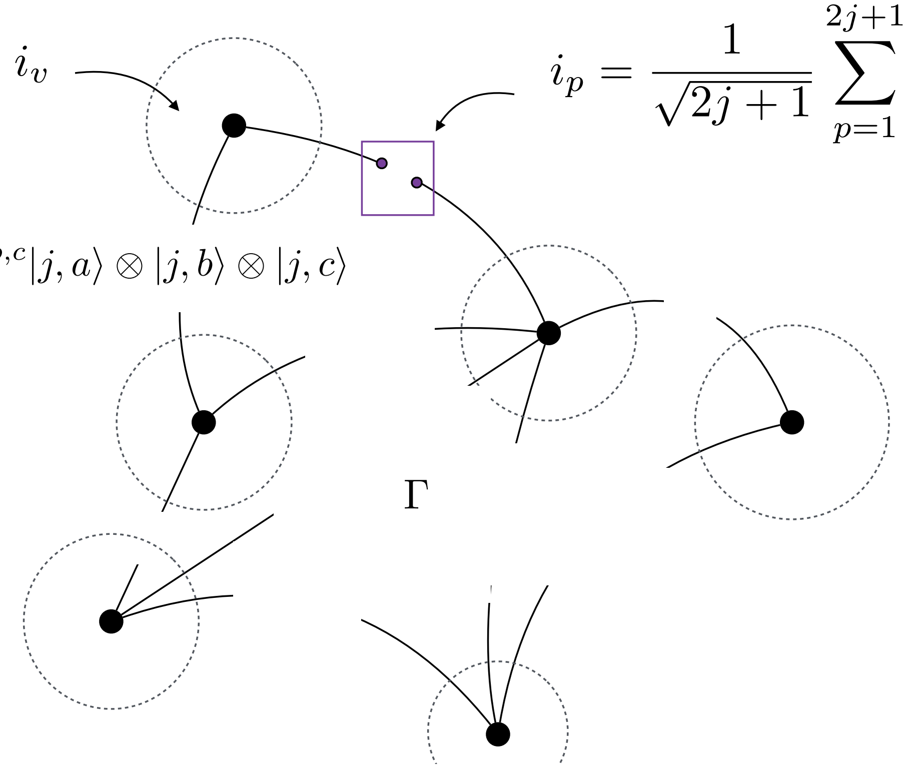

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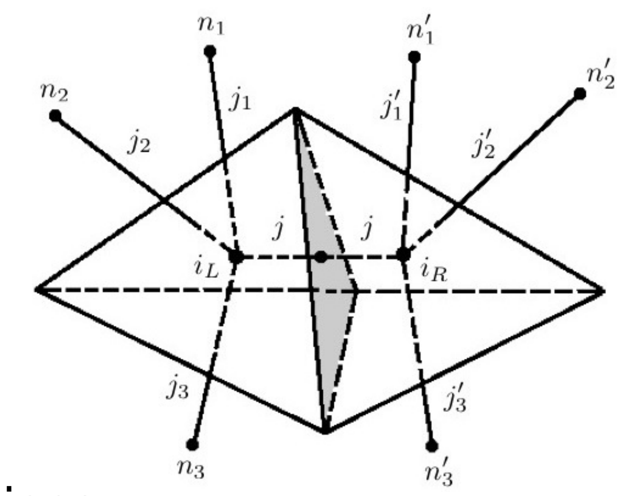


gauge invariance/closure at vertices ~
entanglement of link dofs ~ volume information

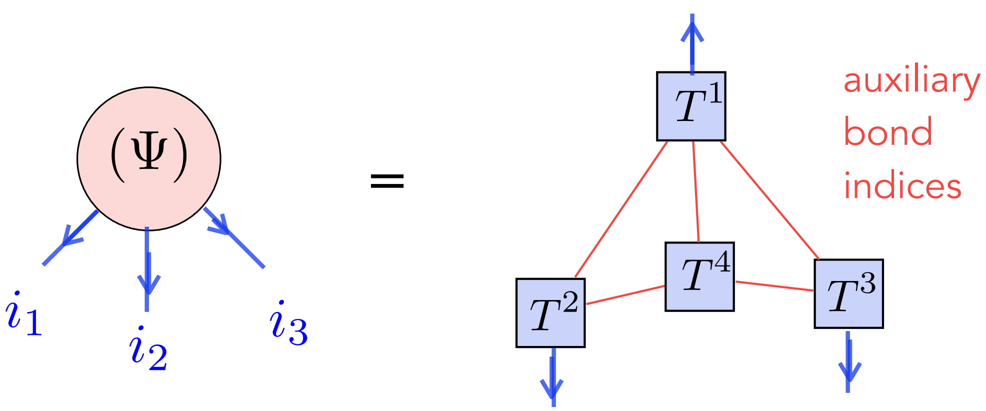
- **entanglement and other QI properties of spin network states** Livine, Terno, ’06; Bianchi et al. ’18; Chirco et al ’14, ’15; Hamma et al. ’15; Anzà, Chirco, ’16, Han et al. ‘16
    - more calculations of entanglement entropy in spin network states and conditions for area law
    - more evidence for “entanglement = spatial connectivity” idea
    - typicality criteria for spin network states, in terms of boundary/bulk separation of 3d regions
    - information-theoretic definition of horizons and entanglement-based “collapse" for spin network states
- **spin foams as tensor networks and their renormalization** Dittrich, Martin-Benito, Steinhaus, Bahr, Livine, ….

GFT, entanglement, tensor networks and holography

$$|\Psi\rangle \equiv \bigotimes_{<ij>} \langle M_{ij}| \bigotimes_{v}^{N} |T_v\rangle$$

Spin networks (for fixed and equal spins) are a special case of tensor networks (with local gauge symmetry)

S. Singh, R. Pfeifer, G. Vidal, '09; G. Evenbly, G. Vidal, '11; S.Singh, G. Vidal, '12; S. Singh, N. McMahon, G. Brennen, '17



Group field theory states are a field-theoretic generalization of random tensor networks - GFT dynamics is probability measure for tensor

probability measure (GFT dynamics): $\frac{1}{Z} d\nu(\varphi)$

Table B	GFT network	Spin Tensor Network	Tensor Network
node	$\varphi(\vec{g}) \equiv \varphi(g_1, g_2, g_3, g_4)$	$\varphi^{\mathbf{j}}_{\{m\}} \propto \sum_{\{k\}} \hat{\varphi}^{\mathbf{j}}_{\{m\}\{k\}}\, i^{\mathbf{j}\{k\}}$	$T_{\{\mu\}}$
link	$M(g_1^\dagger g_\ell g_2)$	M^j_{mn}	$M_{\lambda_1 \lambda_2}$
sym	$\varphi(h\vec{g}) = \varphi(\vec{g})$	$\prod_s^v D^j_{m_s m'_s}(g) i^i_{m'_1 \cdots m'_v} = i^i_{m_1 \cdots m_v}$	$\prod_s^v U_{\mu_s \mu'_s} T_{\mu'_1 \cdots \mu'_v} = T_{\mu_1 \cdots \mu_v}$
state	$\lvert\Phi^{g_\ell}_\Gamma\rangle \equiv \bigotimes_\ell \langle M_{g_\ell}\rvert \bigotimes_n \lvert\psi_n\rangle$	$\lvert\Psi^{\mathbf{ji}}_\Gamma\rangle \equiv \bigotimes_\ell \langle M^{j_\ell}\rvert \bigotimes_n \lvert\phi_n^{\mathbf{j}_n i_n}\rangle$	$\lvert\Psi_\mathcal{N}\rangle \equiv \bigotimes_\ell^L \langle M_\ell\rvert \bigotimes_n^N \lvert T_n\rangle$
indices	$g_i \in G$, $\lvert g_i\rangle \in \mathbb{H} \simeq L^2[G]$	$m_i \in \mathbb{H}_j$, SU(2) spin-j irrep.	$\mu_i \in \mathbb{Z}_n$, nth cyclic group
dim	∞	$\dim \mathbb{H}_j = 2j+1$	$\dim \mathbb{Z}_n = n$

G. Chirco, DO, M. Zhang, '17; '18;
G. Chirco, A. Goessmann, DO, M. Zhang, '19

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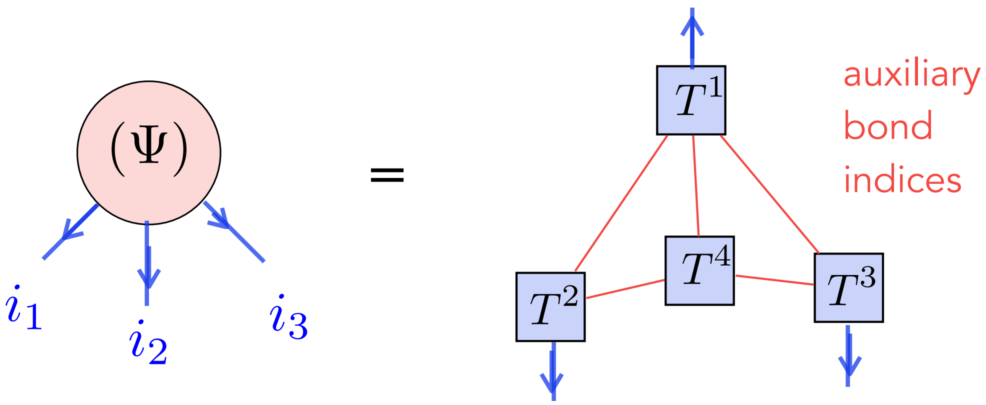

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G. Chirco, DO, M. Zhang, '17; '18;
G. Chirco, A. Goessmann, DO, M. Zhang, '19

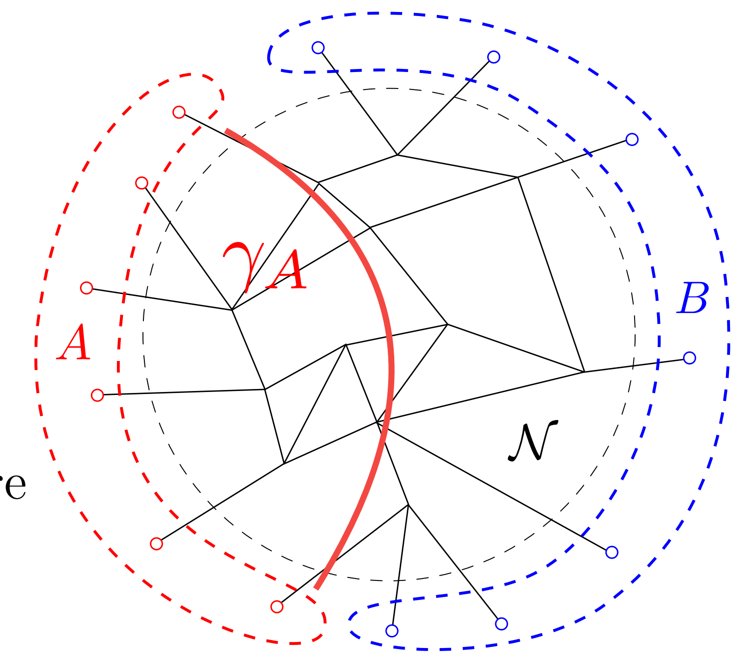
opportunity to:

study TN holographic properties and encoding of entanglement, in full QG, incl. fundamental dynamics

control efficiently entanglement properties of dynamical QG states

Towards Ryu-Takanayagi formula in full QG

Hayden et al. '16 Chirco, DO, Zhang, '17



Goal: compute entanglement entropy for QG (GFT) state defined by generalised Tensor Network

1. Tensors are generalized to group fields, from a finite dimensional object to a square integrable L^2 function, mapping from group manifolds to the complex numbers $\mathbb{C}$.

2. A gauge symmetry of the group field associated to each vertex as a vertex wave function is introduced in order to fit our setup more to the context of the quantum gravity theory.

3. The average over the N-replica of the wave functions (generalised tensors) associated to each network vertex is reinterpreted as a N-point correlation function of a (simple) GFT model, which turns the averaged Rényi entropy into an amplitude in GFT.

(topological BF)

entanglement entropy defined via Renyi entropy, and computed using replica trick

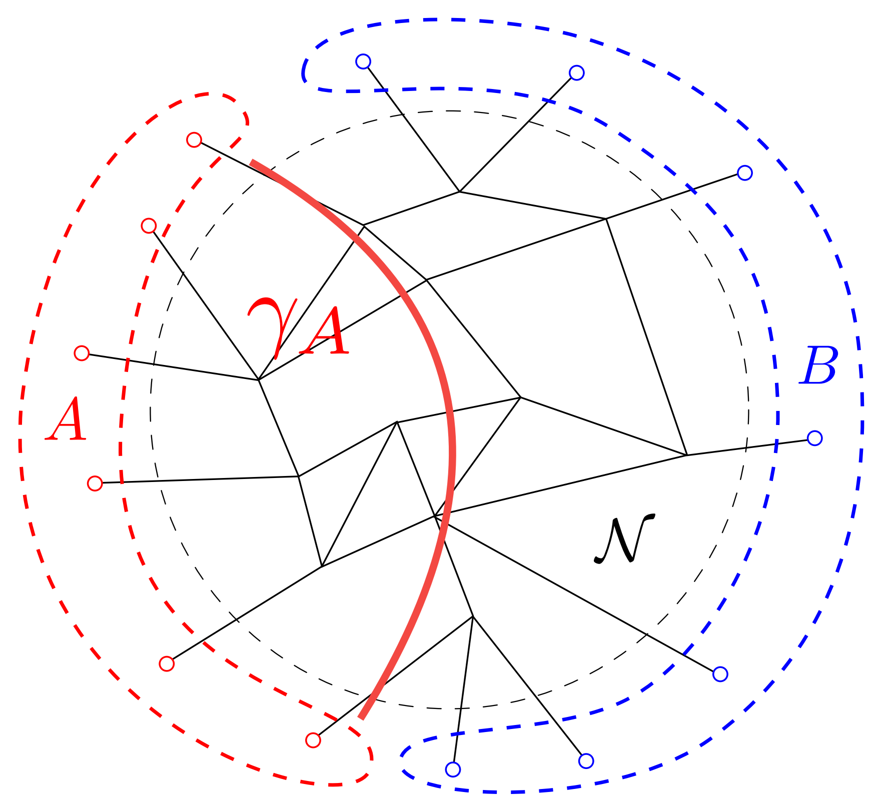
Towards Ryu-Takanayagi formula in full QG

Hayden et al. '16 Chirco, DO, Zhang, '17

——> entropy from evaluation of sum over lattice BF amplitudes (on "bulk lattices")
~ GFT Feynman amplitudes - result dominated by most divergent ones

Freidel, Gurau, DO '09, Bonzom, Smerlak '10-'12

! this results from choice of specific GFT dynamics/model!



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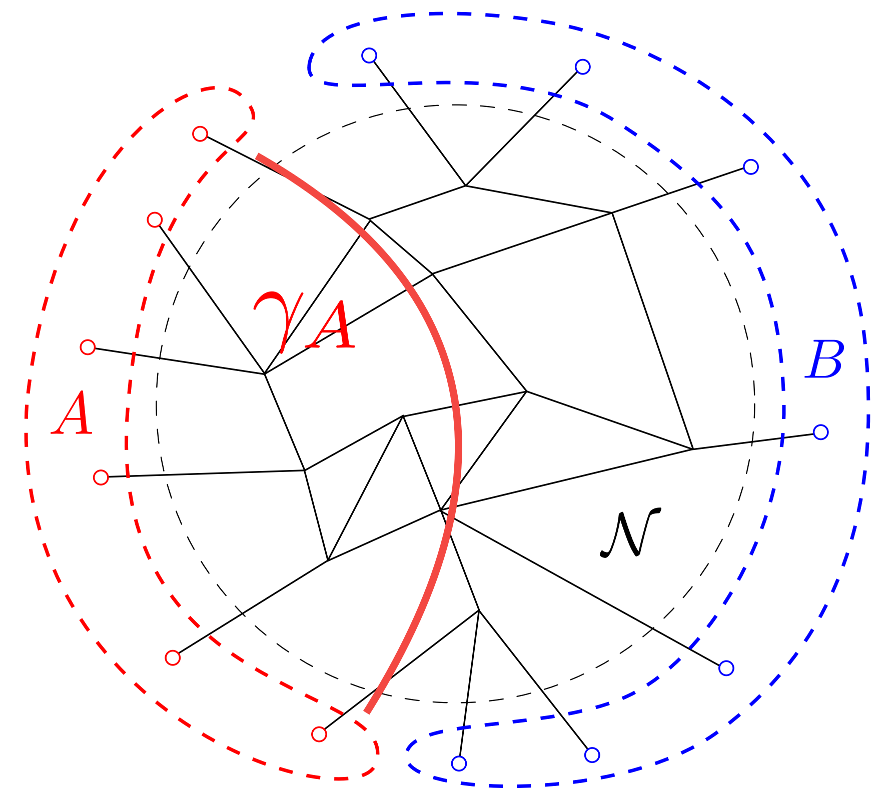
Freidel, Gurau, DO '09, Bonzom, Smerlak '10-'12

result:

$$S_{\rm EE} = \min(\#_{\ell \in \partial_{AB}}) \ln \delta(\mathbb{1})$$

entropy proportional to area of minimal "bulk" surface (number of crossing links) (it is surface in 3d region associated to spin network, though)

(Ryu-Takanayagi-like formula)



Towards Ryu-Takanayagi formula in full QG

Hayden et al. ‘16 Chirco, DO, Zhang, ‘17

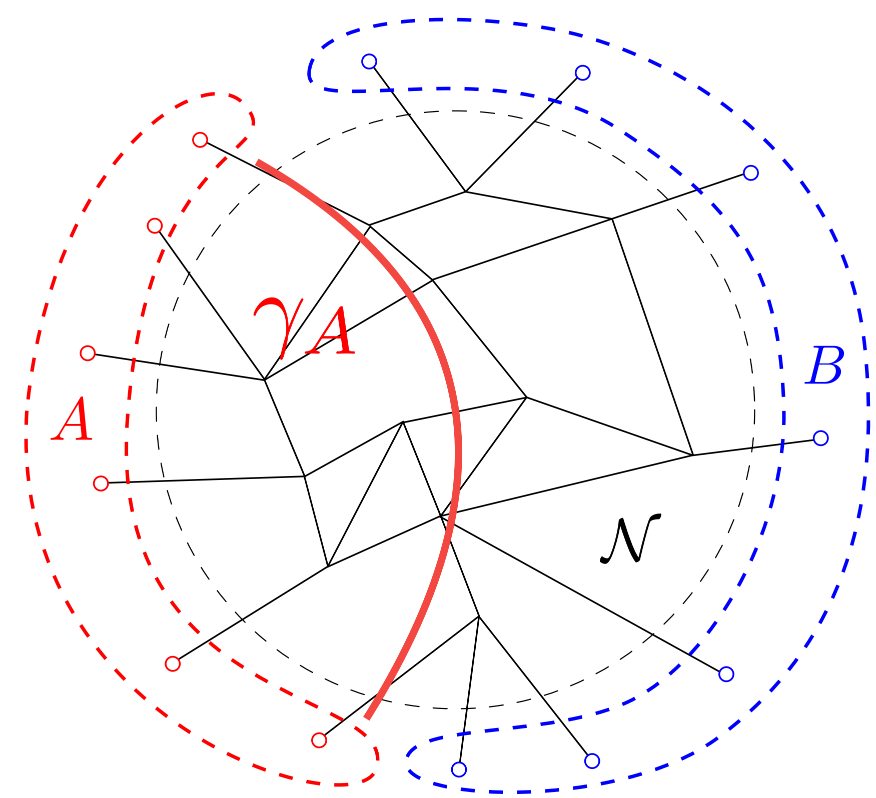
—–> entropy from evaluation of sum over lattice BF amplitudes (on “bulk lattices”)
~ GFT Feynman amplitudes - result dominated by most divergent ones

Freidel, Gurau, DO ’09, Bonzom, Smerlak ’10-’12

! this results from choice of specific GFT dynamics/model!

result:

$$S_{\mathrm{EE}} = \min(\#_{\ell\in\partial_{AB}}) \ln \delta(\mathbb{1})$$



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can compute (perturbative) QG (GFT) corrections…..

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Boulatov model possibly too trivial to have non-trivial effects (topological field theory)

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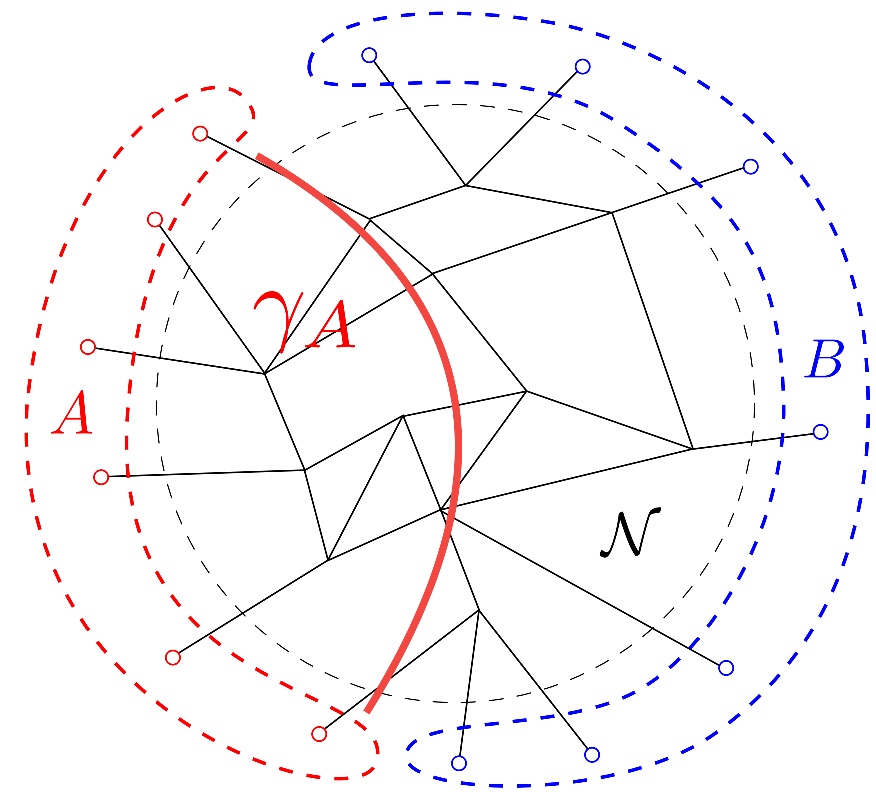

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key points remain: 1) potential of formalism to compute entanglement entropy including QG dynamics; 2) entanglement entropy is measure of connectivity between two regions, via "bulk" geometry

GFT condensate cosmology

GFT (condensate) cosmology

S. Gielen, DO, L. Sindoni, G. Calcagni, M. Sakellariadou,
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Quantum GFT condensates are continuum homogeneous (quantum) spaces

described by single collective wave function ("wave-function homogeneity")
(depending on homogeneous anisotropic geometric data)

superposition of infinitely many spin networks dofs,
"gas"of tetrahedra, all associated with same state

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infinite superposition of Feynman diagrams
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cosmology as QG hydrodynamics

GFT (condensate) cosmology

Gielen, DO, Sindoni, ’13; Calcagni, De Cesare, Gielen, DO, Pithis, Sakellariadou, Sindoni, Wilson-Ewing, …

Simple GFT condensates as homogeneous continuum geometries (not encoding any topological information)

$$|\sigma\rangle := \exp(\hat{\sigma})\,|0\rangle$$

$$\hat{\sigma} := \int d^4g\;\sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I)$$

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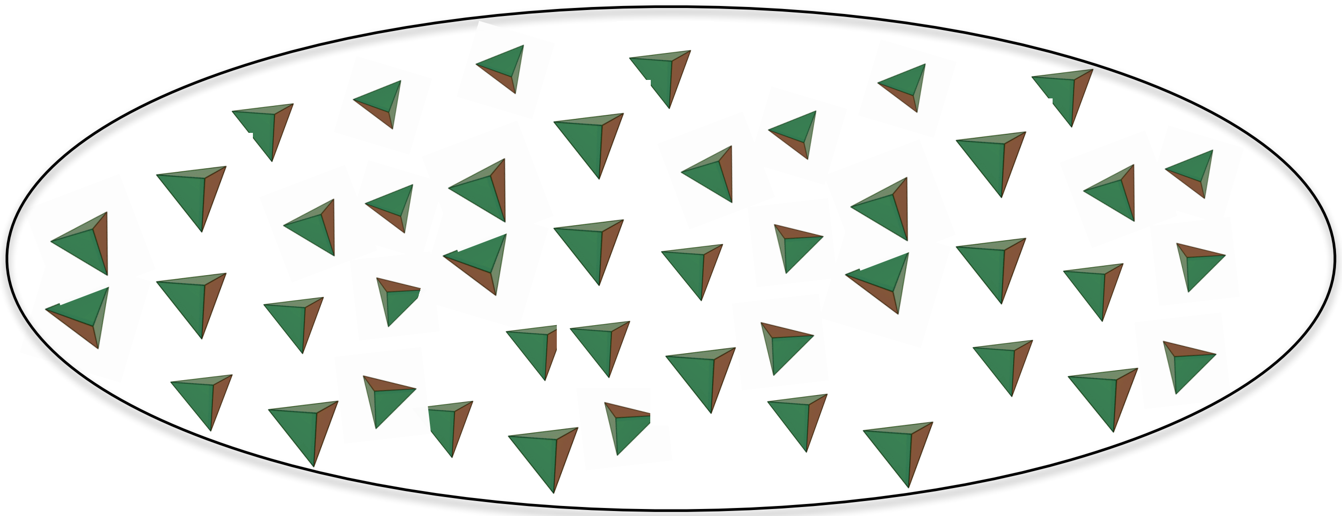

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$$\sigma(\mathcal{D}) \qquad \mathcal{D} \simeq \{\text{geometries of tetrahedron}\} \simeq$$
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$$\simeq \text{minisuperspace of homogeneous geometries}$$

Gielen, ‘14

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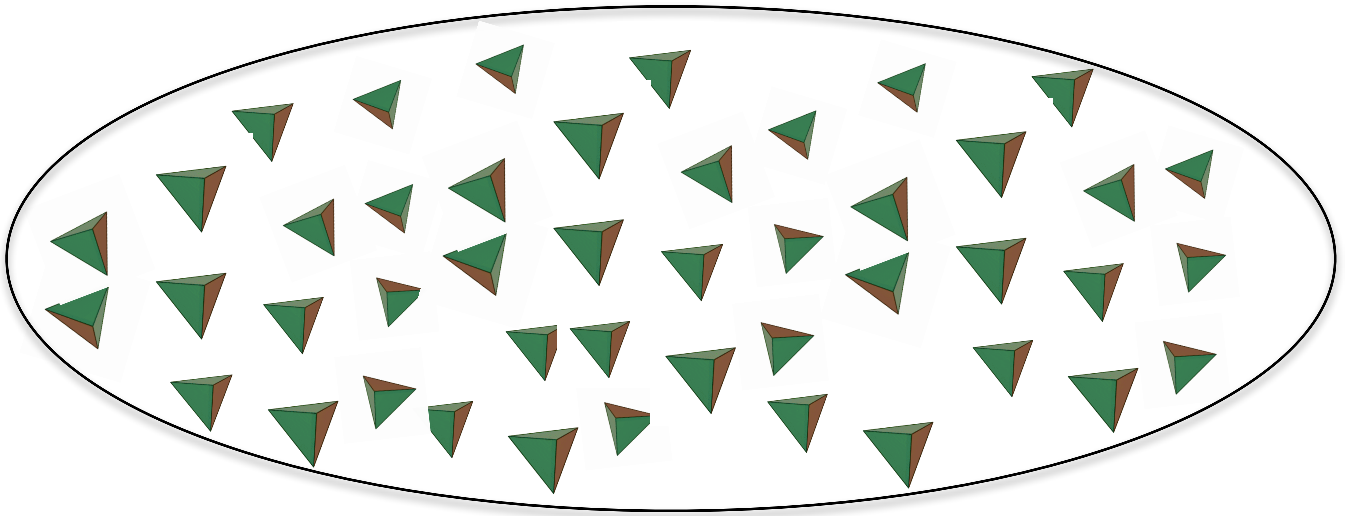

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Gielen, ‘14

Effective dynamics (from inserting state in fundamental quantum eqns: GFT eqns of motion

$$\int [dg'_i]\,\tilde{\mathcal{K}}(g_i, g'_i)\sigma(g'_i) + \lambda \frac{\delta\tilde{\mathcal{V}}}{\delta\varphi(g_i)}|_{\varphi\equiv\sigma} = 0$$

i.e. mean field (Gross-Pitaevskii) hydrodynamics

non-linear and non-local extension of quantum cosmology-like equation for “collective wave function”

no perturbative (spin foam) expansion -

infinite superposition of Feynman diagrams
(infinite sum over discrete “spacetime” lattices)

formally similar to quantum cosmology, but:

no Hilbert space structure (no superposition of “states of universe”, no “collapse of cosmological wave function

Emergent bouncing cosmology from full QG

DO, Sindoni, Wilson-Ewing, '16

- (generalised) EPRL model for 4d Lorentzian QG with SU(2) data, coupled to (discretised) (pre-)scalar field

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- coupling of free massless scalar field

$\hat{\varphi}(g_v) \rightarrow \hat{\varphi}(g_v, \phi)$

$$|\sigma\rangle \sim \exp\left(\int \mathrm{d}g_v \mathrm{d}\phi \, \sigma(g_v, \phi)\hat{\varphi}^\dagger(g_v, \phi)\right)|\mathbf{0}\rangle$$

want to use effective scalar field variable as "physical clock" to define "time"

dynamics chosen so that Feynman amplitudes are discrete path integrals for gravity coupled to free massless scalar field

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$$S = K^{(0)} + K^{(2)} + V + V^\dagger$$

$$K^{(0)} = \int \mathrm{d}\phi \sum_{j_i, m_i, \iota} \bar{\varphi}^{j_v \iota}_{m_v}(\phi) \varphi^{j_v \iota}_{m_v}(\phi) \, (\mathscr{K}_2^{(0)})^{j_v \iota}_{m_v} \qquad K^{(2)} = \int \mathrm{d}\phi \sum_{j_i, m_i, \iota} \bar{\varphi}^{j_v \iota}_{m_v}(\phi) \frac{\partial^2}{\partial \phi^2} \varphi^{j_v \iota}_{m_v}(\phi) \, (\mathscr{K}_2^{(2)})^{j_v \iota}_{m_v}$$

$$V = \sum_{j_i, m_i, \iota_i} \Big[\varphi^{j_1 j_2 j_3 j_4 \iota_1}_{m_1 m_2 m_3 m_4}(\phi) \varphi^{j_4 j_5 j_6 j_7 \iota_2}_{m_4 m_5 m_6 m_7}(\phi) \varphi^{j_7 j_3 j_8 j_9 \iota_3}_{m_7 m_3 m_8 m_9}(\phi) \varphi^{j_9 j_6 j_2 j_{10} \iota_4}_{m_9 m_6 m_2 m_{10}}(\phi) \varphi^{j_{10} j_8 j_5 j_1 \iota_5}_{m_{10} m_8 m_5 m_1}(\phi)$$
$$\times \tilde{\mathscr{V}}_5(j_1, \ldots, j_{10}; \iota_1, \ldots, \iota_5) \Big], \qquad (3$$

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$$\times \tilde{\mathscr{V}}_5(j_1, \ldots, j_{10}; \iota_1, \ldots, \iota_5) \Big], \qquad (3$$

- reduction to isotropic condensate configurations (depending on single variable j): $\quad \sigma(g_v, \phi) \to \sigma_j(\phi)$

Emergent bouncing cosmology from full QG

DO, Sindoni, Wilson-Ewing, ‘16

- effective condensate hydrodynamics (non-linear quantum cosmology):

$$\boxed{A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \sigma_j(\phi)^4 = 0}$$

functions A, B, w define the details of the EPRL model

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also regime in which simple condensate has good chance to be good approximation of true solution of quantum dynamics

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- key relational observables (expectation values in condensate state) with scalar field as clock:

universe volume (at fixed "time") $$V(\phi) = \sum_j V_j \bar{\sigma}_j(\phi)\sigma_j(\phi) = \sum_j V_j \rho_j(\phi)^2 \qquad V_j \sim j^{3/2}\ell_{\mathrm{Pl}}^3$$

momentum of scalar field (at fixed "time") $$\pi_\phi = \langle\sigma|\hat{\pi}_\phi(\phi)|\sigma\rangle = \hbar \textstyle\sum_j Q_j$$

energy density of scalar field (at fixed "time") $$\rho = \frac{\pi_\phi^2}{2V^2} = \frac{\hbar^2(\sum_j Q_j)^2}{2(\sum_j V_j \rho_j^2)^2}$$

observables defined in fundamental Hilbert space; intuition comes from discrete geometric interpretation of fundamental dofs; full continuum geometric interpretation emerges at collective, hydrodynamic level

Emergent bouncing cosmology from full QG

DO, Sindoni, Wilson-Ewing, '16

effective dynamics for volume - generalised Friedmann equations: (GFT interaction terms sub-dominant)

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j\,\rho_j\sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2$$

$$\frac{V''}{V} = \frac{2\sum_j V_j\left[E_j + 2m_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}$$

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$$\rightarrow \quad \boxed{\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j\, m_j\, \rho_j^2}{3\sum_j V_j \rho_j^2}\right)^2} \qquad \boxed{\frac{V''}{V} = \frac{4\sum_j V_j m_j^2 \rho_j^2}{\sum_j V_j \rho_j^2}}$$

approx. classical Friedmann eqns if $m_j^2 \approx 3G_N$

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approx. classical Friedmann eqns if $m_j^2 \approx 3G_N$

$\rightarrow$ $\exists j \;/\; \rho_j(\phi) \neq 0 \;\forall\phi$ $\rightarrow$

$$V = \sum_j V_j\rho_j^2$$

remains positive at all times (with single turning point)

generic quantum bounce (solving classical singularity)!

M. De Cesare, M. Sakellariadou, '16

Special case: single spin condensate

cosmological dynamics entirely due to growth (in relational time) of number of "atoms of space"

simple condensate:

$$\sigma_j(\phi) = 0, \text{ for all } j \neq j_o$$

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{V_{j_o} E_{j_o}}{9V}$$

$$\rho_c = 6\pi G\hbar^2/V_{j_o}^2 \sim (6\pi/j_o^3)\rho_{\rm Pl}$$

LQC-like modified dynamics!

DO, Sindoni, Wilson-Ewing, '16

interactions are also much simpler to study, for such simple condensates

dominance of single-spin condensate realised in several contexts:

A. Pithis, M. Sakellariadou, P. Tomov, '16

- mean field analysis of static GFT models in isotropic restriction: vacua strongly peaked on single spin
- mean field analysis of evolution (in relational time) of isotropic models: single spin dominates at late times

S. Gielen, '16 A. Pithis, M. Sakellariadou, '16

Assorted recent results in GFT cosmology

- effect of GFT interactions:

effect of fundamental GFT interactions

M. De Cesare, A. Pithis, M. Sakellariadou, ‘16

for phenomenological isotropic model with: simplified interaction kernels and only single spin component

one gets effective GFT hydro equations:

$$\partial_\phi^2 \rho - \frac{Q^2}{\rho^3} - \frac{B}{A}\rho - \frac{w}{A}\rho^{n-1} - \frac{w'}{A}\rho^{n'-1} = 0$$

$w' > 0.$

for stability of underlying GFT

compatibility with late time cosmology: $\mu \equiv -\frac{w'}{A} > 0$

$A < 0$

compatibility with free case: $m^2 \equiv \frac{B}{A} > 0$

from this, one gets effective dynamics of universe volume:

$$H^2 = \frac{8\hbar^2 Q^2}{9}\left[\frac{\varepsilon_m}{V^2} + \frac{\varepsilon_E}{V^3} + \frac{\varepsilon_Q}{V^4} + \frac{\varepsilon_\mu}{V^{3-n'/2}}\right]$$

depending from various parameters in the model

$$\varepsilon_E = V_j E,$$

$$\varepsilon_m = \frac{m^2}{2},$$

$$\varepsilon_Q = -\frac{Q^2}{2}V_j^2,$$

$$\varepsilon_\mu = -\frac{\mu}{n'}V_j^{1-n'/2}$$

quite some room for model building, but embedded in fundamental theory

Accelerated phase after bounce: QG inflation?

following the bounce, we have accelerated expansion

issue is: number of e-folds

$$N = \frac{2}{3}\log\left(\frac{\rho_{\rm end}}{\rho_{\rm bounce}}\right)$$

M. De Cesare, M. Sakellariadou, '16

M. De Cesare, A. Pithis, M. Sakellariadou, '16

- in effective cosmological dynamics neglecting GFT interactions:

$$0.119 \lesssim N \lesssim 0.186$$

acceleration is too short-lived to be physically useful

- including effects of GFT interactions (in phenomenological way):

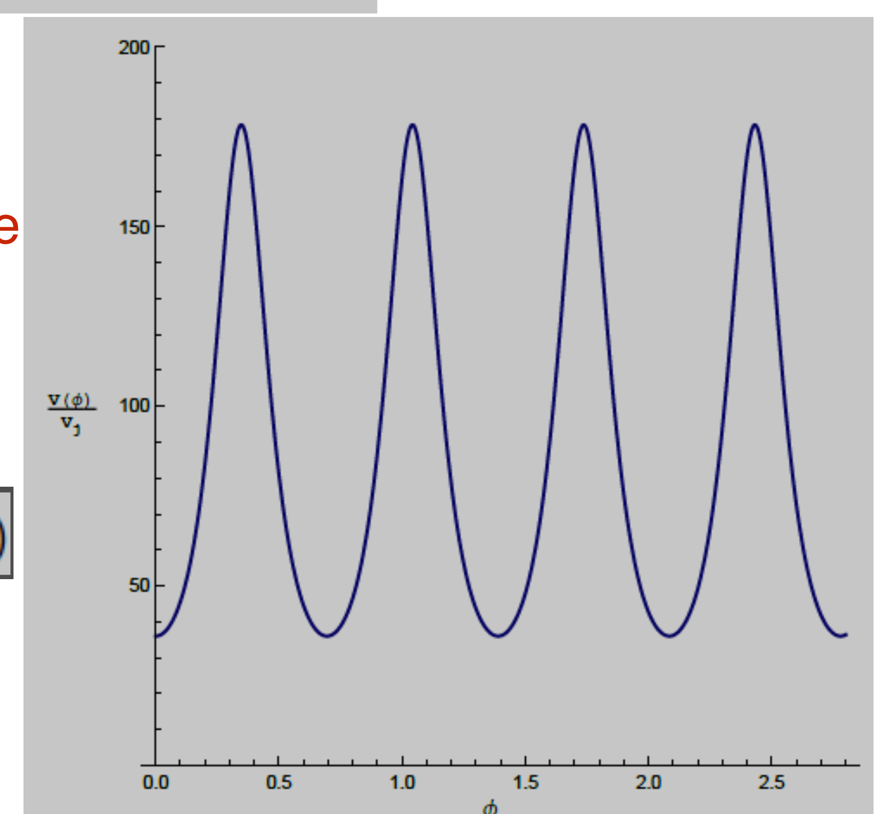
$$\sigma = \rho\, e^{i\theta}$$

$$\partial_\phi^2\rho - m^2\rho - \frac{Q^2}{\rho^3} + \lambda\rho^{n-1} + \mu\rho^{n'-1} = 0$$

one finds:

- bounce
- accelerated expansion following bounce
- decelerated phase and recollapse

$\longrightarrow$ cyclic universe



moreover:

- N at least ~ 60
- no intermediate deceleration between beginning and end of accelerated phase

$\longrightarrow$ $\boxed{\lambda < 0 \text{ and } n \geq 5 \ (n' > n)}$

QG-inflation from GFT condensates

Assorted recent results in GFT cosmology

- effect of GFT interactions

  M. De Cesare, A. Pithis, M. Sakellariadou, '16

  new cosmological terms modifying background dynamics, QG-inflation, ...

- anisotropies

  A. Pithis, M. Sakellariadou, '16

  M. De Cesare, DO, A. Pithis, M. Sakellariadou, '17

  anisotropies (using non-equilateral tetrahedra) only play role close to quantum bounce

- effective cosmological dynamics = specific limiting curvature mimetic gravity

  M. De Cesare, '18, '19

  potentially useful also for QG explanation of cosmological dark sector

- effective Friedmann from generalised hydrodynamics (beyond mean field)

  S. Gielen, A. Polaczek, '19

  effective cosmological dynamics can be derived under more general conditions from relational Hamiltonian evolution, with very similar features

- cosmological perturbations

  S. Gielen, DO, '17 S. Gielen, '18 F. Gerhardt, DO, E. Wilson-Ewing, '18

  formalism can be extended to inhomogeneities; extension of general GFTs to include matter rods; separate universe approach; scale invariant power spectrum follows in some generality, and corrections can be computed

Cosmological perturbations from full QG

S. Gielen, DO, '17

GFT for 4d gravity coupled to 4 free massless scalar fields used as clock and rods

+

isotropic reduction of geometric sector

$$\sigma(g_I, \phi^J) = \sum_{j=0}^{\infty} \sigma_j(\phi^J)\, \mathbf{D}^j(g_I)$$

GFT hydrodynamics equation for isotropic condensates (weak coupling)

$$\left(-B_j + A_j \partial^2_{\phi^0} + C_j\, \Delta_{\phi^i}\right) \sigma_j(\phi^J) = 0$$

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small perturbations around homogeneous condensate universes

$$\sigma_j(\phi^J) = \sigma^0_j(\phi^0)(1+\epsilon\,\psi_j(\phi^J))$$

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small perturbations around homogeneous condensate universes

$$\sigma_j(\phi^J) = \sigma_j^0(\phi^0)(1 + \epsilon\, \psi_j(\phi^J))$$

volume fluctuations and cosmological power spectrum (similarly for matter density fluctuations)

$$\Delta V(\phi_0, k_i; \Phi_0, K_i) \equiv \langle \hat{\tilde{V}}(\phi^0, k_i) \hat{\tilde{V}}(\Phi^0, K_i) \rangle - \langle \hat{\tilde{V}}(\phi^0, k_i) \rangle \langle \hat{\tilde{V}}(\Phi^0, K_i) \rangle$$

$$= \delta(\phi^0 - \Phi^0) \sum_j V_j^2\, |\sigma_j^0(\phi^0)|^2 \left[(2\pi)^3 \delta^3(k_i + K_i) + \epsilon\, (\tilde{\psi}_j(\phi^0, k_i + K_i) + \overline{\tilde{\psi}_j}(\phi^0, -k_i - K_i))\right]$$

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$$\sigma_j(\phi^J) = \sigma^0_j(\phi^0)(1 + \epsilon\, \psi_j(\phi^J))$$

volume fluctuations and cosmological power spectrum (similarly for matter density fluctuations)

$$\Delta V(\phi_0, k_i; \Phi_0, K_i) \equiv \langle \hat{\tilde{V}}(\phi^0, k_i) \hat{\tilde{V}}(\Phi^0, K_i) \rangle - \langle \hat{\tilde{V}}(\phi^0, k_i) \rangle \langle \hat{\tilde{V}}(\Phi^0, K_i) \rangle$$
$$= \delta(\phi^0 - \Phi^0) \sum_j V_j^2\, |\sigma^0_j(\phi^0)|^2 \left[(2\pi)^3 \delta^3(k_i + K_i) + \epsilon\, (\tilde{\psi}_j(\phi^0, k_i + K_i) + \overline{\tilde{\psi}_j}(\phi^0, -k_i - K_i))\right]$$

naturally approximate scale invariance

- dominant part (homogeneous condensate) exactly scale invariant
- deviations suppressed as universe expands and when inhomogeneities are small

Cosmological perturbations from full QG

S. Gielen, DO, '17

GFT for 4d gravity coupled to 4 free massless scalar fields used as clock and rods

+

isotropic reduction of geometric sector

$$\sigma(g_I, \phi^J) = \sum_{j=0}^{\infty} \sigma_j(\phi^J)\, \mathbf{D}^j(g_I)$$

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small relative amplitude

$$\frac{\Delta V(\phi_0, k_i; \Phi_0, K_i)}{\langle \hat{V}(\phi_0)\rangle^2}$$

- dominant term ~ 1/N ~ 1/V
- perturbations further suppressed as universe expands
- if accelerated phase, further suppression of deviations from scale invariance

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- if accelerated phase, further suppression of deviations from scale invariance

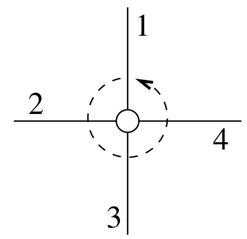
different approximation (same starting point): separate universe framework $\longrightarrow$ QG-corrected eqns for long-wavelength perturbations

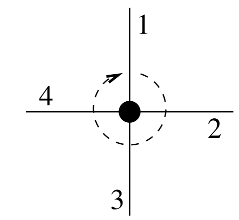
F. Gerhardt, DO, E. Wilson-Ewing, '18

GFT microscopic description of spherically symmetric geometries and horizons

GFT microstates for spherically geometries/horizons

DO, Pranzetti, Sindoni, ’15-‘18

- basic operators: $\hat{\varphi}^\dagger_W(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

$\hat{\varphi}^\dagger_B(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

colored GFT tensors, to control topology, with simplicial geometric variables, to control geometric properties

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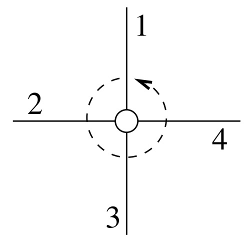
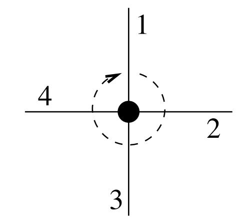
- basic operators: $\hat{\varphi}^{\dagger}_{W}(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

$\hat{\varphi}^{\dagger}_{B}(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

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- quantum homogeneity: homogeneity = vertices with the same wave function
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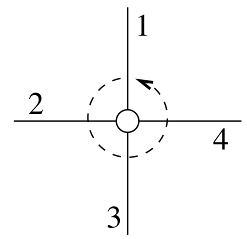
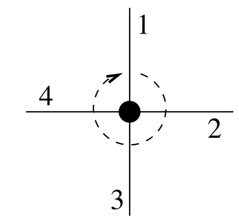
- basic operators: $\hat{\varphi}^\dagger_W(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

$\hat{\varphi}^\dagger_B(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

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Strategy:

1. Start with a (combinatorially simple) seed state for the desired topology
2. Act iteratively with a refinement operator which is topology-preserving and maintains homogeneity of the vertex wave-functions

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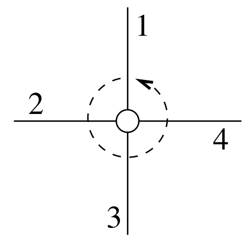
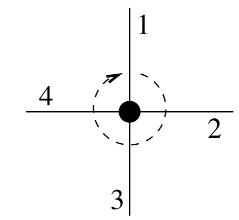
- basic operators: $\hat{\varphi}^{\dagger}_{W}(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

$\hat{\varphi}^{\dagger}_{B}(g_1, g_2, g_3, g_4)\,|0\rangle =$ 

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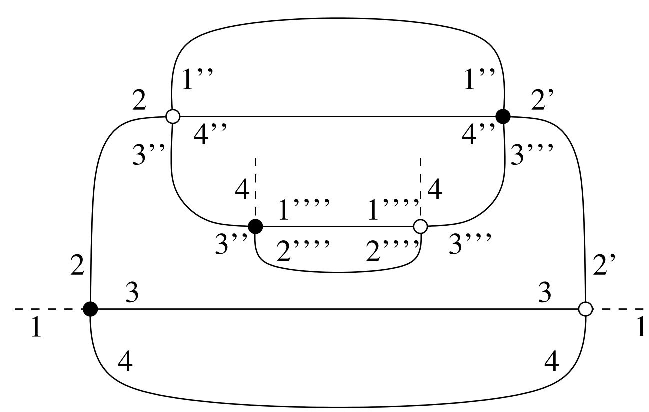
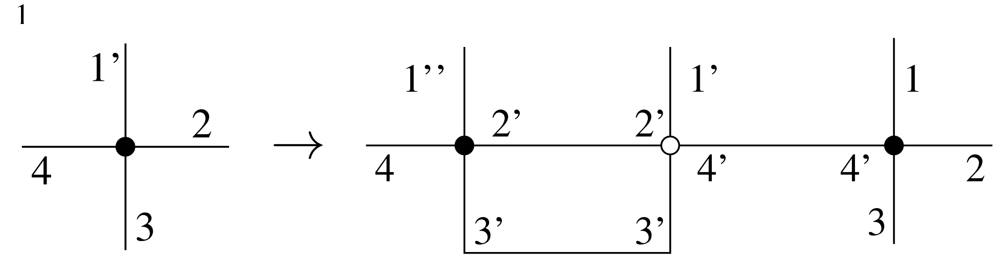
Spherical shell:

Simple seed state with two boundaries:



Refinement operator (dipole insertion):



$$\widehat{\mathcal{M}}_{r,B+} \equiv \int \mathrm{d}k_2 \mathrm{d}k_3 \mathrm{d}k_4 \mathrm{d}h'_4 \mathrm{d}h'_2 \mathrm{d}h'_3$$
$$\hat{\sigma}^{\dagger}_{r,B+}(e, h'_2, h'_3, k_4)\hat{\sigma}^{\dagger}_{r,W+}(e, h'_2, h'_3, h'_4)\hat{\sigma}^{\dagger}_{r,B+}(e, k_2, k_3, h'_4)\hat{\sigma}_{r,B+}(e, k_2, k_3, k_4)$$

GFT microstates for spherically geometries/horizons

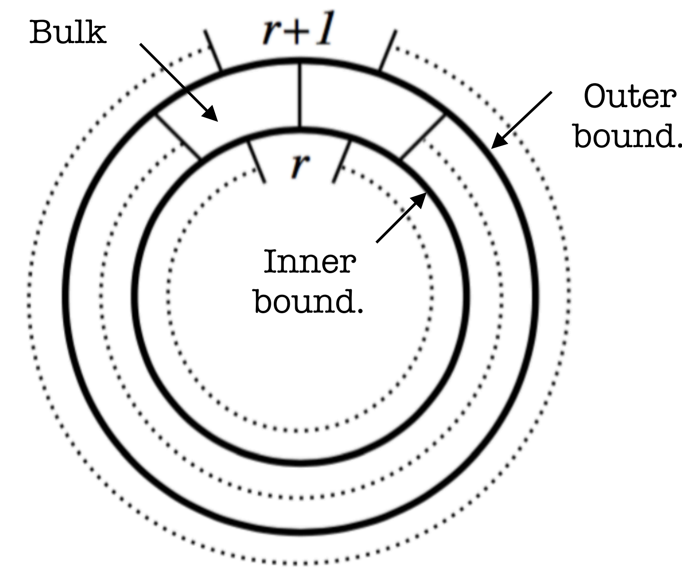
DO, Pranzetti, Sindoni, ’15-‘18

Quantum state for (approximately) continuum homogeneous spherical shell:

(for some high-order polynomial operator function F)

$$|\Psi_r\rangle = F_r(\widehat{\mathcal{M}}_{r,Bs}, \widehat{\mathcal{M}}_{r,Ws})|\tau\rangle$$

large (infinite) superpositions of arbitrarily complex spin network states



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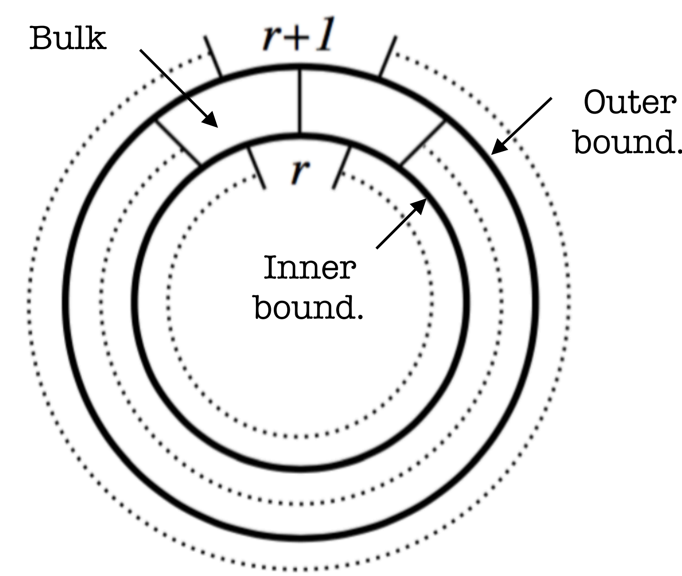

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Quantum state obtained by product of shell states, sharing boundary data, refined in a coordinated way - large (potentially infinite) superposition of spherically symmetric cellular complexes, from gluing homogeneous shells

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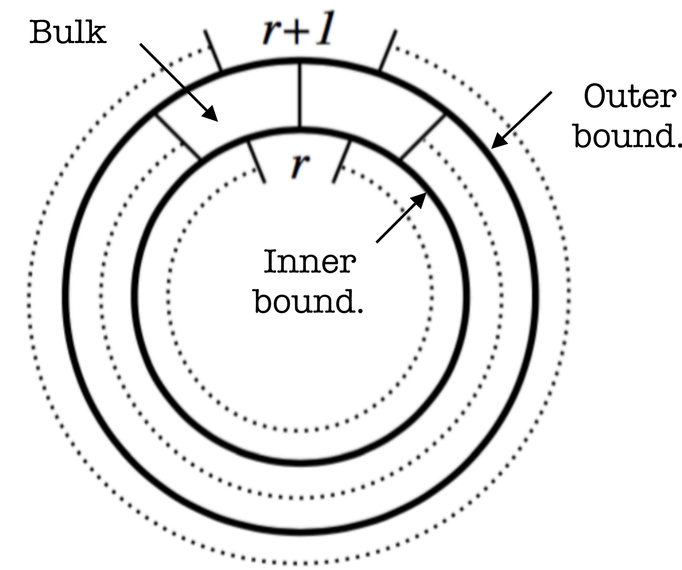

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can define and compute geometric quantities e.g. area operator (for shell r and boundary s)

$$\hat{\mathbb{A}}_{Jr,s} \equiv \sum_{t=B,W} \int (dg)^4 \hat{\sigma}^{\dagger}_{r,ts}(g_I) \sqrt{E^i_J E^j_J \delta_{ij}} \hat{\sigma}_{r,ts}(g_I) \qquad \text{where} \quad E^i_J \triangleright f(g_I) := \lim_{\epsilon \to 0} \mathrm{i} \frac{d}{d\epsilon} f(g_1, \dots, e^{-\mathrm{i}\epsilon\tau^i} g_J, \dots, g_4)$$

$$\langle \hat{\mathbb{A}}_{Jr,s} \rangle = \langle \widehat{n}_{r,s} \rangle a_{Jr,s}$$

expectation value of the area for a single dual J−link

number operator : $\widehat{n}_{r,s} = \sum_{t=B,W} \int dh_I \, \hat{\sigma}^{\dagger}_{r,ts}(h_I) \hat{\sigma}_{r,ts}(h_I)$

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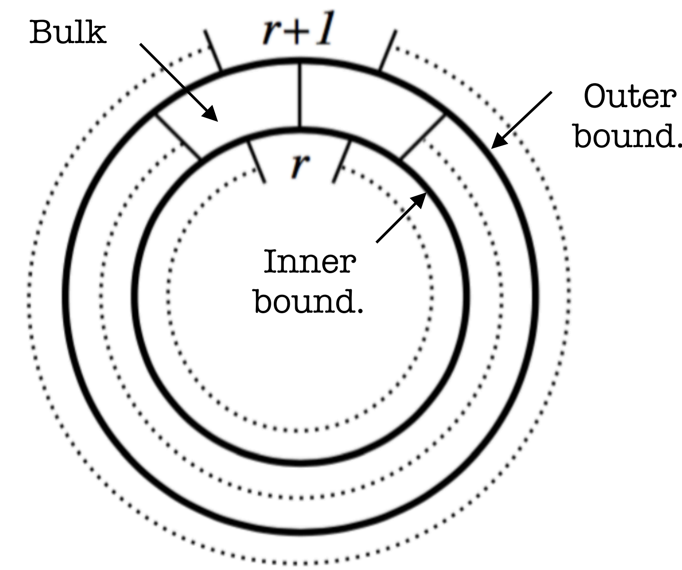

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horizon density matrix (by tracing over all other shells (inside and outside)):

$$\rho^{(n)}_{red-tot} = \frac{1}{\mathcal{N}}\sum_{s=1}^{\mathcal{N}} \rho^{(n)}_{red}(\Gamma_s)$$

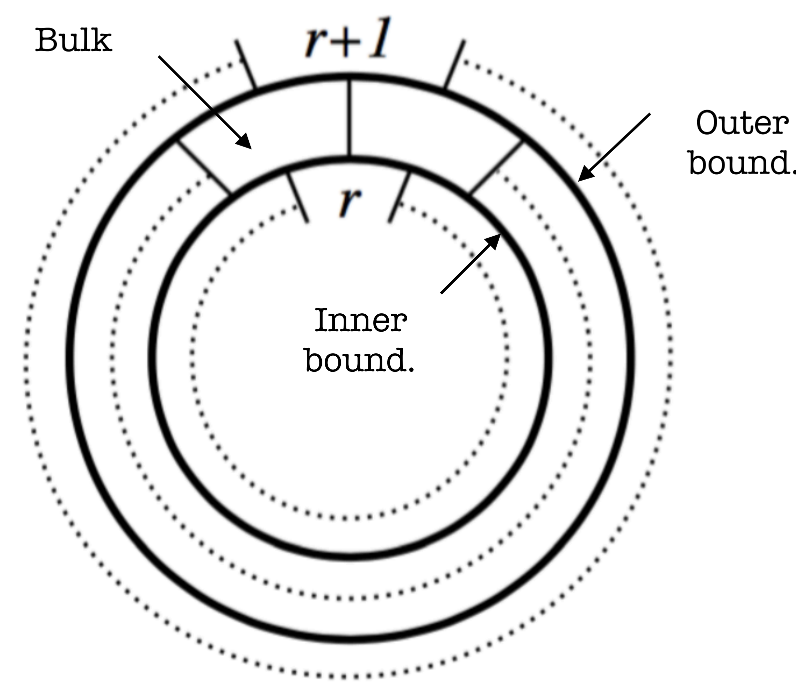
nice features of our GFT states ("weak holographic principle"):

all information about traced-out shells is lost; complete set of eigenstates can be found, labelled by total number of graphs at given number of vertices

Horizon entropy calculation

DO, Pranzetti, Sindoni, ’15-‘18

- large area, small shell volume, small fluctuations (many vertices)
- require maximal entropy ——> “proxy” for “horizon conditions”



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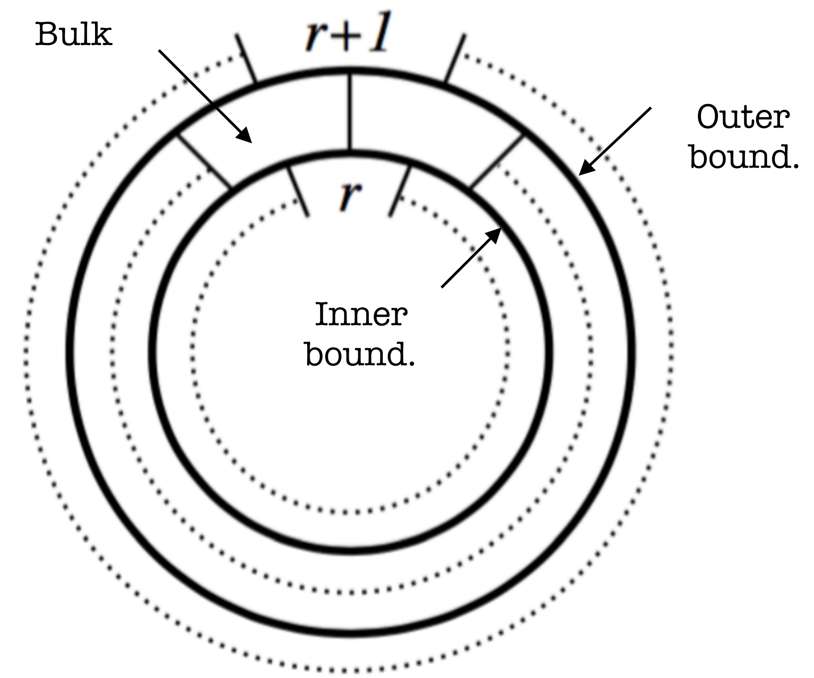

Entanglement (von Neumann) entropy = Boltzmann entropy

(counting horizon graphs; graphs can be counted explicitly)

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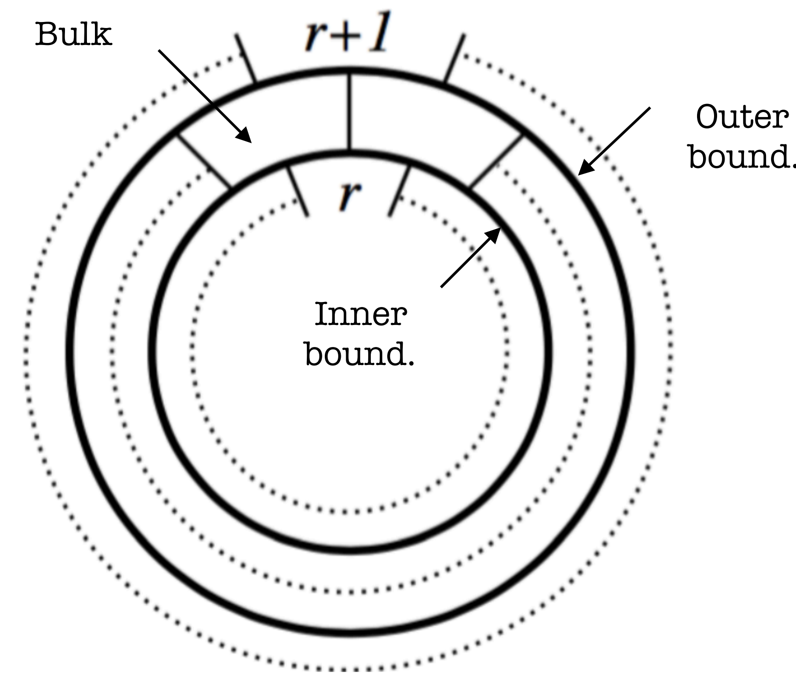

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degeneracy of the single vertex Hilbert space

number of components independently refined

$$S(n,a) = \log\left(\mathcal{N}(n)\Delta(a)\right) \approx 2\,n\,l\log(2) + \log(\Delta(a)) - \frac{l}{2}\log(n)$$

fixed (large) number n of vertices

expectation value of the area for a single radial link

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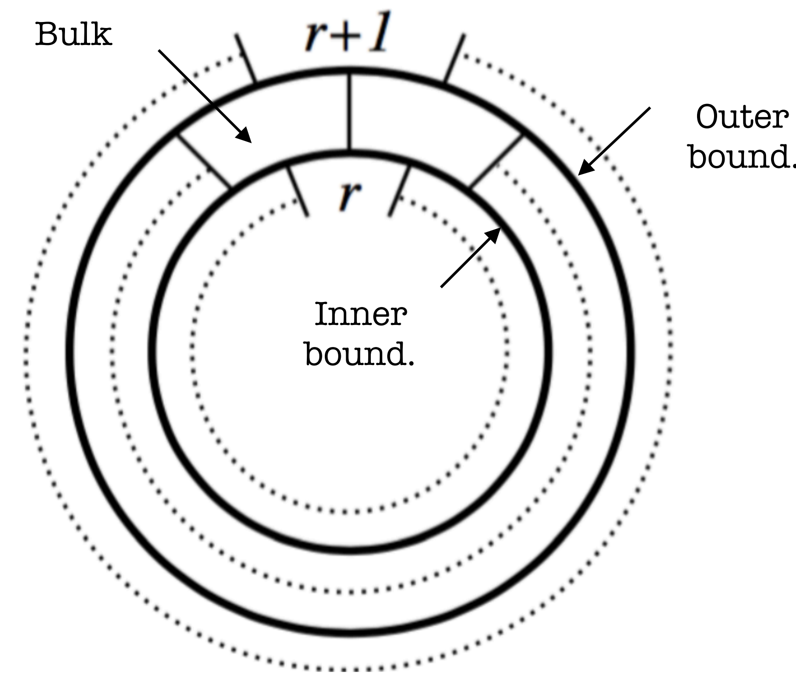

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$$\Sigma(n,a,\lambda) = S(n,a) + \lambda(\mathcal{A}_{IH}/\ell_P^2 - 2an)$$

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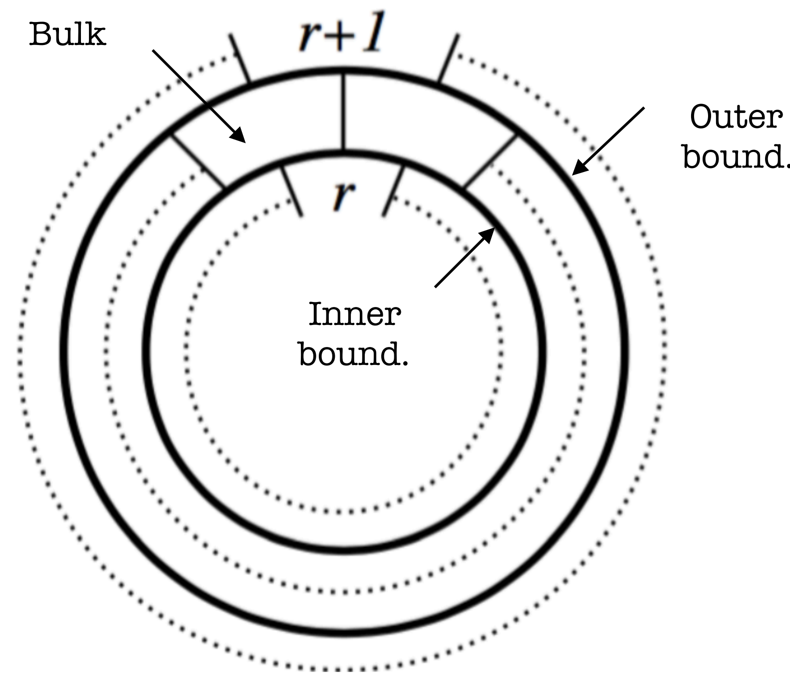

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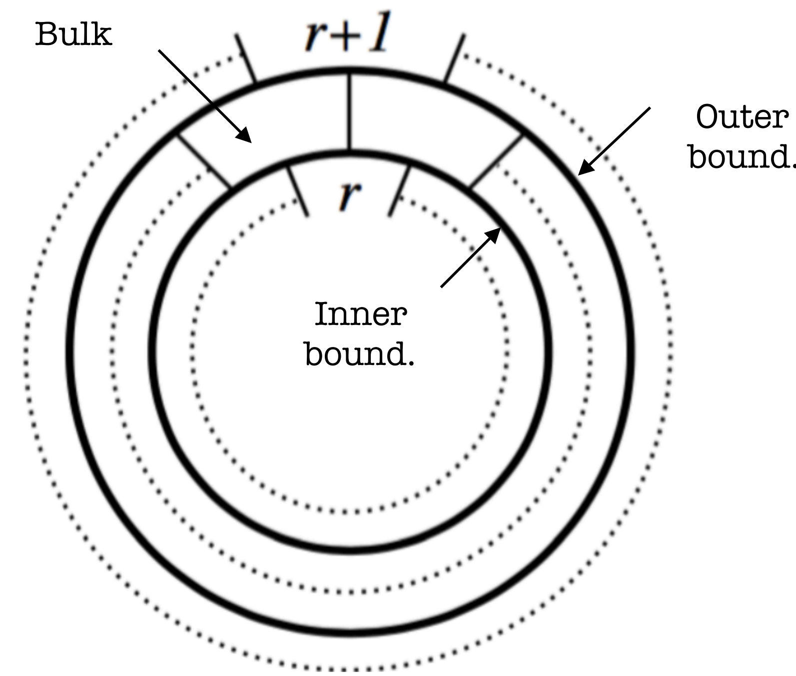
Area law
+ logarithmic corrections

Horizon entropy calculation

DO, Pranzetti, Sindoni, '15-'18



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Frodden, Ghosh, Perez, '11; Pranzetti, '13

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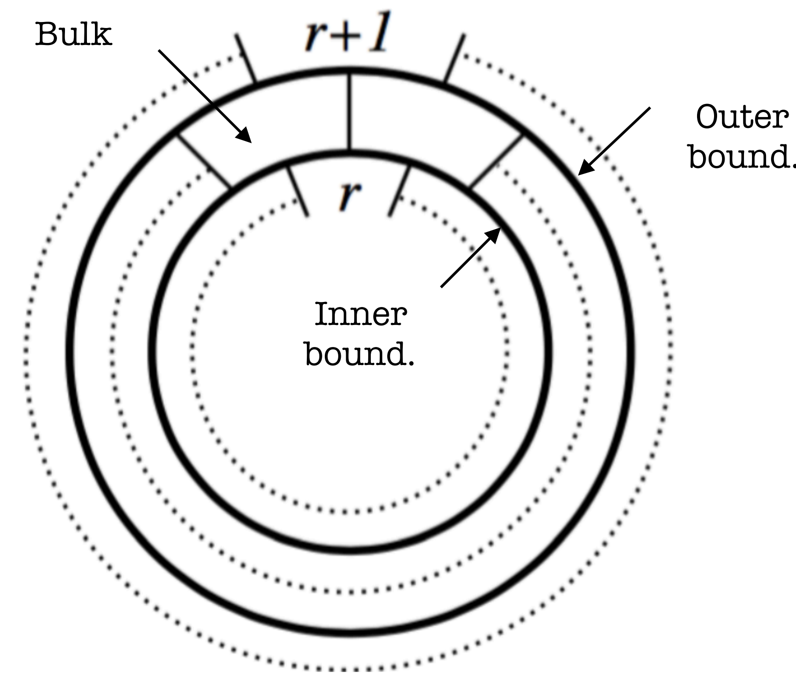
DO, Pranzetti, Sindoni, '15-'18



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Frodden, Ghosh, Perez, '11; Pranzetti, '13

Bekenstein-Hawking entropy from full QG!

no dependence on Immirzi parameter

Thank you for your attention