# Complex Regge Action: Causality Violations and Applications to Quantum Cosmology

José J. Padua-Argüelles[1,a]
jpaduaarguelles@perimeterinstitute.ca

[1]Perimeter Institute for Theoretical Physics, Canada

March 8º, 2022

[a]Work with Bianca Dittrich and Seth Asante: 2112.15387 Collaboration with Ding Jia.

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## Highly oscillatory path integrals and failure of Monte-Carlo

- Holomorphic gradient flow

(Alexandru, Basar et al. 2020) (Jia 2021)

- Lefschetz thimbles

(Feldbrugge, Lehners, Turok 2017) (Han, Huang, et al. 2020)

## Spatial topology change

(Louko Sorkin, 1995)

## Analytical continuation of spin foams and signature mixing

(Han, Liu 2021)

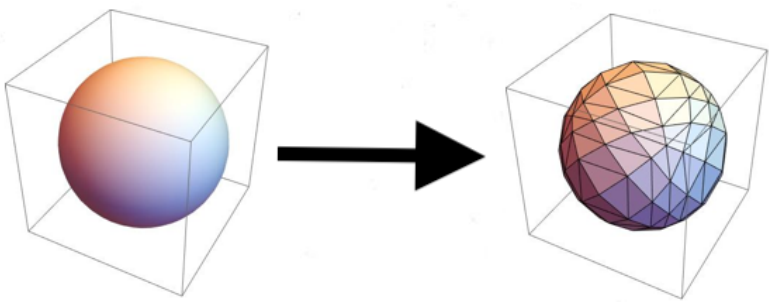
## Path integral over complex metrics

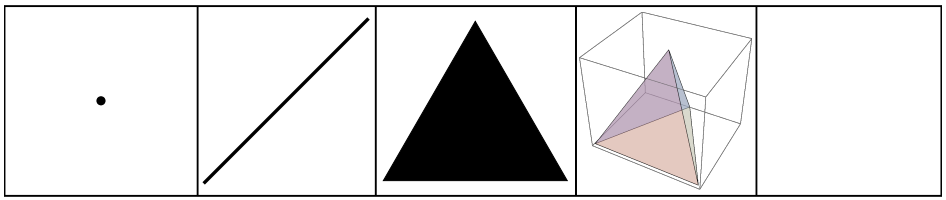
(Witten 2021) (Jonas, Lehners, Meyer, 2021)

# Regge calculus

## Discretize GR

We truncate degrees of freedom by triangulating regions of spacetime with simplices





Metric's information is encoded in segments' lengths.

## Pseudo-ansatz

$$Z_{\triangle} \sim \int\!\!\!\!\!\!\diagup \, d\mu e^{W_{\triangle}}$$

# Length Regge action

Bone: Sub-simplex of co-dimension 2.

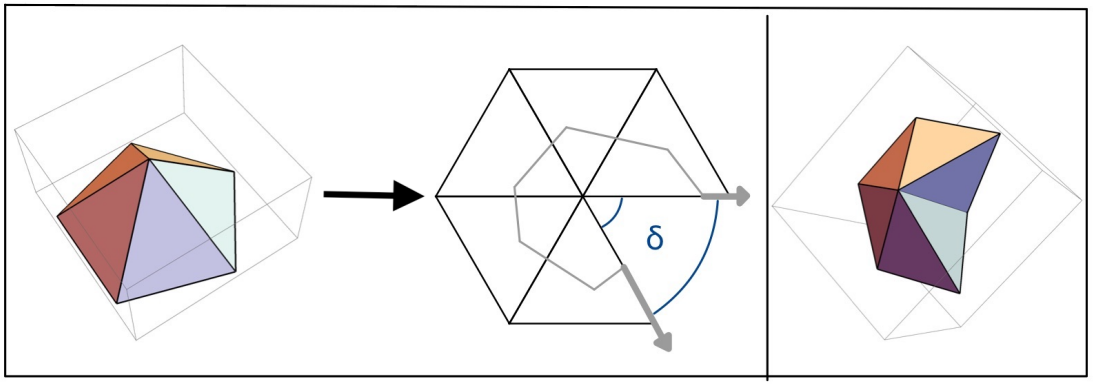
## Regge Action

Curvature as deficit angle about bones.

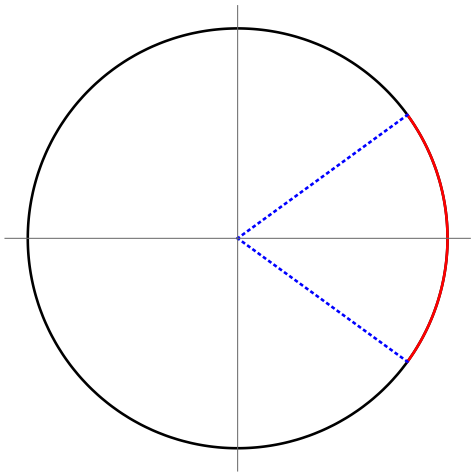
$$S[g]_{EH} = \int_{\mathcal{M}} d^4x\sqrt{-g}R + \text{Bdry}$$

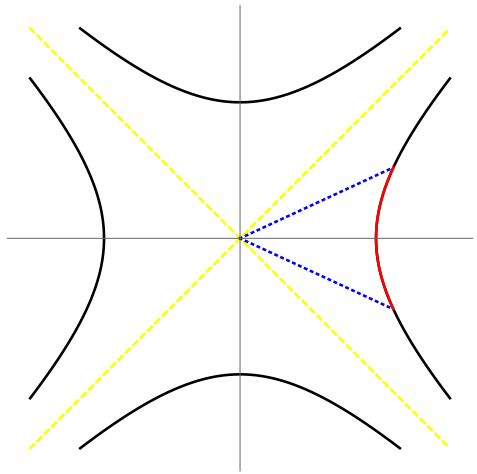
$$\downarrow$$

$$S[\text{lengths}]_R = \sum_{b\in\text{Triangles}} \text{Area}(b)\delta\phi(b) + \text{Bdry}$$



# Lorentzian angles





- $\theta_{I,I} = \operatorname{arccosh}\left(\frac{a\cdot b}{|a||b|}\right) = \log\frac{\mathcal{Z}(a,b)}{|a||b|}$, with
  $\mathcal{Z}(a,b) := a\cdot b + \sqrt{(a\cdot b)^2 - (a\cdot a)(b\cdot b)}.$
  - $\theta_{I,II} := \log\frac{\mathcal{Z}(a,b)}{|a||b|} = \log\frac{\mathcal{Z}(a,b)}{||a||||b||} \mp \frac{i\pi}{2}.$

The Minkowski plane spans a Minkowskian angle of $\mp 2\pi\imath$

(Sorkin, 2019).

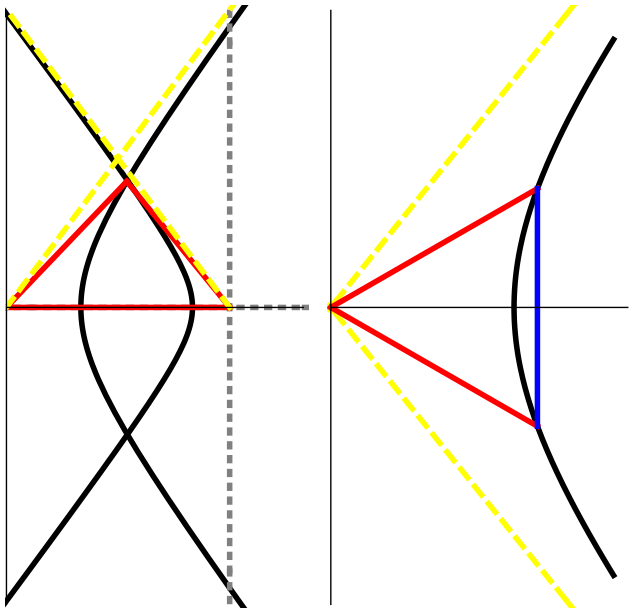
# Hinge-causality violations

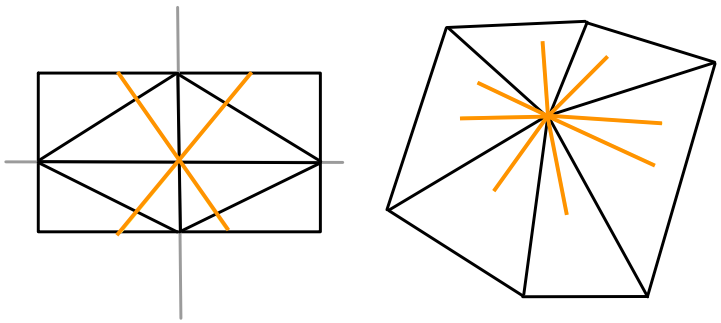
## Deficit angle about a space-like bone

$$\epsilon_t = -2\pi\imath - \sum_{\sigma \supset t} \theta_{t \subset \sigma}$$

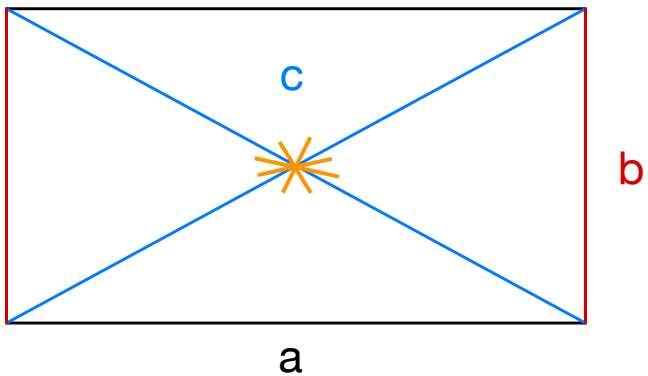
## Complex action

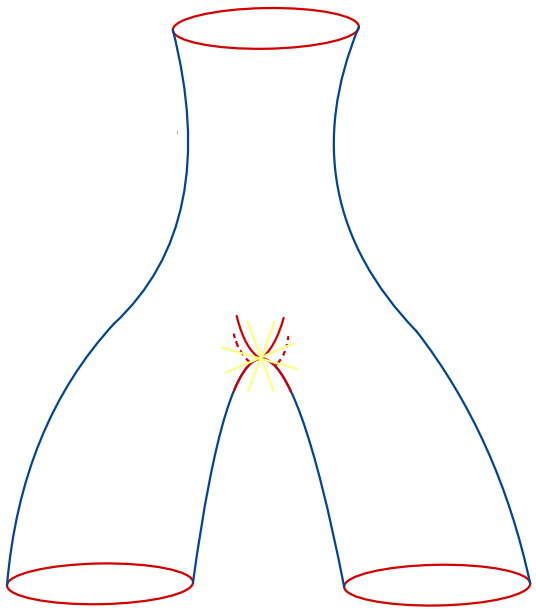
A bone might have more or less than two light-cones attached (irregular light-cone structure).





# Trouser-like violation

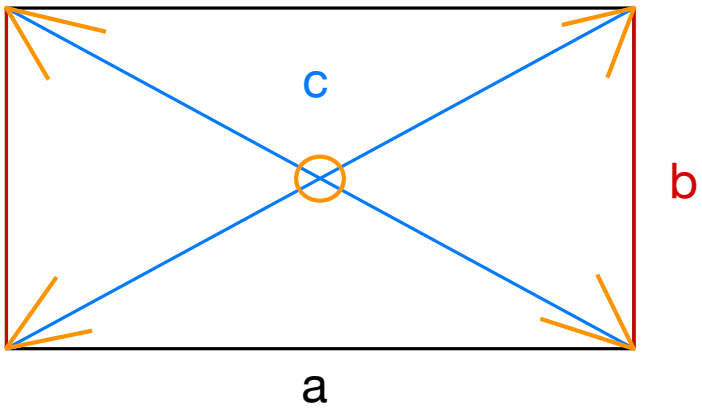


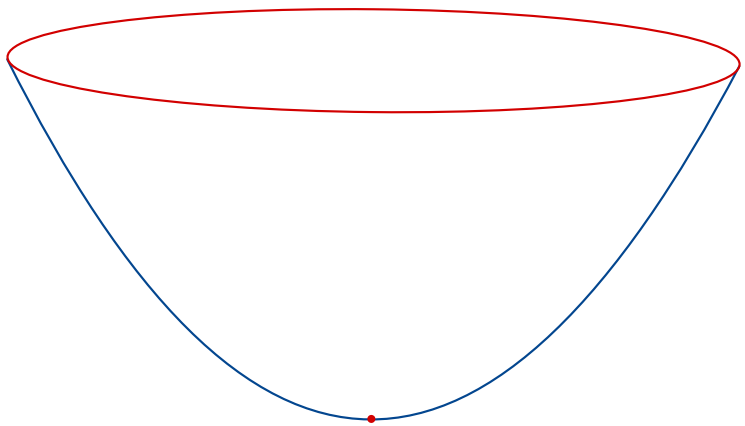



- All edges are space-like
- $a, b > 2c$

The center bone has four light cones ‘attached’!

# Yarmulke-like violation



$c-$edges are time-like

The center bone has zero light cones ‘attached’!

# Spatial topology change

- Frozen topology framework leads to inconsistencies?
- Singularities of matter fields at the crotch of a trouser spacetime.

**Louko-Sorkin**

Suppress trouser-like configurations

(Dowker, Surya 1998)

## Complex metric

$$(\vec{u},\vec{v}) \in \mathbb{C}^2 \times \mathbb{C}^2, \quad \vec{u} \star \vec{v} := u_0 v_0 + u_1 v_1$$

## Generalized Wick rotation

$$(\vec{u},\vec{v}) \in \mathbb{R}^2 \times \mathbb{R}^2, \quad \vec{u} \star \vec{v} := e^{\imath\phi} u_0 v_0 + u_1 v_1$$

$$\theta^{\pm} = -\imath \log_{\mp} \left( \frac{a \star b + \imath \sqrt{\mp((a \star a)(b \star b) - (a \star b)^2)}}{\sqrt{\pm a \star a}\sqrt{\pm b \star b}} \right)$$

## Complex length variables

$$\theta^{\pm}(s_a, s_b, s_c) = -\imath \log_{\mp} \frac{\frac{1}{2}(s_a + s_b - s_c) + 2\sqrt{\mp - \mathbb{A}(s_a, s_b, s_c)}}{\sqrt{\pm s_a}\sqrt{\pm s_b}}$$

For $\phi \in (0, \pi)$,

$$\lim_{\phi \to 0\downarrow} \theta^+ = -\psi_E \quad \text{and} \quad \lim_{\phi \to \pi\uparrow} \theta^+ = -\imath\psi_{L+}. \tag{1}$$

For $\phi \in (-\pi, 0)$,

$$\lim_{\phi \to 0\uparrow} \theta^- = +\psi_E \quad \text{and} \quad \lim_{\phi \to -\pi\downarrow} \theta^- = -\imath\psi_{L-}. \tag{2}$$

$$\ell_{\mathrm{P}}^2\, W^{\pm} = \sum_t \sqrt{\pm \mathbb{V}_t}\,(2\pi \pm \sum_{\sigma \supset t} \theta^{\pm}_{\sigma,t}) - \Lambda \sum_{\sigma} \sqrt{\pm \mathbb{V}_\sigma}. \tag{3}$$

For causally regular configurations

$$\hbar\, W^{+} \;=\; \hbar\, W^{-} = \begin{cases} -iS^{L-}, & \phi = -\pi; \\ -S^{E}, & \phi = 0; \\ +iS^{L+}, & \phi = \pi; \\ +S^{E}, & \phi = 2\pi. \end{cases} \tag{4}$$

Irregular light-cone structure leads to $S^{L-} \neq S^{L+}$.

# Summary

- The Lorentzian Regge action has a sign ambiguity associated to wedges containing light-rays.
- When complexifying the ambiguity is associated to branch-cuts.
- This ambiguity can introduce an imaginary part to the action for irregular light-cone structures. Thus, this gives (exponential) enhancements/suppressions.
- Irregular light-cone structures can be associated with spatial topology change.

Should causally irregular configurations/topology changing processes contribute in the path integral?

# The no-boundary proposal I

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2(\mathrm{d}\chi^2 + \sin(\chi)^2(\mathrm{d}\theta^2 + \sin(\theta)^2\mathrm{d}\phi^2)$$

**Mini super-space path integral leads to**

$$G(a_0, a_1) = \sqrt{\frac{3\pi\imath}{2}} \int_0^\infty \frac{\mathrm{d}\mathcal{N}}{\sqrt{\mathcal{N}}} e^{2\pi^2\imath S_0},$$
$$S_0 = \mathcal{N}^3 \frac{\Lambda^2}{36} + \mathcal{N}\left(-\frac{\Lambda}{2}(a_0^2 + a_1^2) + 3k\right) + \frac{1}{\mathcal{N}}\left(-\frac{3}{4}(a_1^2 - a_0^2)^2\right).$$

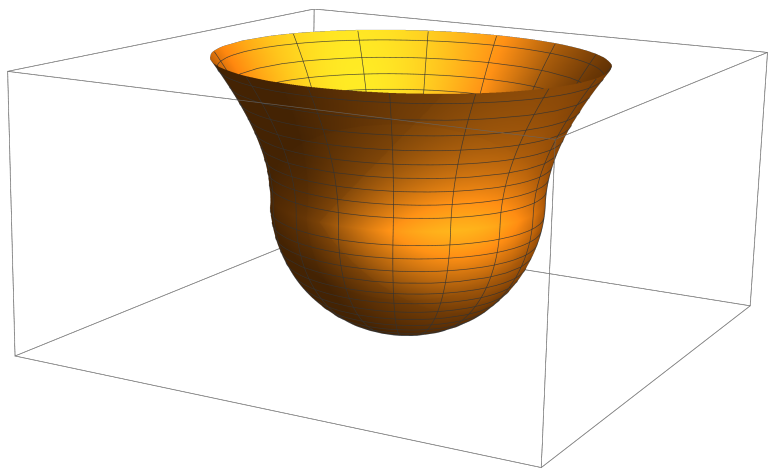
(Feldbrugge, Lehners, Turok, 2017)

# The no-boundary proposal II

## Stationary points

$$\mathcal{N} = \frac{3}{\Lambda}\left(\pm\sqrt{\frac{\Lambda}{3}a_0^2 - 1} \pm \sqrt{\frac{\Lambda}{3}a_1^2 - 1}\right), \quad a_\Lambda := \sqrt{\frac{3}{\Lambda}}$$

- Euclidean-Euclidean
- Euclidean-Lorentzian
- Lorentzian-Lorentzian

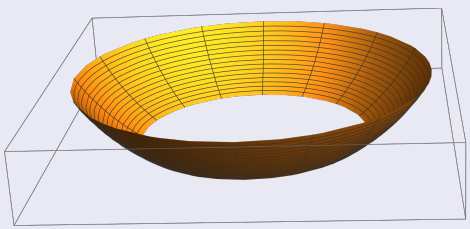


(Dittrich, Gielen, Schander, 2021)

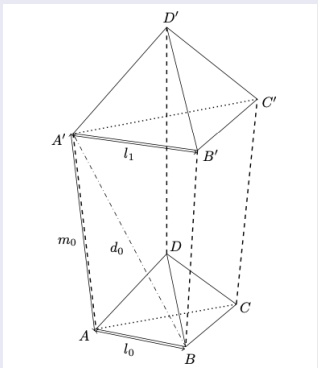
# Simplicial model

## Spherical shells

Glue triangulations of the 3-sphere

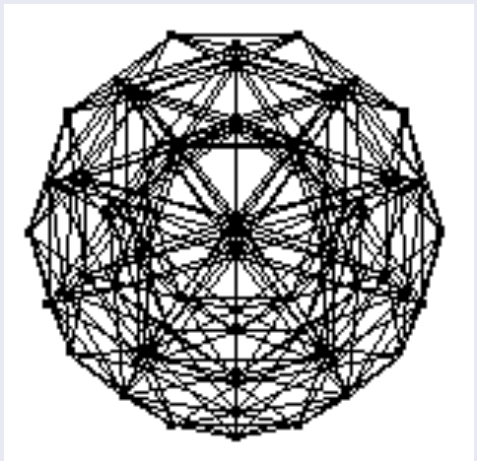


## Building blocks




## Triangulation of $S^3$

Boundaries of 4-dimensional convex polytopes which are triangulations, e.g. 600-cell.



WolframWorld

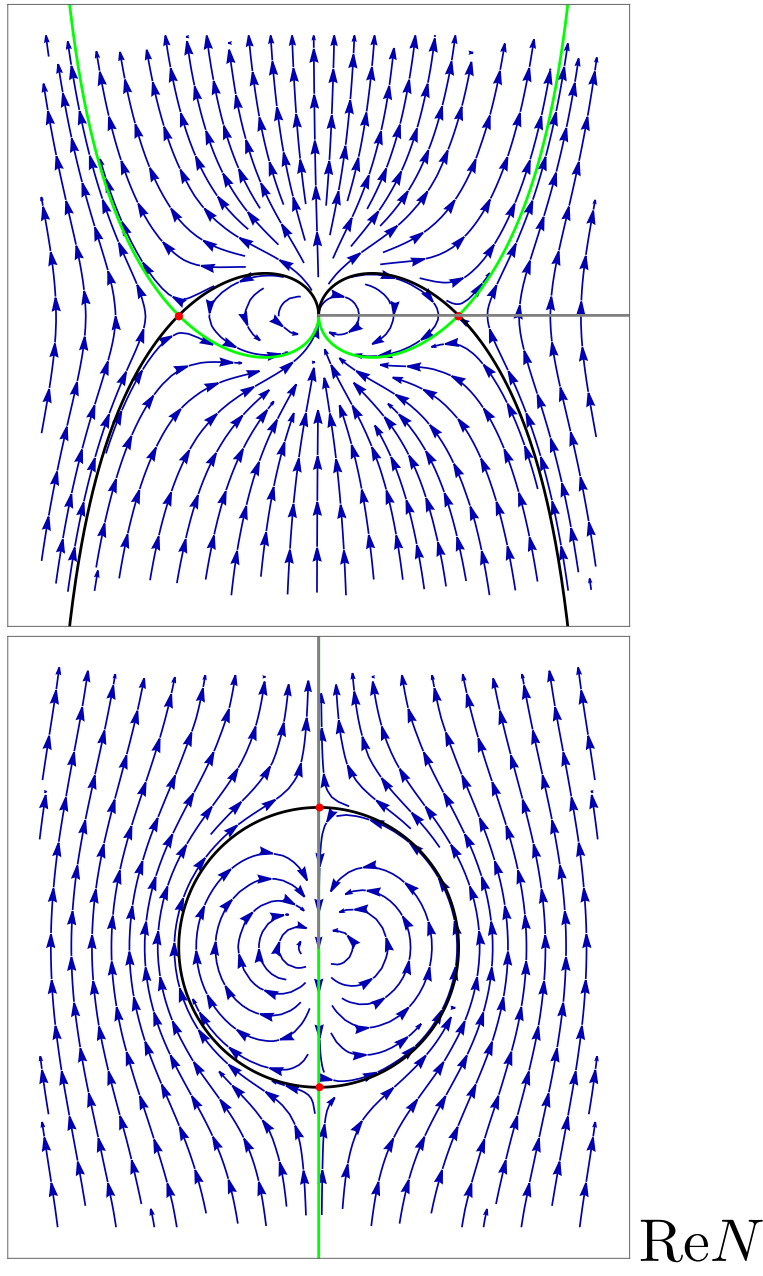
(Dittrich, Gielen, Schander, 2021)

Express the integration contour as a combination of Lefschetz thimbles:

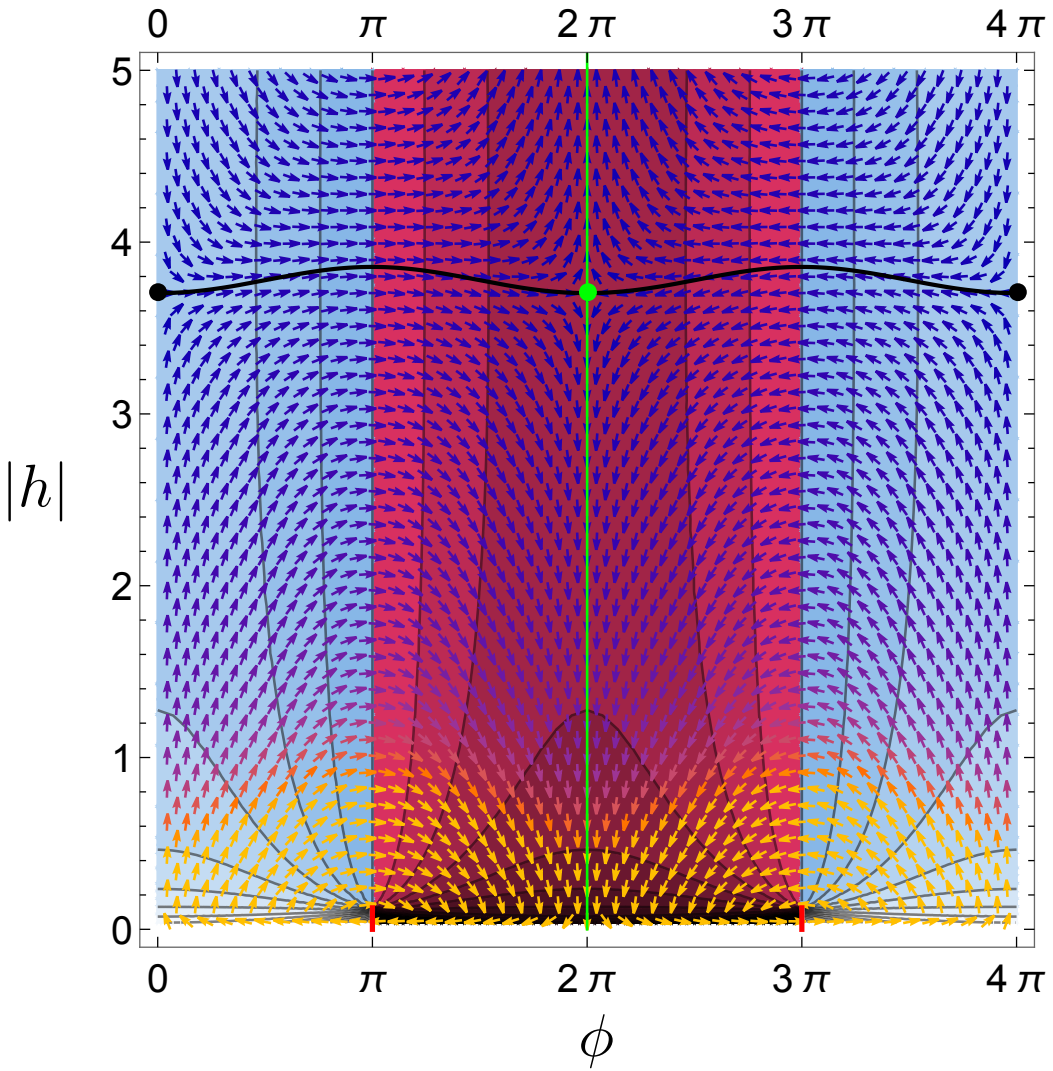
$$-\nabla \mathrm{Re}\left[W(z =: x + iy)\right] = \frac{\mathrm{d}z}{\mathrm{d}t},$$

$$\lim_{t \to -\infty} = z_*,$$

- Constant imaginary part
- Steepest descent for real part



# Flow and thimbles

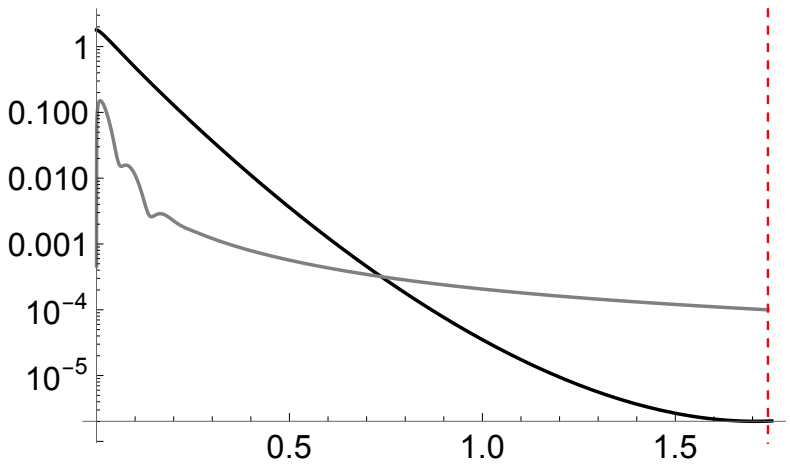


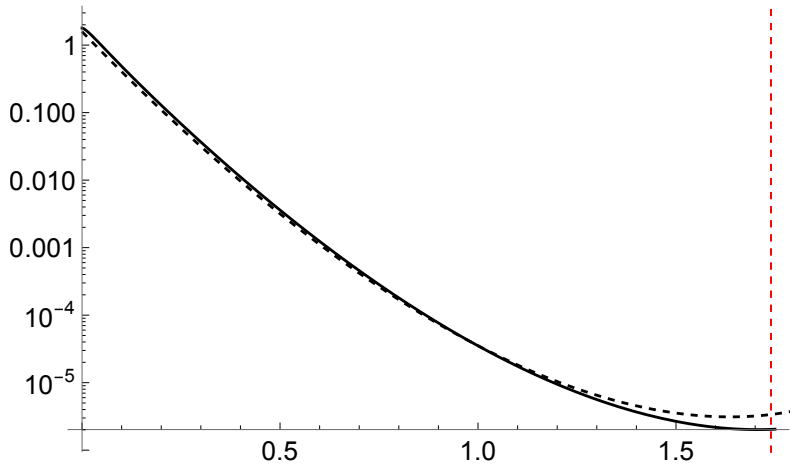

**Agreement with the continuum**

Thimble contributing corresponds to the Vilenkin euclidean branch (not Hartle-Hawking!).

# Path integral results

$Z_i$

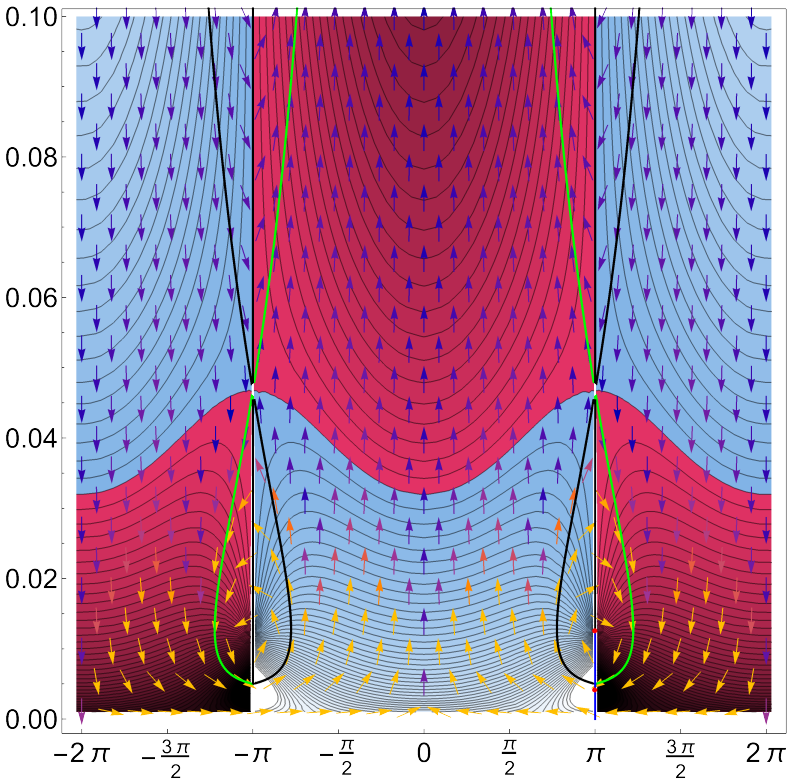


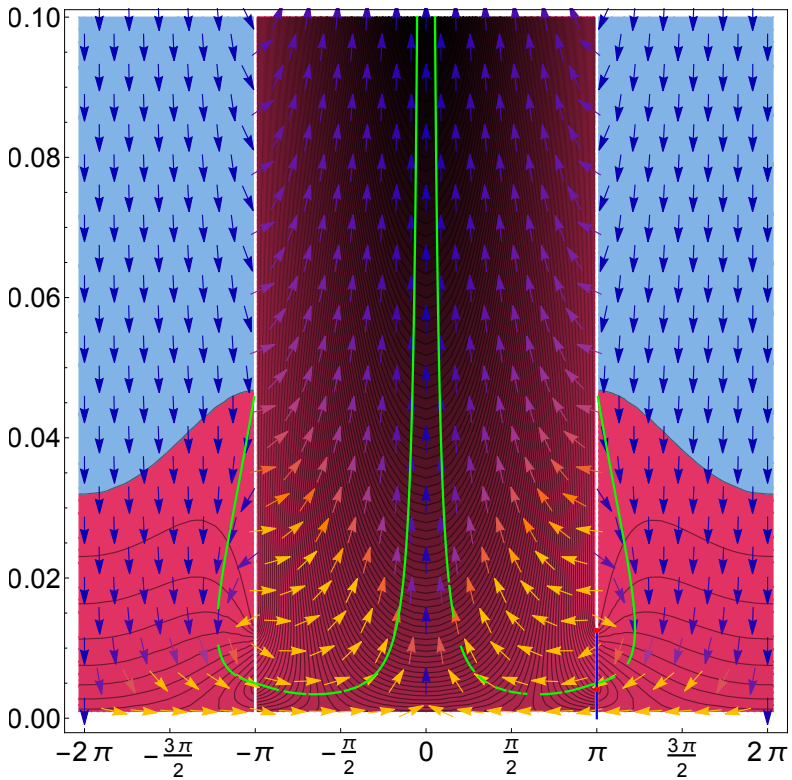




$s_l \sim a_1^2$

Exclusion of the irregular region seems to lead to worse results!

# Flow and thimbles (Lorentzian-Lorentzian)




Thimble crosses the branch-cut! Here we get the 'Hartle-Hawking sign'.

## Complex Regge action

- Lefschetz thimbles
- Holomorphic gradient flow
- Unified framework for mixing signatures

Understand infinite integration ranges (spikes?)

## Hinge causality violations and spatial topology change

Mechanism that produces suppressing contributions?⇒Numerical efficiency!

Which configurations should be summed over in Lorentzian path integrals?

Application to (effective) spin foams?