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Complex Regge Action: Causality Violations and Applications to Quantum Cosmology

José J. Padua-Argüelles^{1,a} jpaduaarguelles@perimeterinstitute.ca

¹Perimeter Institute for Theoretical Physics, Canada

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^aWork with Bianca Dittrich and Seth Asante: 2112.15387 Collaboration with Ding Jia.

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- Picard-Lefschetz theory
- Ball model
- Shell model
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Why comple	ex actions?			

Highly oscillatory path integrals and failure of Monte-Carlo

• Holomorphic gradient flow

(Alexandru, Basar et al. 2020) (Jia 2021)

• Lefschetz thimbles

(Feldbrugge, Lehners, Turok 2017) (Han, Huang, et al. 2020)

Spatial topology change

(Louko Sorkin, 1995)

Analytical continuation of spin foams and signature mixing

(Han, Liu 2021)

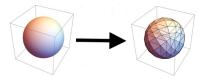
Path integral over complex metrics

(Witten 2021) (Jonas, Lehners, Meyer, 2021)

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Regge action				
Regge	calculus			

Discretize GR

We truncate degrees of freedom by <u>triangulating</u> regions of spacetime with simplices





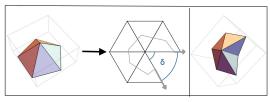
Metric's information is encoded in segments' lengths.

Pseudo- <u>ansatz</u>	
$Z_{ riangle} \sim \oint d\mu e^{W_{ riangle}}$	
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Regge action				
Length	Regge action			

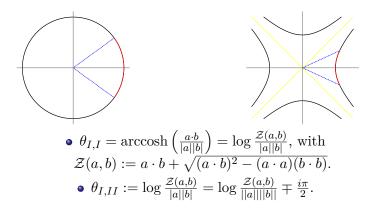
Bone: Sub-simplex of co-dimension 2.

Regge Action Curvature as deficit angle about bones. $S[g]_{EH} = \int_{\mathcal{M}} d^4x \sqrt{-g}R + Bdry$ \downarrow $S[lengths]_R = \sum_{b \in \text{Triangles}} Area(b)\delta\phi(b) + Bdry$



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Lorentzian	angles			
Lorent	tzian angles			



The Minkowski plane spans a Minkowskian angle of $\mp 2\pi i$ (Sorkin, 2019).

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Hinge-causa	lity violations and spatial topo	logy change.		

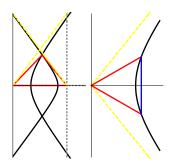
Hinge-causality violations

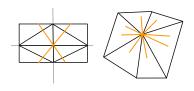
Deficit angle about a space-like bone

$$\epsilon_t = -2\pi i - \sum_{\sigma \supset t} \theta_{t \subset \sigma}$$

Complex action

A bone might have more or less than two light-cones attached (irregular light-cone structure).

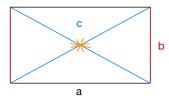


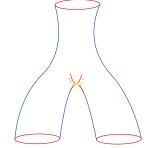


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Hinge-causa	lity violations and spatial topo	ology change.		
Trouse	er-like violation			



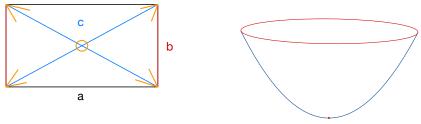


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- All edges are space-like
- a, b > 2c

The center bone has four light cones 'attached'!

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Yarmu	ılke-like violatio	a		



c-edges are time-like

The center bone has zero light cones 'attached'!

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Hinge-causa	lity violations and spatial topo	logy change.		
Spatia	l topology chang	ge		

- Frozen topology framework leads to inconsistencies?
- Singularities of matter fields at the crotch of a trouser spacetime.

Louko-Sorkin

Suppress trouser-like configurations

(Dowker, Surya 1998)

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Complex metric

$$(\vec{u}, \vec{v}) \in \mathbb{C}^2 \times \mathbb{C}^2, \quad \vec{u} \star \vec{v} := u_0 v_0 + u_1 v_1$$

Generalized Wick rotation

$$(\vec{u}, \vec{v}) \in \mathbb{R}^2 \times \mathbb{R}^2, \quad \vec{u} \star \vec{v} := e^{i\phi} u_0 v_0 + u_1 v_1$$

$$\theta^{\pm} = -\imath \log_{\mp} \left(\frac{a \star b + \imath \sqrt{\mp} \left((a \star a)(b \star b) - (a \star b)^2 \right)}{\sqrt{\pm} a \star a} \sqrt{\frac{\pm}{2} b \star b} \right)^2$$

Complex length variables

$$\theta^{\pm}(s_a, s_b, s_c) = -i \log_{\mp} \frac{\frac{1}{2}(s_a + s_b - s_c) + 2\sqrt{\pi} - \mathbb{A}(s_a, s_b, s_c)}{\sqrt{\pm} s_a \sqrt{\pm} s_b}$$

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For
$$\phi \in (0, \pi)$$
,

$$\lim_{\phi \to 0\downarrow} \theta^+ = -\psi_E \quad \text{and} \quad \lim_{\phi \to \pi\uparrow} \theta^+ = -i\psi_{L+}. \quad (1)$$
For $\phi \in (-\pi, 0)$,

$$\lim_{\phi \to 0\uparrow} \theta^- = +\psi_E \quad \text{and} \quad \lim_{\phi \to -\pi\downarrow} \theta^- = -i\psi_{L-}. \quad (2)$$

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Complex Re	gge action			

$$\ell_{\rm P}^2 W^{\pm} = \sum_t \sqrt{{}_{\pm} \mathbb{V}_t} \left(2\pi \pm \sum_{\sigma \supset t} \theta_{\sigma,t}^{\pm} \right) - \Lambda \sum_{\sigma} \sqrt{{}_{\pm} \mathbb{V}_{\sigma}}.$$
 (3)

For causally regular configurations

$$\hbar W^{+} = \hbar W^{-} = \begin{cases} -iS^{L-}, & \phi = -\pi; \\ -S^{E}, & \phi = 0; \\ +iS^{L+}, & \phi = \pi; \\ +S^{E}, & \phi = 2\pi. \end{cases}$$
(4)

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Irregular light-cone structure leads to $S^{L_{-}} \neq S^{L_{+}}$.

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Complex Re	egge action			
Summ	lary			

- The Lorentzian Regge action has a sign ambiguity associated to wedges containing light-rays.
- When complexifying the ambiguity is associated to branch-cuts.
- This ambiguity can introduce an imaginary part to the action for irregular light-cone structures. Thus, this gives (exponential) enhancements/suppressions.
- Irregular light-cone structures can be associated with spatial topology change.

Should causally irregular configurations/topology changing processes contribute in the path integral?

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$$ds^{2} = -N(t)^{2}dt^{2} + a(t)^{2}(d\chi^{2} + \sin(\chi)^{2}(d\theta^{2} + \sin(\theta)^{2}d\phi^{2})$$

Mini super-space path integral leads to

$$\begin{aligned} G(a_0, a_1) &= \sqrt{\frac{3\pi i}{2}} \int_0^\infty \frac{\mathrm{d}\mathcal{N}}{\sqrt{\mathcal{N}}} e^{2\pi^2 i S_0}, \\ S_0 &= \mathcal{N}^3 \frac{\Lambda^2}{36} + \mathcal{N} \left(-\frac{\Lambda}{2} (a_0^2 + a_1^2) + 3k \right) + \frac{1}{\mathcal{N}} \left(-\frac{3}{4} (a_1^2 - a_0^2)^2 \right). \end{aligned}$$

(Feldbrugge, Lehners, Turok, 2017)

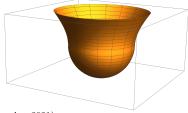
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Stationary points

$$\mathcal{N} = \frac{3}{\Lambda} \left(\pm \sqrt{\frac{\Lambda}{3}a_0^2 - 1} \pm \sqrt{\frac{\Lambda}{3}a_1^2 - 1} \right), \quad a_\Lambda := \sqrt{\frac{3}{\Lambda}}$$

- Euclidean-Euclidean
- Euclidean-Lorentzian
- Lorentzian-Lorentzian



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(Dittrich, Gielen, Schander, 2021)

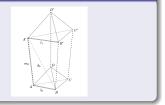
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Simpli	icial model			

Spherical shells

Glue triangulations of the 3-sphere

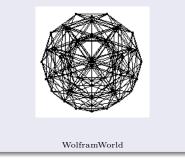


Building blocks



Triangulation of S^3

Boundaries of 4-dimensional convex polytopes which are triangulations, e.g. 600-cell.



(Dittrich, Gielen, Schander, 2021)

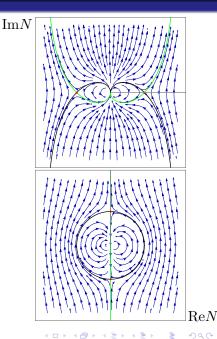
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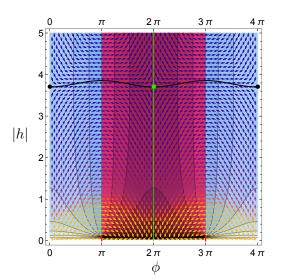
Express the integration contour as a combination of <u>Lefschetz</u> thimbles:

$$-\nabla \operatorname{Re}\left[W(z =: x + iy)\right] = \frac{\mathrm{d}z}{\mathrm{d}t},$$
$$\lim_{t \to -\infty} = z_*,$$

- Constant imaginary part
- Steepest descent for real part



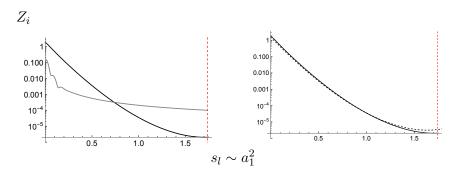
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Ball model				
Flow a	and thimbles			





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Ball model				
Path i	ntegral results			

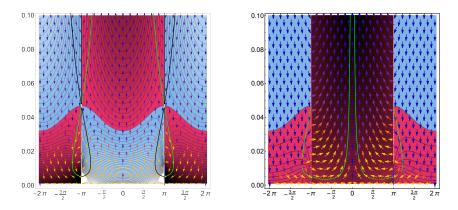


Exclusion of the irregular region seems to lead to worse results!

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Shell model				

Flow and thimbles (Lorentzian-Lorentzian)



Thimble crosses the branch-cut! Here we get the 'Hartle-Hawking sign'.

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Complex Regge action

- Lefschetz thimbles
- Holomorphic gradient flow
- Unified framework for mixing signatures

Understand infinite integration ranges (spikes?)

Hinge causality violations and spatial topology change

Mechanism that produces suppressing contributions? \Rightarrow Numerical efficiency!

Which configurations should be summed over in Lorentzian path integrals?

Application to (effective) spin foams?

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