Big bounce and inhomogeneities: a hybrid approach.

ILQG SEMINAR

16.02.2010

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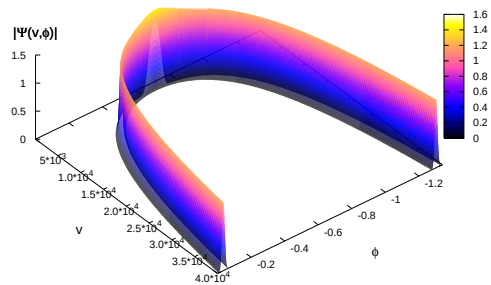
arXiv:0902.0697

Motivation

- Loop quantization of the isotropic/homogeneous cosmological models $\Rightarrow$ changes of the dynamics at near-Planck densities causing the Big Bounce.



- Robustness of the picture:
  - Does it survive in presence of the inhomogeneities?
  - If yes: how the bounce affects the structure of the inhomogeneities?
- Very preliminary answer in context of the Gowdy T^3 model with linearly polarized gravitational waves.

Outline

- The model specification:
  - Definition, partial gauge fixing
  - Degrees of freedom
- The hybrid quantization:
  - Loop quantization of the homogeneous “background”
  - Fock quantization of the “gravitational waves”
- The effective dynamics:
  - Heuristic construction
  - Equations of motion
- Effects of inhomogeneities on the bounce point
  - Effect through the “energy”: finite number of the degrees of freedom
- Asymptotic structure of inhomogeneities
  - Statistical behavior of the energy of the modes.

The Gowdy T^3 model

- The model specification:
  - Manifold structure: $T^3 \times \mathbb{R}$
  - Two spatial Killing fields: hypersurface orthogonal
- Elements of the metric:
  - norm of one of the Killing fields
  - area of the Killing orbits
  - scale factor of 1-metric ind. on the orbit manifold
- Coordinates: $(t, \sigma, \rho, \theta)$ where $\partial_\sigma, \partial_\rho$ – KVF's.
- Gauge fixing: System can be deparametrized completely, here just partial gauge fixing.
  - orbit area and the scale factor are homogeneous.
- Canonical formalism:
  - Most of constraints automatically satisfied
  - The only remaining: spatial averages of
    - the diffeomorphism (in θ) and
    - the Hamiltonian constraint.

Degrees of freedom

- Qualitatively structure of the perturbation theory:
  - Homogeneous:
    - Phase space of Bianchi I model
    - Ashtekar variables: $A^i_a = \frac{c^i}{2\pi}\delta^i_a$, $E^a_i = \frac{p_i}{4\pi^2}\delta^a_i$
  - Inhomogeneous:
    - The metric field $\xi(\theta)$ + canonical momentum P_ξ,
    - Fixing the conformal rescaling freedom of ξ:

      A. Corichi, J. Cortez, G. Mena Marugan, J. Velhinho
      - Upon complete deparametrization ξ can be interpreted as the local scalar field evolving wrt time p_θ.
      - then the standard Fock quantization can be implemented
      - There is unique conformal rescaling ensuring the unitary evolution of that quantum field.
    - Fourier modes $\xi_m, P^m_\xi \rightarrow$

$$a_m = \sqrt{\frac{\pi}{8G|m|}}\left(|m|\xi_m + \frac{4iG}{\pi}P^m_\xi\right)$$

      where: $\{a_m, a^\star_{m'}\} = -i\delta_{m,m'}$.
- Despite the choice of DOF's the complete nonlinear structure of the system implemented.

Metric and the constraints

- Metric: $\mathrm{d}s^2 = \frac{|p_\theta p_\sigma p_\delta|}{4\pi^2}\left[e^{\tilde{\gamma}}\left(-\frac{\tilde{N}^2}{(2\pi)^4}\mathrm{d}t + \frac{\mathrm{d}\theta^2}{{p_\theta}^2}\right) + e^{-\frac{2\pi\tilde{\xi}}{\sqrt{p_\theta}}}\frac{\mathrm{d}\sigma^2}{{p_\sigma}^2} + e^{\frac{2\pi\tilde{\xi}}{\sqrt{p_\theta}}}\frac{\mathrm{d}\delta^2}{{p_\delta}^2}\right],$

  $\tilde{\xi}(\theta) = \sum_{m\neq 0}\frac{\sqrt{G}}{\pi\sqrt{|m|}}(a_m + a^\star_{-m})e^{im\theta},$

  $\tilde{\gamma}(\theta) = \left(\frac{2c_\delta p_\delta}{c_\sigma p_\sigma + c_\delta p_\delta} - 1\right)\frac{2\pi}{\sqrt{|p_\theta|}}\tilde{\xi}(\theta) - \frac{\pi^2}{|p_\theta|}\tilde{\xi}^2(\theta) - \frac{8\pi G\gamma}{c_\sigma p_\sigma + c_\delta p_\delta}\zeta(\theta),$

  $\zeta(\theta) = i\sum_{m,\tilde{m}\neq 0}\mathrm{sgn}(m+\tilde{m})\frac{\sqrt{|m+\tilde{m}||\tilde{m}|}}{m}$
  $\times (a_{-\tilde{m}} - a^\star_{\tilde{m}})(a_{m+\tilde{m}} + a^\star_{-(m+\tilde{m})})e^{im\theta}.$

- Diffeomorphism: $C_\theta = \sum_{m=1}^{\infty} m(a^\star_m a_m - a^\star_{-m}a_{-m}) = 0,$

- Hamiltonian:

$$C = C_H + C_I,$$

$$C_H = -\frac{2}{\gamma V}(\Theta_\theta\Theta_\sigma + \Theta_\theta\Theta_\rho + \Theta_\rho\Theta_\sigma), \qquad \Theta_i = c_i p_i, \qquad V = \sqrt{|p_\theta p_\rho p_\sigma|}.$$

$$C_I = \frac{G}{V}\left[\frac{(\Theta_\rho+\Theta_\sigma)^2}{\gamma^2|p_\theta|}H^\xi_{\mathrm{int}} + 32\pi^2|p_\theta|H^\xi_o\right],$$

$$H^\xi_o = \sum_{m\neq 0}|m|a^\star_m a_m,$$

$$H^\xi_{\mathrm{int}}\sum_{m\neq 0}\frac{1}{2|m|}\left[2a^\star_m a_m + a_m a_{-m} + a^\star_m a^\star_{-m}\right]$$

The Hybrid quantization

- Main idea: **M. Martin-Benito, L. Garay, G. Mena Marugan**
  - Perturbation-like treatment: inhomogeneities treated as "fields" living on the homogeneous background.
  - Different quantization of the homogeneous and inhomogeneous DOFs.
- Loop quantization of the homogeneous DOFs (Bianchi I):
  - Selected original LQC lapse VN
  - C_H expressed as $\int_\Sigma \mathrm{d}^3x e^{-1}\epsilon^{ij}{}_k E^a_i E^b_j F^k_{ab}$ (where $e=\sqrt{|\mathrm{det}E|}$ and $F^k_{ab} := 2\partial_a A^k_b + \epsilon^k{}_{ij} A^i_a A^j_b$).
  - Components reexpressed in terms of holonomies and $\hat{p}_i$.
  - Lengths λ_i of holonomies fixed by setting the area of the rectangular loop used for rewriting F to equal *area gap* Δ. Used old prescription in which $\lambda_i = \lambda_i(p_i)$. Not correct for noncompact models.
  - Applied factor ordering in which the triad orientations always separate.
  - C_H densitized: lapse N restored on the quantum level

$$C_H \to \mathcal{C}_H = (\widehat{1/V})^{-1/2} C_H (\widehat{1/V})^{-1/2}.$$

The Hybrid quantization (2)

- Fock quantization of the inhomogeneous DOFs:
  - Selected complex structure naturally associated with the identification of $a_m, a_m^\star$ as creation and annihilation operators.
  - Constructed Fock space
  - $a_m, a_m^\star$ in C_θ and C_I promoted to operators
  - The quantization of the homogeneous elements of C_θ and C_I transferred from the previous point
  - densitization of C_I analogous to C_H
- Final form:

$$\hat{\mathcal{C}} = \hat{\mathcal{C}}_H + \hat{\mathcal{C}}_I$$

$$\hat{\mathcal{C}}_H = -\frac{2}{\gamma^2}[\hat{\Theta}_\theta\hat{\Theta}_\sigma + \hat{\Theta}_\theta\hat{\Theta}_\rho + \hat{\Theta}_\rho\hat{\Theta}_\sigma]$$

$$\hat{\Theta}_i = (3/2)\hat{v}_i \sin(\hat{b}_i), \qquad \hat{v}_i = 2/(3\sqrt{\Delta})\hat{p}_i\sqrt{|\hat{p}_i|}$$

where $b_i := \sqrt{\Delta}c_i/\sqrt{p_i}$ is a canonical momentum of v_i.

$$\hat{\mathcal{C}}_I = \ell_{\rm Pl}^2\left[\frac{(\hat{\Theta}_\rho+\hat{\Theta}_\sigma)^2}{\gamma^2}\left(\widehat{\frac{1}{\sqrt{|p_\theta|}}}\right)^2\hat{H}^\xi_{\rm int} + 32\pi^2|\hat{p}_\theta|\hat{H}^\xi_o\right]$$

The effective dynamics

- Heuristic method:
  - the operators $\hat{v}_i$, $\sin(\hat{b}_i)$, $\hat{\Theta}$, $\hat{a}_m$, $\hat{a}_m^\dagger$ replaced by their expectation values,
  - neglected regularization of $\widehat{1/\sqrt{|p_\theta|}}$
- Why we hope it can work:
  - successfully tested with homogeneous models (one field) – expected to work with finite number,
  - countable number of "matter" DOFs,
  - specific structure – coupled only in pairs and through the total energy.
- The effective constraint: $\mathcal{C} = \mathcal{C}_H + \mathcal{C}_I$

$$\mathcal{C}_H = -\frac{2}{\gamma^2}[\Theta_\theta\Theta_\sigma + \Theta_\theta\Theta_\rho + \Theta_\rho\Theta_\sigma], \qquad \Theta_i = (3/2)v_i \sin(b_i),$$

$$\mathcal{C}_I = \ell_{\rm Pl}^2 \left[\frac{(\Theta_\rho+\Theta_\sigma)^2}{\gamma^2|p_\theta|} H_{\rm int}^\xi + 32\pi^2|p_\theta| H_o^\xi\right].$$

Equations of motion

- Hamilton's equations:

$$\dot{v}_\theta = \frac{24\pi G}{\gamma}\left(\Theta_\rho + \Theta_\sigma\right) v_\theta \cos b_\theta,$$

$$\dot{b}_\theta = -\frac{24\pi G}{\gamma}\left(\Theta_\rho + \Theta_\sigma\right)\sin b_\theta - \frac{16\pi G^2}{3\gamma}\left(\frac{2}{3\sqrt{\Delta}}\right)^{2/3}\left(\Theta_\rho + \Theta_\sigma\right)^2 \frac{\mathrm{sgn}(v_\theta)}{|v_\theta|^{5/3}} H^\xi_{\mathrm{int}}$$

$$+\frac{512}{3}\pi^3 G^2 \gamma \left(\frac{3M}{2}\right)^{2/3} \frac{\mathrm{sgn}(v_\theta)}{|v_\theta|^{1/3}} H^\xi_0,$$

$$\dot{a}_m = -32\pi^2 i G|m| \left(\frac{3\sqrt{\Delta}}{2}\right)^{2/3} |v_\theta|^{2/3} a_m$$

$$-\frac{iG}{\gamma^2 |m||v_\theta|^{2/3}}\left(\frac{2}{3\sqrt{\Delta}}\right)^{2/3}\left(\Theta_\rho + \Theta_\sigma\right)^2 \left(a_m + a^*_{-m}\right),$$

$$\dot{v}_\rho = \frac{24\pi G}{\gamma}\left(\Theta_\theta + \Theta_\sigma - \left(\frac{2}{3\sqrt{\Delta}}\right)^{2/3} G\left(\Theta_\rho + \Theta_\sigma\right)\frac{H^\xi_{\mathrm{int}}}{|v_\theta|^{2/3}}\right) v_\rho \cos b_\rho,$$

$$\dot{b}_\rho = -\frac{24\pi G}{\gamma}\left(\Theta_\theta + \Theta_\sigma - \left(\frac{2}{3\sqrt{\Delta}}\right)^{2/3} G\left(\Theta_\rho + \Theta_\sigma\right)\frac{H^\xi_{\mathrm{int}}}{|v_\theta|^{2/3}}\right)\sin b_\rho.$$

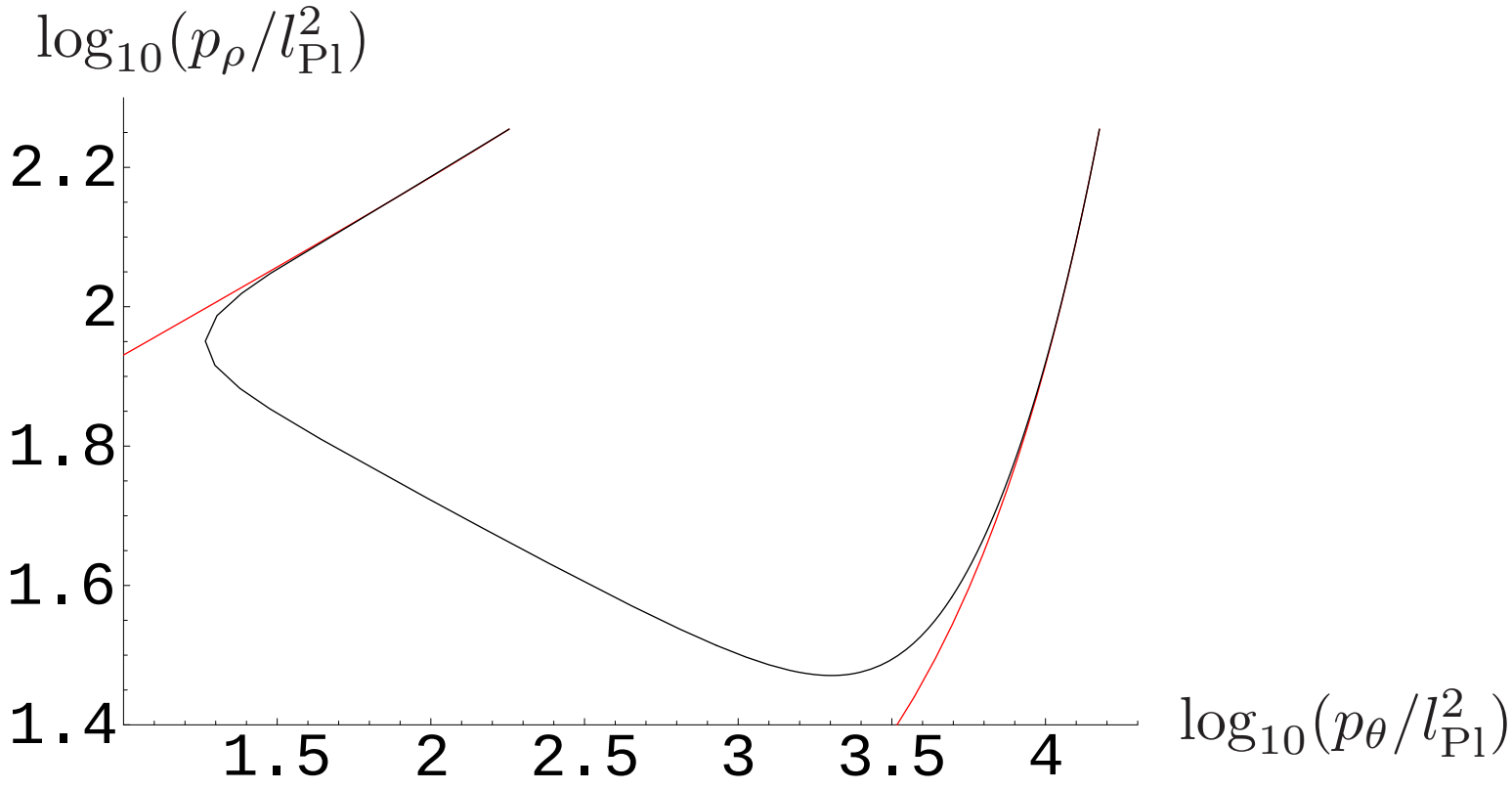
  Reduce to Bianchi I for $a_m = 0$.

- Θ_ρ, Θ_σ – constants of motion.

- Effective diffeomorphism constraint: $\mathcal{C}_\theta = \sum_{m=1}^{\infty} m K_m$
  where $K_m \equiv a_m a^*_m - a_{-m} a^*_{-m}$ – const. of motion.

The dynamical trajectories

An example of a dynamical trajectory – the solution to EOMs



The bounce

- For $a_m = 0$: position of the bounce: $v_i^b = (2/3)\Theta_i$.
- The directions σ, ρ: from eq. for $\dot{v}_\rho$, at the bounce $\cos(b_\rho) = 0$ thus (see def. Θ_ρ) value of v_ρ^b unchanged.
- The direction θ: more complicated ...
  - Maximal solution to:

$$\frac{9}{4}(\Theta_\rho + \Theta_\sigma)^2 {v_\theta^b}^2 - \left[F(v_\theta^b) - \Theta_\rho\Theta_\sigma\right]^2 = 0$$

$$F(v_\theta) = 16\pi^2\gamma^2 G \left(\frac{3\sqrt{\Delta}}{2}\right)^{2/3} v_\theta^{2/3} H_0^\xi$$

$$+\frac{G}{2}\left(\Theta_\delta + \Theta_\sigma\right)^2 \left(\frac{2}{3\sqrt{\Delta}}\right)^{2/3} \frac{H_{\rm int}^\xi}{v_\theta^{2/3}}$$

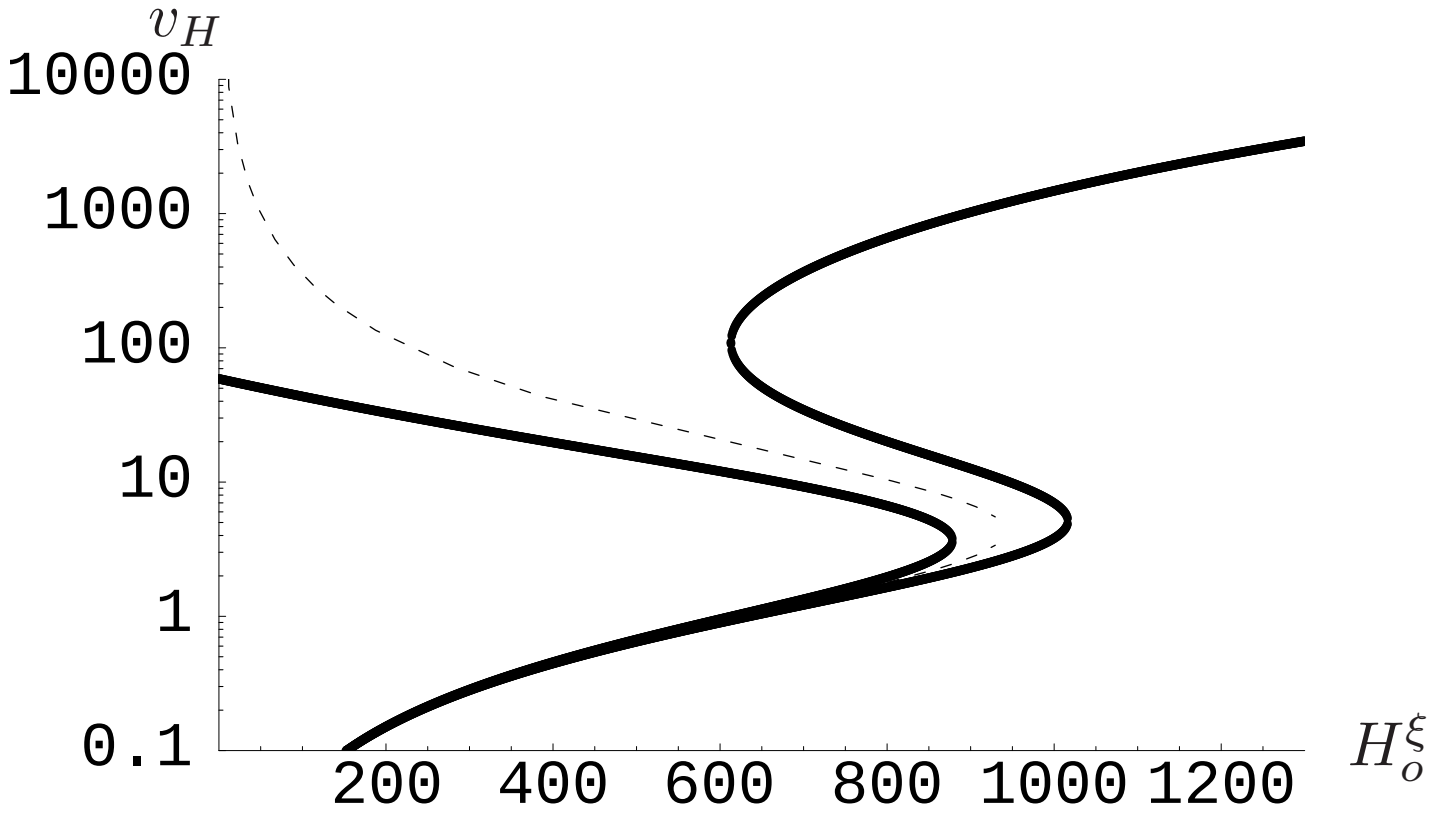
    (root of the polynomial of the 5th order).
  - Two cases:
    - For $\Theta_\rho\Theta_\sigma < 0$: inhomogeneities increase $[\ldots]$ – $v_\theta^b > (2/3)\Theta_\theta$.
    - For $\Theta_\rho\Theta_\sigma > 0$: the terms inside $[\ldots]$ compete – numerical studies necessary.

Numerical studies of the bounce

- Inhomogeneities enter only through H_o^ξ, $H_{\rm int}^\xi$ – finite number of DOFs!
- Partial compactification: reparametrization
  $H_{\rm int}^\xi \;=:\; \alpha H_0^\xi, \quad \alpha \in [0,2]$,
  $\Theta_\delta \;=:\; \frac{3}{2}\frac{v_H}{\beta}, \quad \Theta_\sigma \;=:\; \frac{3}{2}\frac{v_H}{1-\beta}, \quad \beta \in [0,1]$.
  Only two dimensions noncompact: H_o^ξ, v_H.
- Numerical methodology:
  - For each point $(v_H, H_o^\xi, \alpha, \beta)$ numerically finding v_θ^b as the maximal root of the 5th order polynomial.
  - $v_\theta^b(v_H, H_o^\xi, \alpha, \beta) \to v_\theta^b(v_H, \alpha, \beta) = \min_{H_o^\xi} v_\theta^b(v_H, H_o^\xi, \alpha, \beta)$. Domain necessary for numerical scanning determinable analytically.
  - Result scanned over the compact α, β.
  - Also verified the limit $v_H \to \infty$ for fixed α, β.
- THE RESULT: $v_\theta^b \geq 0.05 v_H$!!!

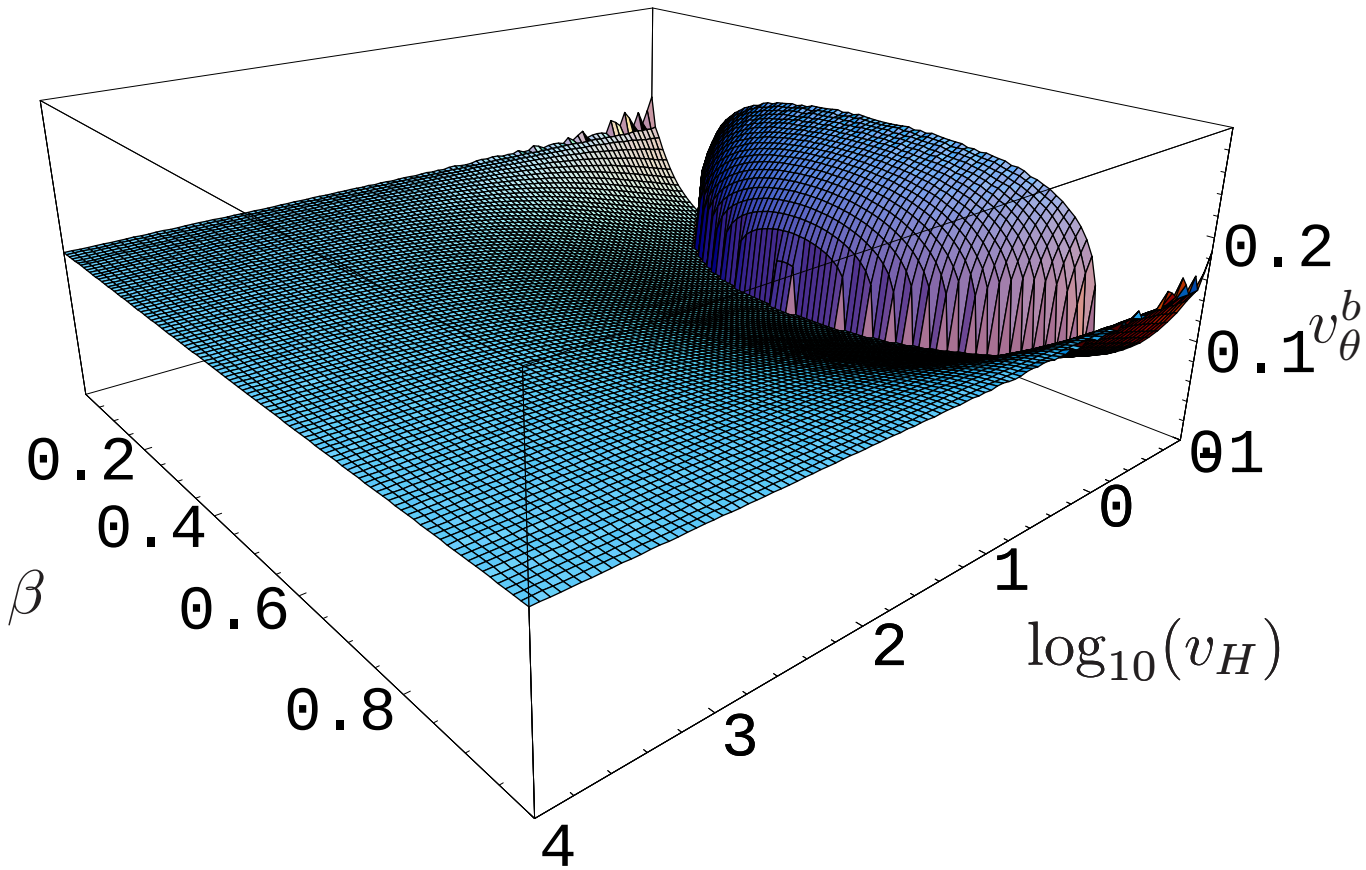
Numerical scan of the bounce (1)

Roots for fixed v_H, α, β.



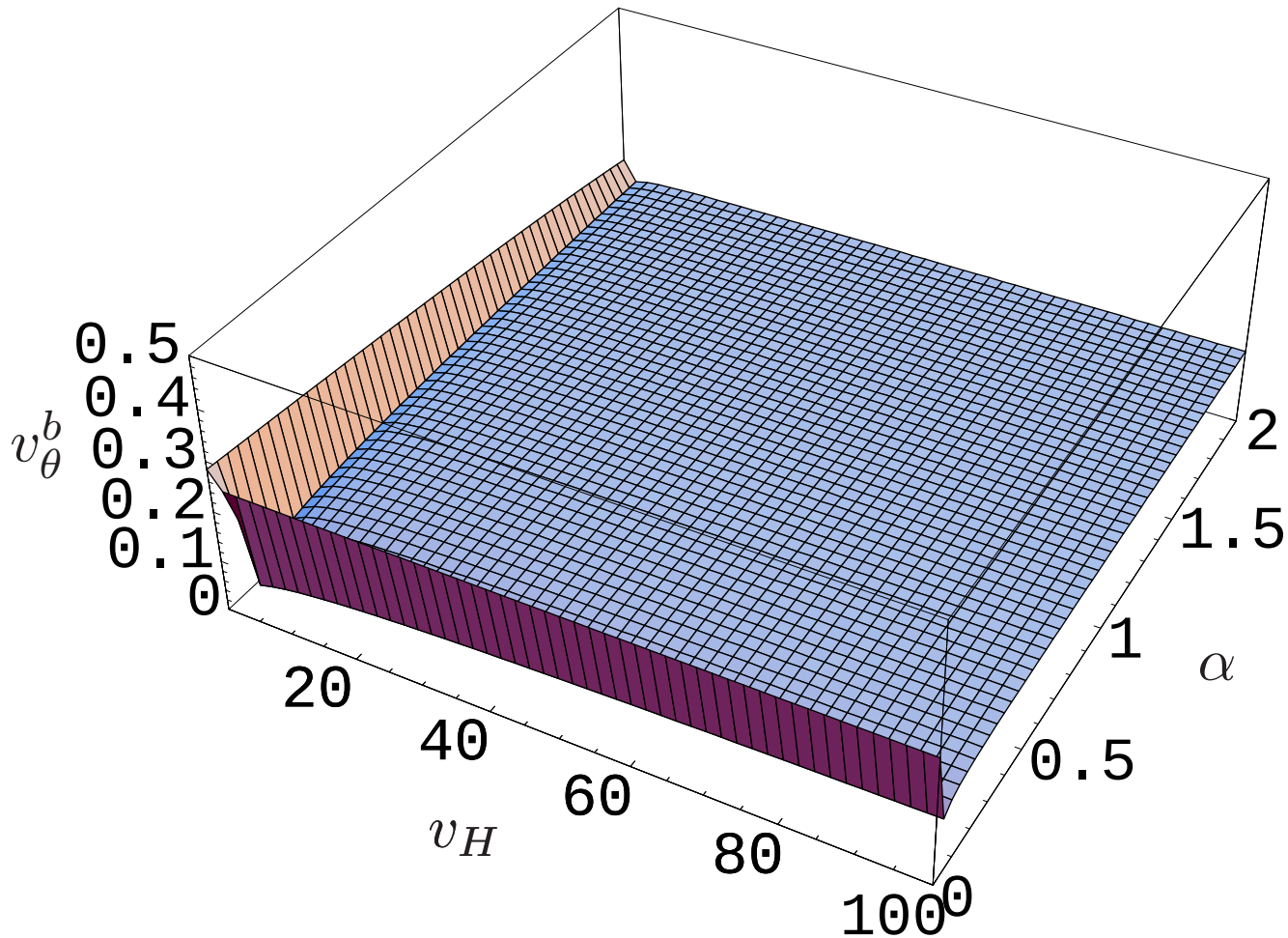
Numerical scan of the bounce (2)

Minimum $v_\theta^b(v_H, \alpha, \beta)$ for fixed α.



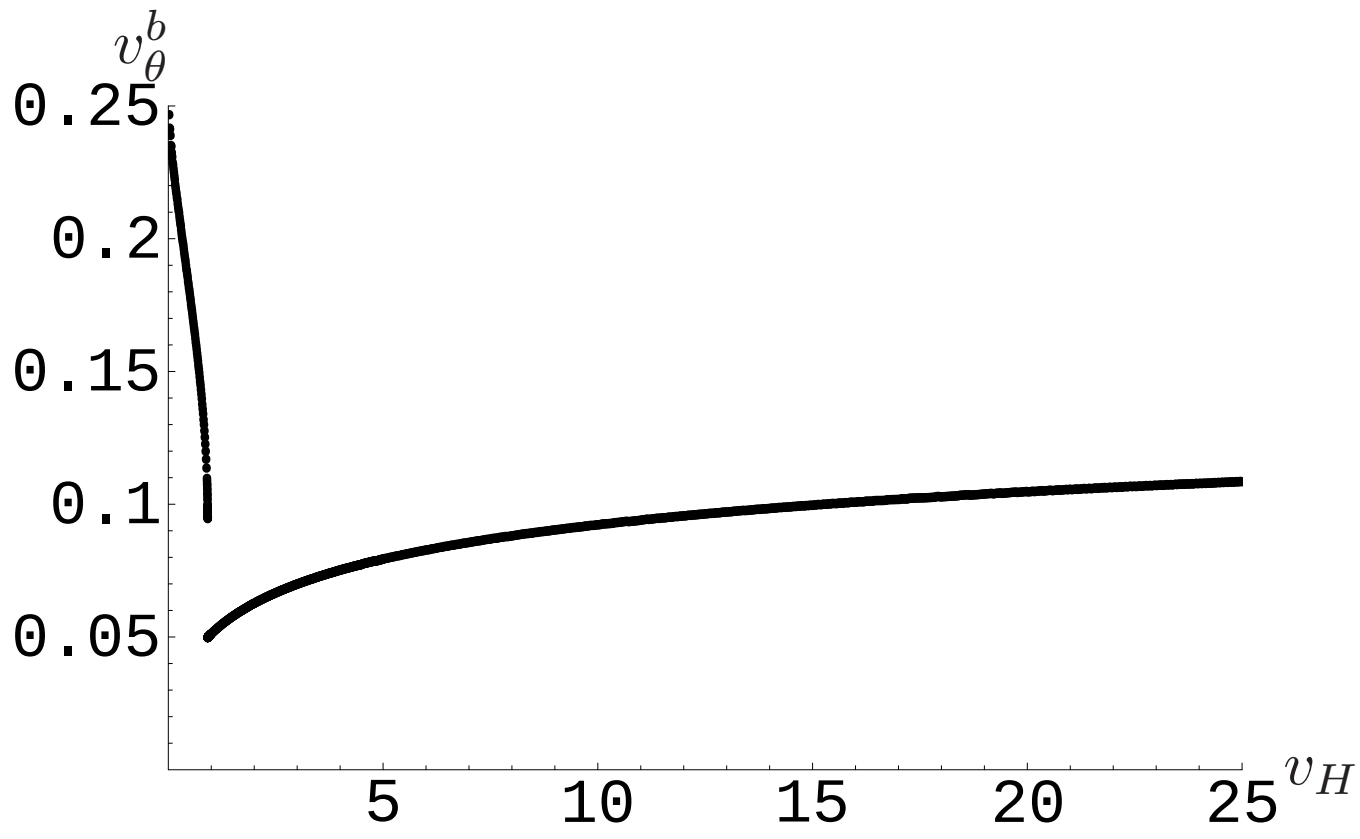
Numerical scan of the bounce (3)

Minimum $v_\theta^b(v_H, \alpha, \beta)$ for fixed β.



Numerical scan of the bounce (4)

The slice of v_H, α, β for fixed α, β.

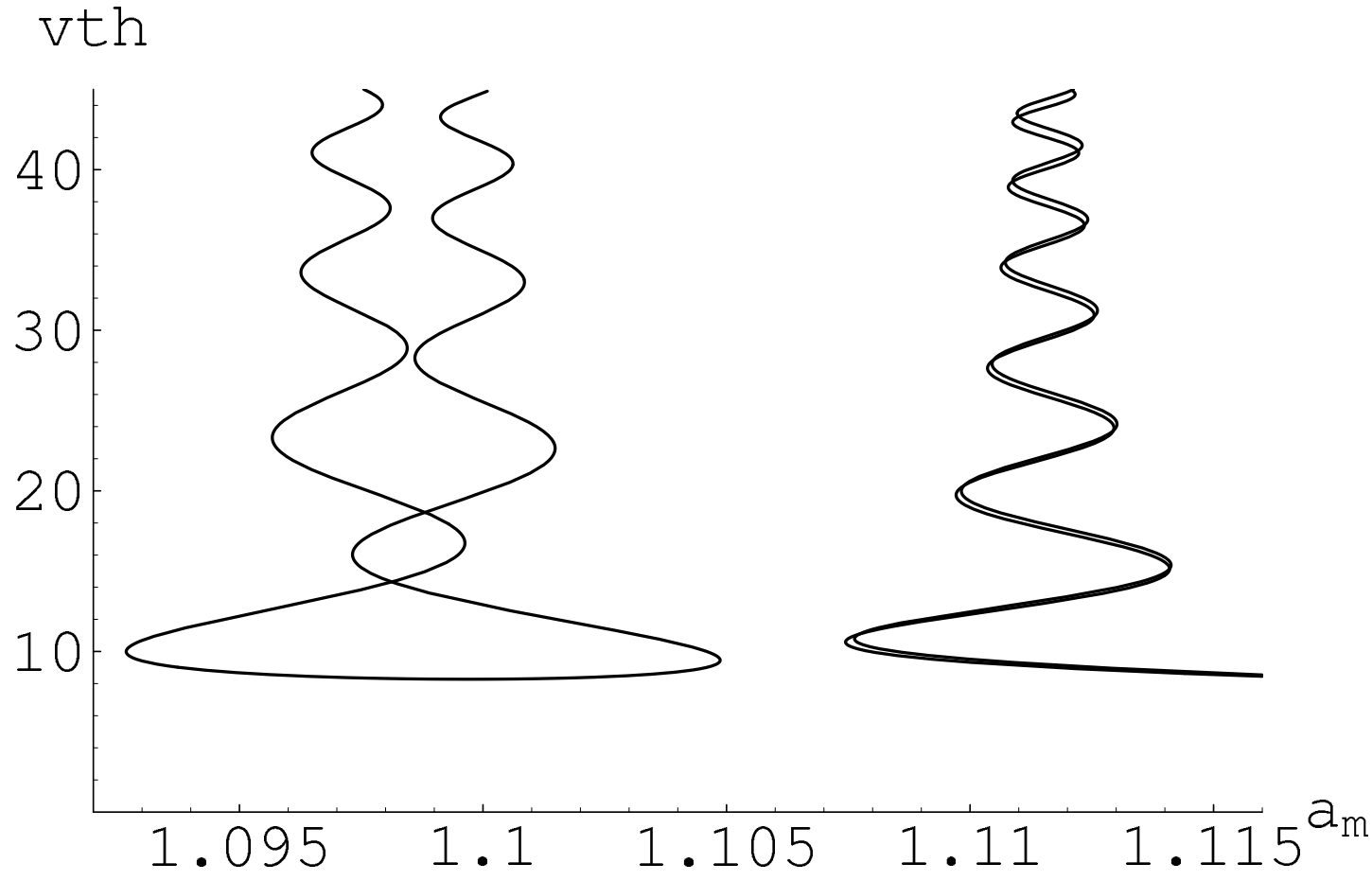


Caveats

- On dynamical trajectories $H_o^\xi \neq \mathrm{const}$ – oscillations!
  However...
  - oscillations are small – small change of v_θ^b in the range of achieved values of H_o^ξ for given trajectory.
- All the roots are reflective boundaries. Thus in principle possible for the trajectory to go "down the throat" to the singularity!
  However...
  - The process requires an infinite number of reflections, thus fine tuning (critical phenomena).
  - Only very small region of the parameter space can violate the predicted bound on v_θ^b.
- Conclusions:
  - The bounce is generic.
  - There is a upper 5% bound on the position of the bounce, which is violated only in a very small region of the parameter space.

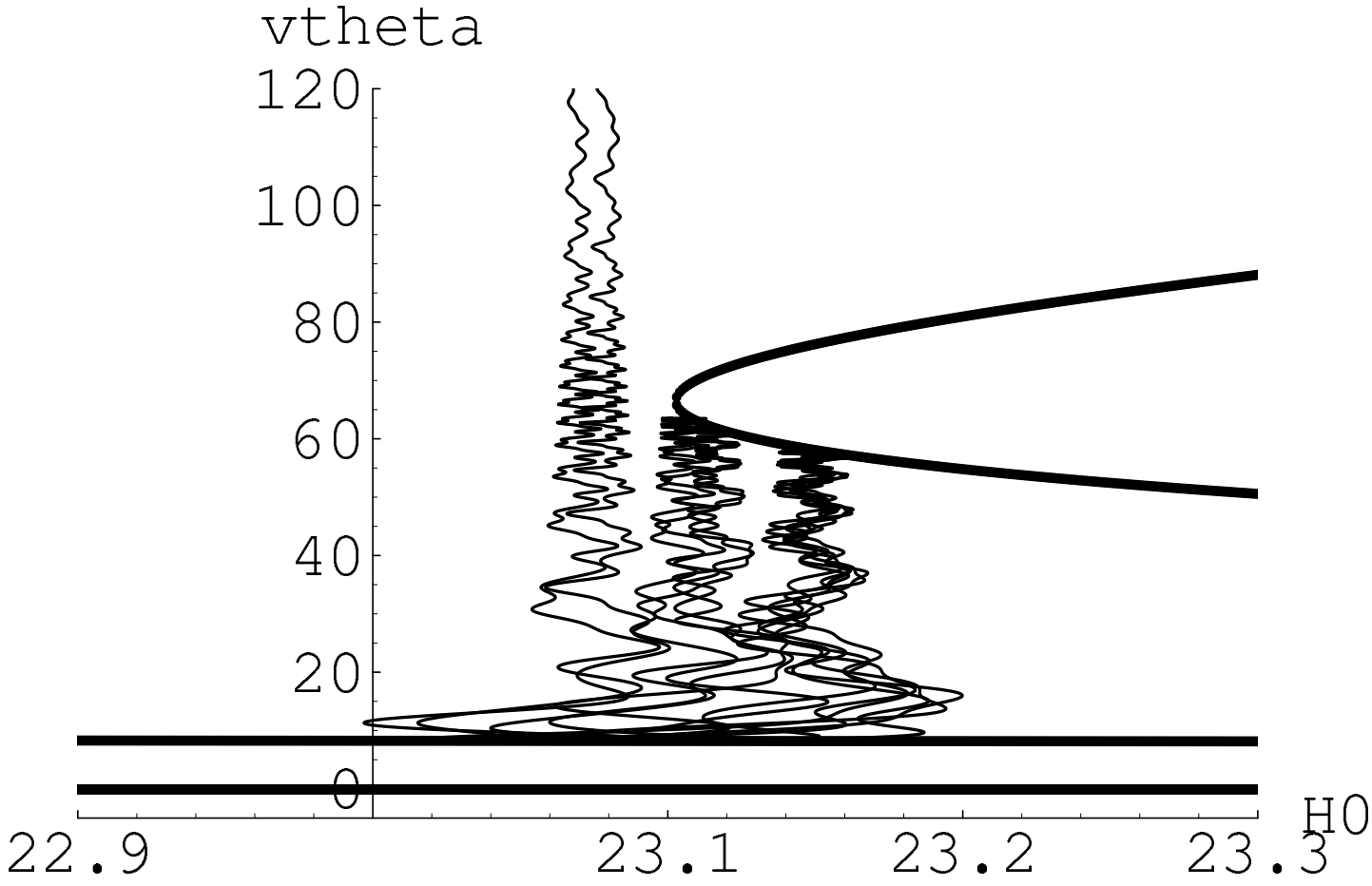
Dynamical trajectories (1)

An illustration of the oscillations of a_m in the time evolution.



Dynamical trajectories (2)

Near-critical dynamical trajectory (multiple bounces and recollapses).

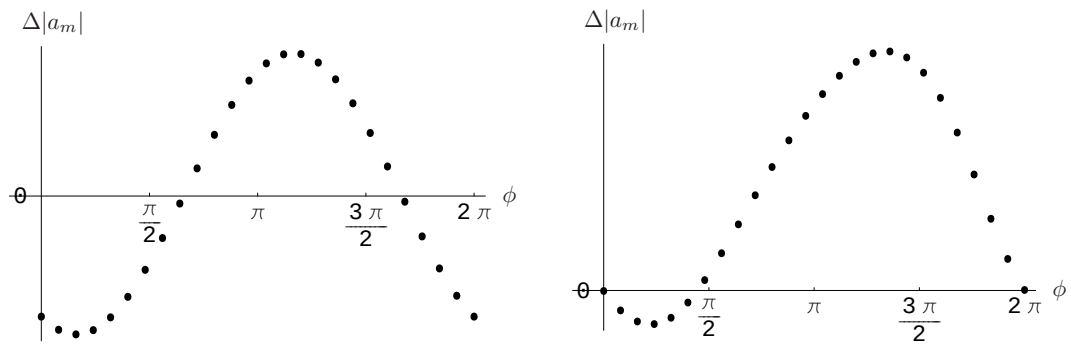


The energy of inhomogeneities

- H_o^ξ – convenient measure of the energy of the inhomogeneities,
  - For large v_θ approaches constants different for the future and the past.
  - Contribution of a single mode encoded in $|a_m|^2$.
  - Change between asymptotic values of $|a_m|^2$ depends on the phases between a_m and a_{-m}.
- Idea: neglect the information about the phases: The deterministic system becomes statistical.
- Question: How energy distribution changes through the bounce?
- Answer: Through the numerical Monte-Carlo methods.
  - Distribution of the energies: values of $|a_m|$ with Gaussian probability.
  - Distribution of the phases: uniform distribution.

The evolution of the modes

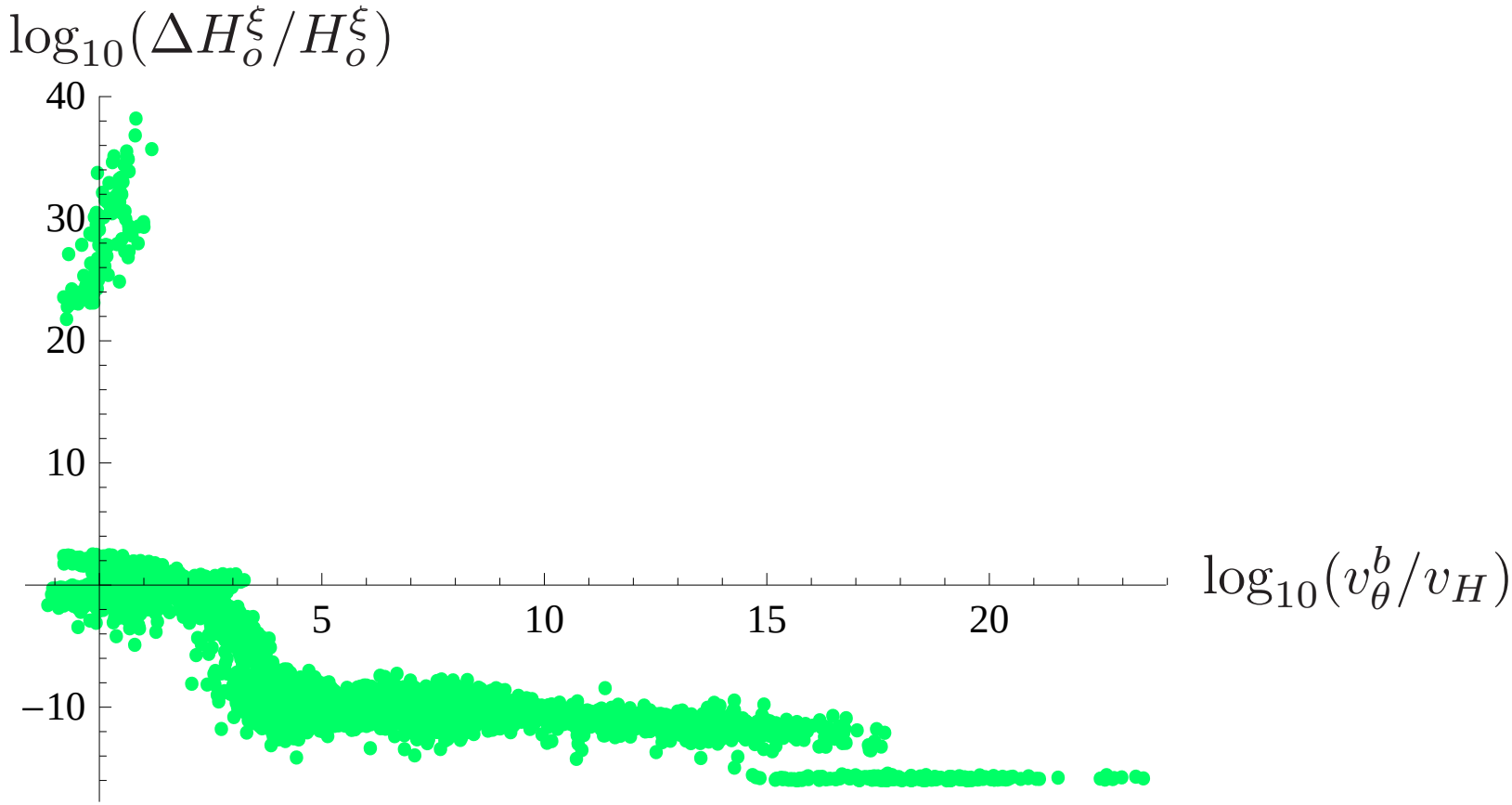
- Observation: Fix all other parameters and probe the amplitude change $\Delta|a_m|$ between asymptotic past and future as a function of the phase ϕ.



- Two domains:
  - Inhomogeneity dominated: The average of $\Delta|a_m|$ over ϕ vanishes. Statistical preservation.
  - Near vacuum: The average is positive: amplification of the inhomogeneities.

The energy change distribution

The relative change in the energy of a single mode as a function of v_θ^b



Conclusions

- The fate of the bounce:
  - In considered model the bounce is generic.
  - The position v of the bounce in the Killing directions is not affected by the inhomogeneities.
  - In the "inhomogeneous" direction the bounce position v_θ^b is bounded by an analogous value of the homogeneous universe $v_\theta^b \geq 0.05 v_H$. The bound can be violated only in some very small region of the parameter space near the critical solutions.
- The evolution of the inhomogeneities:
  - Once we neglect the phases between a_m and a_{-m} the change of the energies of the inhomogeneities between $t \to -\infty$ and $t \to +\infty$ attains the probabilistic character.
  - Two regions of a distinct qualitative behavior:
    - inhomogeneity dominated: where the amplitudes of the gravitational wave modes are statistically preserved, and
    - near vacuum, where the amplitudes are statistically amplified.