How ubiquitous is entanglement in QFT?

arXiv: 2302.13742

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In collaboration with:

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IN·DEI·NOMINE·FELICITER

Outline of the talk

Motivation:

A surprising property of QFT

The Reeh-Schlieder theorem I

Let $\hat{\phi}$ be a free scalar field in $(1+D)$-Minkowski spacetime, and $\mathcal{H}$ its Hilbert space

Consider field observables localized in spacetime:

$$\hat{\phi}_F \equiv \int \mathrm{d}^4x \, F(x)\hat{\phi}(x) \quad \text{Smeared field}$$

with $F(x)$ smooth and of compact support.

- Fact (intuitively reasonable):
  States of the form $\hat{\phi}_{F_1}\hat{\phi}_{F_2}\dots\hat{\phi}_{F_n}\,|0\rangle$ are sufficient to generate the Hilbert space, $\mathcal{H}$

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- Reeh-Schlieder thm. (intuitively not reasonable):

  The previous statement remains true if the functions $F_i(x)$ are all restricted to be supported in an arbitrarily small region of spacetime $\mathcal{V}$

Not surprised?

Graphic (and dramatic) example

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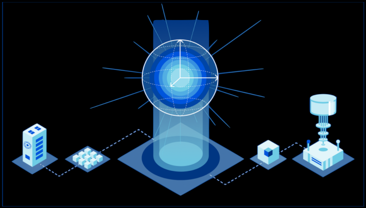

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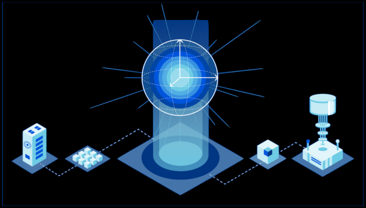

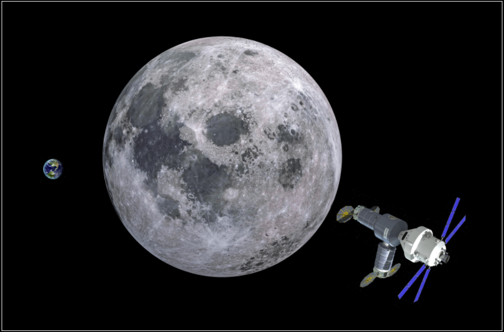

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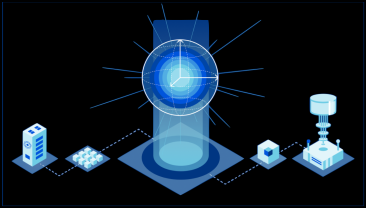

Houston, we've got a problem

Intuitive explanation: Entanglement

This behavior is reminiscent of something well-known in Quantum Mechanics

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Let $\mathcal{H}_A$ and $\mathcal{H}_B$ be Hilbert spaces of two systems, both of dimension n. Any state in $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as

$$|\Psi\rangle = \sum_{i=1}^{n} c_i \, |i\rangle_A \otimes |i\rangle_B \ , \quad \text{(Schmidt form)}$$

with $\{|i\rangle_A\}_{i=1}^n$ and $\{|i\rangle_B\}_{i=1}^n$ bases in $\mathcal{H}_A$ and $\mathcal{H}_B$

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If $c_i \neq 0 \quad \forall i \quad \rightarrow \quad |\Psi\rangle$ is fully entangled

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If $|\Psi\rangle$ is fully entangled, then

$$\boxed{\text{Any state in } \mathcal{H}_A \otimes \mathcal{H}_B \text{ can be written as } \hat{O}_A \otimes \hat{I}_B \, |\Psi\rangle}$$

Entanglement in QFT

Lesson from the Reeh-Schlieder Theorem:

Entanglement is ubiquitous in QFT

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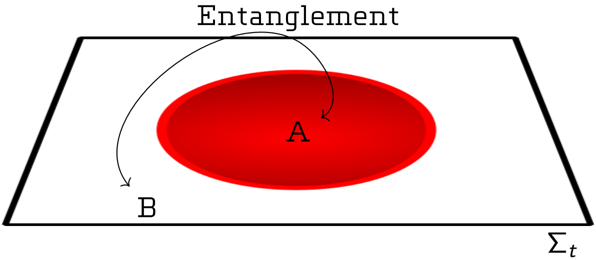

- The d.o.f. within A are entangled with those in B

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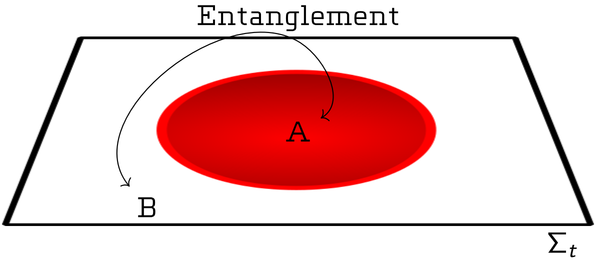

$S_{\mathrm{ent}} \to \infty$

Infinitely many d.o.f.

- The d.o.f. within A are entangled with those in B
- Quantifier: Geometric entanglement entropy

Questions

Lesson from the Reeh-Schlieder Theorem:

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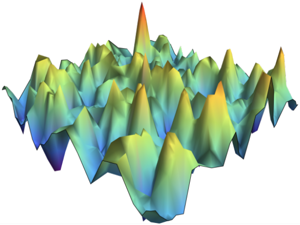

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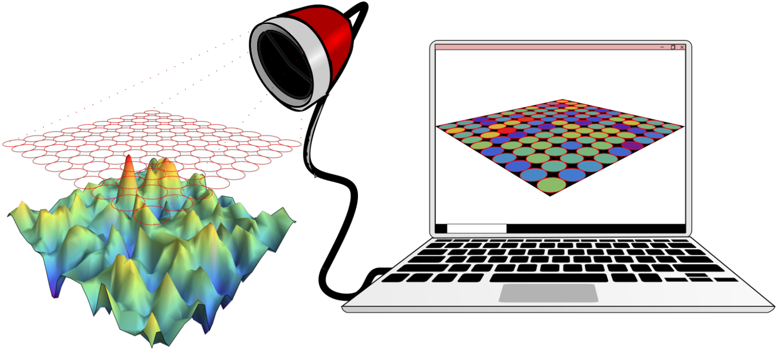


Questions

Lesson from the Reeh-Schlieder Theorem:

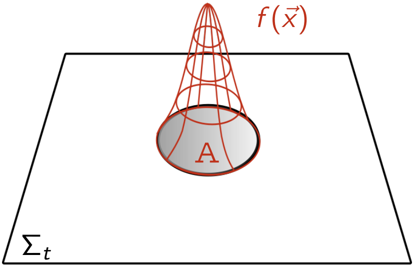
Entanglement is ubiquitous in QFT

- Entanglement is a property of a state AND a choice of subsystems

  ⟨see, e.g., Agullo, Bonga, and Ribes Metidieri⟨ 2022⟩, *JCAP*⟩

Is there entanglement between 'natural' subsystems?

A possible choice of subsystems in QFT I



$f(\vec{x})$: function of compact support within region A

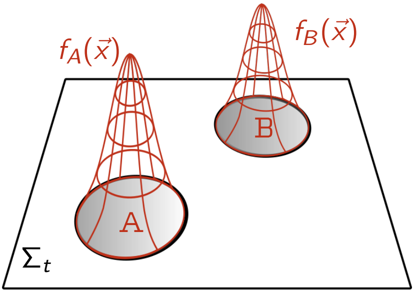
($\sim$ sensitivity of a one-pixel detector)

$$\hat{\Phi}[f] \equiv \int \mathrm{d}^3x\, f(\vec{x})\,\hat{\phi}(\vec{x}) \qquad \hat{\Pi}[f] \equiv \int \mathrm{d}^3x\, f(\vec{x})\,\hat{\pi}(\vec{x})$$

$$\left[\hat{\Phi}[f], \hat{\Pi}[f]\right] = i \int \mathrm{d}^3x (f(\vec{x}))^2 = i$$

$(\hat{\Phi}[f], \hat{\Pi}[f])$ pair of canonically conjugate operators: System with 1 d.o.f.

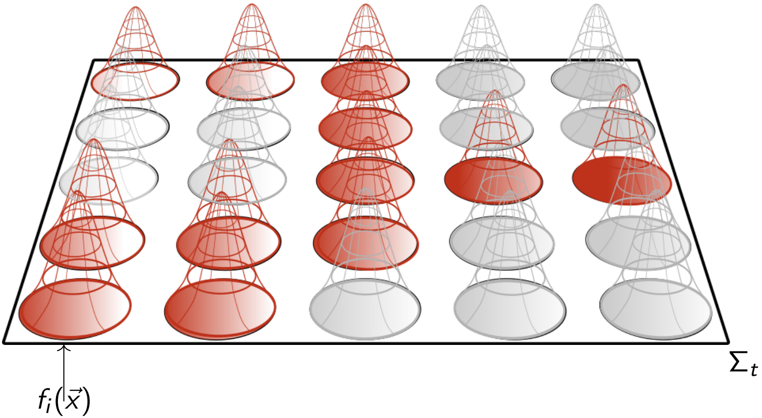
A possible choice of subsystems in QFT II



Question:

Entanglement between $(\hat{\Phi}[f_A], \hat{\Pi}[f_A])$ and $(\hat{\Phi}[f_B], \hat{\Pi}[f_B])$?

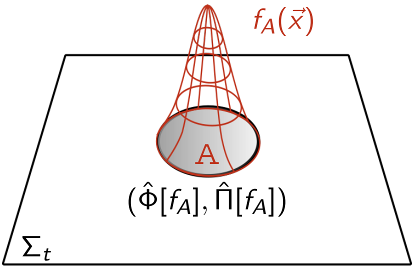
A possible choice of subsystems in QFT III



Question:

Entanglement between A and B?

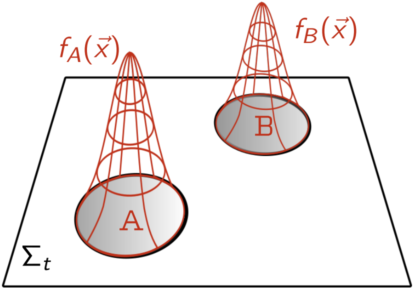
How do we compute entanglement?



- $|0\rangle$ is a Gaussian state (with zero average)
- $(\hat{\Phi}[f_A], \hat{\Pi}[f_A])$ is a Gaussian subsystem $\longrightarrow$ $\hat{\rho}_A^{\mathrm{red}}$ is a Gaussian state

Therefore, $\hat{\rho}_A^{\mathrm{red}}$ is fully determined by its covariance matrix:

$$\sigma_A^{\mathrm{red}} = \begin{pmatrix} 2\langle \hat{\Phi}_A^2 \rangle & \langle \hat{\Phi}_A \hat{\Pi}_A + \hat{\Pi}_A \hat{\Phi}_A \rangle \\ \langle \hat{\Phi}_A \hat{\Pi}_A + \hat{\Pi}_A \hat{\Phi}_A \rangle & 2\langle \hat{\Pi}_A^2 \rangle \end{pmatrix}$$



Similarly, for 2 d.o.f.:

$$\hat{\rho}_{AB}^{\mathrm{red}} \longleftrightarrow \sigma_{AB}^{\mathrm{red}} = \begin{pmatrix} \sigma_A^{\mathrm{red}} & C \\ C^{\mathrm{T}} & \sigma_B^{\mathrm{red}} \end{pmatrix} \qquad \det C < 0$$

- $\hat{\rho}_{AB}^{\mathrm{red}}$ is always mixed $\rightarrow$ The von Neumann entropy of $\hat{\rho}_{AB}^{\mathrm{red}}$ is not a quantifier for the entanglement between A and B
- We use Logarithmic Negativity (Log Neg)

Log Neg

- Faithful for systems of 1 vs N modes (pure and mixed)

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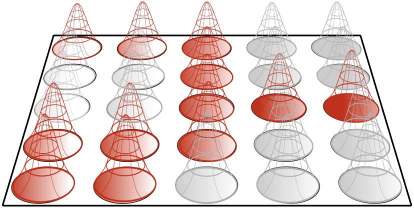

$$E_{\mathcal{N}} = \sum_{j}^{N_A+N_B} \max\{0, -\log_2 \tilde{\nu}_j\}\,,$$

with $\tilde{\nu}_j$ are the symplectic eigenvalues of $\tilde{\sigma}$

$N_A + N_B$

$$\tilde{\sigma} = T\sigma T\,, \quad T = (\oplus_{i=1}^{N_A} I_2) \oplus (\oplus_{j=1}^{N_B} \sigma_z)$$

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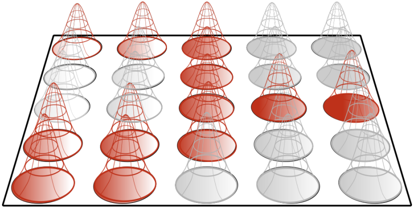

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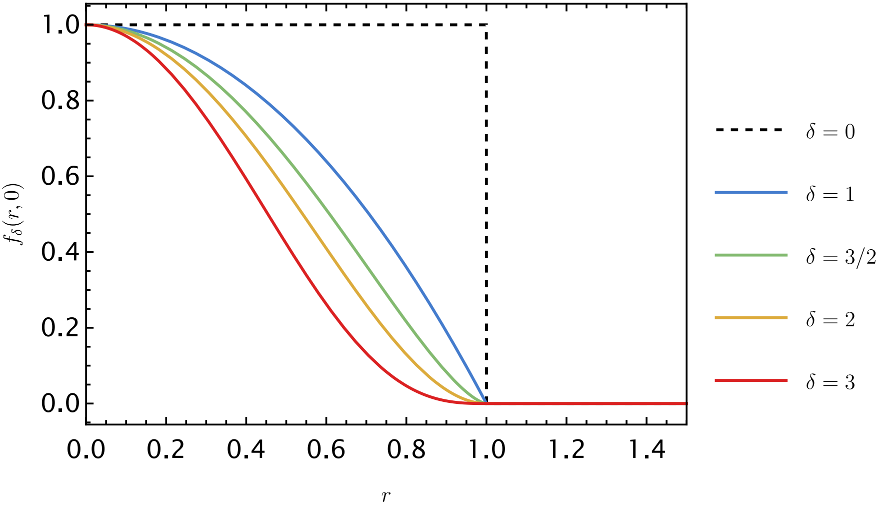
- **Physical meaning:**
  - Lower bound for Distillable entanglement
  - For Gaussian states $\equiv$ Entanglement Cost.

Family of functions of compact support

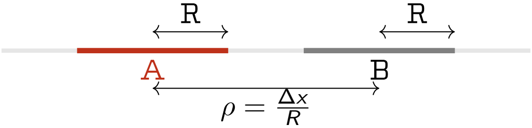
$$f_\delta(r, r_0) = A_\delta \, \Theta\left(1 - \frac{|r - r_0|}{R}\right)\left(1 - \left(\frac{r - r_0}{R}\right)^2\right)^\delta, \qquad \delta > 0$$



Results

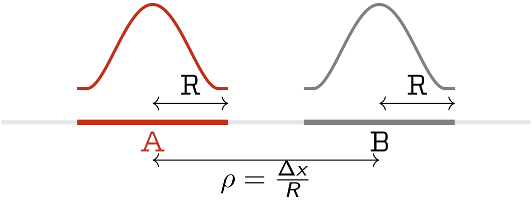
Entanglement between two degrees of freedom

Entanglement between 2 d.o.f.



$D = 1$

Entanglement between 2 d.o.f.



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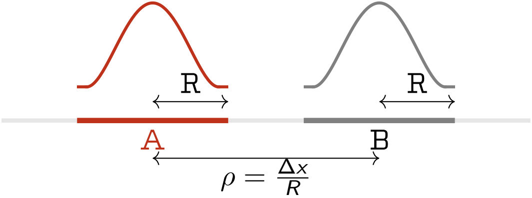

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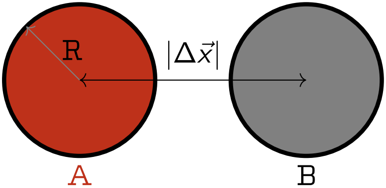
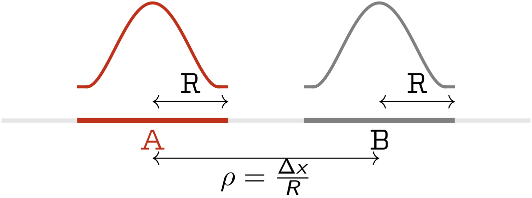


$D = 2$

Entanglement between 2 d.o.f.



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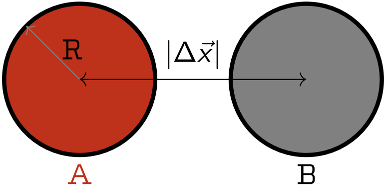

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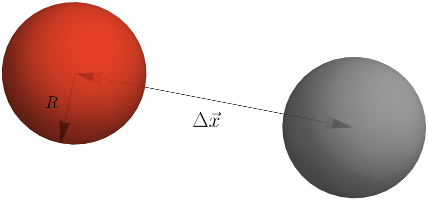


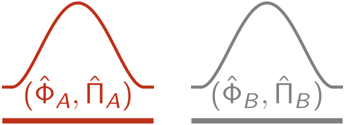
$D = 3$

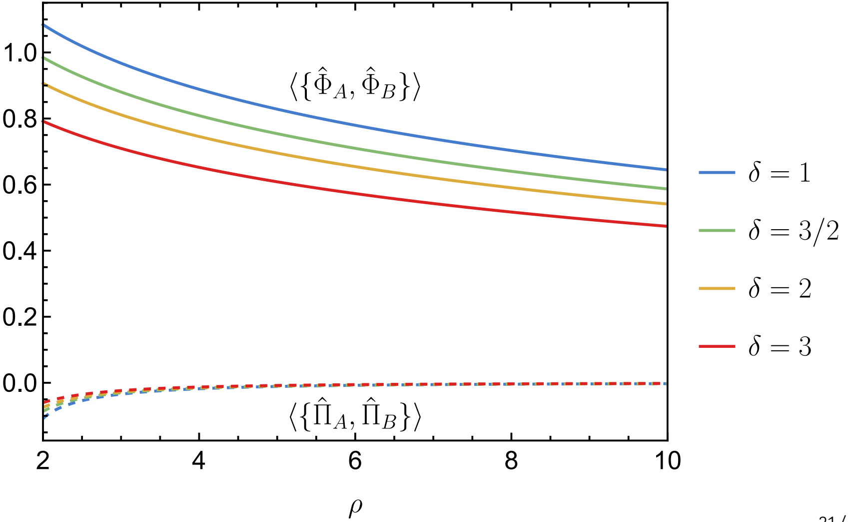
Entanglement between 2 d.o.f.

$D = 1$ $\dashrightarrow$ Mass needed to regularize IR divergences
$\rightarrow$ numerical results

$$\left.\begin{array}{l} D = 2 \\ \\ D = 3 \\ \vdots \end{array}\right\}$$ Massless scalar field $\rightarrow$ analytic results

Entanglement between 2 d.o.f.: $D = 1$





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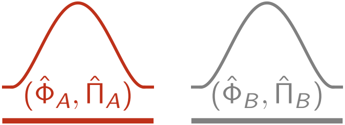
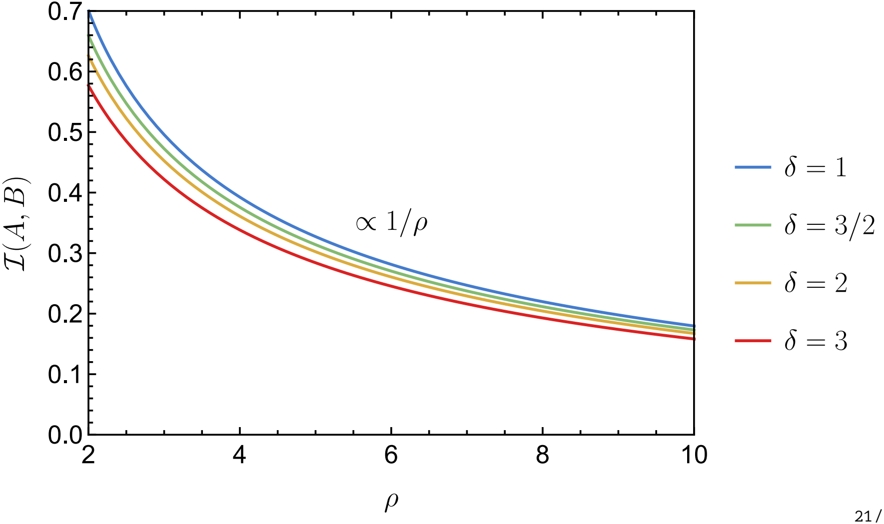


Mutual Information = Classical + Quantum correlations



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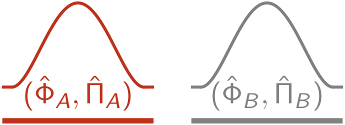
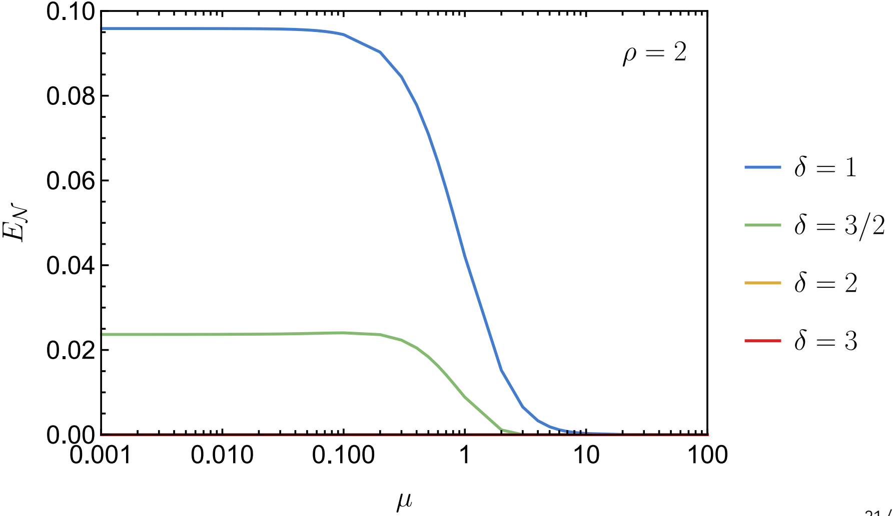


$\mu \equiv m\,R$



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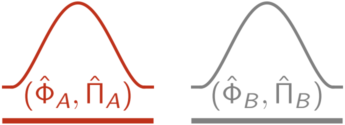

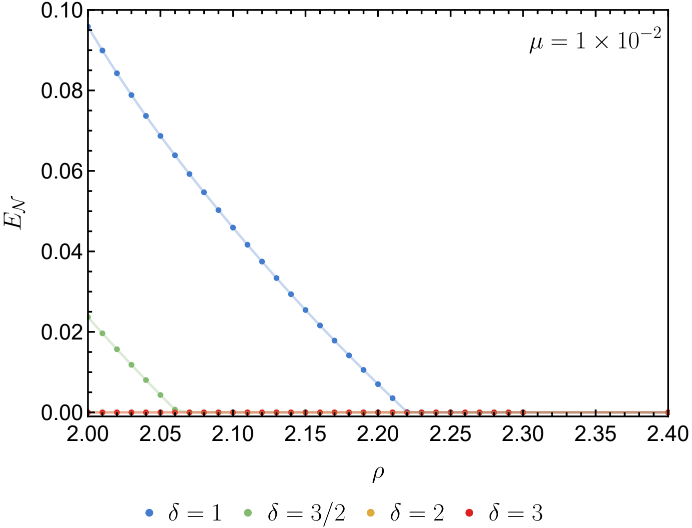

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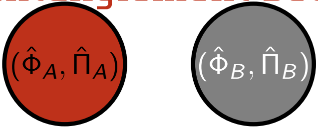

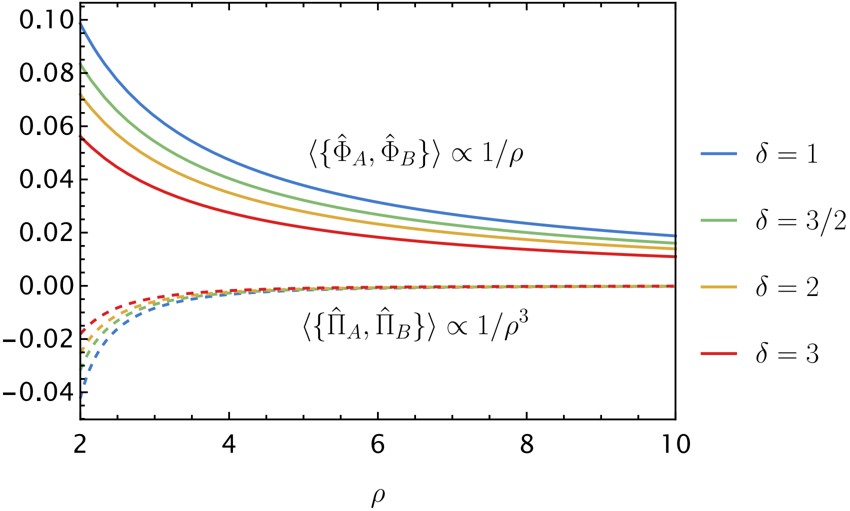

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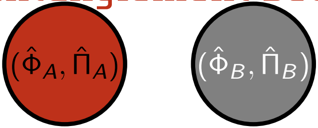

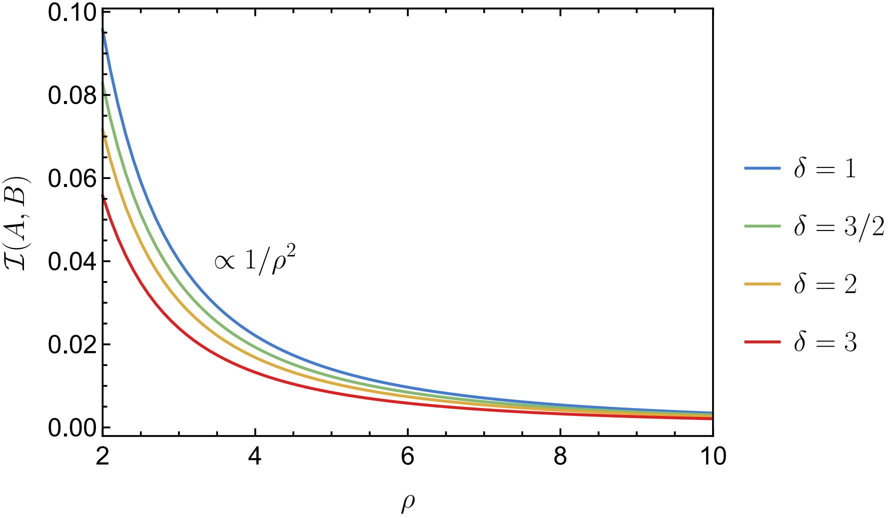

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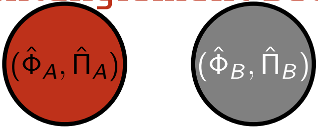
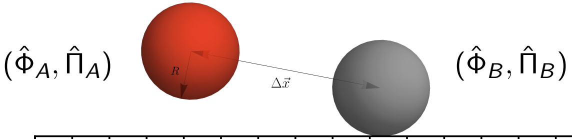
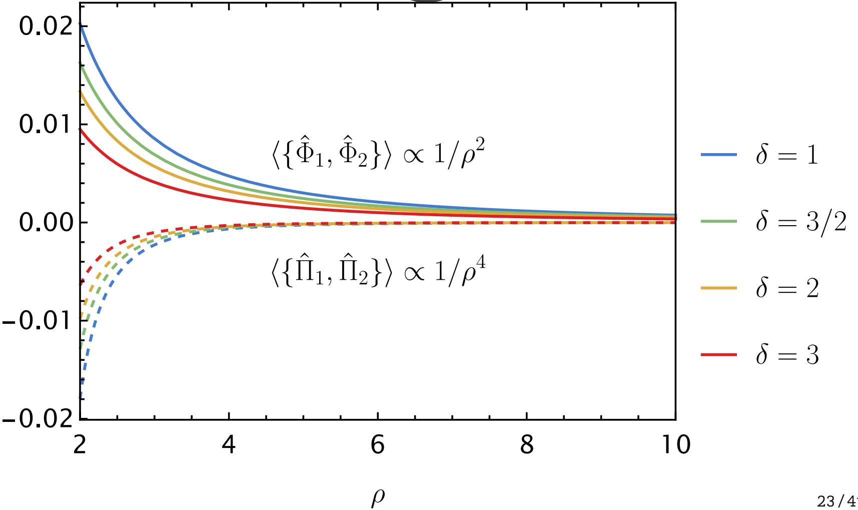


$$\boxed{\text{But no entanglement for any value of } \delta!}$$

Entanglement is more 'sparse' than in $(1+1)$-dimensions

Entanglement between 2 d.o.f.: $D = 3$





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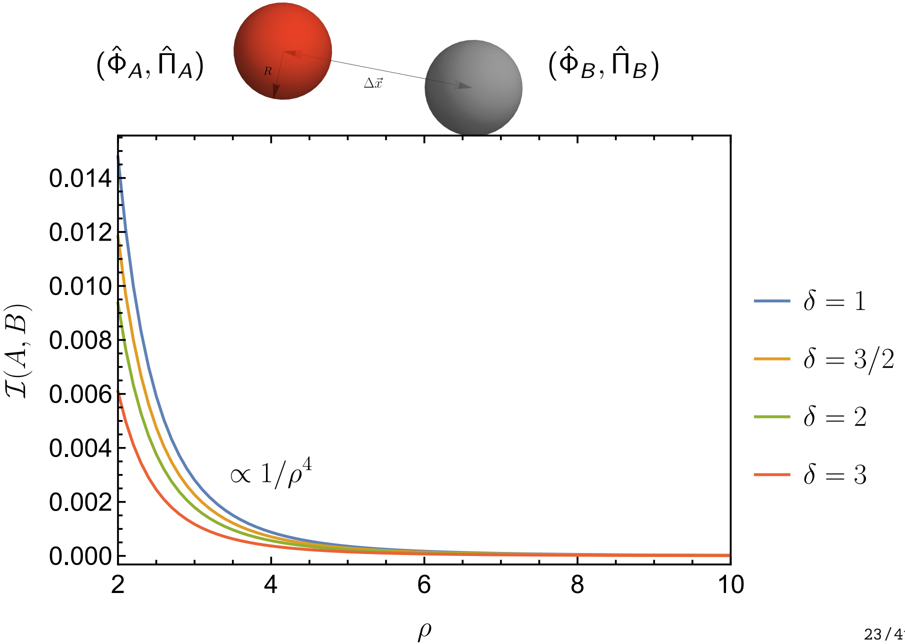

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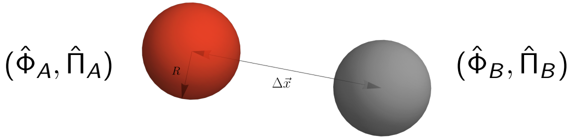

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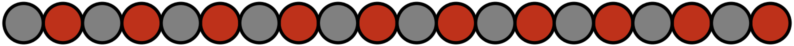
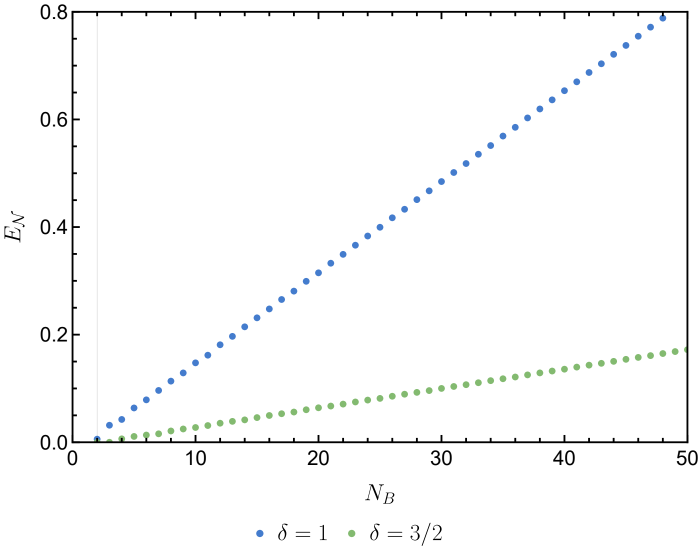
Other configurations

Other configurations: $D = 2$





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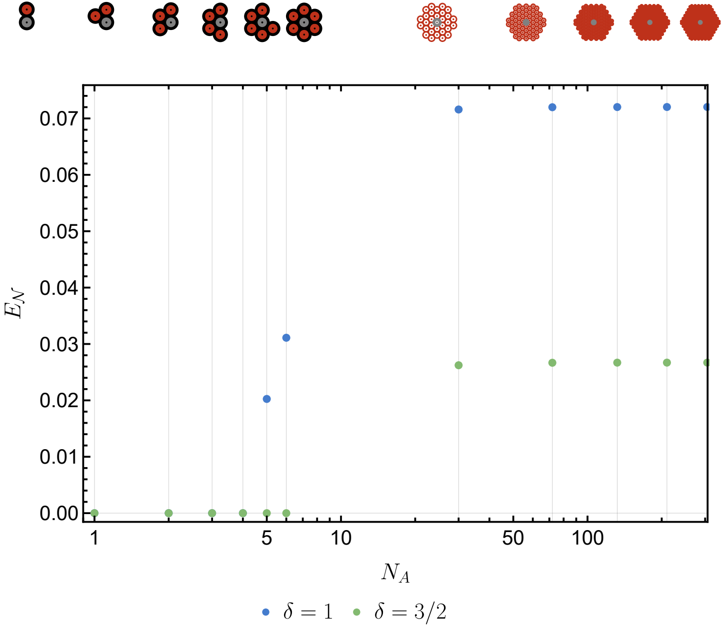
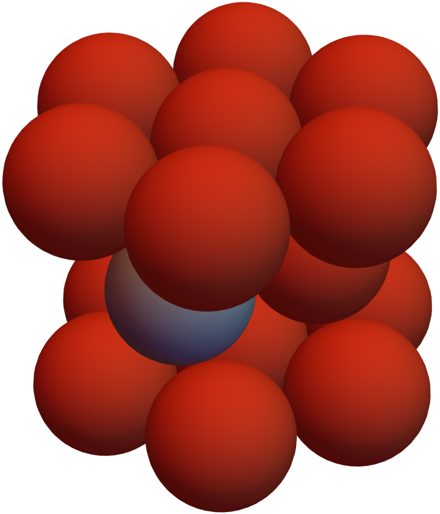


Other configurations: $D = 3$

When $D = 3$, entanglement is quite difficult to find!

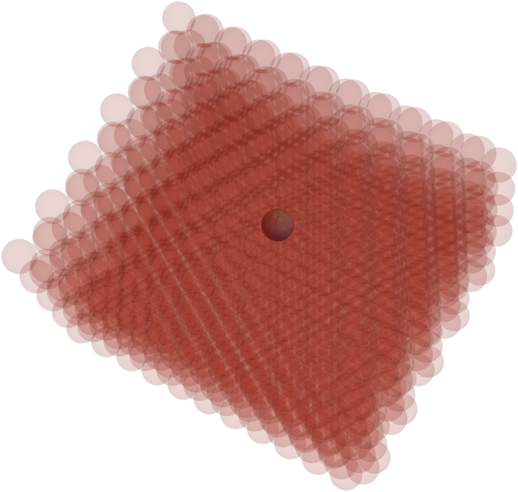
Other configurations: $D = 3$



$$N_A = 16$$

$$N_B = 1$$

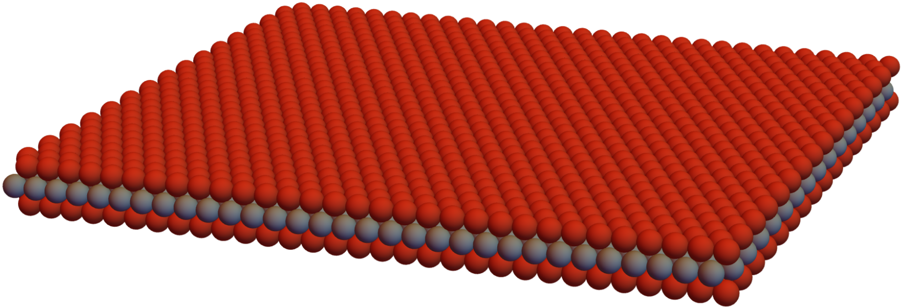
Other configurations: $D = 3$



$$N_A = 1088$$

$$N_B = 1$$

Other configurations: $D = 3$



$$N_A = 1922$$

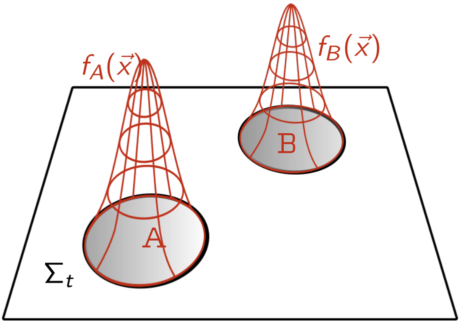
$$N_B = 961$$

Other calculations we have done

- Different smearing functions for field and momentum
- Non-positive smearing functions
- Linear combinations of field and momentum
- Other families of smearing functions
- ...

$\boxed{\text{Entanglement is difficult to find!}}$

Are we saying that there is no entanglement?

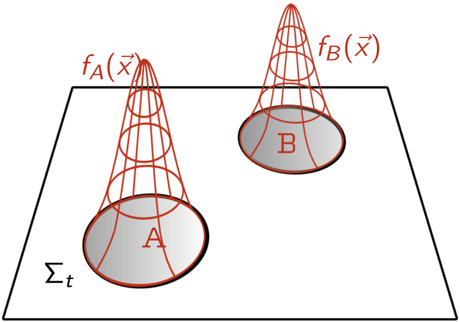


Subsystem A: $(\hat{\Phi}_A, \hat{\Pi}_A)$

Subsystem B: $(\hat{\Phi}_B, \hat{\Pi}_B)$

Are we saying that there is no entanglement?

Absolutely not!



Subsystem A: $(\hat{\Phi}_A, \hat{\Pi}_A)$

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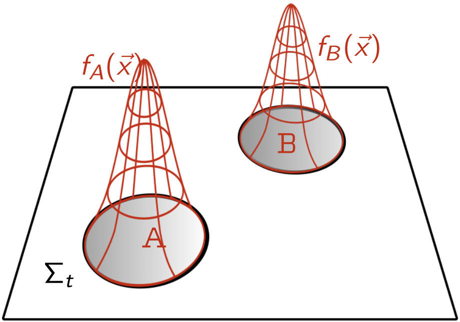

Subsystem A: $(\hat{\Phi}_A, \hat{\Pi}_A)$

Subsystem B: $(\hat{\Phi}_B, \hat{\Pi}_B)$

Change of basis:

$$\hat{\Phi}'_1 = \cosh(r)\,\hat{\Phi}_A + \sinh(r)\,\hat{\Phi}_B \quad \hat{\Phi}'_2 = \cosh(r)\,\hat{\Phi}_B + \sinh(r)\,\hat{\Phi}_A$$
$$\hat{\Pi}'_1 = \cosh(r)\,\hat{\Pi}_A - \sinh(r)\,\hat{\Pi}_B \quad \hat{\Pi}'_2 = \cosh(r)\,\hat{\Pi}_B - \sinh(r)\,\hat{\Pi}_A$$

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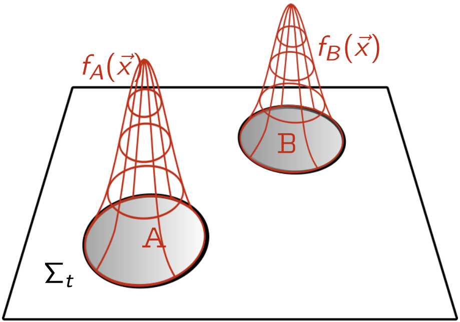
Absolutely not!



Subsystem A: $(\hat{\Phi}_A, \hat{\Pi}_A)$

Subsystem B: $(\hat{\Phi}_B, \hat{\Pi}_B)$

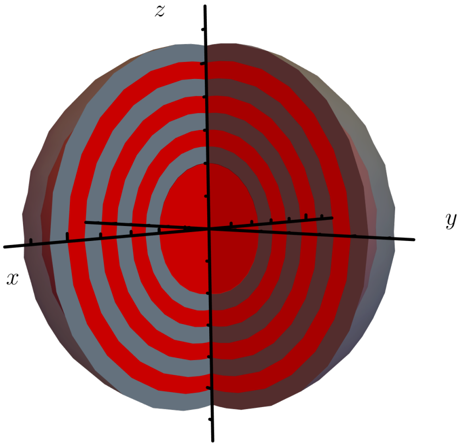
Non-Local!

Change of basis:

$$\hat{\Phi}'_1 = \cosh(r)\,\hat{\Phi}_A + \sinh(r)\,\hat{\Phi}_B \quad \hat{\Phi}'_2 = \cosh(r)\,\hat{\Phi}_B + \sinh(r)\,\hat{\Phi}_A$$
$$\hat{\Pi}'_1 = \cosh(r)\,\hat{\Pi}_A - \sinh(r)\,\hat{\Pi}_B \quad \hat{\Pi}'_2 = \cosh(r)\,\hat{\Pi}_B - \sinh(r)\,\hat{\Pi}_A$$

Subsystems $(\hat{\Phi}'_1, \hat{\Pi}'_1)$ and $(\hat{\Phi}'_2, \hat{\Pi}'_2)$ are entangled $\forall r > r_{\rm min}$

Other configurations: $D = 3$

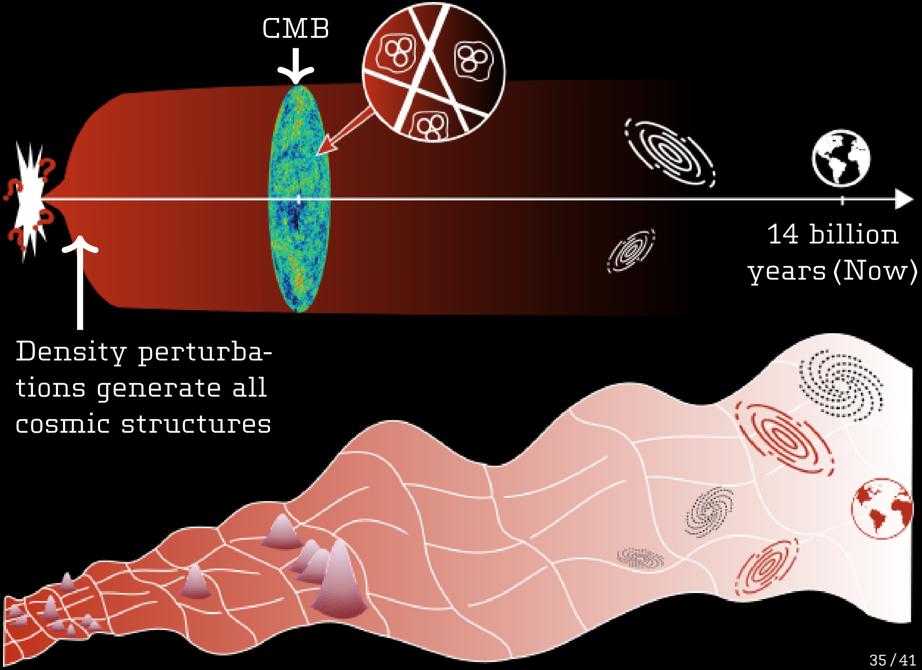


Here we find entanglement! But requires fine-tuning!

Application:

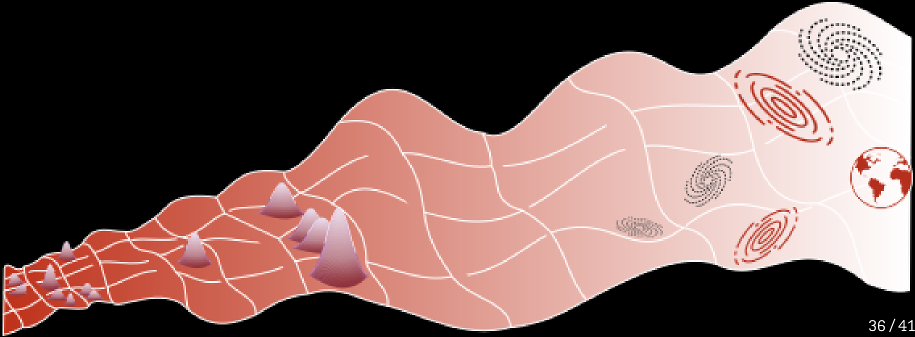
Entanglement in the early universe

The early universe and Inflation



The early universe and Inflation

- Can we test the QUANTUM ORIGIN of the cosmological density perturbations at the end of inflation?
- Same strategy as in Minkowski spacetime
- Toy model: scalar field in de Sitter spacetime



Entanglement in de Sitter spacetime I

- 2 degrees of freedom in (1+3)-dimensional de Sitter spacetime (Poincaré patch)

- More correlations

- Calculation more subtle due to the IR divergences!

- One can show that the Log Neg is finite and independent of the regulator

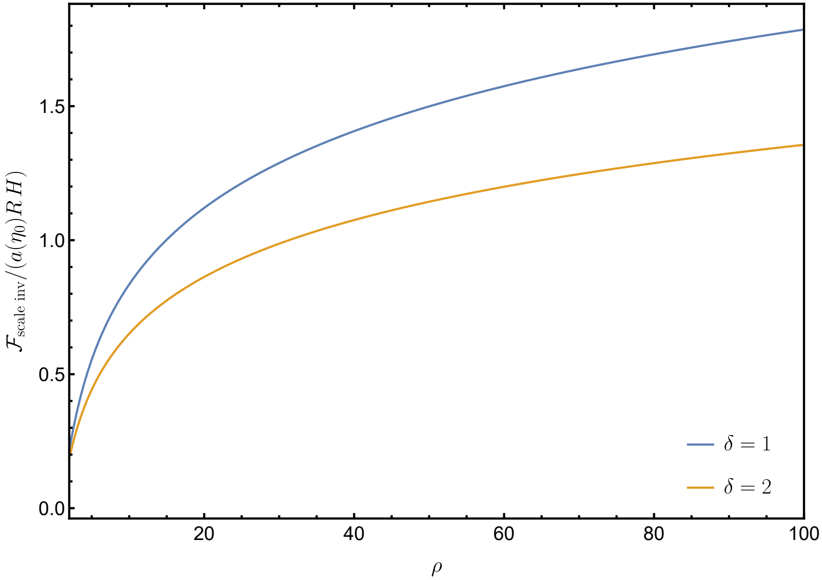
Entanglement in de Sitter spacetime I

- 2 degrees of freedom in (1+3)-dimensional de Sitter spacetime (Poincaré patch)
- More correlations
- Calculation more subtle due to the IR divergences!
- One can show that the Log Neg is finite and independent of the regulator

We find no entanglement for any of the functions f_δ!

Entanglement in de Sitter spacetime II

$$\tilde{\nu}_-^{\mathtt{dS}} = \sqrt{(\tilde{\nu}_-^{\mathtt{Mink}})^2 + \mathcal{F}_{\mathtt{scale\ inv}}}\,,$$



Conclusions

Conclusions

- Entanglement is there, but it is distributed in a subtle manner

- It can be found by either:

  (i) Involving infinitely many d.o.f. (experimentally impossible)

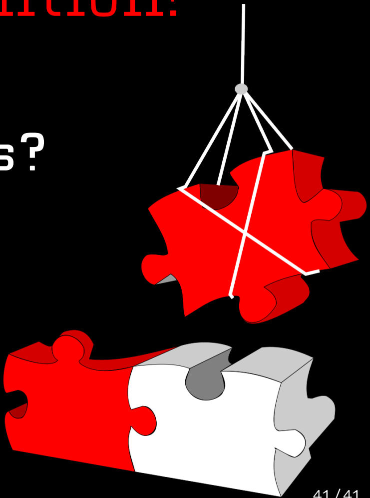
  (ii) Carefully selecting the d.o.f. (experimentally difficult)

- Harder to find with increasing spacetime dimension

- In de Sitter spacetime we find more correlations, but they do not contain entanglement

Thank you very much for your attention!

Questions?



Want more details?

Back-up slides

The Reeh-Schlieder Theorem

The Reeh-Schlieder Theorem

The Reeh-Schlieder theorem states that one can generate the full Hilbert space of a free QFT, $\mathcal{H}_0$, by restricting attention to the set of smearing functions compactly supported in an arbitrary small open set $V \subset \Sigma$, and a corresponding small neighborhood $\mathcal{U}_V$ of V in spacetime.

Violation of causality?

Back to QM and fully entangled states: If $|\Psi\rangle$ is fully entangled, any state in $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as $\hat{O}_A \otimes \hat{I}_B |\Psi\rangle$.

There is no violation of causality because, if we restrict $\hat{O}_A$ to be unitary, then

$$\langle\Psi|(\hat{U}_A^\dagger \otimes \hat{I}_B)\hat{O}_B(\hat{U}_A \otimes \hat{I}_B)|\Psi\rangle = \langle\Psi|\hat{O}_B\hat{U}_A^\dagger U_A)|\Psi\rangle = \langle\Psi|\hat{O}_B|\Psi\rangle$$

True for all operators $\hat{O}_B$

How do we compute stuff?

The von Neumann entropy of Gaussian states

$$S(\hat{\rho}|_A) = -\mathrm{Tr}(\hat{\rho}|_A \log_2 \hat{\rho}|_A) = \sum_{i=1}^{N} f(\nu_i)\,,$$

where

$$f(\nu) = \frac{\nu+1}{2}\log_2\left(\frac{\nu+1}{2}\right) - \frac{\nu-1}{2}\log_2\left(\frac{\nu-1}{2}\right)$$

Mutual Information

- 2 smeared degrees of freedom

$$\mathcal{I}(A,B) = S(\hat{\rho}^{(A)}_{N=1}) + S(\hat{\rho}^{(B)}_{N=1}) - S(\hat{\rho}_{N=2})\,,$$

where

- $S(\hat{\rho})$ is the von Neumann entropy
- $\hat{\rho}^{(A)}_{N=1}$ is the reduced density matrix of the first degree of freedom
- $\hat{\rho}^{(B)}_{N=1}$ is the reduced density matrix of the second degree of freedom
- $\hat{\rho}_{N=2}$ is the total density matrix.

The covariance matrix

- The covariance matrix: $\sigma = \mathrm{Tr}[\{(\hat{r} - \langle\hat{r}\rangle), (\hat{r} - \langle\hat{r}\rangle)^{\mathrm{T}}\}\hat{\rho}]$
- For 2 'smeared' d.o.f.'s in Minkowski spacetime:

$$\sigma = 2\begin{pmatrix} \langle\hat{\Phi}_1^2\rangle & 0 & \langle\{\hat{\Phi}_1, \hat{\Phi}_2\}\rangle & 0 \\ 0 & \langle\hat{\Pi}_1^2\rangle & 0 & \langle\{\hat{\Pi}_1, \hat{\Pi}_2\}\rangle \\ \langle\{\hat{\Phi}_1, \hat{\Phi}_2\}\rangle & 0 & \langle\hat{\Phi}_2^2\rangle & 0 \\ 0 & \langle\{\hat{\Pi}_1, \hat{\Pi}_2\}\rangle & 0 & \langle\hat{\Pi}_2^2\rangle \end{pmatrix},$$

- The symplectic eigenvalues are the (absolute value of the) eigenvalues of the matrix $i\,\Omega\,\sigma$, where Ω is the symplectic form
- In this case $\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. If $\langle\hat{\Phi}_1^2\rangle = \langle\hat{\Phi}_2^2\rangle = \langle\hat{\Phi}^2\rangle$

$$\nu_\pm = 2\sqrt{(\langle\hat{\Phi}^2\rangle \pm \langle\{\hat{\Phi}_1, \hat{\Phi}_2\}\rangle)(\langle\hat{\Pi}^2\rangle \pm \langle\{\hat{\Pi}_1, \hat{\Pi}_2\}\rangle)} \geq 1$$

The partially transposed covariance matrix

- In the previous example, the partially transposed covariance matrix is

$$\tilde{\sigma} = 2\begin{pmatrix} \langle \hat{\Phi}_1^2 \rangle & 0 & \langle \{\hat{\Phi}_1, \hat{\Phi}_2\} \rangle & 0 \\ 0 & \langle \hat{\Pi}_1^2 \rangle & 0 & -\langle \{\hat{\Pi}_1, \hat{\Pi}_2\} \rangle \\ \langle \{\hat{\Phi}_1, \hat{\Phi}_2\} \rangle & 0 & \langle \hat{\Phi}_2^2 \rangle & 0 \\ 0 & -\langle \{\hat{\Pi}_1, \hat{\Pi}_2\} \rangle & 0 & \langle \hat{\Pi}_2^2 \rangle \end{pmatrix}.$$

- The symplectic eigenvalues of $\tilde{\sigma}$ are given by

$$\tilde{\nu}_\pm = 2\sqrt{(\langle \hat{\Phi}^2 \rangle \pm \langle \{\hat{\Phi}_1, \hat{\Phi}_2\} \rangle)(\langle \hat{\Pi}^2 \rangle \mp \langle \{\hat{\Pi}_1, \hat{\Pi}_2\} \rangle)}$$

Analytical expressions

$$\sigma = 2N_\delta^2 \begin{pmatrix} \mathcal{J}(-1,\delta,\mu) & 0 & \mathcal{L}(-1,\delta,\mu,\rho) & 0 \\ 0 & \mathcal{J}(1,\delta,\mu) & 0 & \mathcal{L}(1,\delta,\mu,\rho) \\ \mathcal{L}(-1,\delta,\mu,\rho) & 0 & \mathcal{J}(-1,\delta,\mu) & 0 \\ 0 & \mathcal{L}(1,\delta,\mu,\rho) & 0 & \mathcal{J}(1,\delta,\mu) \end{pmatrix},$$

where $N_\delta^2 = R^D \frac{2^{2\delta}\Gamma(1+D/2+2\delta)\Gamma(1+\delta)^2}{\Gamma(1+2\delta)\Gamma(D/2)}$. If $D > 1$,

$$\mathcal{J}(\lambda,\delta,\mu=0) = R^{-(D+\lambda)} \frac{\Gamma(1+2\delta-\lambda)\Gamma\left(\frac{D+\lambda}{2}\right)}{2^{1+2\delta-\lambda}\Gamma\left(1+\delta-\frac{\lambda}{2}\right)^2\Gamma\left(1+2\delta+\frac{D-\lambda}{2}\right)},$$

and

$$\mathcal{L}(\lambda,\delta,\mu=0,\rho_{ij}) = R^{-(D+\lambda)}\rho^{-(D+\lambda)} \frac{\Gamma(D/2)\Gamma\left(\frac{D+\lambda}{2}\right)}{2^{1+2\delta-\lambda}\Gamma\left(\frac{D}{2}+1+\delta\right)^2\Gamma\left(-\frac{\lambda}{2}\right)}$$
$$\times\, {}_3F_2\left(1+\frac{\lambda}{2}, \frac{D+\lambda}{2}, \frac{D+1}{2}+\delta; \frac{D}{2}+1+\delta, D+1+2\delta; \frac{4}{\rho_{ij}^2}\right).$$

Entanglement decreases with D

Consider a single degree of freedom. The symplectic eigenvalue is given by

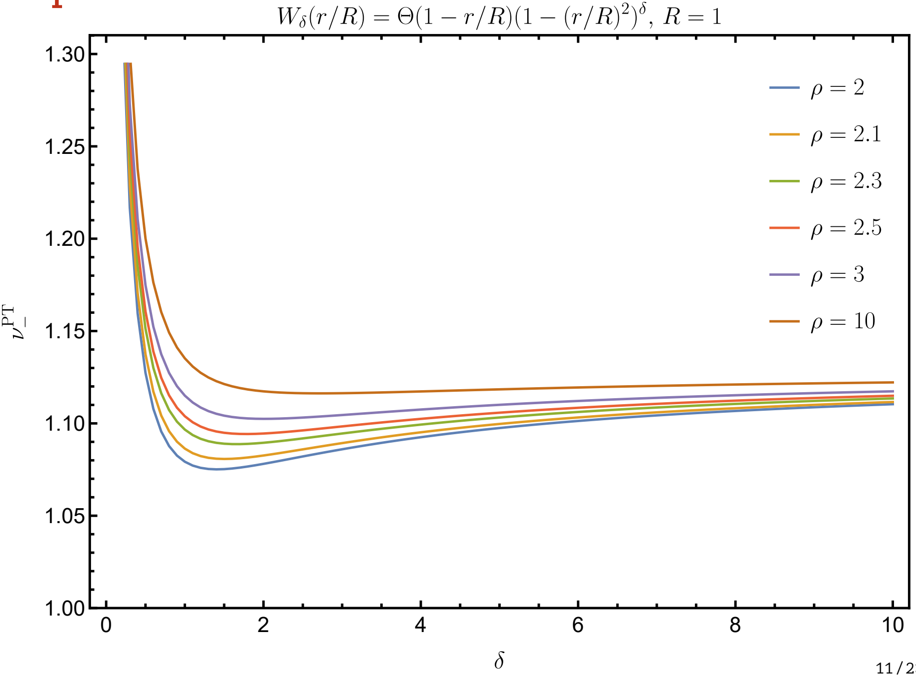
$$\nu = \frac{\Gamma(\delta+1)^2\Gamma\left(\frac{D}{2}+2\delta+1\right)\sqrt{\frac{\Gamma\left(\frac{D-1}{2}\right)\Gamma\left(\frac{D+1}{2}\right)\Gamma(2\delta)\Gamma(2\delta+2)}{\Gamma\left(\frac{1}{2}(D+4\delta+1)\right)\Gamma\left(\frac{1}{2}(D+4\delta+3)\right)}}}{\Gamma\left(\frac{D}{2}\right)\Gamma\left(\delta+\frac{1}{2}\right)\Gamma\left(\delta+\frac{3}{2}\right)\Gamma(2\delta+1)},$$

We study the behavior of ν in the smooth limit $\delta \to \infty$ in a spacetime with $D \to \infty$:

$$\lim_{D\to\infty}\left(\lim_{\delta\to\infty}\nu^2\right) = \lim_{D\to\infty}\frac{\Gamma\left(\frac{D-1}{2}\right)\Gamma\left(\frac{D+1}{2}\right)}{\Gamma\left(\frac{D}{2}\right)^2} = 1$$

$$\lim_{\delta\to\infty}\left(\lim_{D\to\infty}\nu^2\right) = \lim_{\delta\to\infty}\frac{\Gamma(2\delta)\Gamma(\delta+1)^4\Gamma(2\delta+2)}{\Gamma\left(\delta+\frac{1}{2}\right)^2\Gamma\left(\delta+\frac{3}{2}\right)^2\Gamma(2\delta+1)^2} = 1\,.$$

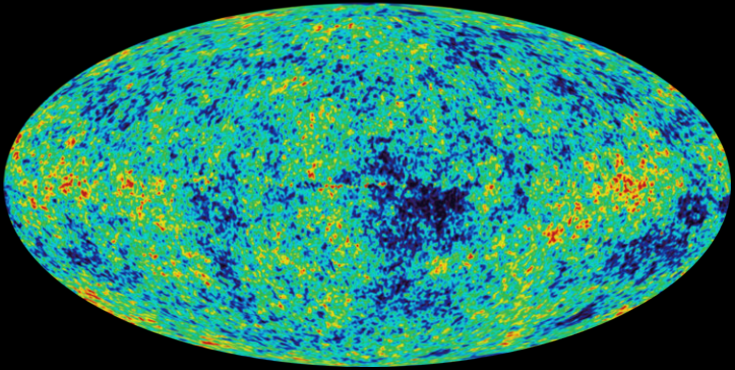
δ-dependence

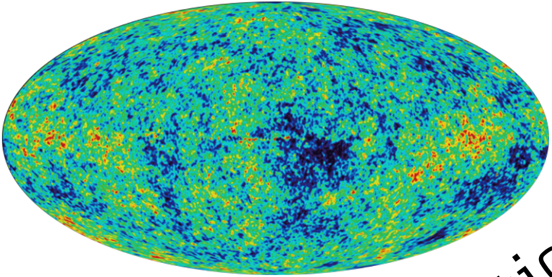


Entanglement in the early universe

The CMB

- The CMB: Cosmic Microwave Background
- Thermal black body spectrum at a temperature $T = 2.73$ K
- Small inhomogeneities: $\Delta T / T \sim 10^{-5}$



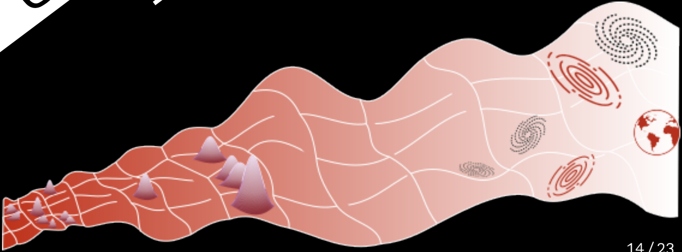


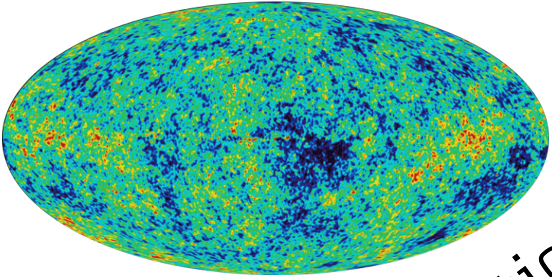
Observations

Theory

Cosmic structures from QUANTUM fluctuations

Density perturbations





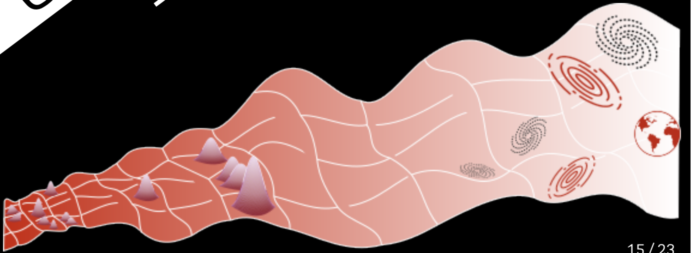
Observations

Quantum what?
All we need is classical physics...

Theory

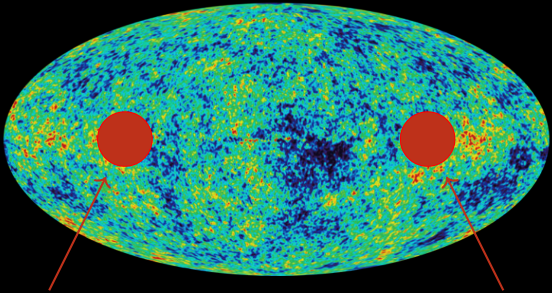
Cosmic structures from QUANTUM fluctuations

Density perturbations



Questions to answer:

- What are the genuinely quantum features in the primordial perturbations at the end of inflation?
- Why does the CMB look so classical?
- Is there a way to test the quantum origin of the primordial perturbations?



$(\hat{\Phi}[\text{pixel1}], \hat{\Pi}[\text{pixel1}])$ $(\hat{\Phi}[\text{pixel2}], \hat{\Pi}[\text{pixel2}])$

(Bi-partite) Entanglement in real space in QFT?[1]

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[1] Martin and Vennin(2021), *JCAP*; Espinosa-Portalés and Vennin(2022), *JCAP*

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