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CPT - Marseille 16/04/2024

LIGHT-CONE THERMODYNAMICS

based on arXiv:2307.12031
in collaboration with Alejandro Perez
and De Lorenzo, Perez 2018

LIGHT-CONE THERMODYNAMICS

Light-cones in 4D Minkowski spacetime are conformal Killing horizons

The conformal group is isomorphic to SO(5,1).

Any generator ξ defines a Conformal Killing Field such that

$$\mathscr{L}_{\xi}\eta_{\mu\nu} = \frac{\psi}{2}\eta_{\mu\nu},$$

$$\psi = \nabla_{\mu}\xi^{\mu}$$

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

LIGHT-CONE THERMODYNAMICS

The only generators that don't contain angular components are

$$D = r\partial_r + t\partial_t$$

$$P_0 = \partial_t$$

$$K_0 = -2tD - (r^2 - t^2)P_0$$

The most general radial MCKF is

$$\xi = -aK_0 + bD + cP_0$$
$$= (av^2 + bv + c)\partial_v + (au^2 + bu + c)\partial_u$$

$$u = t - r, \quad v = t + r$$

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

LIGHT-CONE THERMODYNAMICS
THE CAUSAL STRUCTURE OF ξ

The norm of $\xi = (av^2 + bv + c)\partial_v + (au^2 + bu + c)\partial_u$

is given by $\xi \cdot \xi = -(av^2 + bv + c)(au^2 + bu + c)$

It's null along the light cones defined by

$$u = u_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$$v = v_\pm = u_\pm$$

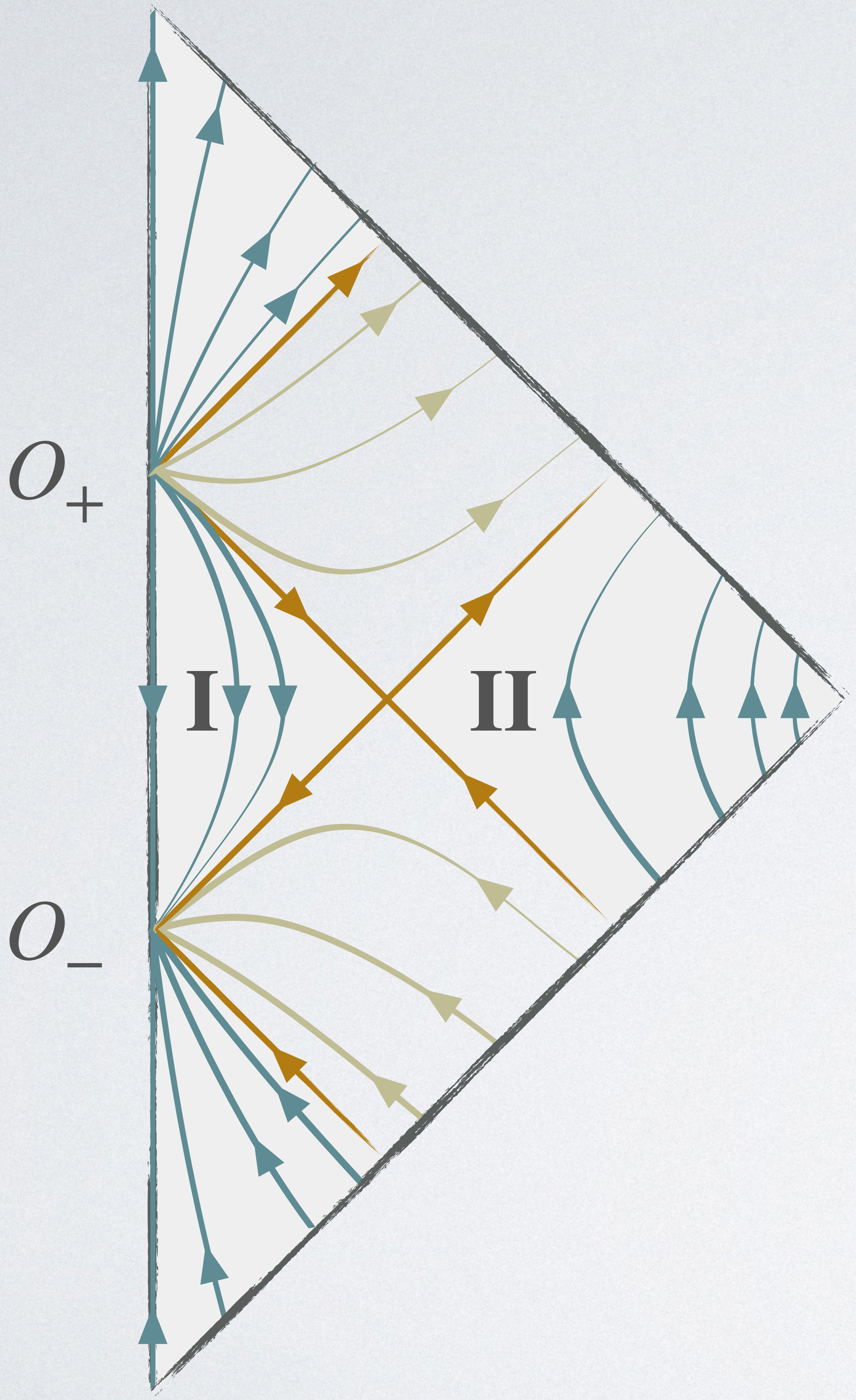
ξ vanishes at the intersection of $u = u_-$, $v = v_+$

$$t_H := -\frac{b}{2a} \qquad r_H := \frac{v_+ - u_-}{2} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

LIGHT-CONE THERMODYNAMICS

Martinetti, Rovelli (2003)
Kay, Wald (1991)
Hislop, Longo (1982)
Jacobson, Visser (2022)

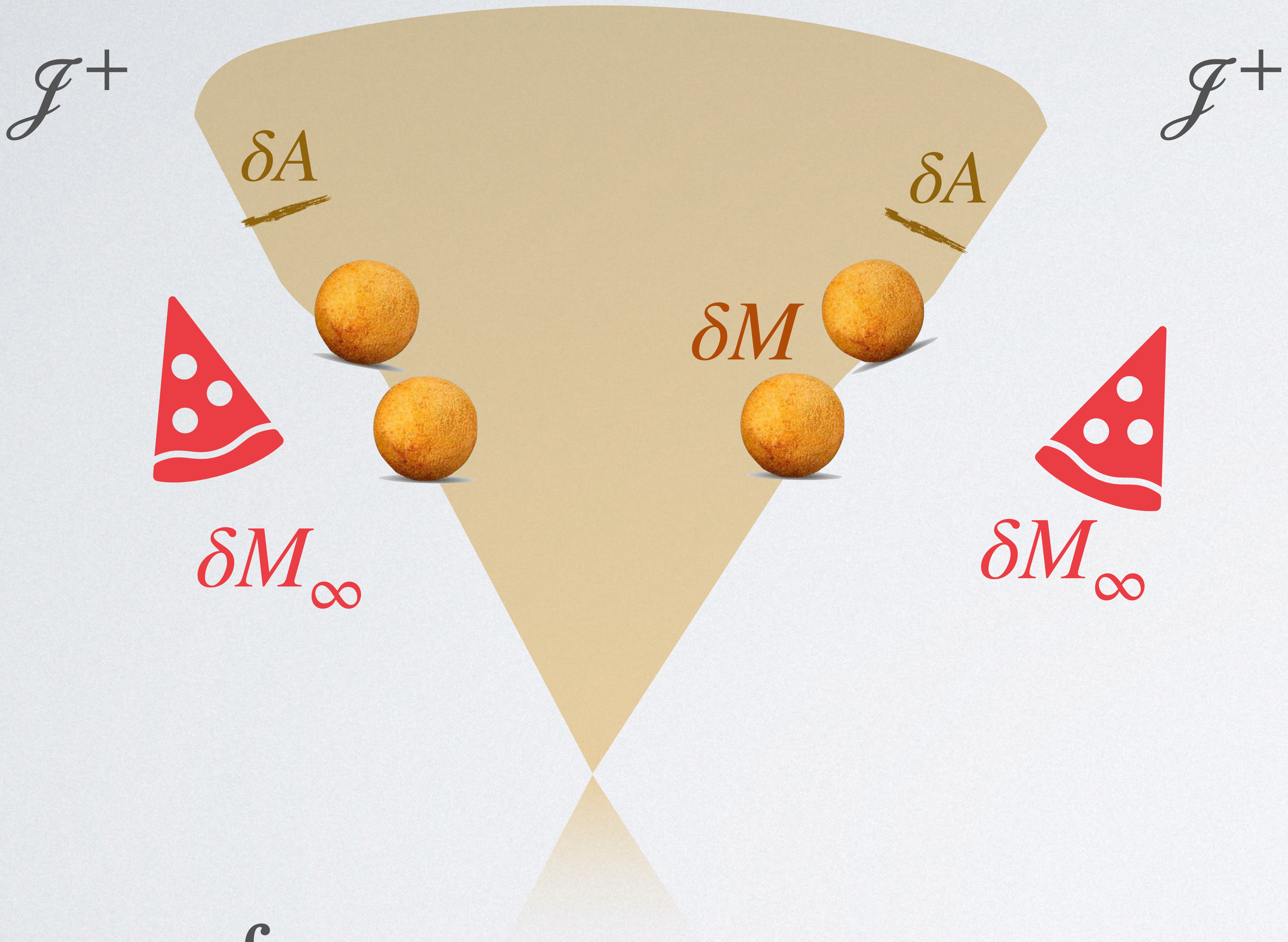


- ξ defines two Conformal Killing Horizons at the past and future light cones of $O_\pm = (t = v_\pm, r = 0)$
- Each horizon has constant (conformally invariant) surface gravity defined via $\nabla_\mu \left(\xi \cdot \xi\right) \hat{=} -2\kappa \eta_{\mu\nu} \xi^\nu$
- Events in spacetime are separated as in a spherical charged black hole, also in the extremal case.
- The topology of the horizons is $S^2 \otimes \mathbb{R}$

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

LAWS OF LIGHT-CONE THERMODYNAMICS

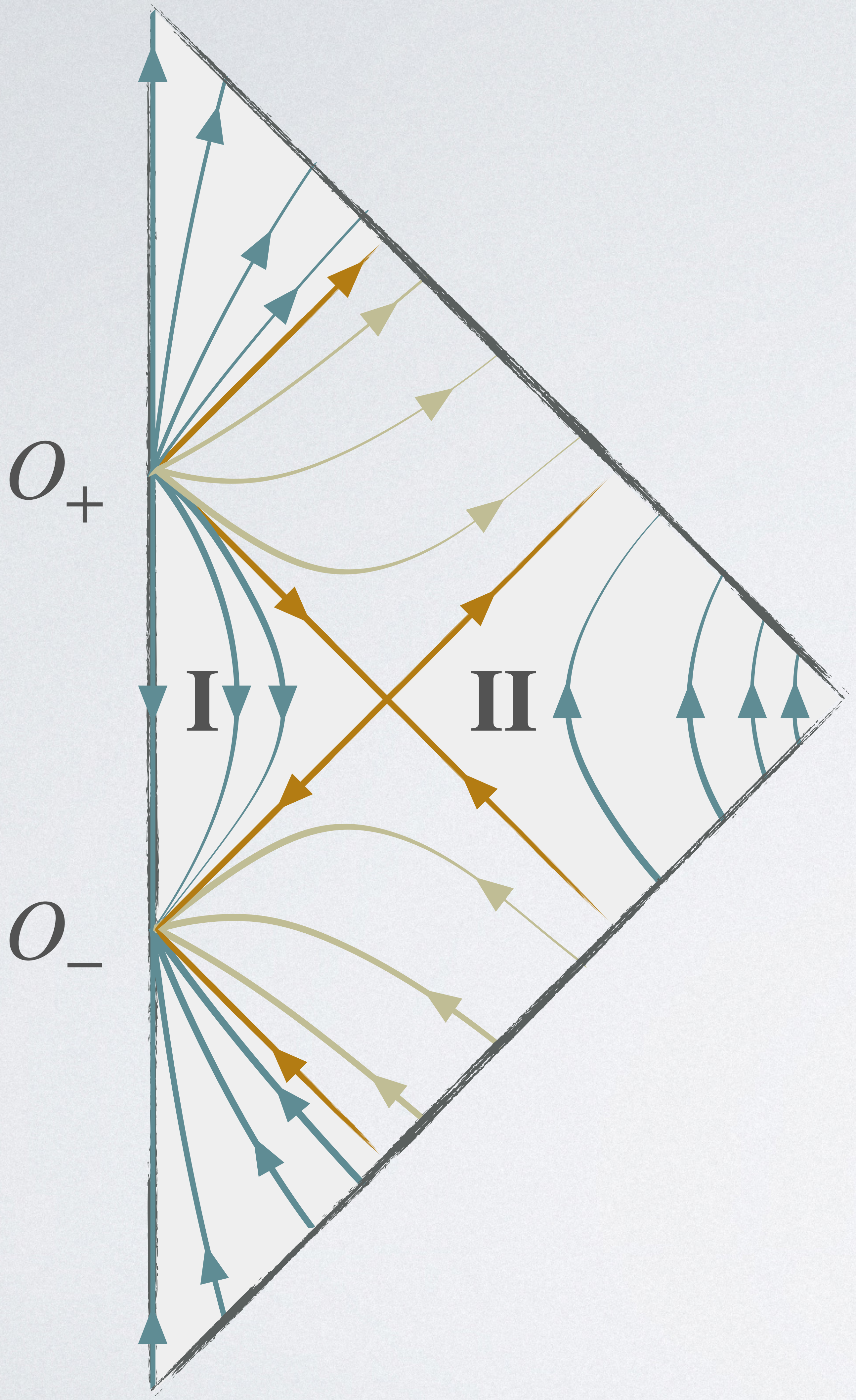
from T De Lorenzo, A Perez (2018)



$$M := \int_\Sigma T_{\mu\nu}\xi^\mu d\Sigma^\nu$$

0. constant surface gravity κ on the conformal Killing horizon
1. under conformally-invariant matter perturbations $\delta M = \frac{\kappa}{2\pi}\frac{\delta A}{4} + \delta M_\infty$
2. $\delta A \geq 0$
3. extremal radial **MCKFs** have vanishing "temperature" and vanishing "entropy"

LIGHT-CONE THERMODYNAMICS



- The integral lines of ξ correspond to observers accelerating radially with constant $a = \kappa \dfrac{r}{r_H\sqrt{\xi_\mu \xi^\mu}}$

- The temperature measured by an Unruh-DeWitt detector (which breaks conformal invariance) will be $a/2\pi$.

- To detect the temperature κ, a scale invariant detector should be built. Its interest rely rather on global features than local measurements.

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

A Perez, SR 2023
Unruh 1976

Goal: writing the Minkowski vacuum $\left|0\right\rangle_M$ as a superposition of particle states associated to ξ

how to characterize positive frequency solutions of the KG equation with respect to inertial time on a light cone?

$$\square \Phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)\Phi = 0$$

$$= \left(-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)\Phi(x)$$

LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

A Perez, SR 2023

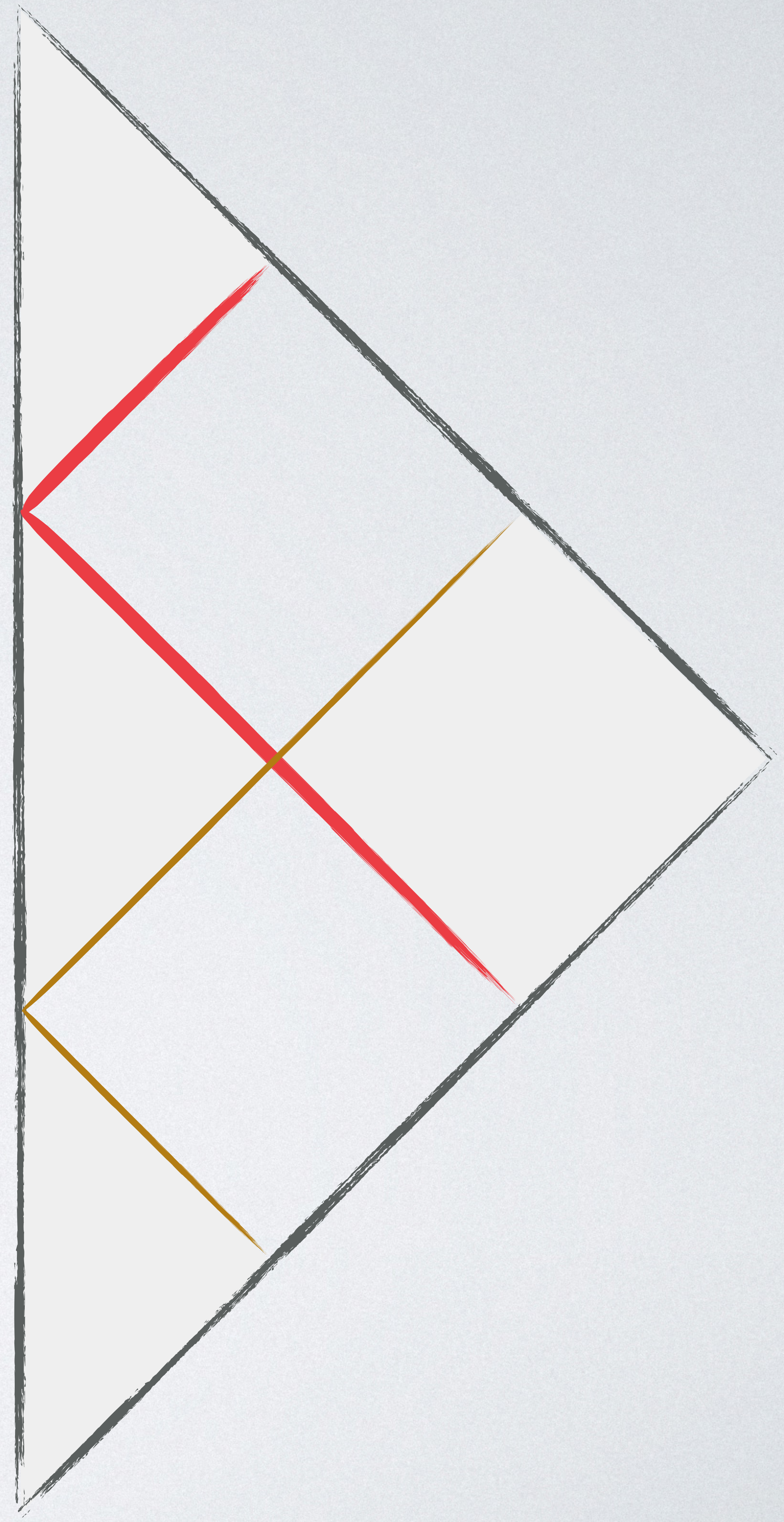
$$\Phi(x) = e^{-i\omega t} Y_{\ell m}(\theta, \varphi) R_\ell(r)$$

KG equation: $$\left(\omega^2 + \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{r} - \frac{\ell(\ell+1)}{r^2}\right) R_\ell = 0$$

solved by the spherical Bessel functions

$$R_\ell(r) = j_\ell(\omega r)$$

Solutions are completely characterized in the union of the past and future light cone.

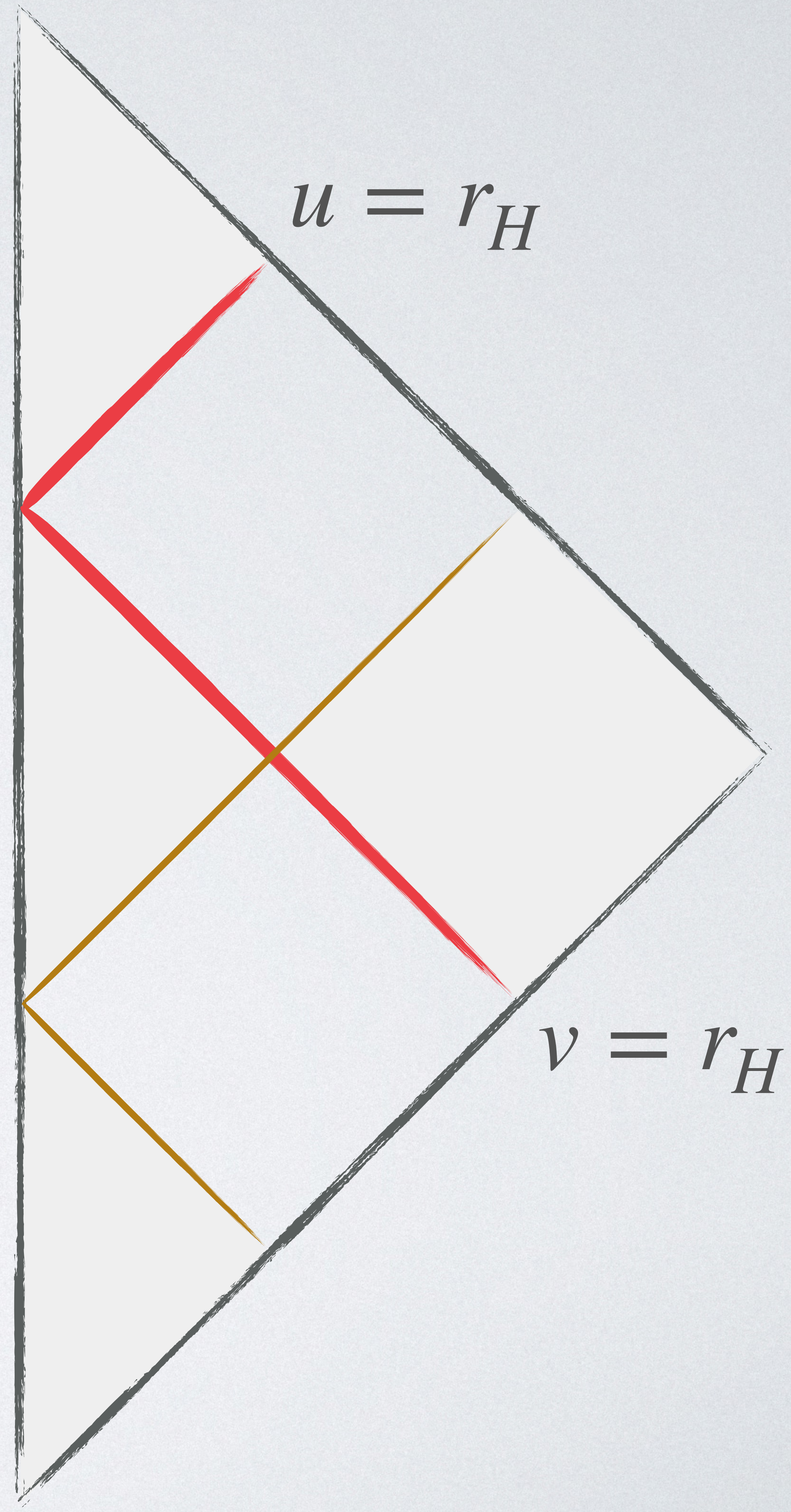


LIGHT-CONE THERMODYNAMICS

A Perez, SR 2023

decomposition of the Minkowski vacuum

$$\Phi(x) = e^{-i\omega t} Y_{\ell m}(\theta, \varphi) j_\ell(\omega r) \qquad t = \frac{v+u}{2}, \quad r = \frac{v-u}{2}$$



By means of a coordinate transformation we can set $b = 0$, $\quad v_\pm := \pm\, r_H$

at $u = r_H$, $\qquad \Phi = e^{-i\omega\left(\frac{v + r_H}{2}\right)} Y_{\ell m} j_\ell \left(\omega \frac{v - r_H}{2}\right), \qquad v > r_H$

at $v = r_H$, $\qquad \Phi = e^{-i\omega\left(\frac{u + r_H}{2}\right)} Y_{\ell m} j_\ell \left(\omega \frac{r_H - u}{2}\right), \qquad u < r_H$

which can be written in terms of a single variable which covers the whole real line.

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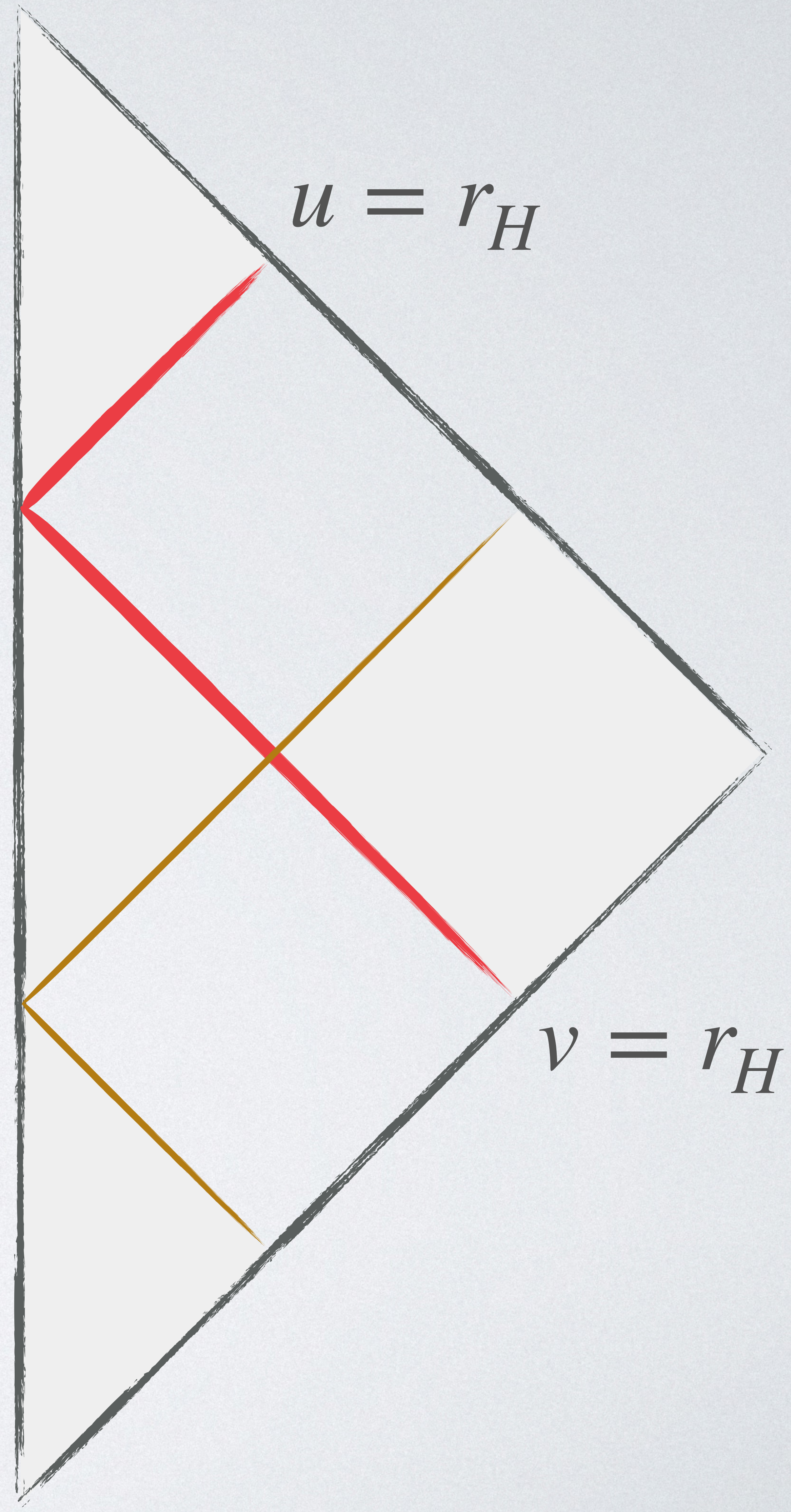
decomposition of the Minkowski vacuum

In terms of a single null variable z spanning from $-\infty$ to $+\infty$, solutions of the KG equation take the form

$$\Phi = e^{-i\omega\left(\frac{z+v_+}{2}\right)} Y_{\ell m} j_\ell\left(\omega \frac{z-v_+}{2}\right),$$



Φ is analytic. We want to characterize $\omega > 0$ solutions.
Let us look where they are bounded. For large $|z|$:

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decomposition of the Minkowski vacuum

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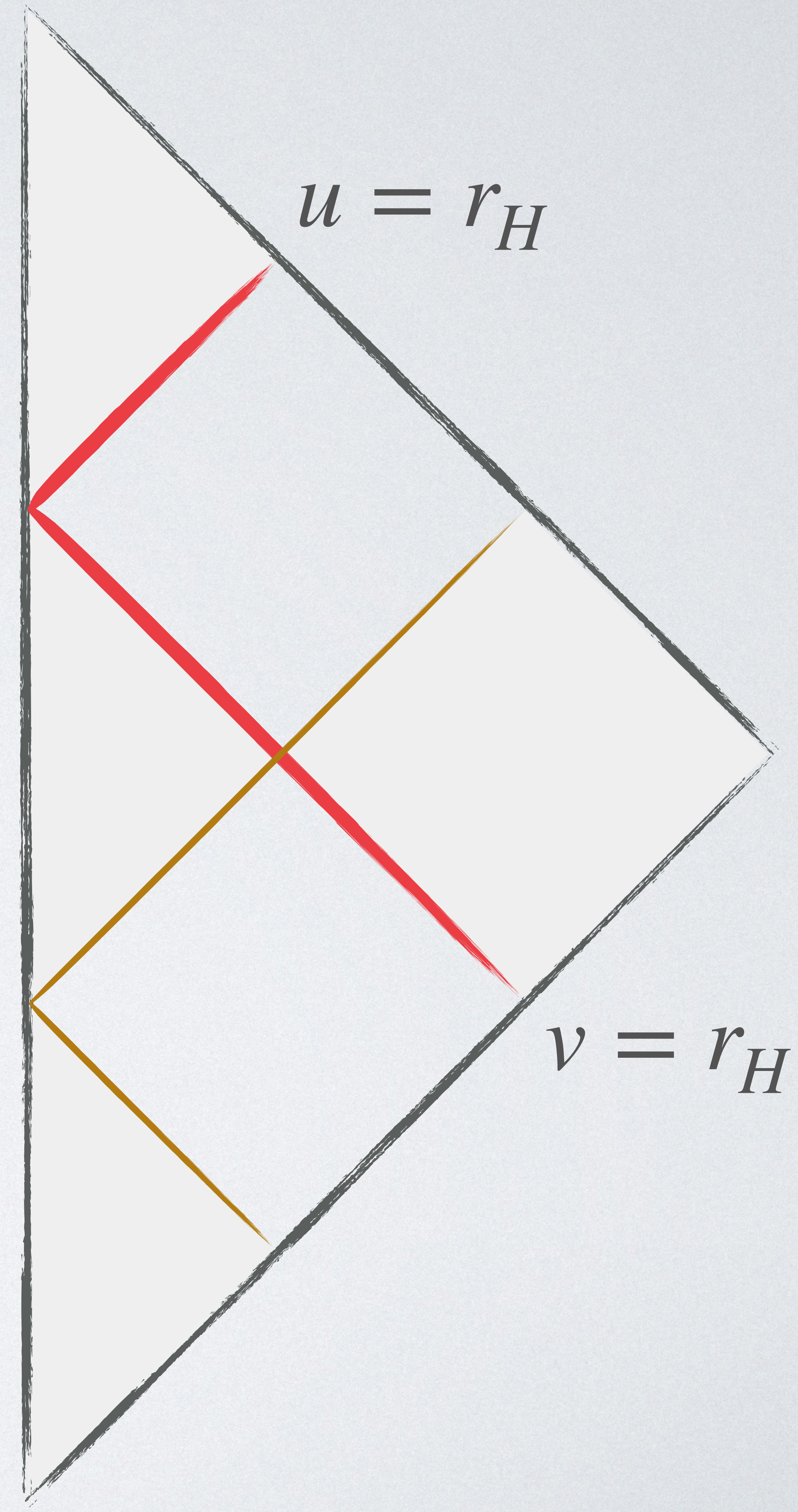
Φ is analytic. We want to characterise $\omega > 0$ solutions.
Let us look where they are bounded. For large $|z|$:

$$\Phi \approx e^{-i\omega\left(\frac{z+r_H}{2}\right)} Y_{\ell m} \sin\left(\omega\frac{z-r_H}{2} - \frac{\ell\pi}{2}\right) / \left(\omega\left(z-r_H\right)\right),$$

$$= Y_{\ell m}\frac{A}{z-r_H}e^{-i\omega z}$$

bounded for $\mathbf{Im}(z) < 0$

$\omega > 0$ solutions are **analytic** functions of z **bounded** in the lower-half complex plane $\mathbf{Im}(z)$



LIGHT-CONE THERMODYNAMICS
decomposition of the Minkowski vacuum

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What are the positive-frequency solutions associated to ξ ?

Consider this coordinate transformation

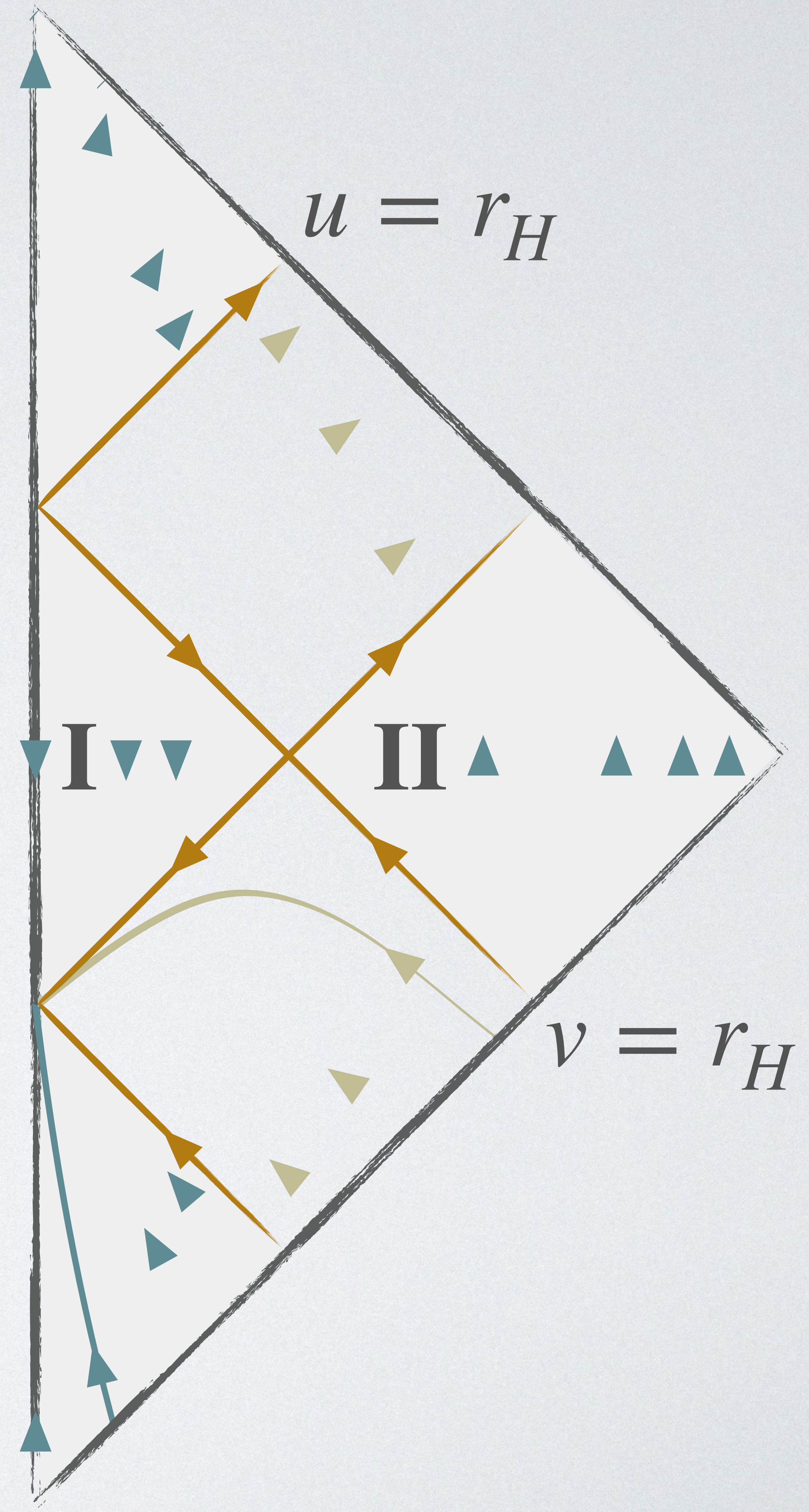
$$t = \frac{r_H \sinh(\kappa\tau)}{\cosh(\kappa\rho) - \cosh(\kappa\tau)}$$

$$r = -\frac{r_H \sinh(\kappa\rho)}{\cosh(\kappa\rho) - \cosh(\kappa\tau)}$$

$$u = -r_H \coth\left(\frac{\kappa\tilde{u}}{2}\right)$$

$$v = -r_H \coth\left(\frac{\kappa\tilde{v}}{2}\right)$$

$$ds_M^2 = \Omega_{\mathbf{II}}(\tau,\rho)\left(-d\tau^2 + d\rho^2 + \kappa^{-2}\sinh^2(\kappa\rho)dS^2\right)$$



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decomposition of the Minkowski vacuum

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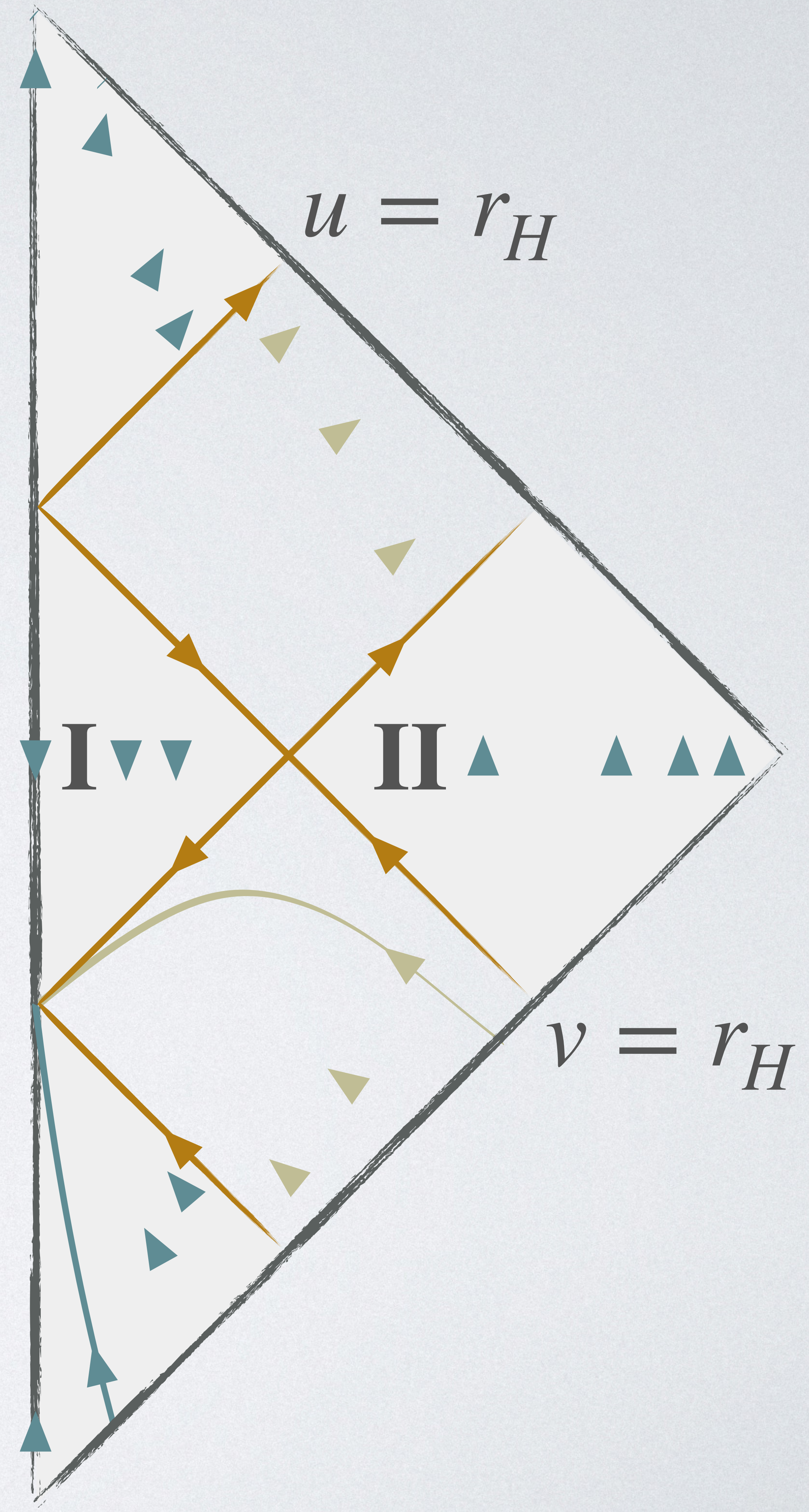
$$ds^2_M = \Omega_{\mathbf{II}}(\tau,\rho)\left(-d\tau^2 + d\rho^2 + \kappa^{-2}\sinh^2(\kappa\rho)dS^2\right)$$

Under a conformal transformation $g_{\mu\nu} \to g'_{\mu\nu} = C^2 g_{\mu\nu}$,

solutions of $\left(\Box - \frac{1}{6}R\right)\Phi = 0$ are mapped via $\Phi \to C^{-1}\Phi$.

We look for solutions in the conformal metric.
The solution takes the form: $Q_{\omega\ell m} = e^{-i\omega\tau}Y_{\ell m}\frac{R_{\omega\ell\pm}(\rho)}{\sinh(\kappa\rho)}$



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decomposition of the Minkowski vacuum

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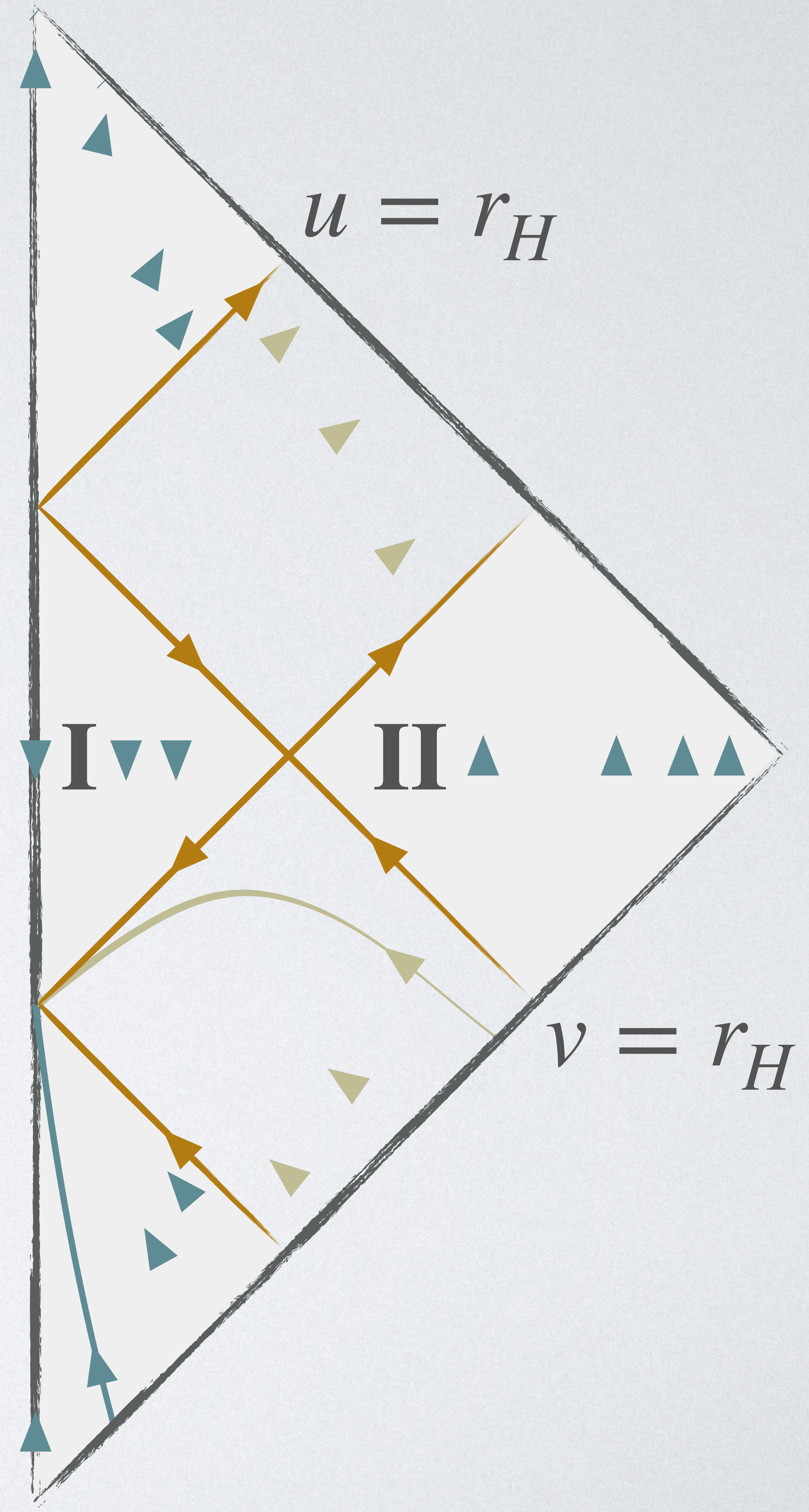
$$Q_{\omega\ell m} = e^{-i\omega\tau} Y_{\ell m} \frac{R_{\omega\ell\pm}(\rho)}{\sinh(\kappa\rho)}$$

KG equation: $$\left(\frac{\partial^2}{\partial\rho^2} + \omega^2 - \frac{\ell(\ell+1)\kappa^2}{\sinh^2(\kappa\rho)}\right) R_{\omega\ell\pm}(\rho) = 0$$

Near the past boundary of region II ($v = r_H$) $\rho \to +\infty$ and the effective potential vanishes. Thus

$$Q_{\omega\ell m} = e^{-i\omega(\tau \pm \rho)} \frac{Y_{\ell m}}{\sinh(\kappa\rho)}$$



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decomposition of the Minkowski vacuum

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studying solutions near the light cone in region **II** (and **III**) gives

$$u = -\, r_H \coth\left(\frac{\kappa \tilde{u}}{2}\right)$$

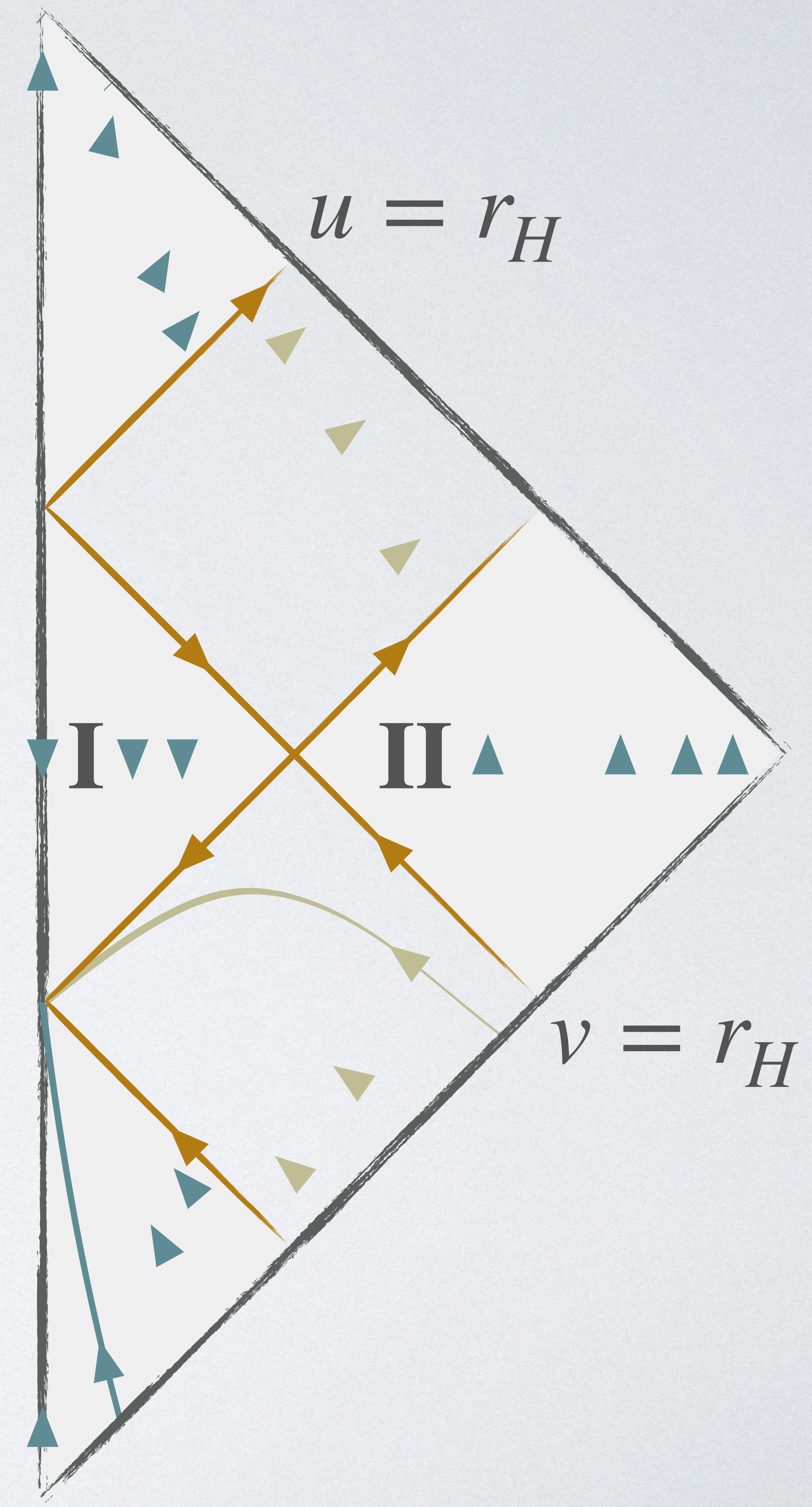
$$\Phi_{\mathbf{II}}^{\omega \ell m} = \Omega_{\mathbf{II}}^{-1} Q_{\omega \ell m} = \frac{1}{r} Y_{\ell m} e^{-i\omega(\tau - \rho)}$$

$$= \frac{1}{r_H - u} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{u - r_H}{u + r_H}\right)}$$

Similarly

$$\Phi_{\mathbf{I}}^{\omega \ell m} = \frac{1}{r_H - u} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{r_H + u}{r_H - u}\right)}$$

at $v = r_H$



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decomposition of the Minkowski vacuum

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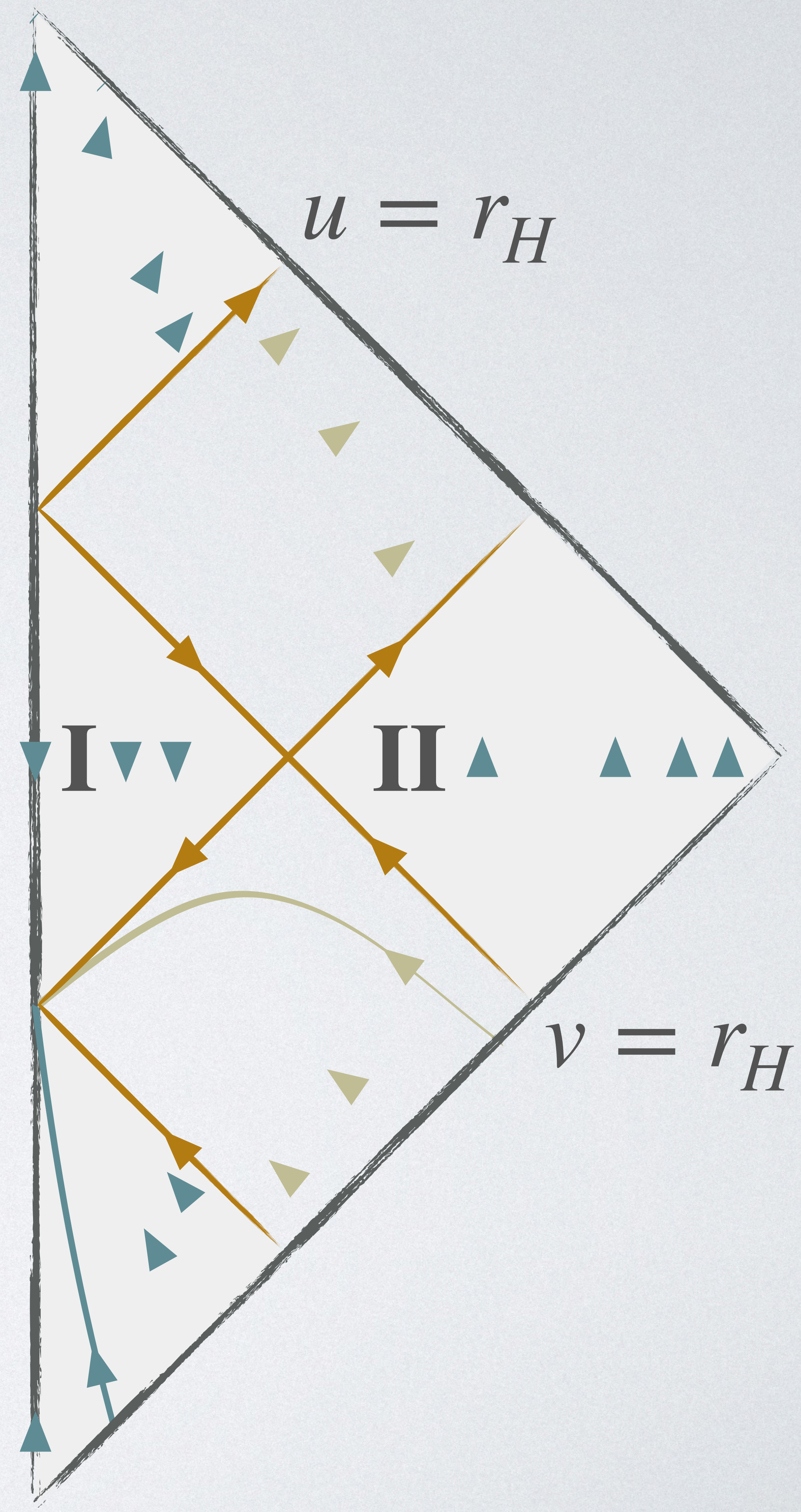
$$\Phi_{\mathbf{II}}^{\omega\ell m} = \frac{1}{r_H - z} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{z-r_H}{z+r_H}\right)} \qquad z < -r_H,\ z > r_H$$

$$\Phi_{\mathbf{I}}^{\omega\ell m} = \frac{1}{r_H - z} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{r_H+z}{r_H-z}\right)} \qquad -r_H < z < r_H$$



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evaluate the complex function $F_\omega = \frac{1}{r_H - z} e^{-i\frac{\omega}{\kappa}\log\left(\frac{z-r_H}{z+r_H}\right)}$

at $z = x - i\epsilon,\ \epsilon > 0$

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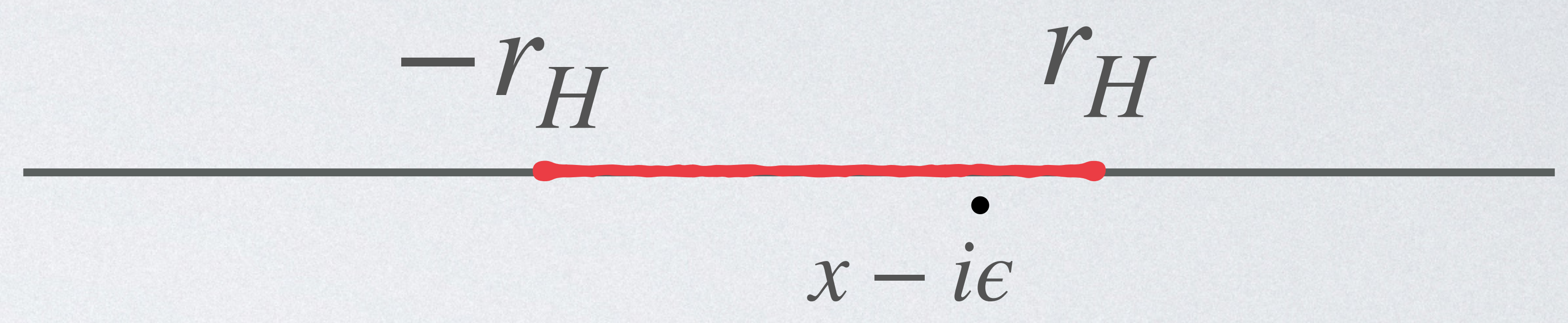
$$F_\omega = \frac{1}{r_H - x + i\epsilon} e^{-i\frac{\omega}{\kappa}\log\left(\frac{x - r_H - i\epsilon}{x + r_H + i\epsilon}\right)}$$



for $-r_H < x < r_H$ we get

$$\log\left(\frac{x - r_H - i\epsilon}{x + r_H - i\epsilon}\right) = \log\left(\frac{r_H - x - i\epsilon}{x + r_H - i\epsilon} e^{-i\left(\pi - \mathcal{O}(\epsilon)\right)}\right) = \log\left(\frac{r_H - x - i\epsilon}{x + r_H - i\epsilon}\right) - i\pi$$

$$F_\omega \approx \frac{e^{-\frac{\pi\omega}{\kappa}}}{r_H - x} e^{-i\frac{\omega}{\kappa}\log\left(\frac{x - r_H}{x + r_H}\right)} = e^{-\frac{\pi\omega}{\kappa}}\overline{\Phi}^{\omega}_{\mathbf{I}}$$

LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

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Fulling 1973, Davies 1975, Unruh 1976

similarly

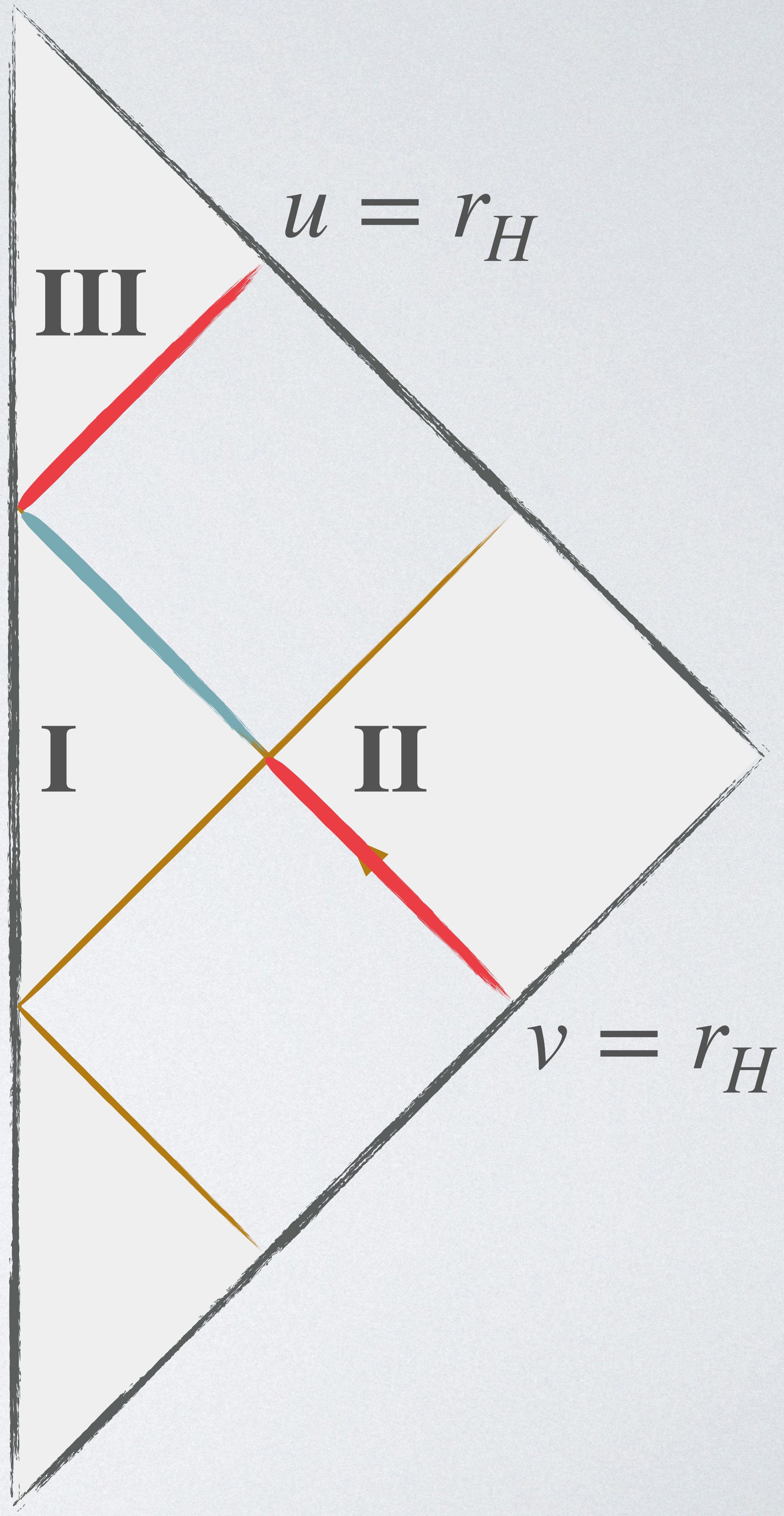
$$F_\omega = \Phi^\omega_{\mathbf{II}} + e^{-\frac{\pi\omega}{\kappa}}\overline{\Phi}^\omega_{\mathbf{I}}$$

$$F'_\omega = \Phi^\omega_{\mathbf{I}} + e^{-\frac{\pi\omega}{\kappa}}\overline{\Phi}^\omega_{\mathbf{II}}$$

are analytic and bounded in the lower-half plane in terms of the Minkowski coordinate, thus **positive frequency solutions**.

$$\left(a_{\mathbf{II}\omega} + e^{-\frac{\pi\omega}{\kappa}} a^\dagger_{\mathbf{I}\omega}\right)|0\rangle_M = 0, \qquad \left(a_{\mathbf{I}\omega} + e^{-\frac{\pi\omega}{\kappa}} a^\dagger_{\mathbf{II}\omega}\right)|0\rangle_M = 0$$

$$|0\rangle_M = \prod_{\omega\ell m}\left(\sum_n e^{-\frac{n\pi\omega_i}{\kappa}} |n,\omega,\ell,m\rangle_{\mathbf{I}} \otimes |n,\omega,\ell,m\rangle_{\mathbf{II}}\right)$$



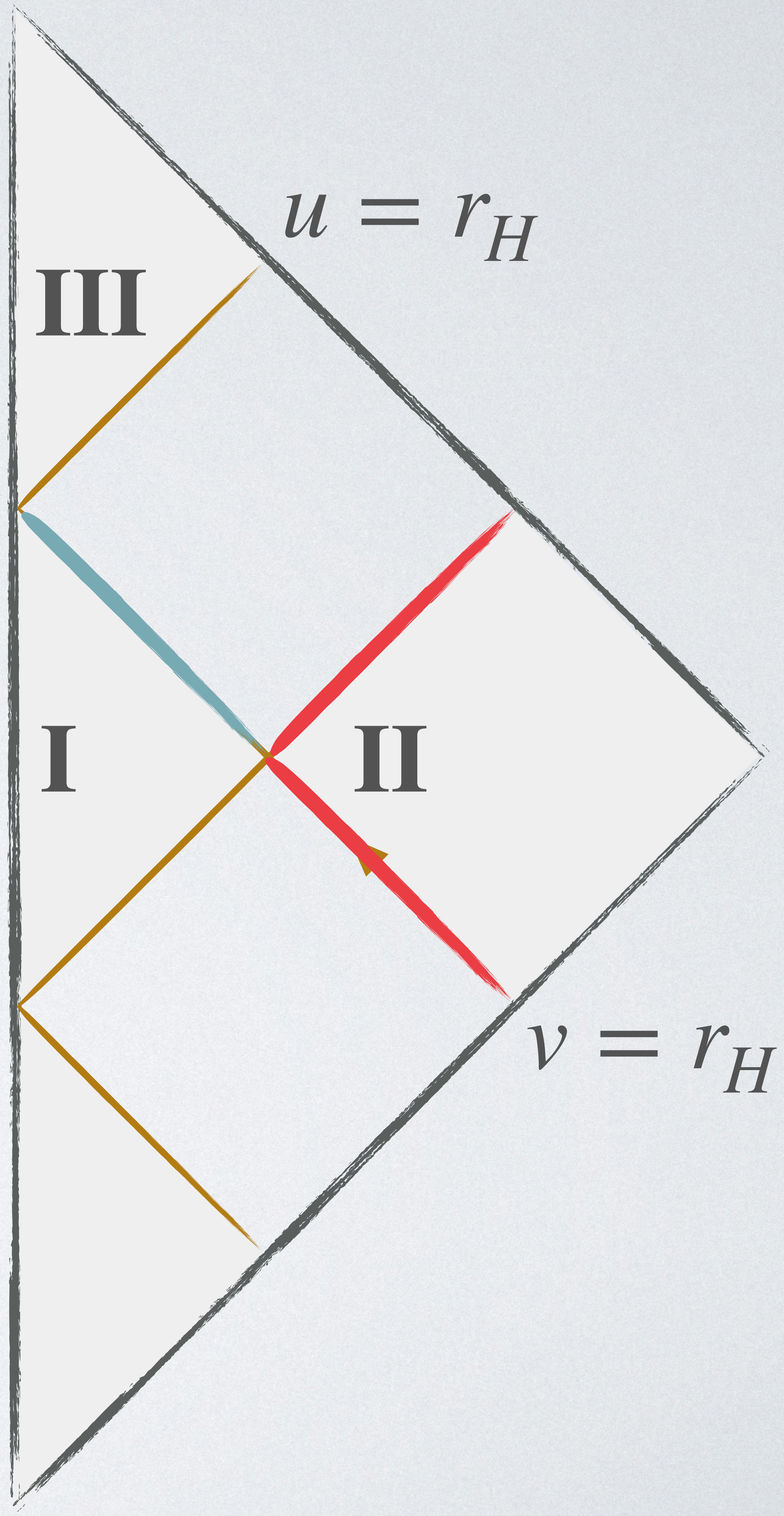
LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

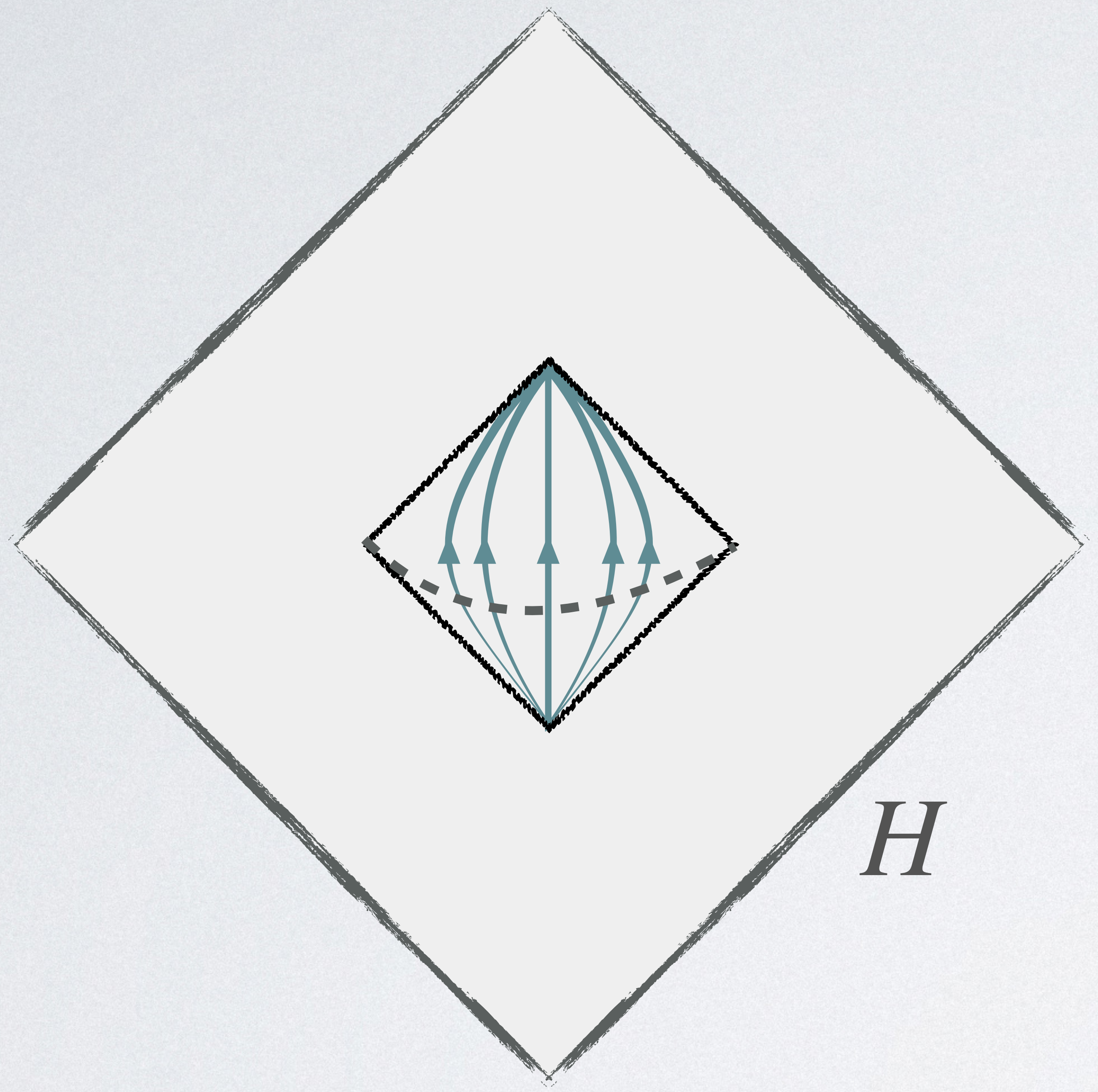
$$|0\rangle_M = \prod_{\omega \ell m} \left(\sum_n e^{-\frac{n\pi\omega_i}{\kappa}} \left|n, \omega, \ell, m\right\rangle_{\mathbf{I}} \otimes \left|n, \omega, \ell, m\right\rangle_{\mathbf{II}} \right)$$

In writing the previous result it was crucial to identify the Hilbert space in region **III** with the one corresponding to the modes in region **II** crossing the future horizon. The identification comes from the analysis of the solutions in the two null boundaries. The result as written without considering the identification is

$$|0\rangle_M = \prod_{\omega \ell m} \left(\sum_n e^{-\frac{n\pi\omega_i}{\kappa}} \left|n, \omega, \ell, m\right\rangle_{\mathbf{I}} \otimes \left(\left|n, \omega, \ell, m\right\rangle_{\mathbf{IIout}} \oplus \left|n, \omega, \ell, m\right\rangle_{\mathbf{III}} \right) \right)$$



PERSPECTIVES



causal diamond in the static patch of deSitter spacetime. The boundary H is the cosmological horizon. In blue the trajectories of the CKF. (Jacobson, Visser 2019)

- the laws of Light-cone thermodynamics can be extended to any conformally flat spacetime. In some of these (Fröb 2023) we can expect a similar decomposition to take place.

- the temperature and entropy of conformal horizons are used (Alonso-Serrano, Liška 2023) to get Weyl-Transverse gravity using a local thermodynamic approach (Jacobson 1995). They evoke that non-conformal matter can be used to source the cosmological constant (Perez, Sudarsky, Bjorken 2018). Quantifying the "violation" of light-cone thermodynamics by non-conformal matter may give insights on the subject.

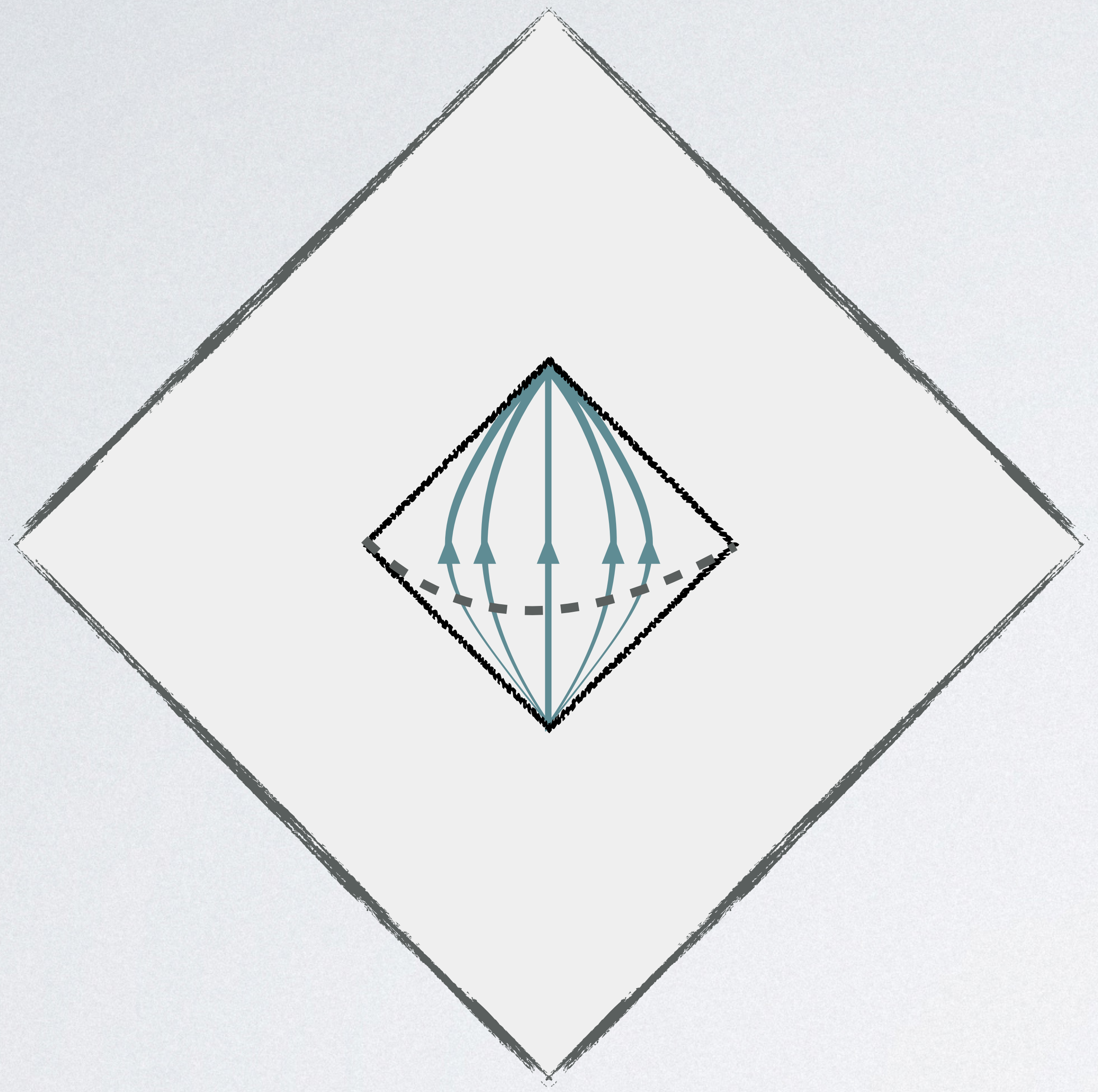
PERSPECTIVES

Trajectory of a moving mirror emitting radiation in Minkowski spacetime.

- In Wald(2019), Tomitsuka Yamaguchi Hotta(2019) moving mirrors in 1+1 Dimensions have been studied as a model for Hawking radiation purified without energy cost. In solving the problem, it is crucial to know the analytic form of the trajectories of Rindler observers. Despite the loss of simplicity going from 1+1D to 3+1D QFT, it is worth trying a generalisation of the previous models to spherically symmetric trajectories in light.

- Works by Arzano (2020,2023) show that this conformal temperature can be found in the context of conformal quantum mechanics, suggesting some sort of relation.

PERSPECTIVES



- In the context of measurements in quantum field theory, the algebra of observables related to apparatuses has compact support in regions of spacetime with the same features as causal diamonds. Studying the problem in light of these results may help making a significant step.

- Angular momentum?

THANK YOU

- *Light-cone Thermodynamics*, De Lorenzo, Perez (2017)
- *Modular Structure of the Local Algebras Associated With the Free Massless Scalar Field Theory*, Hislop, Longo (1981)
- *Radial conformal motion in Minkowski spacetime*, Herrero, Morales (1999)
- *Notes on Black-Hole evaporation*, Unruh (1976)
- *Diamonds's temperature: Unruh effect for bounded trajectories and thermal time hypothesis*, Martinetti, Rovelli (2003)
- *Thermodynamics as a tool for (quantum) gravitational dynamics*, Alonso-Serrano, Liška (2023)
- *Modular Hamiltonian for de Sitter diamonds*, Fröb (2023)
- *Particle and energy cost of entanglement of Hawking radiation with the final vacuum state*, Wald (2019)
- *Spacetime Equilibrium at Negative Temperature and the Attraction of Gravity,* Jacobson, Visser (2019)