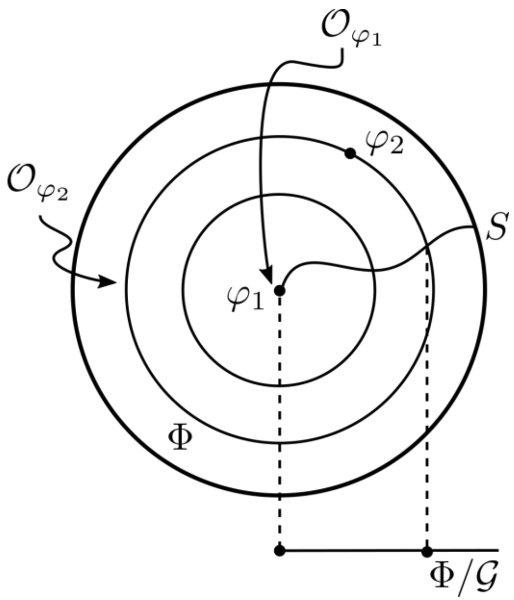
A unified geometric framework for YM charges and dressings in finite regions



Aldo Riello*

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w/ Henrique Gomes & Florian Hopfmüller

ILQGS – 23 October 2018

Plan

The whole talk will be restricted to YM (for important technical reasons)

Part I – introduction

brief philosophical and technical motivation

Part II – the geometry of field space

field space geometry as a principal fiber bundle; introduction of the field-space connection form as a gauge-reference frame ϖ

Part III – gauge invariant symplectic geometry

construction; vanishing of all gauge-charges

Part IV – Singer-DeWitt connection

construction of the connection; global charges; Dirac-like dressings

Part V – Higgs connection

construction; condensates and broken phases

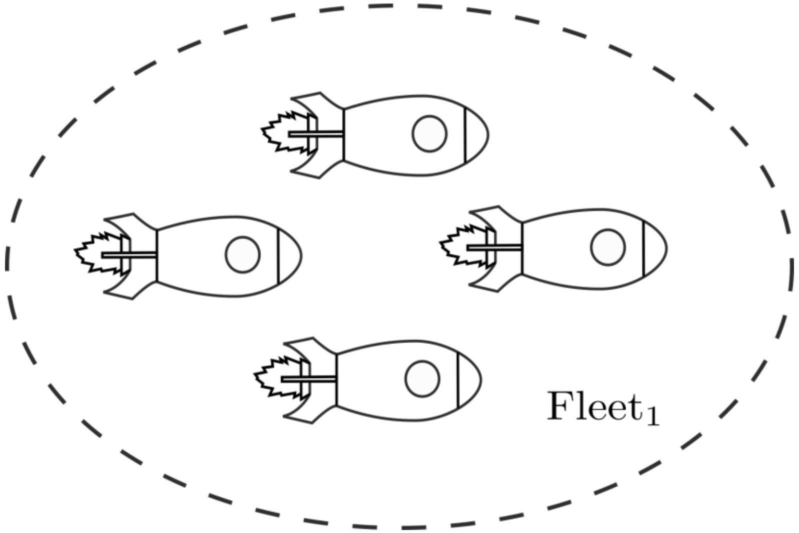
Part I

introduction

Why gauge?*

The true essence of gauge theories has been suggested to be relational

Gauge as a "handle" for subsystem to attach to each other



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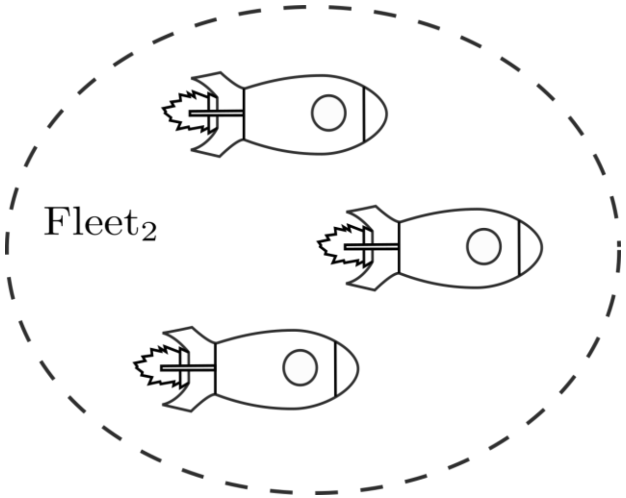

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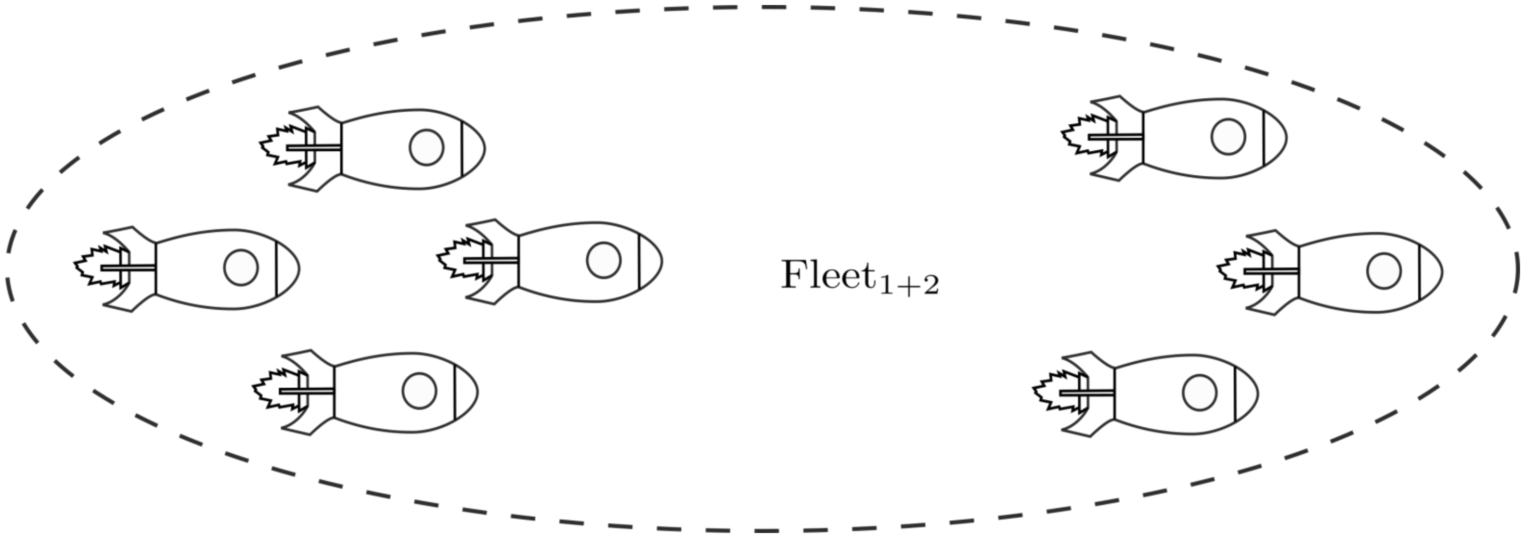

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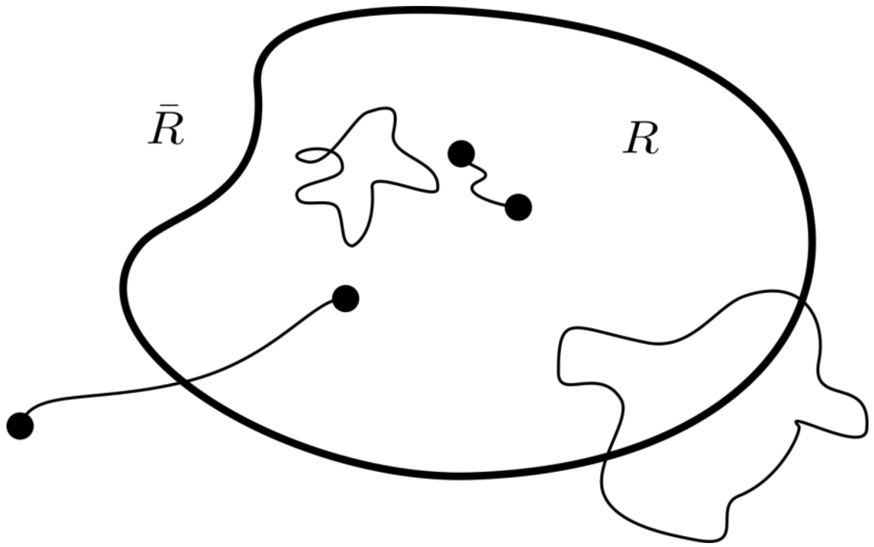
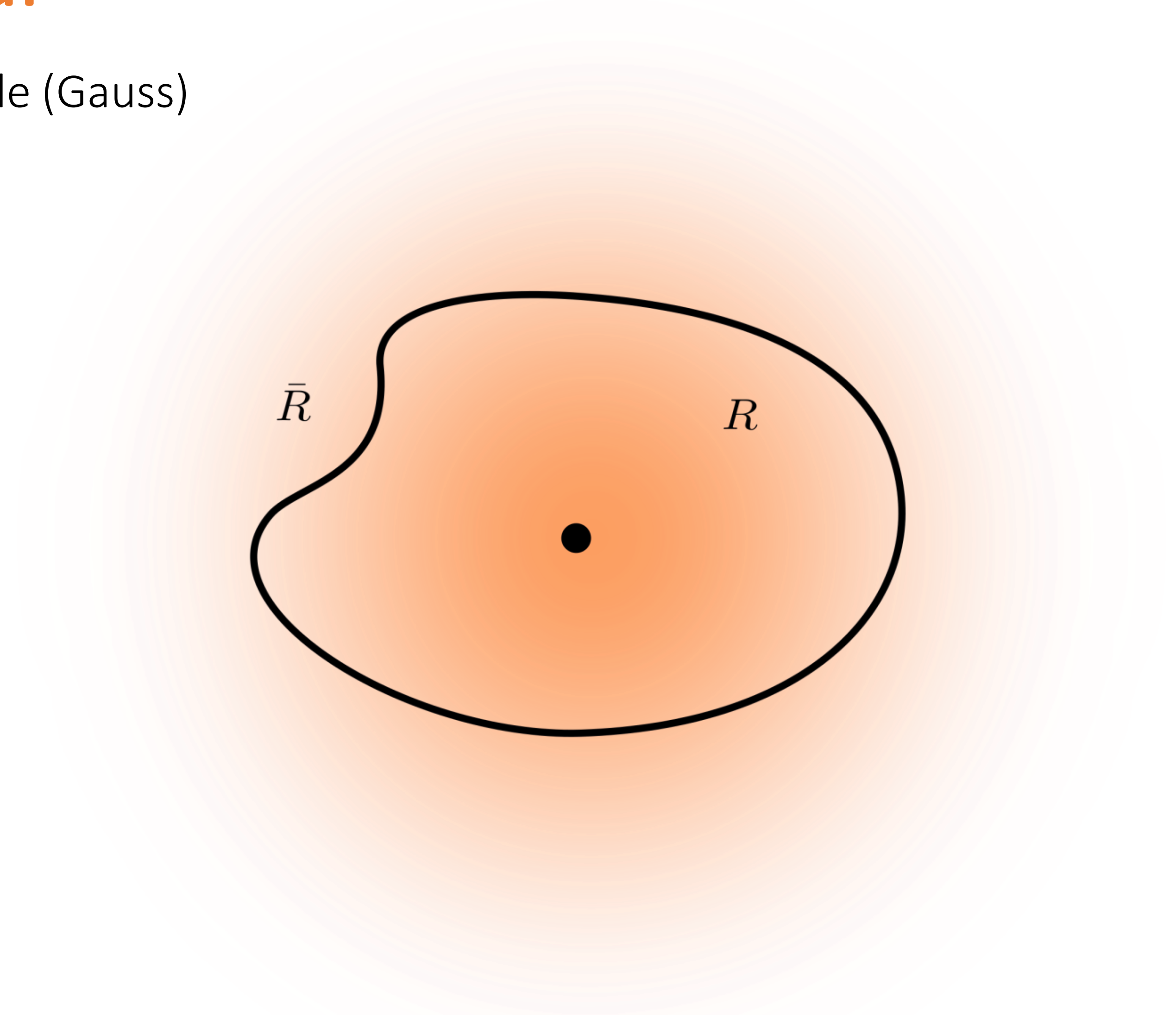
Gauge theories are nonlocal

Gauge invariant degrees of freedom are nonlocalizable (Gauss)

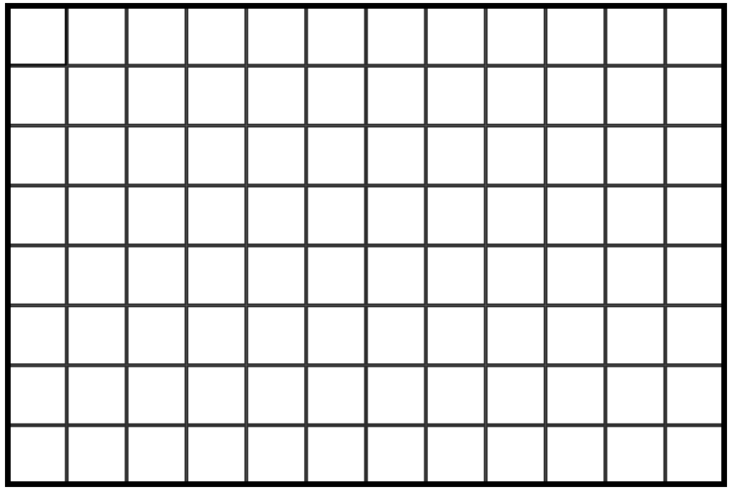




[Dirac; Torre; Giddings; Donnelly; Freidel; …]

The puzzle of edge modes

Lattice
global gauge-invariant state (spinnetwork)



open region:
local gauge-variant states

[Lattice & entanglement entropy: Donnelly ’08 (electric); Delcamp, Dittrich, AR ‘16 (magnetic)]

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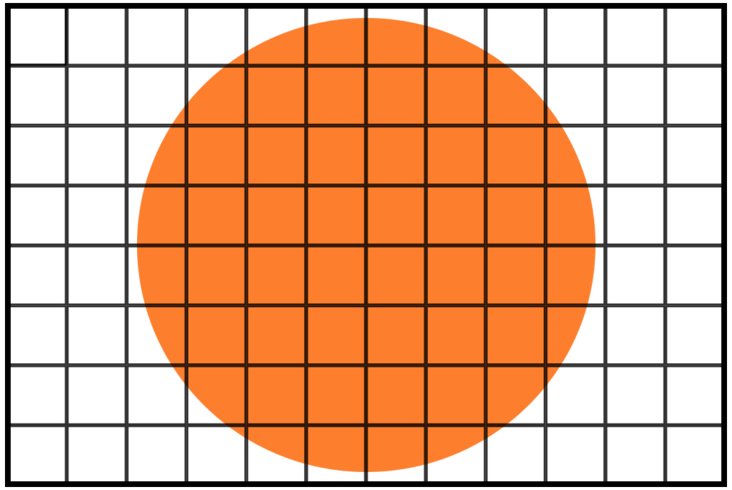

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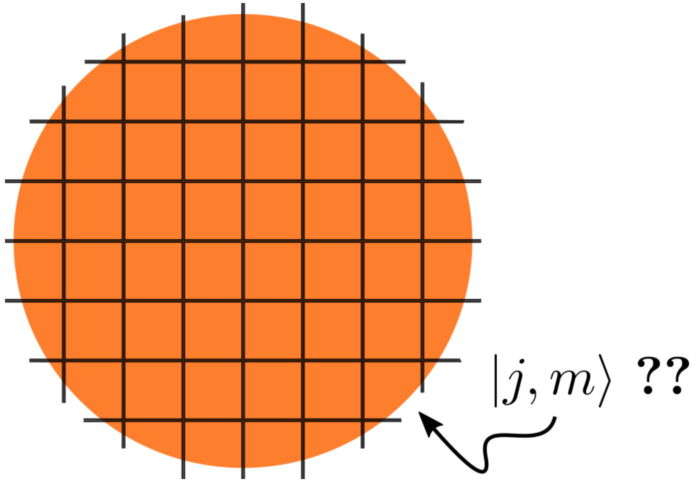

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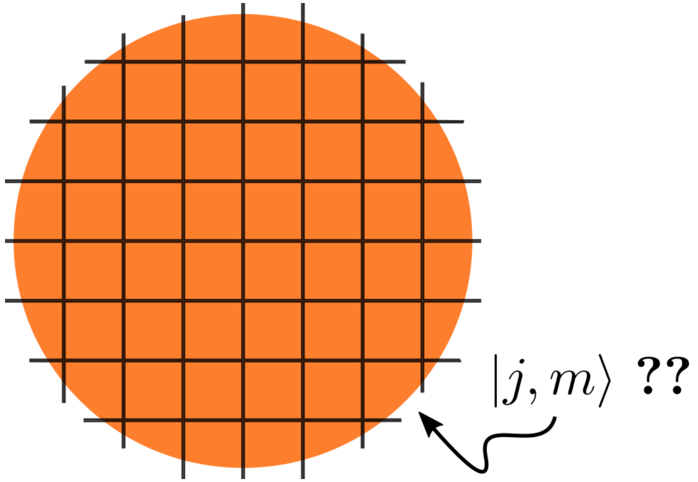


open region:
local gauge-variant states

Extended Hilbert space construction
of regional states

$$\mathcal{H}^{\text{g.inv.}}_{(R\cup\bar{R})} = \left(\mathcal{H}^{\text{ext}}_{R} \otimes \mathcal{H}^{\text{ext}}_{\bar{R}}\right)//G$$

Classical continuous analogue:
extend field space with
G-valued "edge modes" [Donnelly-Freidel]

$$\mathcal{H}^{\text{ext}}_{R} \rightsquigarrow \left\{A(x\in R); h(x \in \partial R)\right\}$$

$$\theta^{\text{ext}} = \text{Tr}\left(E\delta A\right) - \text{d}\,\text{Tr}\left(E\delta h h^{-1}\right)$$

Puzzle: Physical status of "new fields"?

[Lattice & entanglement entropy: Donnelly '08 (electric); Delcamp, Dittrich, AR '16 (magnetic)]
[Extend. field space w/ edge modes: Donnelly & Freidel '16; Geiller '17. But see also Regge & Teitelboim '74!]

The puzzle of gauge charges

The covariant Hamiltonian approach produces (Noether)

infinitely many gauge charges

$$Q[\xi] = \int \theta(\delta_\xi A) \hat{=} \oint \mathrm{Tr}\Big(E(x)\xi(x)\Big)$$

However, they suffer a corner ambiguity

Then, which charges are physical?

$$\theta \mapsto \theta + \mathrm{d}\alpha$$

$$Q[\xi] \mapsto Q[\xi] + \oint \alpha(\delta_\xi A)$$

How to tell
pure gauge transformations
from
physical symmetries
?

Also, gauge charges are related to gauge constraints and must therefore vanish [(!) integration by parts]

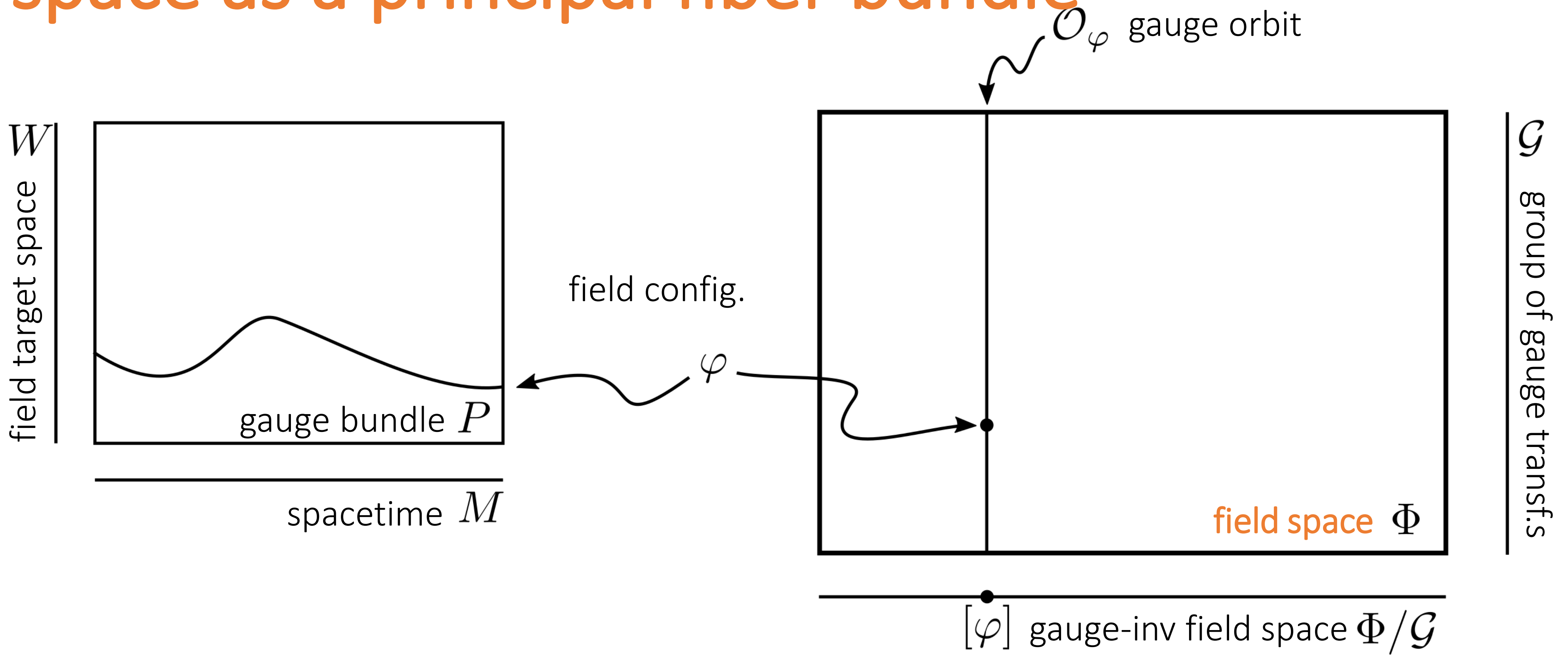
Then, how to characterize charged objects? [e.g. total electric charge in a region]

[Regge & Teitelboim ’74; Balachandran et al ‘94; Lavelle & McMullan 90s; Barnich, Brandt, Henneaux ’00s]

Part II

the geometry of field space

Field space as a principal fiber bundle

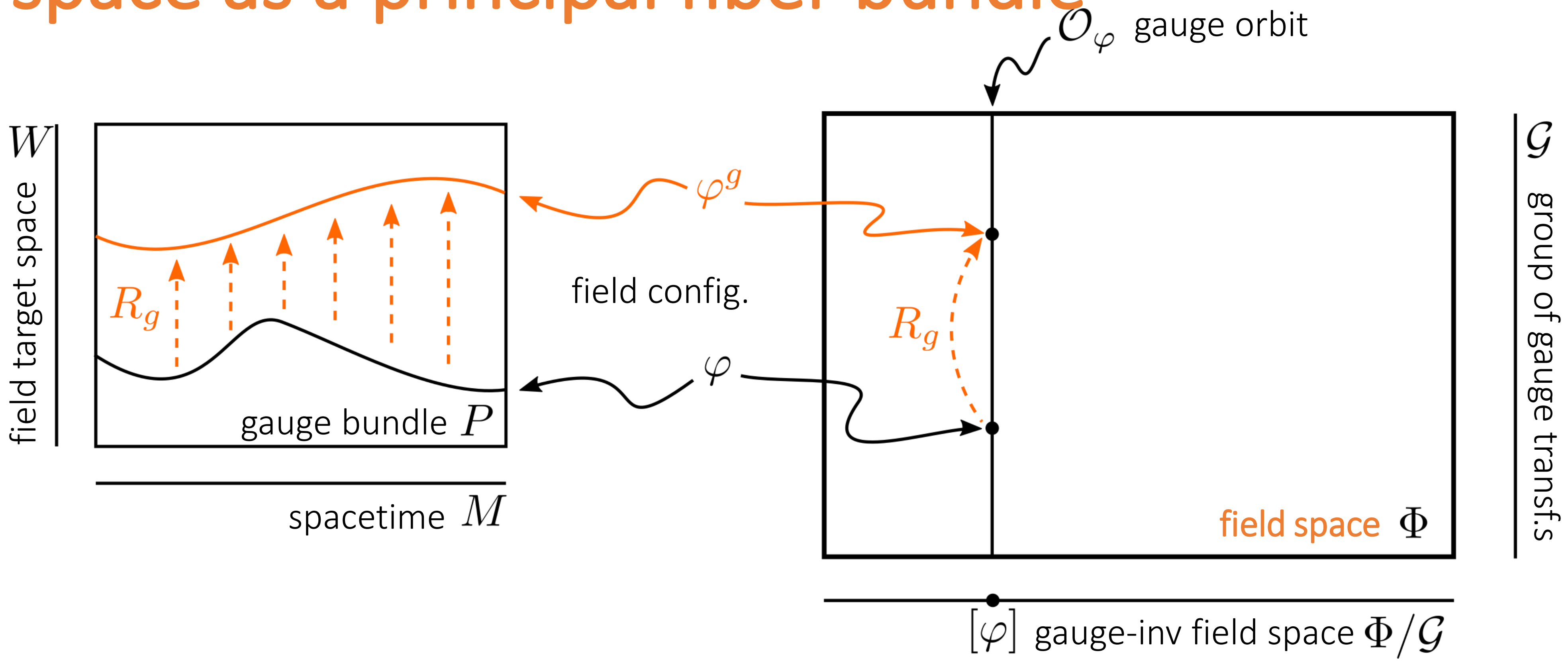


$(g : x \mapsto G) \in \mathcal{G}$ gauge parameter, will be generalized to being *field-dependent* ($\rightsquigarrow$ "action groupoid")

$\xi \in \mathrm{Lie}(\mathcal{G})$ infinitesimal gauge parameter

$\xi^\sharp \in \mathfrak{X}(\Phi)$ corresponding infinitesimal gauge transformation (vector field on field space)

Field space as a principal fiber bundle



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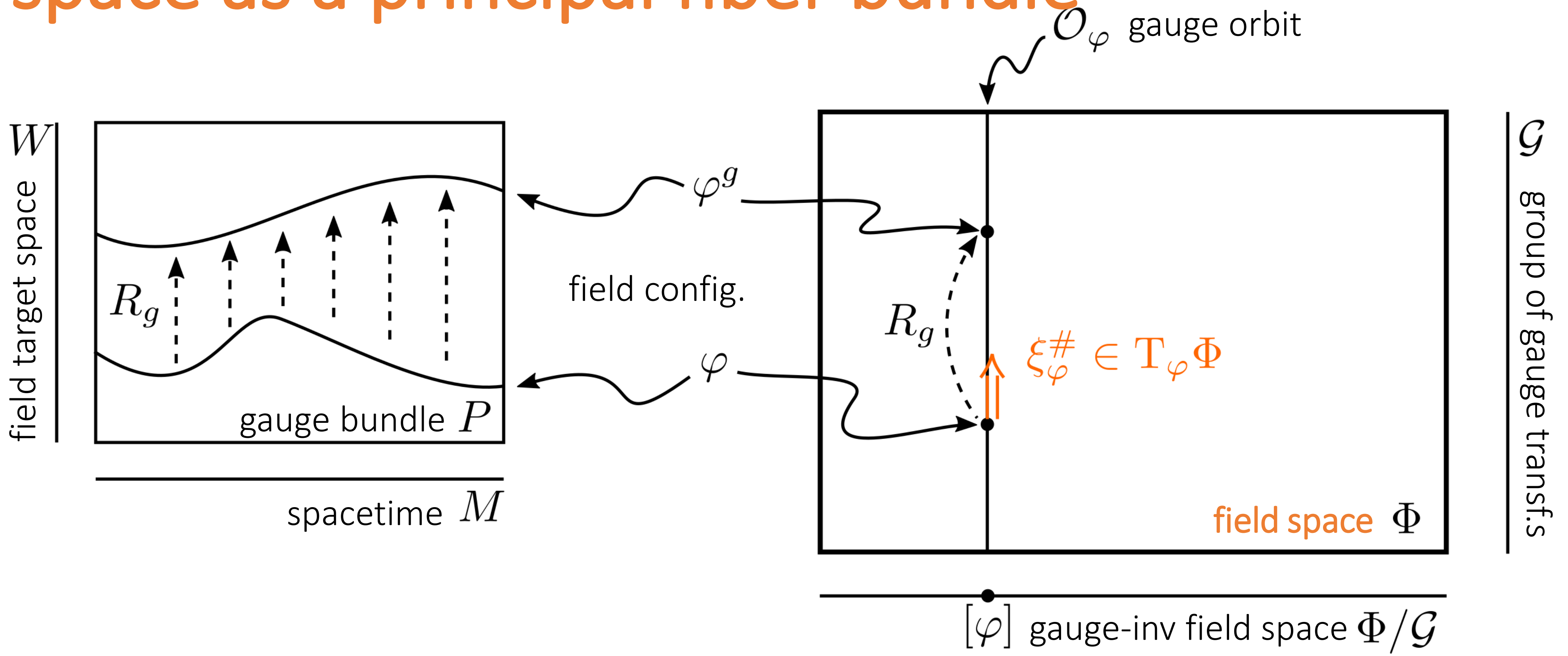

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Gauge v. Physical: a relative notion

The gauge structure identifies
the vertical subspaces, canonically
(ie *pure-gauge* transformations are well-defined)

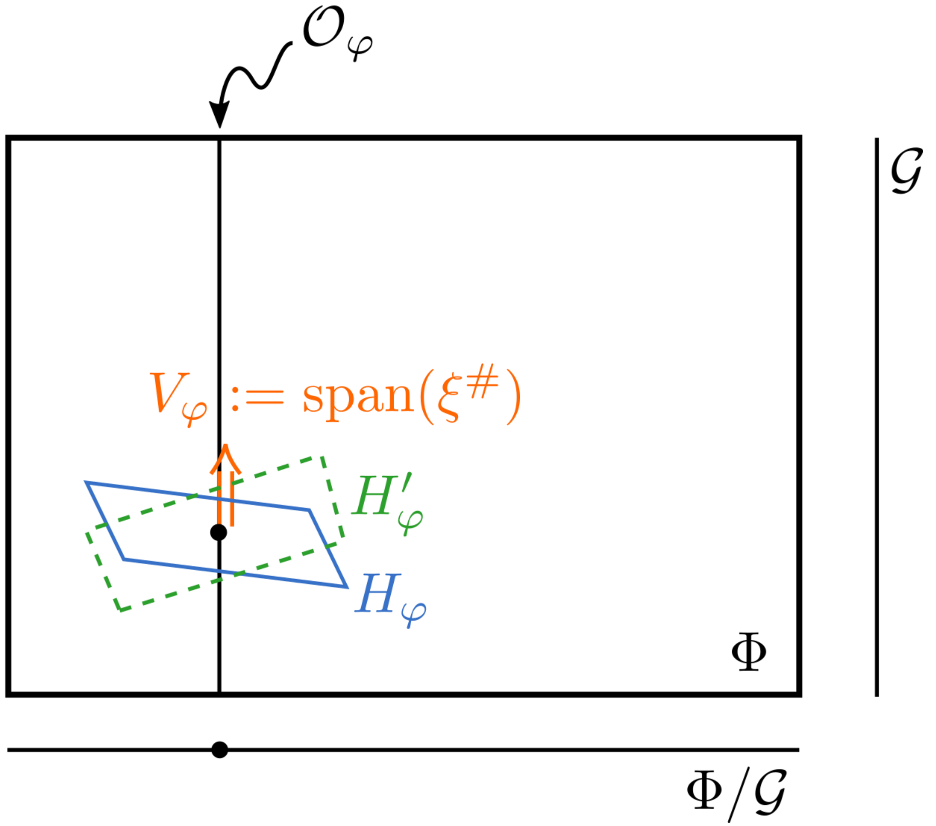
$$V_\varphi = \mathrm{T}_\varphi \mathcal{O}_\varphi$$

Their horizontal complements
identify *physical* variations,
but are *noncanonical*

$$\mathrm{T}_\varphi \Phi = H_\varphi \oplus V_\varphi = H'_\varphi \oplus V_\varphi$$

Thus,
a general field variation $\mathbb{X} \in \mathrm{T}_\varphi \Phi$
cannot be decomposed a priori
into its *pure-gauge* and *physical* parts

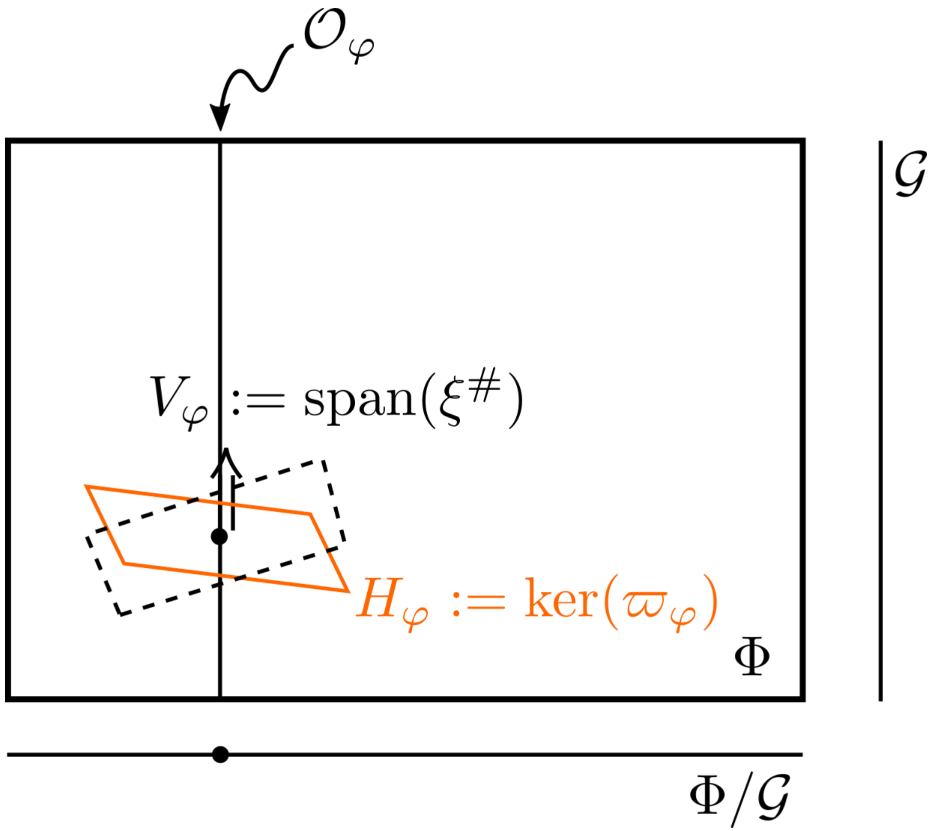
$$\rightsquigarrow \quad \mathbb{X} = \mathbb{h} + \eta^\sharp = \mathbb{h}' + (\eta')^\sharp$$



[if it is double-struck, it is a field-space object]

A covariant gauge-frame: the connection ϖ [Var-Pie]

Covariant *choice* of horizontal (*physical*) subspaces encoded in
a field-space (functional) connection, $\varpi \in \Omega^1\Big(\Phi, \mathrm{Lie}(\mathcal{G})\Big)$



This is a field-space 1-form,
which transforms covariantly under gauge transformations

$\mathbb{i}_{\xi^\sharp}\varpi = \xi$	(horizontality)
$\mathbb{L}_{\xi^\sharp}\varpi = [\varpi, \xi] + \mathbb{d}\xi$	(covariance)

$\mathbb{d}\xi \neq 0$ iff ξ is a field-dependent gauge-transformations

[δ often indicates variations (ie field-space vectors, here $\mathbb{X}$), but sometimes also a field-space differential (here, $\mathbb{d}$)]

The curvature of ϖ

If the Φ is a trivial bundle,

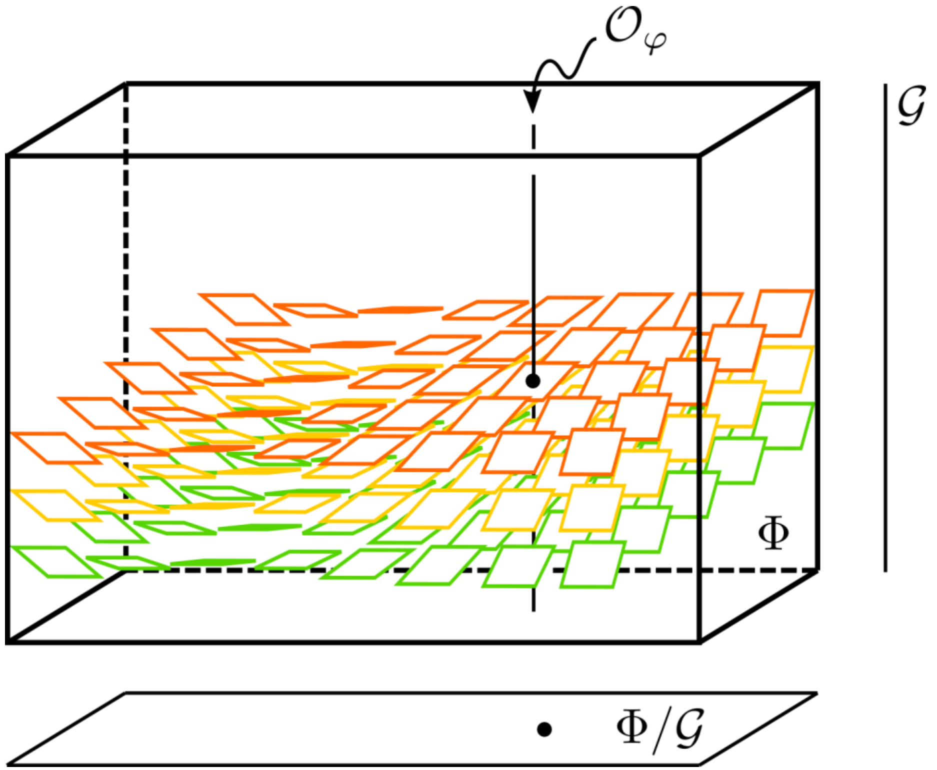
then a global horizontal section exists (gauge fixing)

ie a global notion of physical v. gauge

But generically, Φ is a nontrivial bundle

this is encoded in the curvature of ϖ

$$\mathbb{F} = d\varpi + [\varpi, \varpi] \qquad \text{(curvature)}$$



Therefore, the curvature of ϖ is a proxy for nontrivial, global (*nonperturbative*) structures of Φ

Part III

gauge invariant symplectic geometry

Gauge-invariant symplectic potential and form

In presence of corners,

the "standard" symplectic potential *fails to be invariant*

under *field-dependent* gauge transformations

$$\mathbb{L}_{\xi^\sharp}\theta = Q[\mathbb{d}\xi] \hat{=} \mathrm{d}\,\mathrm{Tr}\Big(E\mathbb{d}\xi\Big) \neq 0$$

$$\mathbb{d}_H A := \mathbb{d}A - \mathrm{D}_A\varpi \qquad \mathbb{d}_H^2 A := -\mathrm{D}_A\mathbb{F}$$

$$\mathbb{d}_H E := \mathbb{d}E + [\varpi, E] \qquad \mathbb{d}_H^2 E := [\mathbb{F}, E]$$

$$\mathbb{d}_H \psi := \mathbb{d}\psi + \varpi\psi \qquad \mathbb{d}_H^2 \psi := \mathbb{F}\psi$$

However, ϖ allows to define a horizontal ("covariant") differential, and thus

a "**horizontal**" **symplectic potential**, **fully gauge-invariant**

$$\theta_H := \mathrm{Tr}\Big(E\mathbb{d}_H A\Big) \hat{=} \theta - \mathrm{d}\,\mathrm{Tr}\Big(E\varpi\Big)$$

$$\Omega_H := \mathbb{d}_H\theta_H \hat{=} \Omega + \mathrm{d}\mathbb{d}\,\mathrm{Tr}\Big(E\varpi\Big) \qquad (\,\mathbb{d}\text{ exact}\,)$$

$\rightsquigarrow$ identification of gauge invariant the regional d.o.f. relatively to ϖ

All gauge invariant charges vanish (...almost!)

Applying Noether's procedure

to the horizontal symplectic potential,

one immediately finds that all "horizontal" (physical) gauge-charges vanish,

even in finite regions:

$$Q_H[\xi] := \theta_H(\delta_\xi \varphi) \equiv \mathbb{i}_{\xi^\sharp}\theta_H = 0$$

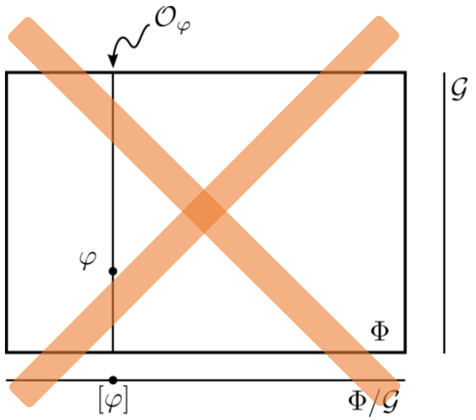
$$\Uparrow$$

$$\mathbb{i}_{\xi^\sharp}\varpi = \xi \quad \Rightarrow \quad \mathbb{i}_{\xi^\sharp}\mathbb{d}_H\varphi = 0$$

Question so where did the electric (or color) charge go?

to answer, we need to amend a mathematical imprecision (...and we need matter fields, too)

Reducible configurations: towards global charges



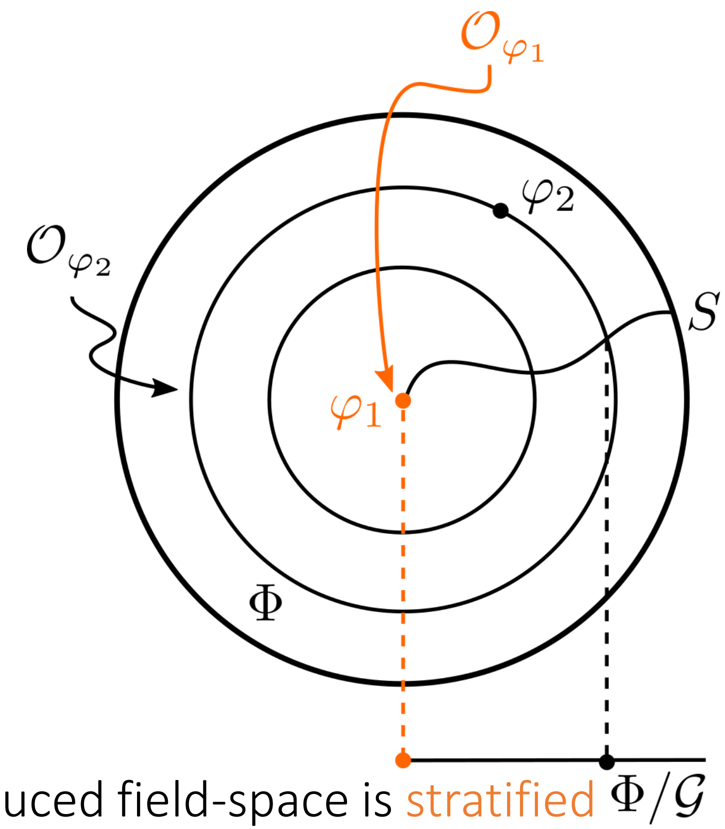
Field-space is **not** a PFB

There are *symmetric* ("reducible") *configurations*

where the fibers degenerate

$$\exists \varphi_1, \chi : \quad \chi^\sharp_{|\varphi_1} = 0$$

(Yang-Mills: $\exists A, \chi : \quad \chi^\sharp_{|A} = (\mathrm{D}_A \chi) \frac{\mathbb{d}}{\mathbb{d}A} = 0$)



reduced field-space is stratified

"Killing" transformations are in the kernel of $\cdot^\sharp$: room for something new to happen at reducible configurations

Part IV

the Singer-DeWitt connection

ϖ from a supermetric

Recall: $\varpi_\varphi \Leftrightarrow H_\varphi \subset \mathrm{T}_\varphi\Phi$

Is there a natural notion of “horizontality”?

PFB structure is *not* enough, but in presence of a supermetric $\mathbb{G}$ on Φ: $H_\varphi := (V_\varphi)^\perp$

Is there a natural supermetric?

Pure gauge-theory: the kinetic term provides the unique gauge-compatible ultralocal supermetric

$$\mathbb{G}_R(\mathbb{X},\mathbb{Y}) = \int_R g^{ij}\mathrm{Tr}\Big(\mathbb{X}_i\mathbb{Y}_j\Big) \quad \rightsquigarrow \quad K = \mathbb{G}_R(\dot{A}_i,\dot{A}_j) = \int_R g^{ij}\dot{A}_i\dot{A}_j$$

$$\Downarrow$$

Singer-DeWitt connection

$$\mathrm{D}^2_A\varpi = \mathrm{D}^i_A \mathbb{d} A_i$$

$$n_i \mathrm{D}^i_A\varpi_{|\partial R} = n^i\mathbb{d}A_{i|\partial R}$$ boundary conditions at ∂R (*new*)

Physics relative to A_i : the SdW connection

The Singer-DeWitt connection provides a gauge-reference frame relative to A_i in a finite regions R

Singer-DeWitt connection

$$\mathrm{D}^2_A \varpi = \mathrm{D}^i_A \mathbb{d} A_i$$

$$n_i \mathrm{D}^i_A \varpi_{|\partial R} = n^i \mathbb{d} A_{i|\partial R}$$

$$\rightsquigarrow \qquad \varpi_{\mathrm{Ab},\infty}(x) = -\int \frac{\mathrm{d}^3 y}{4\pi} \frac{\partial^i \mathbb{d} A_i(y)}{|x-y|}$$

nonlocal, but regional

The non-Abelian SdW connection has curvature: $\mathrm{D}^2_A \mathbb{F} = g^{ij}[\mathbb{d}_H A_i, \mathbb{d}_H A_j]$ (Gribov problem)

Remark: the SdW connection always exists

[without boundaries: Singer '78-'81, Narasimhan & Ramadas '79; Babelon & Viallet '79-'81, Asorey & Mitter '81; DeWitt, e.g. '03]

Global charges from the SdW connection

[similar conclusions to Barnich & Brandt '02; and DeWitt, e.g. '03]

We discussed pure Yang-Mills, what if we add matter (electrons & quarks)?

Remark [*co-rotation principle*]

A gauge transformation acts simultaneously on all fields.

To detect it, in general a connection built out of one field is enough.

$$\rightsquigarrow \quad \xi^\sharp = \int (\mathrm{D}_A\xi)\frac{\mathbb{d}}{\mathbb{d}A} - (\xi\psi)\frac{\mathbb{d}}{\mathbb{d}\psi}$$

$$\theta := \mathrm{Tr}\Big(E\mathbb{d}A\Big) + \bar\psi\mathbb{d}\psi \;\rightsquigarrow\; \theta_{\text{SdW-}H} := \mathrm{Tr}\Big(E\mathbb{d}_H A\Big) + i\bar\psi\gamma_\mu\mathbb{d}_H\psi * \mathrm{d}x^\mu$$ for the SdW's $\mathbb{d}_H$

At reducible configurations of A, $\mathrm{D}_A\chi = 0$ but $\chi^\sharp \neq 0$ (due to the matter field). Thus,

$$Q_{\text{SdW-}H}[\chi] = \mathbb{i}_{\xi^\sharp}\theta_{\text{SdW-}H} = \int \mathrm{Tr}\Big(J\chi\Big) \mathrel{\hat=} \oint \mathrm{Tr}\Big(E\chi\Big)$$

or, in words,

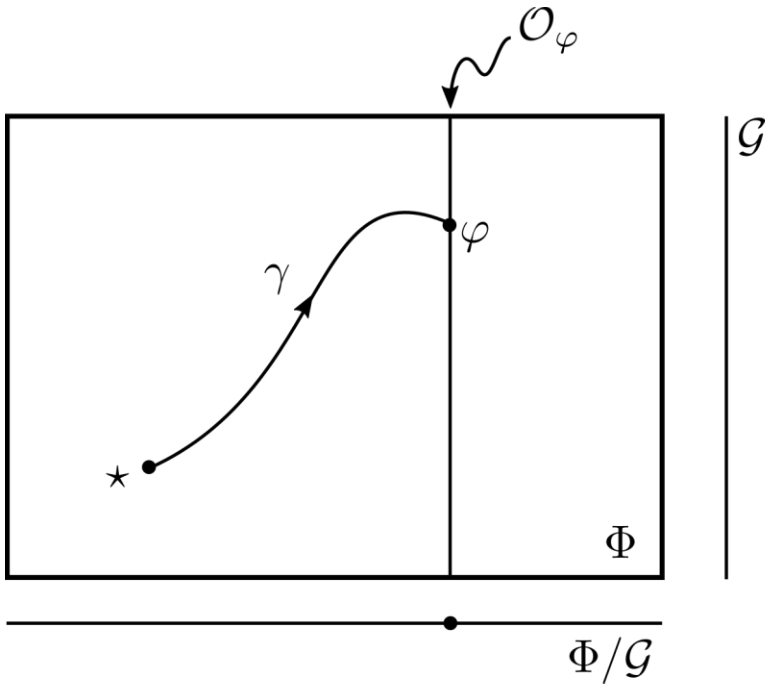
wrt the SdW connection, quarks and electron possess global (Killing) charges on symmetric A_i backgrounds,

while all other "pure-gauge" charges vanish

Dirac dressing from a SdW Wilson-line: Abelian

We have a connection, what about G-valued Wilson lines?

$$h_\gamma(\varphi) = \mathbb{P}\exp\int_{\star\xrightarrow{\gamma}\varphi}\varpi$$



Start from the Abelian theory (in infinite volume):

there, *ϖ is flat* and $h_\gamma(A)$ is independent of γ :

$$h^{\rm SdW}_{{\rm Ab},\infty}(A) = h_{\rm Dirac}(A) = \exp\int \frac{{\rm d}^3y}{4\pi}\frac{\partial^i A_i(y)}{|x-y|}$$

is the Dirac dressing for the electron: $\hat\psi_{\rm Dirac} := h_{\rm Dirac}(A)^{-1}\psi$

while the 'Dirac-dressed photon' is the transverse photon $\hat A_{\rm Dirac} := A - (h^{-1}\partial h)_{\rm Dirac} = A^\perp$

Finally, one has a relation between dressings and horizontal symplectic geometry:

$$\theta(\hat\psi, \hat A, \mathbb{d}\hat\psi, \mathbb{d}\hat A) \equiv \theta_{{\rm SdW}\text{-}H}(\psi, A, \mathbb{d}\psi, \mathbb{d}A)$$

Dirac dressing from a SdW Wilson-line: non-Abelian

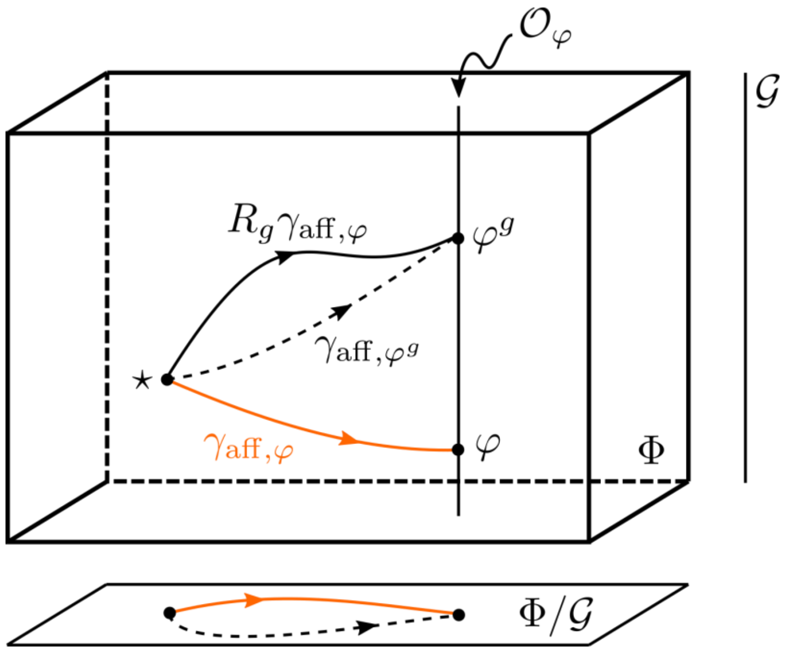
In the non-Abelian theory the dressing factor

$$h_\gamma(\varphi) = \mathbb{P}\exp\int_{\star\xrightarrow{\gamma}\varphi}\varpi$$

does depend on γ, since ϖ has curvature

For $h_\gamma(\varphi)$ to be a valuable dressing factor, one has to demand the *dressing paths to be gauge-compatible*

$$R_g\gamma_\varphi = \gamma_{\varphi^g} \quad \forall g\in\mathcal{G}$$



Gauge compatibility is satisfied by Vilkovisky-DeWitt geodetic paths (but not by affine paths)

But: global issues with existence and uniqueness of geodesics $\Longleftrightarrow$ Gribov problem

good news: *the horizontal symplectic geometry requires only infinitesimal dressings*

[Vilkovisky 1984: *The unique effective action* & *The gospel according to DeWitt*; our setup links Vilko & DeWitt's work to Lavelle & McMullan's]

Part V

the Higgs connection

Fundamental matter gives a flat ultralocal ϖ

The SdW connection represents physics in the gauge frame of A_i. Which physics in a matter gauge-frame?

Consider a scalars field ϕ, the kinetic term gives again a viable ultralocal supermetric

$$\mathbb{G}_R(\mathbb{X},\mathbb{Y}) = \int_R \Re\left(\mathbb{X}^\dagger \mathbb{Y}\right) \quad \rightsquigarrow \quad K = \mathbb{G}_R(\dot\phi,\dot\phi) = \int_R \dot\phi^\dagger \dot\phi$$

Assume that ϕ is in the fundamental representation of $\mathrm{U}(1)$ or $\mathrm{SU}(2)$ (a fundam. irrep. is free)

$$\varpi_{\mathrm{U}(1)} = \frac{1}{2\phi^\dagger\phi}\Big((\mathbb{d}\phi)^\dagger\phi - \phi^\dagger(\mathbb{d}\phi)\Big) \quad \text{and} \quad \varpi_{\mathrm{SU}(2)} = \frac{1}{2\phi^\dagger\phi}\Big((\mathbb{d}\phi)^\dagger\sigma^a\phi - \phi^\dagger\sigma^a(\mathbb{d}\phi)\Big)\sigma_a$$

This is *ultralocal* (ϕ transforms without derivatives), but is only defined at configurations where $\langle\phi^\dagger\phi\rangle \neq 0$

Not always available: requires a spontaneously broken phase $\rightsquigarrow$ "*Higgs connection*"

[condensate as gauge-frame: e.g. Superconductor [see Susskind '15 *Electromagnetic memory*], Higgs]

It turns out that the Higgs connection is *flat* (where defined)

Higgs connections v. edge modes

Parametrize $\phi = \phi(h, \rho) = \rho h v_o$

$$\rightsquigarrow \quad \varpi = -\mathbb{d}h h^{-1}$$

Therefore, the Higgs connection is described by the *would-be Goldstone modes* of the field

The horizontal symplectic form relative to the Higgs connection,

$$\theta_{\text{Higgs-}H} \hat{=} \operatorname{Tr}\Big(E \mathbb{d} A\Big) - \mathrm{d}\operatorname{Tr}\Big(E \mathbb{d} h h^{-1}\Big)$$

All gauge-charges vanish, no global charge is left-over (free action, no stabilizer, true PFB)

Which looks a lot like the Donnelly-Freidel symplectic potential,

but it has a very different interpretation: since the h's are *not* new fields, just coordinates on field space

Higgs connections in a spontaneously broken phase

In the generic case, one finds formally

$$\varpi = \mathcal{E}^{ab}(\varphi)\Big((\mathrm{d}\phi)^\dagger \tau^a \phi - \phi^\dagger \tau^a (\mathrm{d}\phi)\Big)\tau_b \text{ , where } \quad \mathcal{E}^{ab} := \Big(\mathcal{D}_{ab}(\phi)\Big)^{-1} \quad \text{with} \quad \mathcal{D}_{ab}(\phi) := \phi^\dagger [\tau_a, \tau_b]_+ \phi$$

Now, if ϕ is a *non-free* representation,

$\mathcal{D}_{ab}(\phi)$ is **degenerate**.

In fact $\mathcal{D}_{ab}(\phi)$ is the **mass-matrix** of the vector bosons in a minimally coupled YM-Higgs theory

Therefore, *the matter condensate can only dress* (or *serve as a gauge-frame for*) *the broken gauge modes*

For the unbroken ones, one must resort to an extra SdW connection

Global charges are automatically associated only to unbroken gauge symmetries

The vielbein-ϖ and Lorentz invariance

A similar setup to the Higgs connections is found

in Einstein-Cartan gravity: $L_{\mathrm{EC}}[e,\omega] = \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}[\omega]$

where one might ask the question of whether it is fully equivalent to the metric formulation

(e.g. Noether charges with spurious Lorentz contribution don't give back usual black hole mechanics)

In this case we can easily find a "Higgs-connection" for the Lorentz symmetry, the *vielbein connection*

$$\mathbb{G}(\mathbb{X},\mathbb{Y}) = \int g^{\mu\nu}\eta_{IJ}\mathbb{X}^I_\mu \mathbb{Y}^J_\nu \quad \rightsquigarrow \quad \varpi^{IJ} = e^{\mu[I}\mathbb{d}e^{J]}_\mu \quad \text{and} \quad \mathbb{F}^{IJ} = -\tfrac{1}{4}e^{\mu I}e^{\rho J}g^{\nu\sigma}\mathbb{d}g_{\mu\nu}\mathbb{d}g_{\rho\sigma}$$

$$\theta_H \hat{=} \theta - \mathrm{d}\left(\epsilon_{IJKL}e^I \wedge e^J e^\mu \mathbb{d}e^L_\mu\right) = \theta_{\text{DePaoli-Speziale}} \cong \theta_{\text{metric}}$$

(Can also define the Lorentz-invariant (Lorentz-horizontal) action of diffeos: recover a proposal of Jacobson's)

In the Ashtekar-Barbero framework, a similar corner term was proposed by Freidel & Perez

[Jacobson & Mohd '15; De Paoli & Speziale '18; Perez & Freidel '15]

Part VI

summary and conclusions

Summary: ϖ for relational gauge theories in finite regions

- Reviewed the PFB-like geometry of field space

  Vertical = gauge + Horizontal = physical (but no canonical split)

- For a split one needs to make a choice

  Choice of a field-space (functional) connection form ϖ – constructed using “kinetic” supermetrics

- The connection form provides *a gauge-reference*

  It can be built out of different fields, with different properties and interpratations

  Its Wilson-lines provide natural dressing factors

  - Singer-DeWitt connection (SdW) from A field:

    curved in non-Abelian theories; gives global “Killing” charges at reducible configurations;

    Dirac-like dressings (only perturbatively in the curved non-Abelian case); related to Vilkovisky’s construction

  - Higgs connection from matter field ϕ:

    defined only where vev does not vanish (condensate); flat; reproduces phenomenology of broken phases.

There is much more!

More results that I did not cover:

- Horizontal projections do not commute with regional restrictions
- Time dependent gauge transformations and the role of A_0
- “Historical dressings” (non-perturbative, but history dependent)
- Relations to “geometric” BRST [see my ILQGS (Dec 2016)]

More questions (wip and discussions with: Gomes, Herczeg, Hopfmüller, Duarte, François, Schiavina):

- Beyond YM
- Gravity, diffeomorphisms, and the relationally defined boundaries [see my ILQGS (2016) for some initial thoughts]
- Fermions and chiral matter (necessary non-local?)
- Relations to BV-BRST?

References

Comprehensive

Gomes, Hopfmüller, & AR (2018). [arxiv:1808.02074]
A unified geometric framework for boundary charges and dressings: non-Abelian theory and matter.

Basic ideas, in short

Gomes & AR (2018). Phys. Rev. D 98, 025013
A unified geometric framework for boundary charges and dressings.

For the relation to BRST, and more on relationalism, see also

Gomes & AR (2017). JHEP 05, 017
The Observer's Ghost: a field-space connection-form and its application to gauge theories and general relativity

And there's more to come…

Thank you