
Guido Scarabottolo

Quasilocal holography from quantum gravity
in 3 dimensions

Aldo Riello

Largely based on works in collaboration with
Bianca Dittrich
Christophe Goeller
Etera Livine

Perimeter Institute for Theoretical Physics
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Introduction

Guido Scarabottolo

Finite & Quantum

Holography from non-perturbative Quantum Gravity ?

Finite (v. asymptotic) boundaries

Quantum (v. classical) boundary conditions

Questions

- What are the dual degrees of freedom?
- What is their dynamics?
- How does it compare with “standard” holography?
- And to (semi)classical computations
  e.g. Hamilton-Jacobi for linearized GR, Regge calculus [Dittrich et al]

… in 3d

We need a nonperturbative theory of Quantum Gravity

And in 3d (Euclidean, Λ=0) we have one: the **Ponzano-Regge** model

- it is *under mathematical control*
- it provides a clear picture of *quantum geometry*
- it has an *exact* realization of quantum *diffeomorphism* symmetry (in the bulk: not at the boundary!)
- it is *topological* (no bulk-local dof)

Talk (mostly) based on:
Dittrich, Goeller, Livine, AR "Quasi-local holographic dualities in non-perturbative 3d quantum gravity" series (NPB, CQG 2018)
AR "Quantum edge modes in 3d gravity and 2+1d topological phases of matter" PRD 2018
Goeller, Livine, AR "Non-Perturbative 3D Quantum Gravity: Quantum Boundary States & Exact Partition Function" to appear

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Congratulations Dr. Christophe !

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Take home messages

3d QG is the ideal framework to investigate boundaries and holography in QG

We have full control of the theory at the quantum/classical/geometric/finite/asymptotic levels

Ideal testbed to try and test ideas about holography, edge modes, continuum v. classical limit (renormalization, criticality, etc…)

Possibility to use tools and insights from other fields
E.g. AdS/CFT, spin chains, 2d stat models, integrability, condensed matter…

First…

- derivation of holographic duality from non-perturbative q-Gravity/q-Geometry model
- computation of extended spinfoam amplitude (check e.g. orientation “problem”)
- matching of a QG computation with a QFT one
  + proposal for nonperturbative QG effects/corrections
    (winding numbers ~ curvature resonances)

A lot to do, to clarify, and to play with!

Boundaries

Part of a larger project: understand quasilocal dof of gauge theories & gravities

Setup: consider finite & bounded regions, and how gauge & diffeos interact with boundaries

Advertisement: in the symplectic context, I have been developing new tools for YM theory:
many new results on factorization/gluing of YM dof, symmetries, and SSS [1]
and on the fate of symmetries in the asymptotic limit [2]

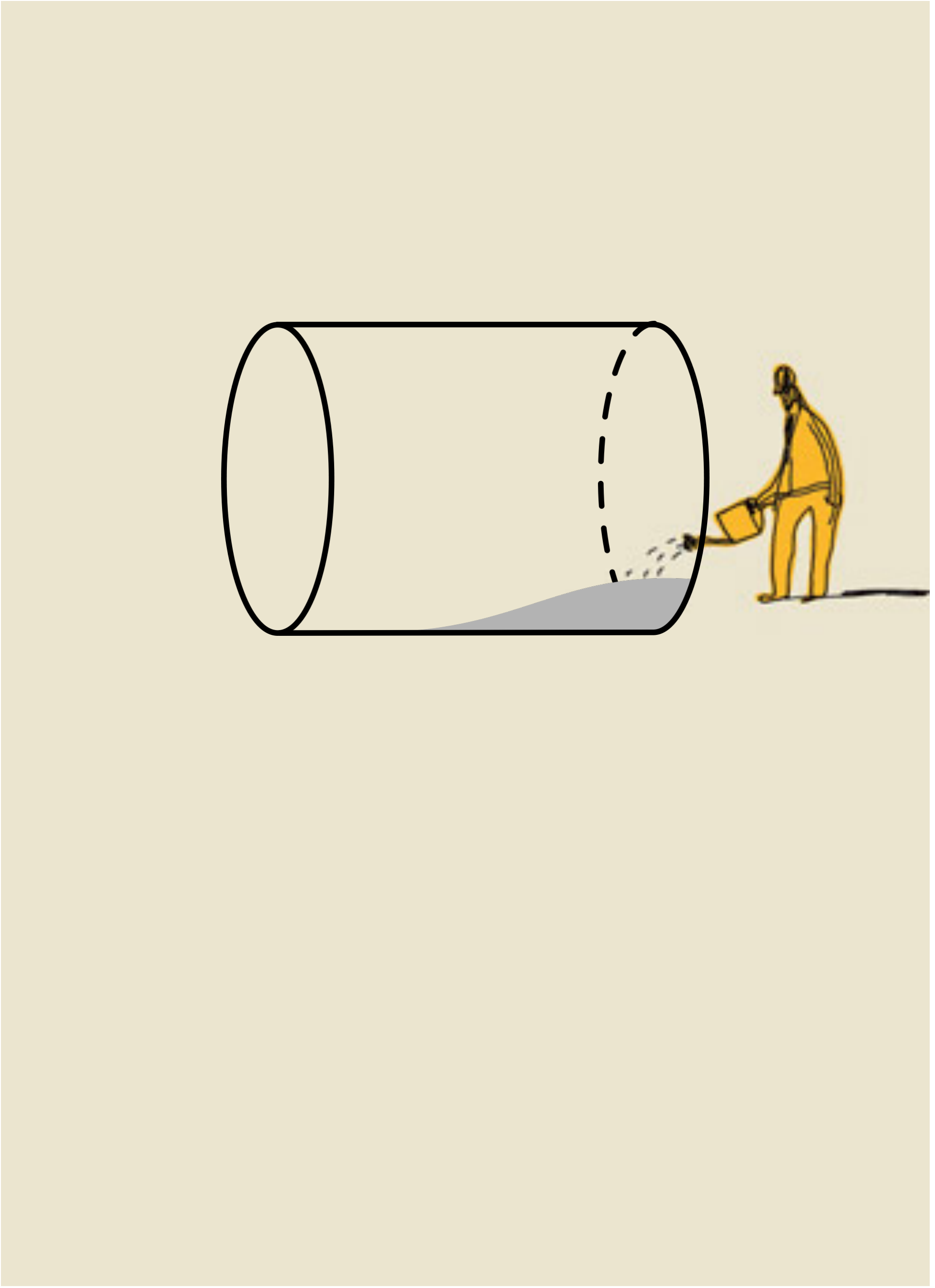
This is not only my interest!

Boundaries are alive and well in the LQG community (already 2 other ILQGS this year)…
[Bodendorfer, Corichi, Dittrich, Freidel, Geiller, Livine, Perez, Pranzetti, Wieland …]

…as well as in the larger theoretical/mathematical physics community!
[holography, asymptotic symmetries, condensed matter, entanglement entropy, extended tqft, BV-BFV, …]

[1] Gomes & AR, "The quasilocal degrees of freedom of Yang-Mills theory" arXiv:1910.04222
[2] AR, "Soft charges from the geometry of field space" arXiv:1904.07410

Part I - Preliminaries



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A quick review of

3d gravity

$$S_{\mathrm{EC}}[e,\omega] = \int_M \mathrm{tr}\big(e \wedge F[\omega]\big)$$

First order 3d gravity = SU(2) BF **topological field theory**

EoM : flatness $F[\omega] = \mathrm{d}\omega + \omega \wedge \omega = 0$

torsion-freeness $T[\omega, e] = \mathrm{d}e + \omega \wedge e = 0$

$\rightarrow$ $\begin{cases} \omega = g^{-1}\mathrm{d}g \\ e = g^{-1}(\mathrm{d}\lambda)g \end{cases}$ on-shell (trivial topology)

Pullback of EoM on a Cauchy hypersurface gives the constraints

flatness $\rightarrow$ BF shift symmetry $\begin{cases} \delta_\lambda e = \mathrm{d}_\omega \lambda \\ \delta_\lambda \omega = 0 \end{cases}$

torsion-freeness $\rightarrow$ “Lorentz” symmetry $\begin{cases} \delta_\xi e = [e, \xi] \\ \delta_\xi \omega = \mathrm{d}_\omega \xi \end{cases}$

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Pullback of EoM on a Cauchy hypersurface gives the constraints

flatness → BF shift symmetry $\begin{cases} \delta_\lambda e = \mathrm{d}_\omega \lambda \\ \delta_\lambda \omega = 0 \end{cases}$

residual diffeos in PR

torsion-freeness → "Lorentz" symmetry $\begin{cases} \delta_\xi e = [e,\xi] \\ \delta_\xi \omega = \mathrm{d}_\omega \xi \end{cases}$

"conjugate" symmetries (Poisson-Lie)

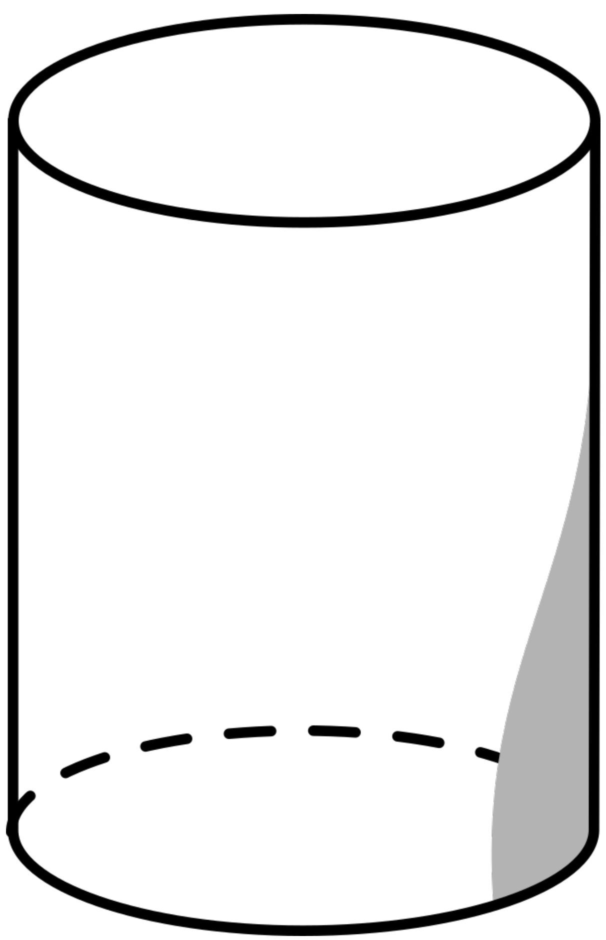
$G = \mathrm{T}^*\mathrm{SU}(2)$

Upon quantization [1]

Drinfel'd-double sym

[1] E.g.
Meusburger, Noui, "The Hilbert space of 3d gravity: quantum group symmetries and observables", ATMP 14 (2010)
Delcamp, Dittrich, AR "Fusion basis for lattice gauge theory and loop quantum gravity", JHEP 02 (2016)

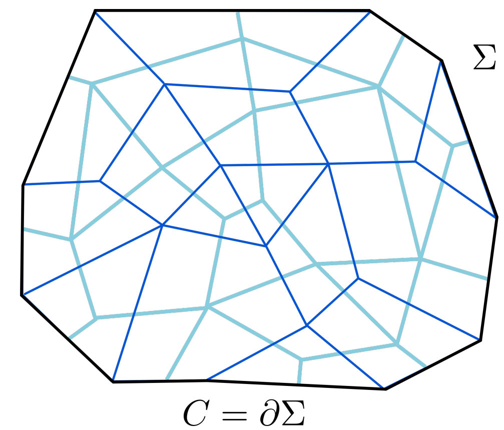
Holographic setup #1



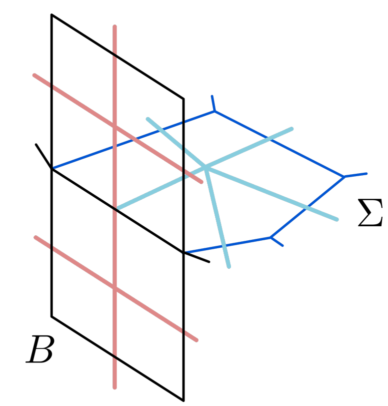
Final state $|\phi_{\mathrm{f}}\rangle$

Quantum
boundary conditions
ψ_B

Initial state $|\phi_{\mathrm{i}}\rangle$



Spin-network state (on-shell depends only on edge states)
[to be idenified #1]

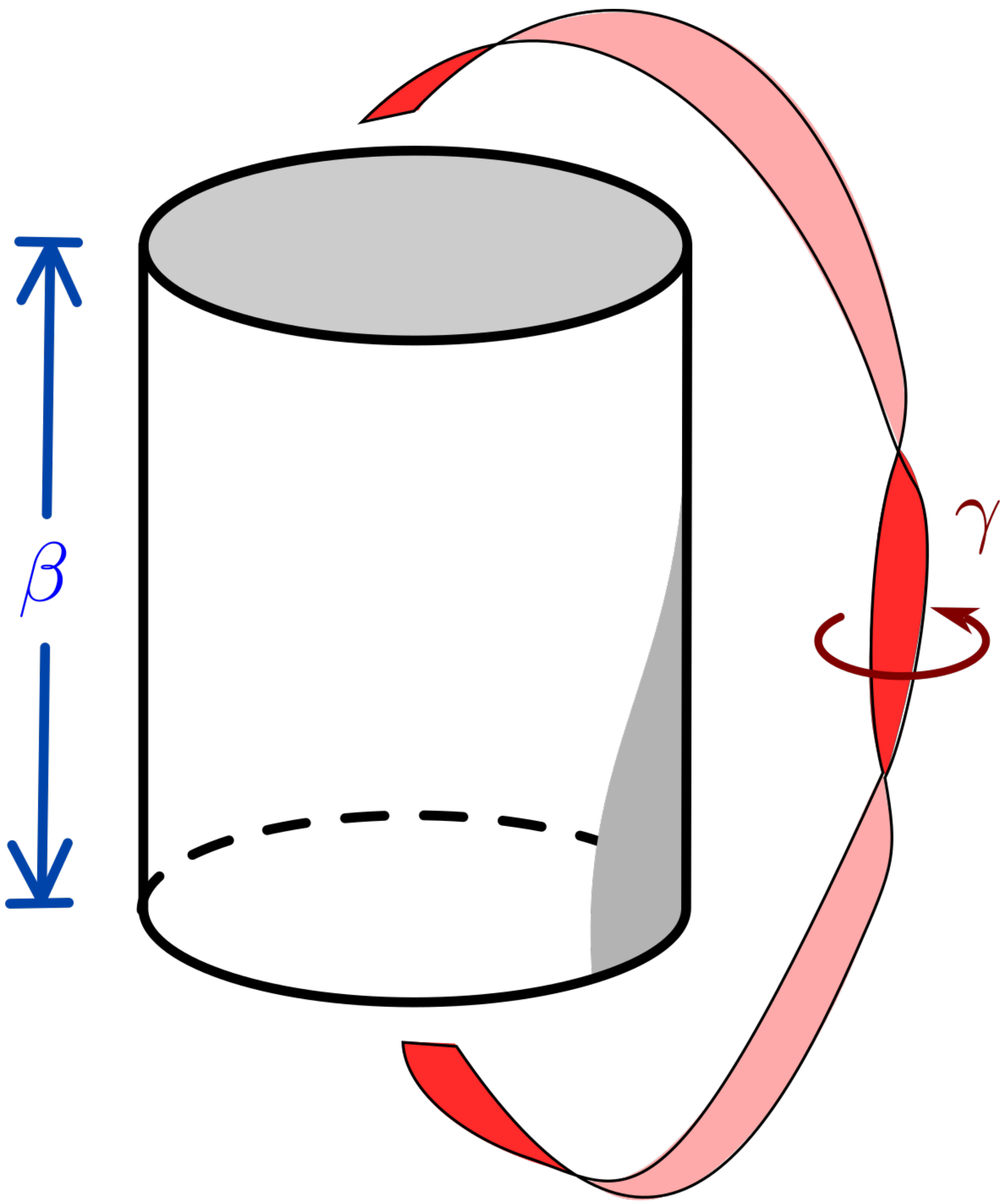


Spin-network that fixes quantum boundary metric
[to be chosen #2]

$$\langle \phi_{\mathrm{f}} | W_{\mathrm{PR}}[\psi_B] | \phi_{\mathrm{i}} \rangle =$$

Ponzano-Regge dynamics defines 3d QG transition amplitudes at fixed (quantum) boundary conditions

Holographic setup #2



Identify initial and final states up to a twist γ
[curvature around non contractible cycle]

Thermal/torus partition function
[to be computed #3]
compare to semiclassical/continuous results

$$Z_{\mathrm{PR}}[\psi_B,\gamma] := \sum_{\phi} \langle \phi | R[\gamma] W_{\mathrm{PR}}[\psi_B] | \phi \rangle$$

Holographic dual theory? [to be determined #4]
i.e. can we interpret this amplitude as the partition function for the edge dof on a certain background?

$$\int_{g_{\partial M}=h} \mathcal{D}g \, e^{-\mathrm{i}S_{\text{GR-3d}}[g]} \stackrel{?}{=} \int \mathcal{D}\phi \, e^{-\mathrm{i}S_{\text{2d}}[\phi|h]}$$

Some results [a very incomplete list]

AdS$_3$
$\Lambda<0$

Brown, Henneaux 86 : weaken AdS3 b.c. to obtain Virasoro sym at scri
Bañados et al 90s : BTZ + applications of Chern-Simons / WZW
Carlip 05 : identifies normal diffeos at scri as boundary dof
Maloney, Witten 07 : compute partition function via rep. theory
Giombi, Maloney, Yin 08 : compute partition function @ 1-loop from GR
Cotler, Jensen 19: new interpretation of boundary dof + Schwarzian action; and resolution of some puzzle in Maloney-Witten (on modular invariance)

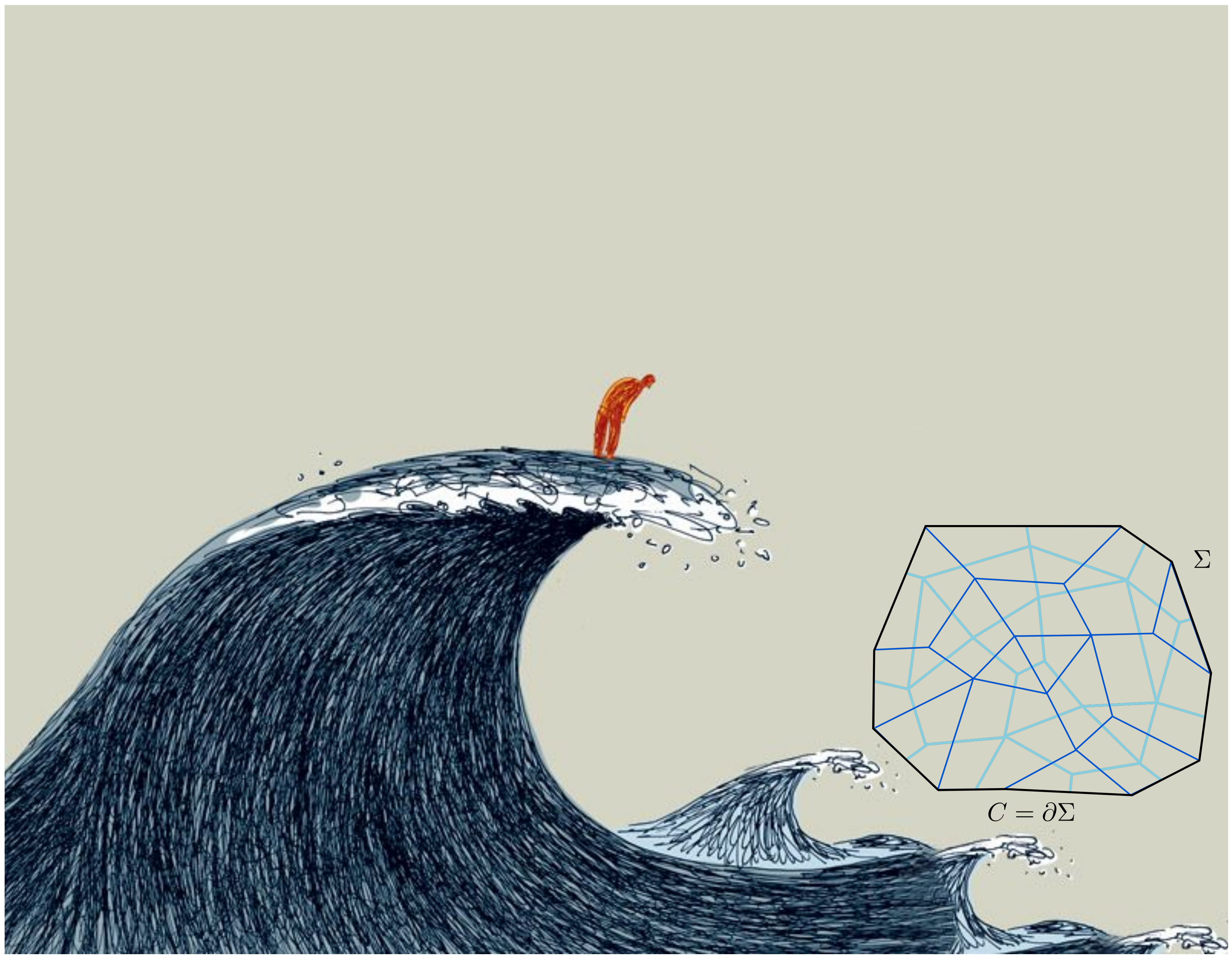
[+ many many others]

Interesting parallels and differences

Minkowski$_3$
$\Lambda=0$

Oblak 15 : Characters of BMS3 = flat limit of Virasoro
Barnich et al 15 : compute partition function @ 1-loop from GR
Bonzom, Dittrich 15 : compute partition function @ 1-loop in quantum Regge calculus: agreement modulo discrete truncation
Dittrich, Goeller, Livine, AR 17-19 : quantum PR computations & dualities
AR 17 : quantum edge states for PR, dualities & 3d QG symmetries
Castro, Dittrich 19 : point particles leads to massive BMS3 character
Asante, Dittrich, Hopfmüller 19 : linearized HJ theory for general boundaries
Asante, Haggard, Dittrich 18 : "flat-Regge" (KBF) generalization to 4d
AR, Artigas-Guimarey 19 : PR with Wilson-line observables [wip]

Part II - Edge States



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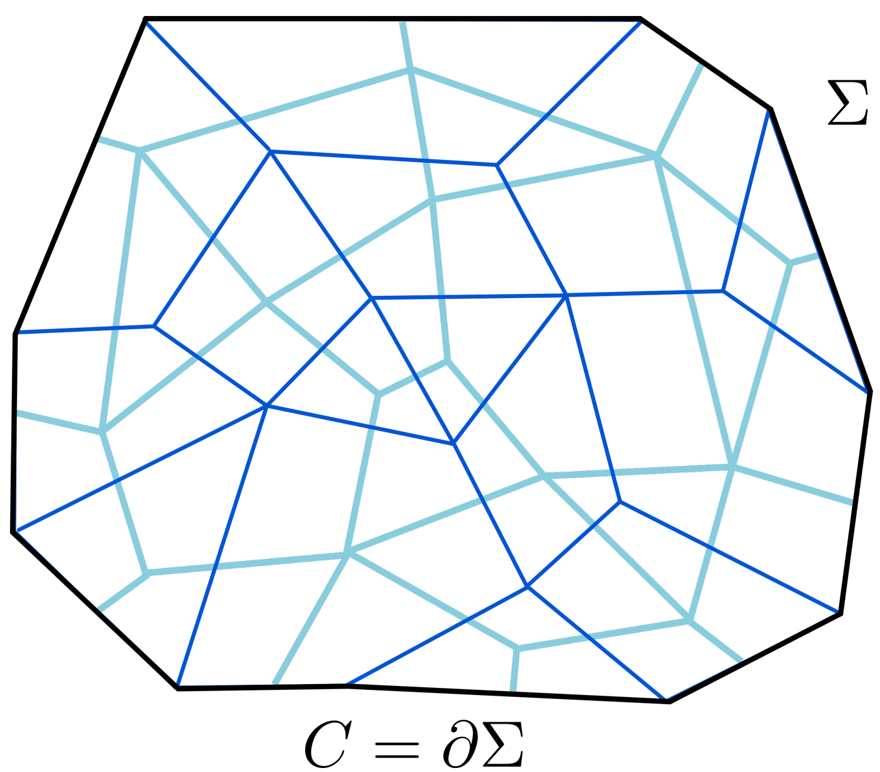
Balachandran, Chandar, Momen, NPB 461 (1996);
Donnelly, PRD 77 (2008); Donnelly, Freidel JHEP 09 (2016);
Casini, Huerta, Rosabal PRD 89 (2014);
Delcamp, Dittrich, AR JHEP 11 (2016); AR, PRD 98 (2018)

Kinematic states

Dark blue = cellular decomposition Δ of hypersurface Σ

Light blue = Poincaré dual graph Γ (spin-network)



$$\phi(g_\ell) = \phi(u_t^{-1} g_\ell u_s) \qquad \text{bulk}$$

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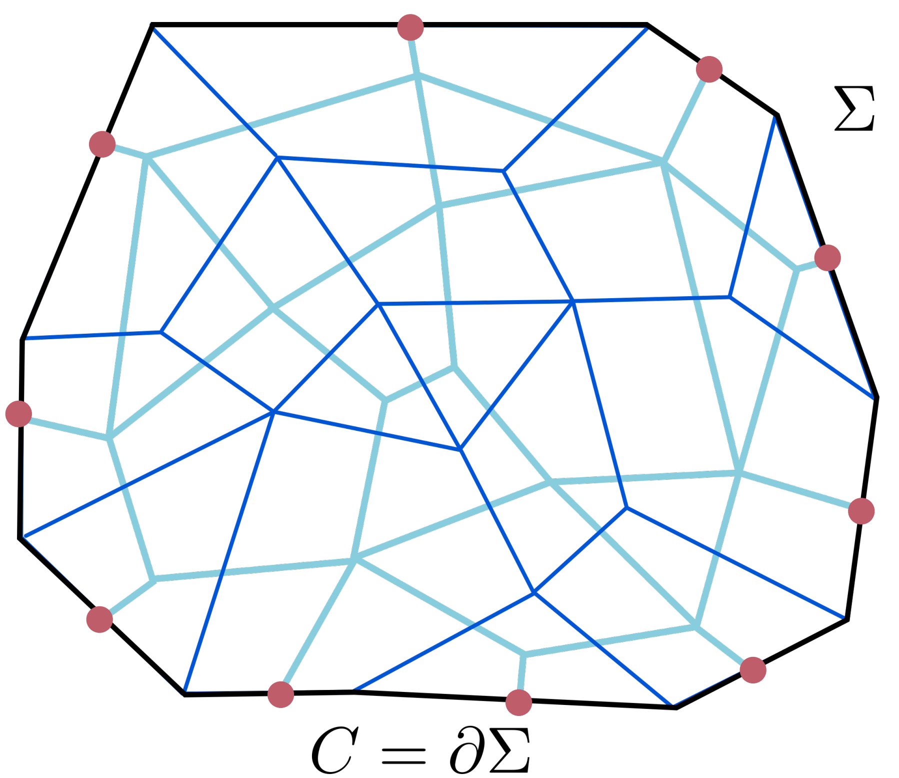


$$\phi(g_\ell) = \phi(u_t^{-1} g_\ell u_s) \qquad \text{bulk}$$

Boundary breaks gauge
need for "compensating" fields:
Carlip's "would-be-gauge" dofs, or "edge modes"

↓

$$\phi(g_\ell, g_{n_\partial}) = \phi(u_t^{-1} g_\ell u_s, u_{n_\partial}^{-1} g_{n_\partial})$$

RMK this choice of compensating fields, focuses on Lorentz symmetry
it corresponds to the "electric center" prescription of CHR14

Kinematic states

Planchharel

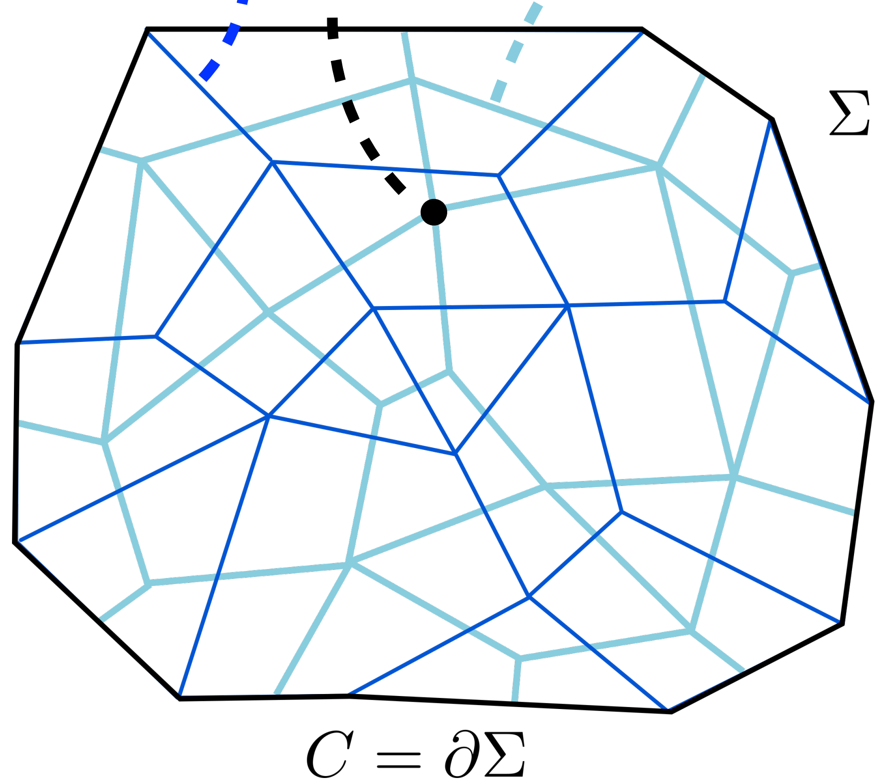
develop over Spin-network basis = “Group Fourier transform”

$$\phi(g_\ell) = \sum_{j_\ell} \hat{\phi}(j_\ell, I_n)\,\mathrm{tr}_\Gamma\Big(\bigotimes_n I^{m\dots}_{p\dots} \bigotimes_\ell D^{j_\ell}(g_\ell)^p{}_m\Big)$$

bulk spin
= q. bulk metric

bulk
intertwiner

bulk
holonomy



Kinematic states

Planchernel

develop over Spin-network basis = “Group Fourier transform”

$$\phi^{[j_{\ell_\partial}]}(g_\ell, g_{n_\partial}) = \sum_{j_\ell, m_{n_\partial}} \hat{\phi}^{[j_{\ell_\partial}]}(j_\ell, I_n, m_{n_\partial}) \mathrm{tr}_\Gamma \Big(\bigotimes_n I^{m\dots}_{p\dots} \bigotimes_\ell D^{j_\ell}(g_\ell)^p{}_m \bigotimes_{n_\partial} D^{j_{\ell_\partial}}(g_{n_\partial})^m{}_{m_{n_\partial}} \Big)$$

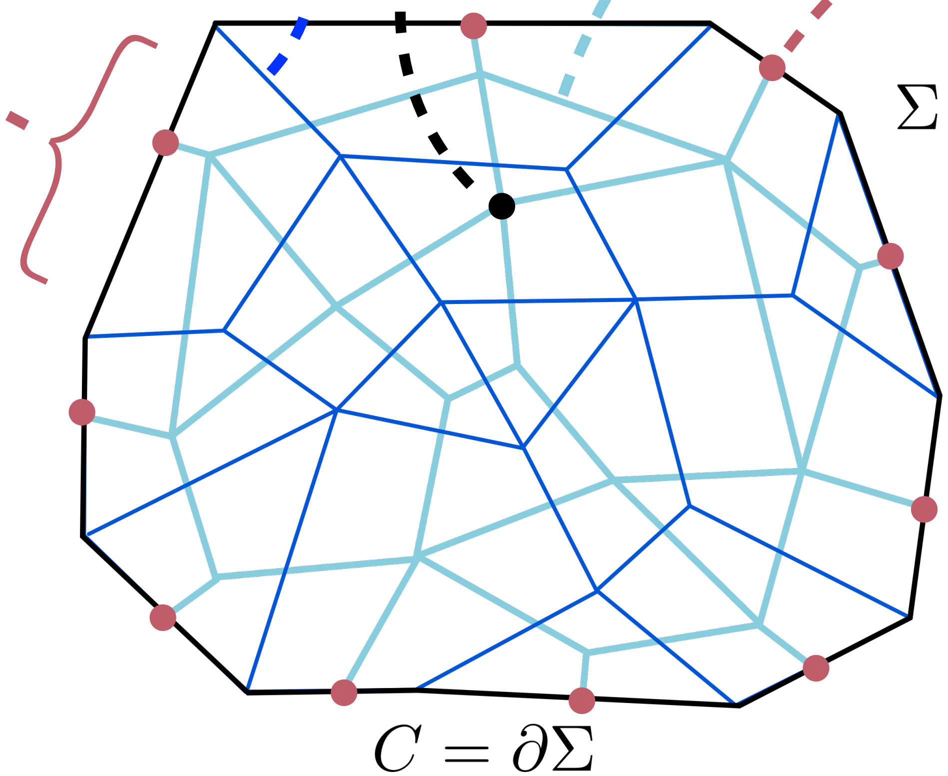
boundary spin
= induced q. metric
at the boundary
[superselected]

bulk spin
= q. bulk metric

bulk
intertwiner

bulk
holonomy

boundary field
= boundary Lorentz
reference frame
~
Analogous to WZW
field in CS



Boundary dof

group elements as boundary dof

develop over Spin-network basis = “Group Fourier transform”

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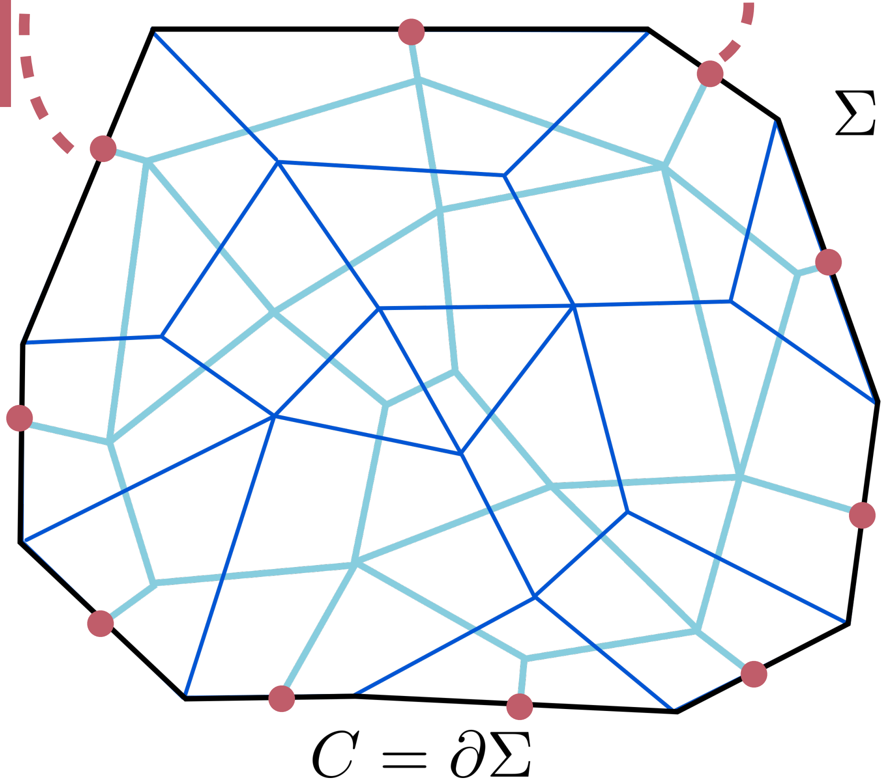
Partial Fourier Transform $\longrightarrow \check{\phi}^{[j_{\ell_\partial}]}(g_\ell, m_{n_\partial}) \longleftrightarrow \tilde{\phi}^{[j_{\ell_\partial}]}(g_\ell, \xi_{n_\partial})$

spinors as boundary dof

magnetic indices as boundary dof



Boundary dof - summary

group elements as boundary dof	magnetic indices as boundary dof	spinors as boundary dof

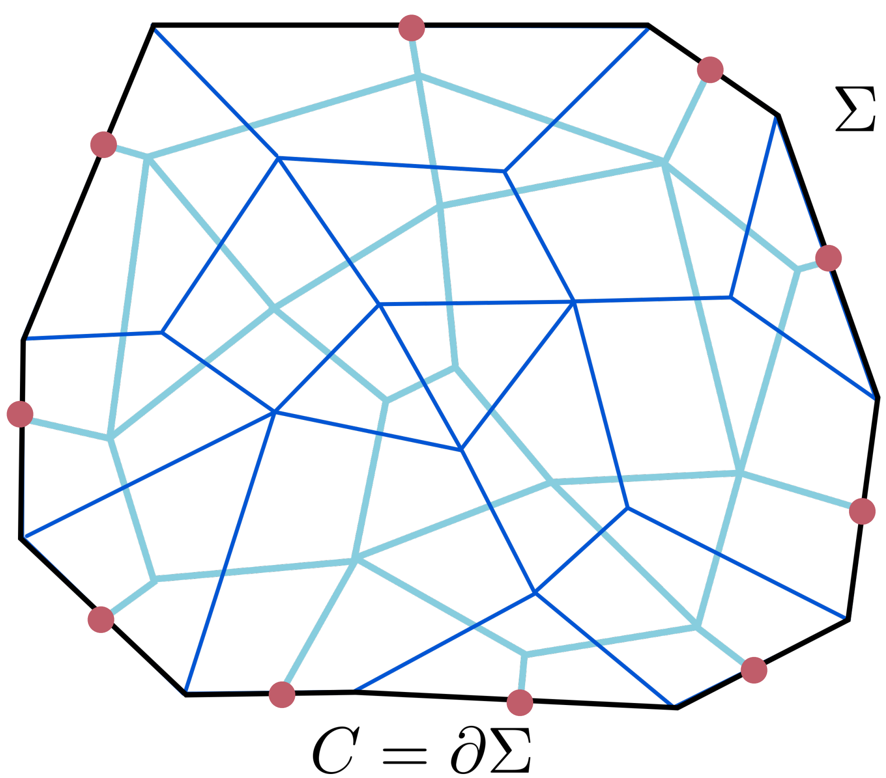
$$\phi^{[j_{\ell_\partial}]}(g_\ell, g_{n_\partial}) \longrightarrow \check{\phi}^{[j_{\ell_\partial}]}(g_\ell, m_{n_\partial}) \longleftrightarrow \tilde{\phi}^{[j_{\ell_\partial}]}(g_\ell, \xi_{n_\partial})$$

in analogy with: ~ WZW ~ Wieland

$$\eta_{\text{Wie}} \sim j\xi$$

The induced (quantum) **boundary metric is externally given** (superselected)

Mathematically, it is encoded in the **boundary spins**



The **boundary dof** geometrically represent the orientation of the reference frame at/**orientation of the boundary** (~extrinsic curvature)

Mathematically, this is encoded either in a group element, a spinor, or a magnetic index

Kinematic states

Boundary dof - summary

group elements as boundary dof	magnetic indices as boundary dof	spinors as boundary dof

$$\phi^{[j_{\ell_\partial}]}(g_\ell, g_{n_\partial}) \longrightarrow \check{\phi}^{[j_{\ell_\partial}]}(g_\ell, m_{n_\partial}) \longleftrightarrow \tilde{\phi}^{[j_{\ell_\partial}]}(g_\ell, \xi_{n_\partial})$$

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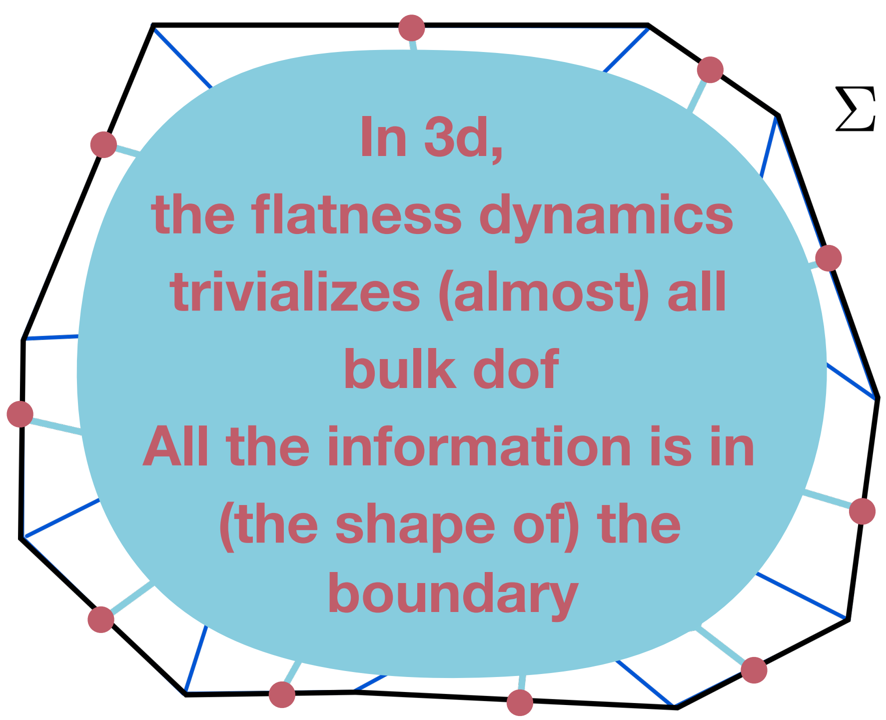

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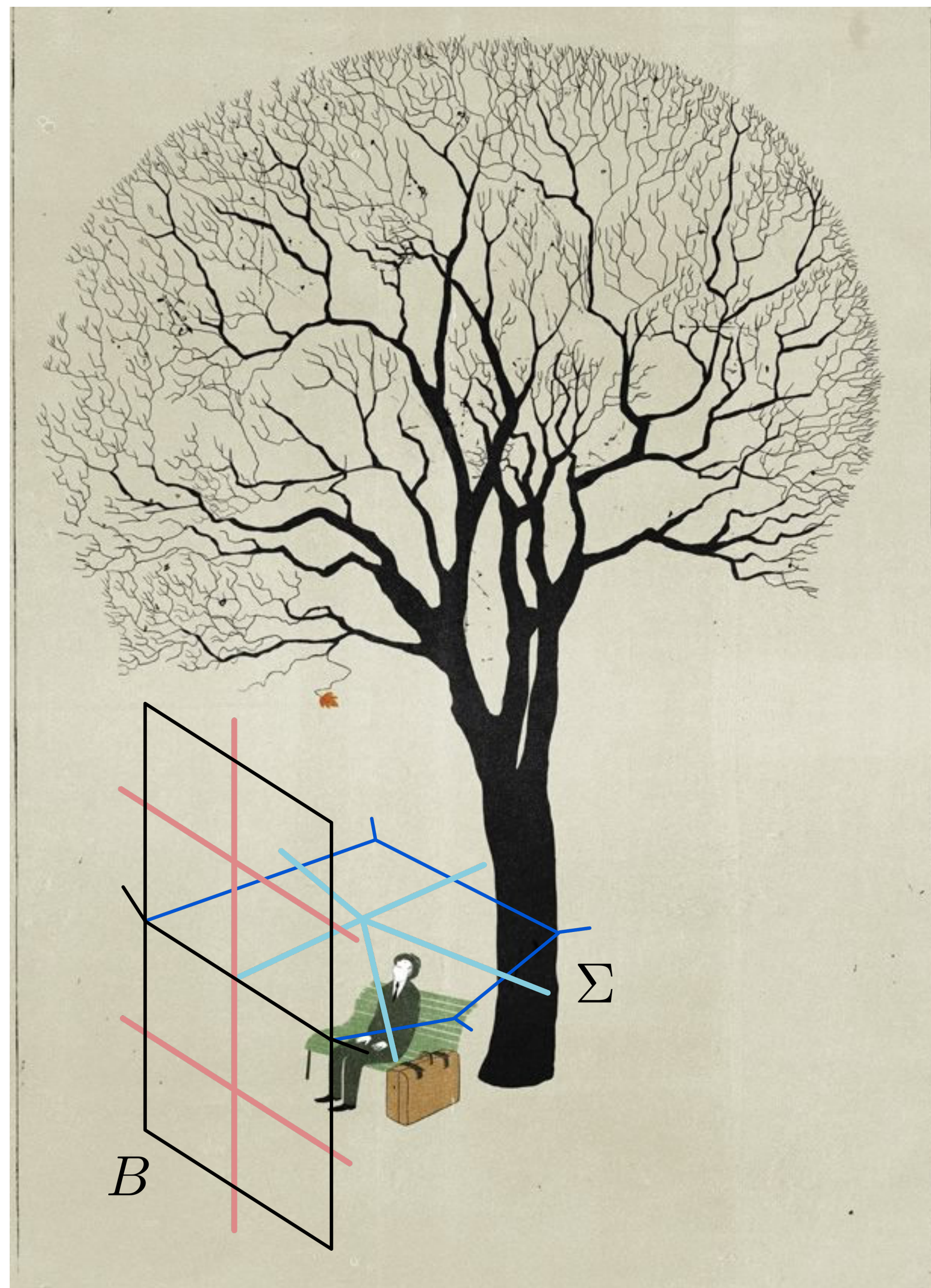
Part III - Boundary dynamics



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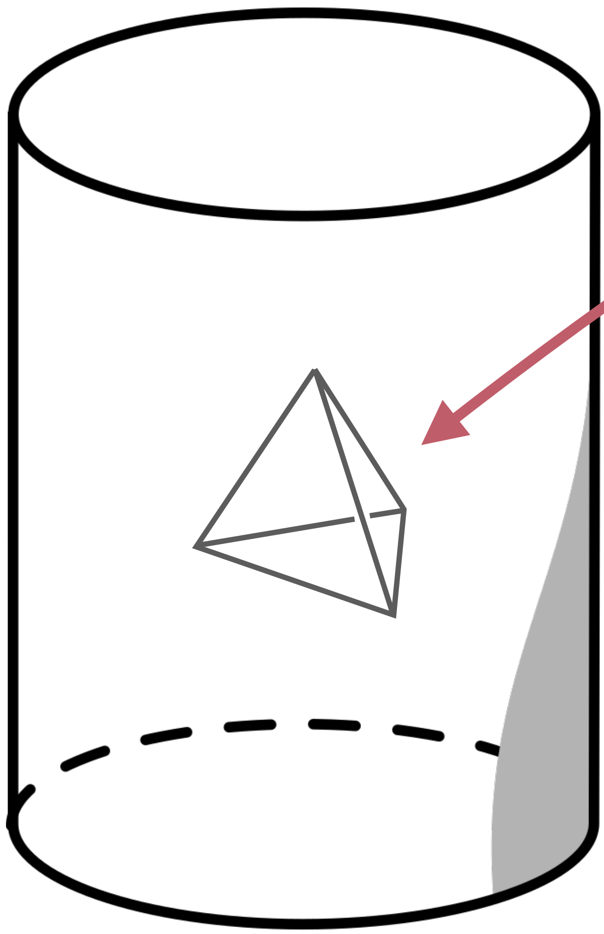
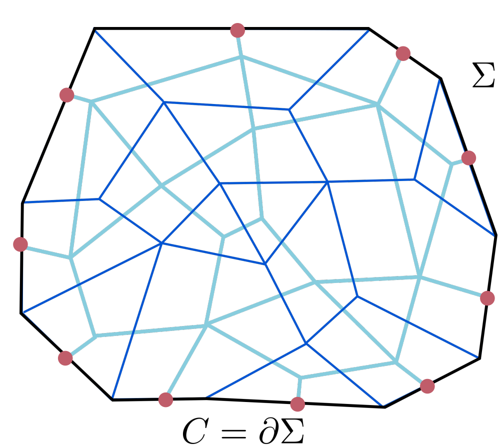
From bulk to boundary dynamics



$$\langle \phi_{\mathrm{f}} | W_{\mathrm{PR}} | \phi_{\mathrm{i}} \rangle = \sum_{\{j\}} \prod_{e} (-1)^{2j_e} (2j_e + 1) \prod_{\text{tets}} \{6j\} \hat{\phi}^*_{\mathrm{f}}(j^{\mathrm{f}}_{\partial}) \hat{\phi}_{\mathrm{i}}(j^{\mathrm{i}}_{\partial})$$

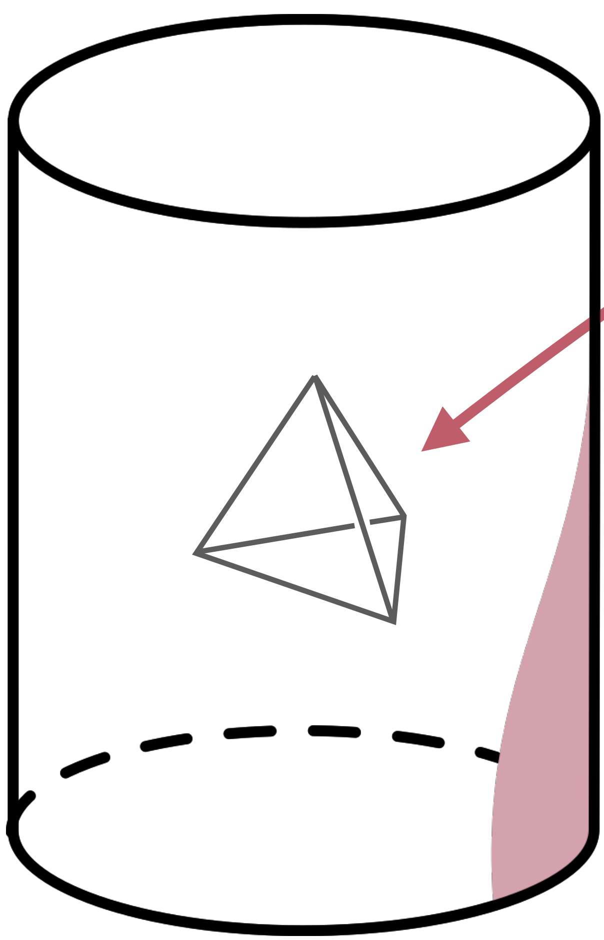
$$= \int \mathrm{d}g \prod_{f} \delta\Big(\overleftarrow{\prod} g_\ell\Big) \phi^*_{\mathrm{f}}(g^{\mathrm{f}}_{\partial}) \phi_{\mathrm{i}}(g^{\mathrm{i}}_{\partial})$$

(Almost) trivial bulk dynamics:
PR dynamics projects onto flat states



$|\phi_{\mathrm{i}}\rangle$

From bulk to boundary dynamics



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(Almost) trivial bulk dynamics:
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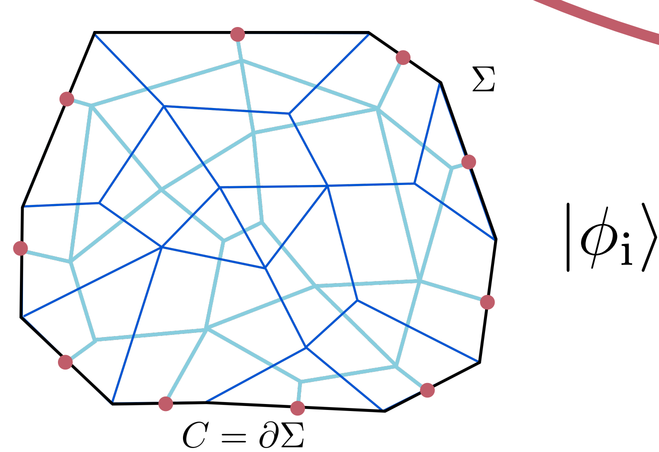
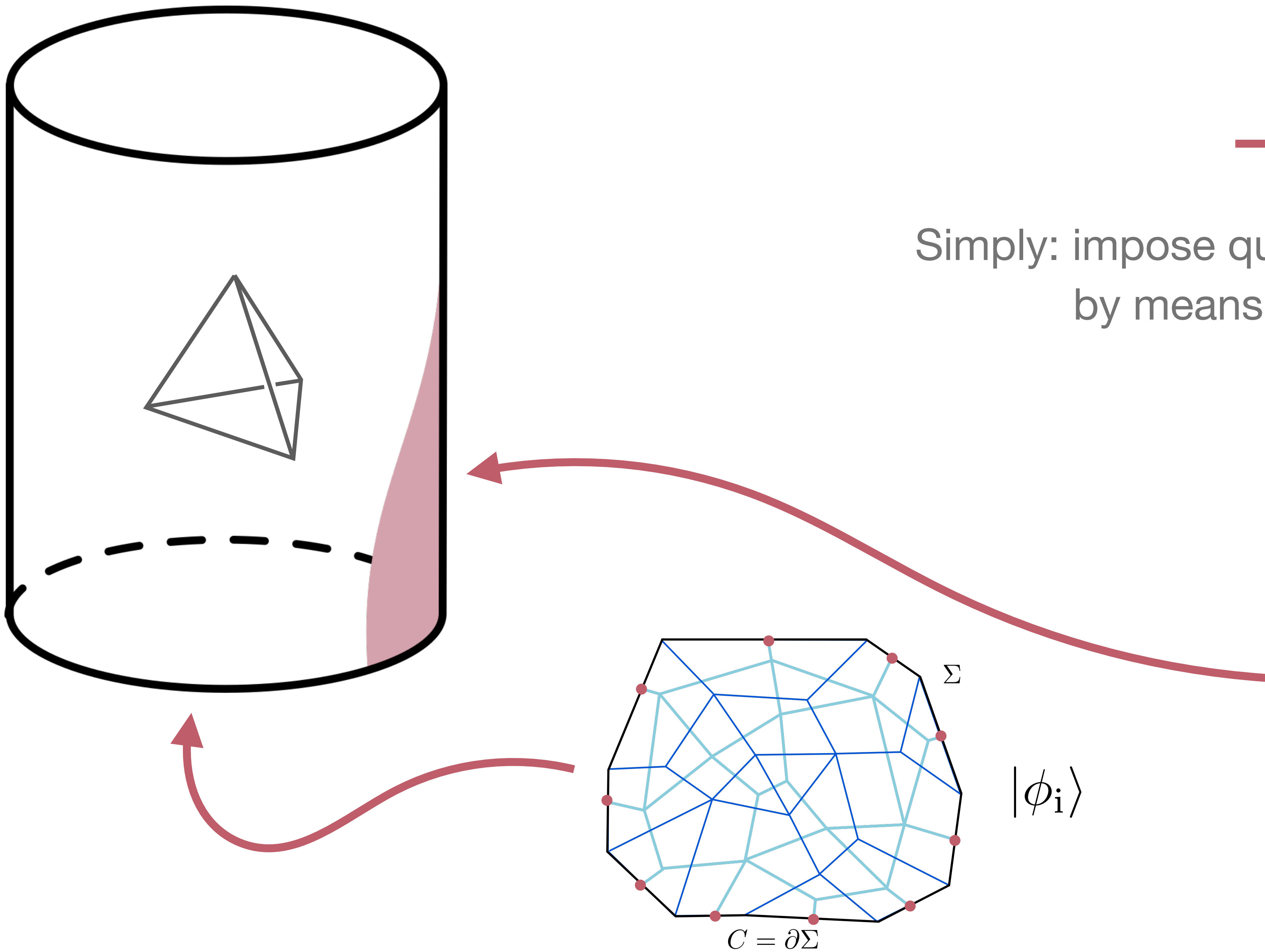
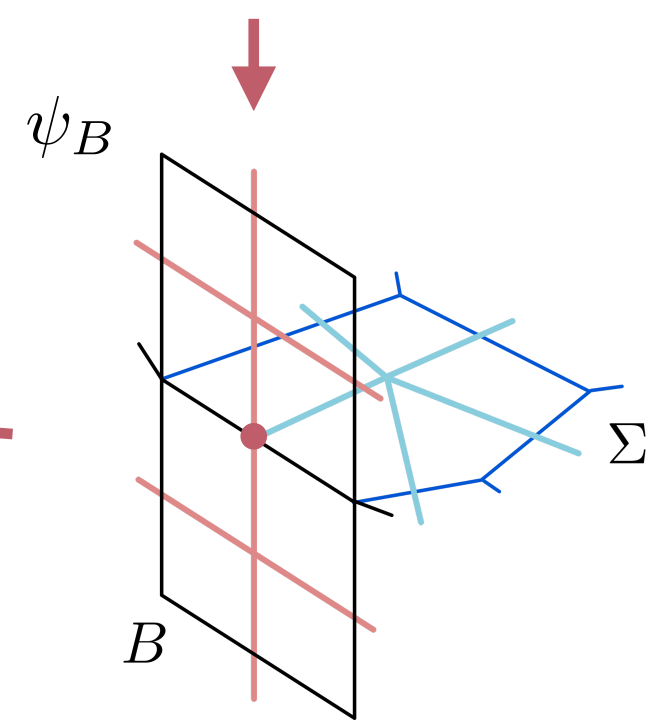
How to treat the “time-like” boundary?



From bulk to boundary dynamics

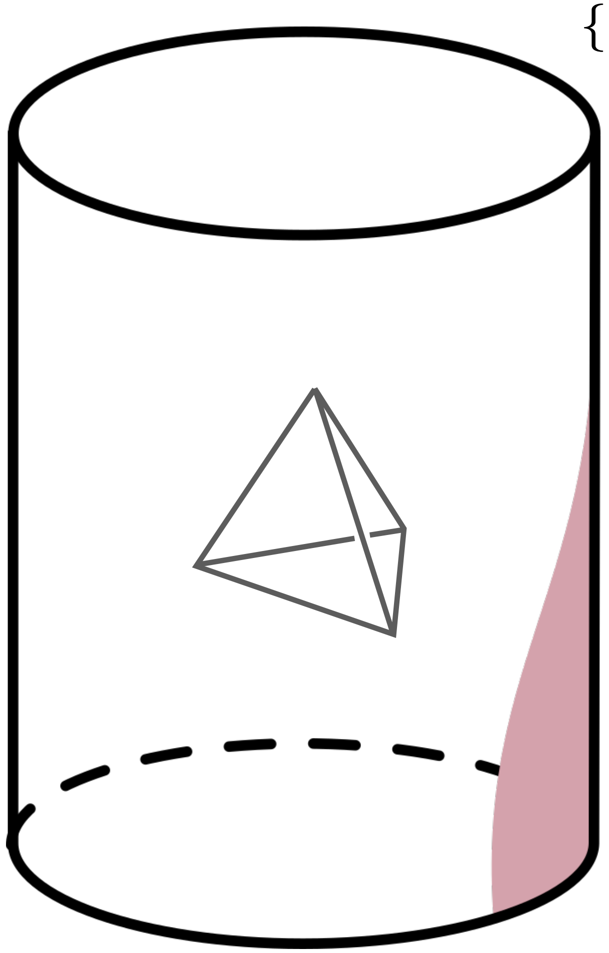


Simply: impose quantum boundary conditions
by means of a boundary “state”



From bulk to boundary dynamics

$$\langle \phi_{\mathrm{f}} | W_{\mathrm{PR}}[\psi_B] | \phi_{\mathrm{i}} \rangle = \sum_{\{m_\partial\}} \int \mathrm{d}g \prod_f \delta\Big(\overleftarrow{\prod} g_\ell\Big) \phi_{\mathrm{f}}^*(g_\partial^{\mathrm{f}}, m_\partial^{\mathrm{f}}) \psi_B(g_\partial^{\mathrm{B}}, m_\partial^{\mathrm{f}}, m_\partial^{\mathrm{i}}) \phi_{\mathrm{i}}(g_\partial^{\mathrm{i}}, m_\partial^{\mathrm{i}})$$



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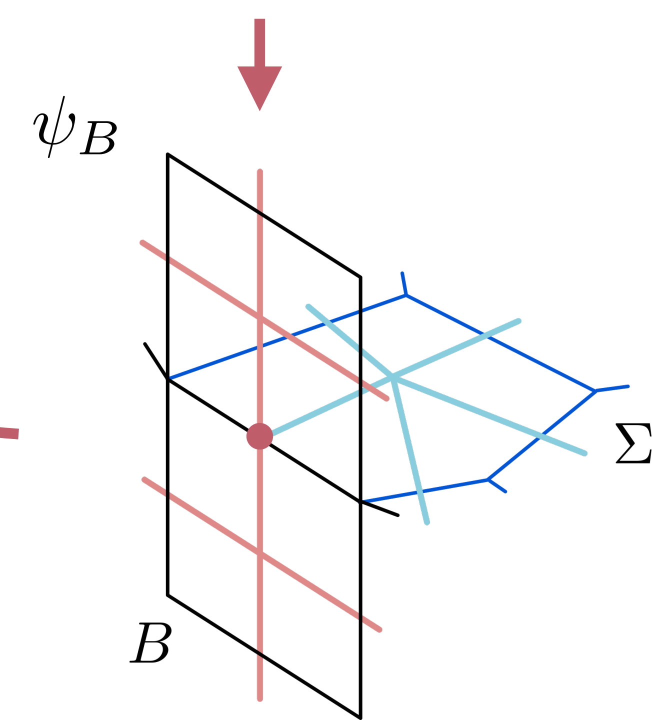

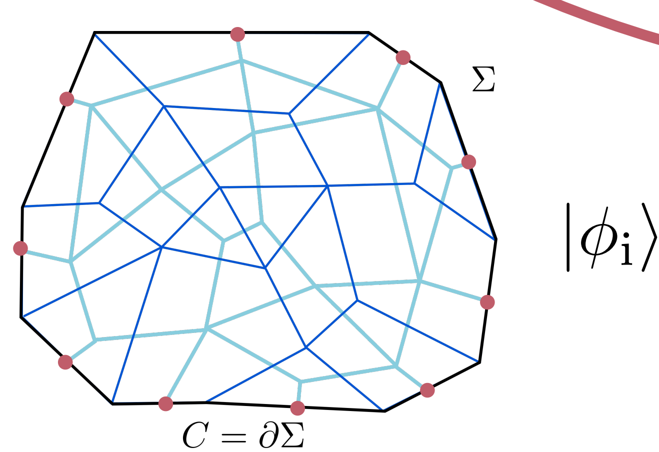

From bulk to boundary dynamics

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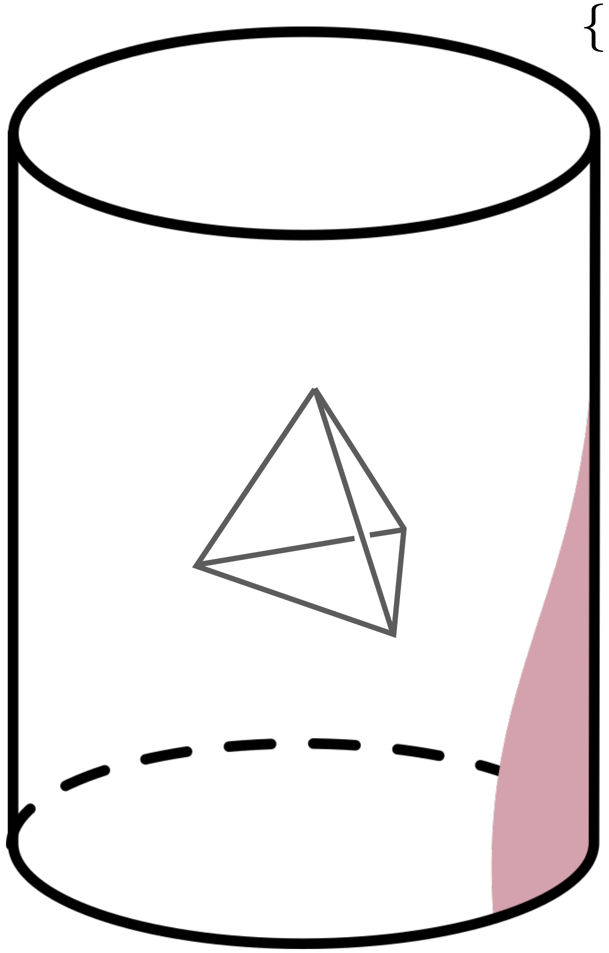
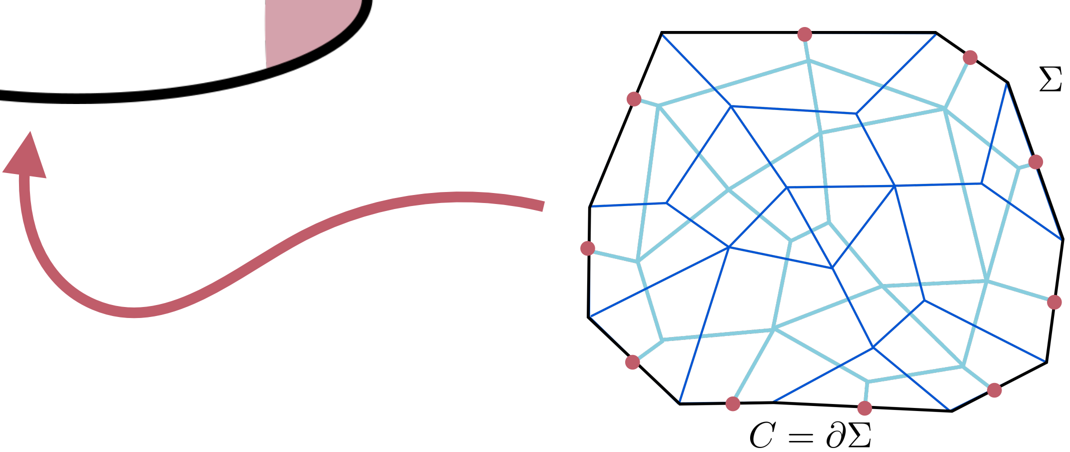
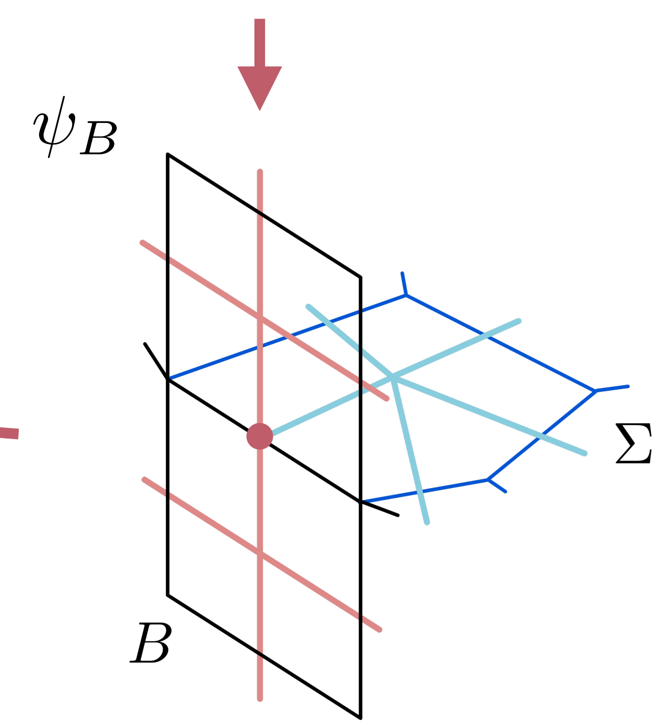
$$= \sum_{\{m_\partial\}} \phi^*_{\mathrm{f}}(\mathbb{1}, m^{\mathrm{f}}_\partial) \underline{\psi_B(\mathbb{1}, m^{\mathrm{f}}_\partial, m^{\mathrm{i}}_\partial)} \phi_{\mathrm{I}}(\mathbb{1}, m^{\mathrm{I}}_\partial)$$

Trivial Topology

Simply: impose quantum boundary conditions by means of a boundary "state"

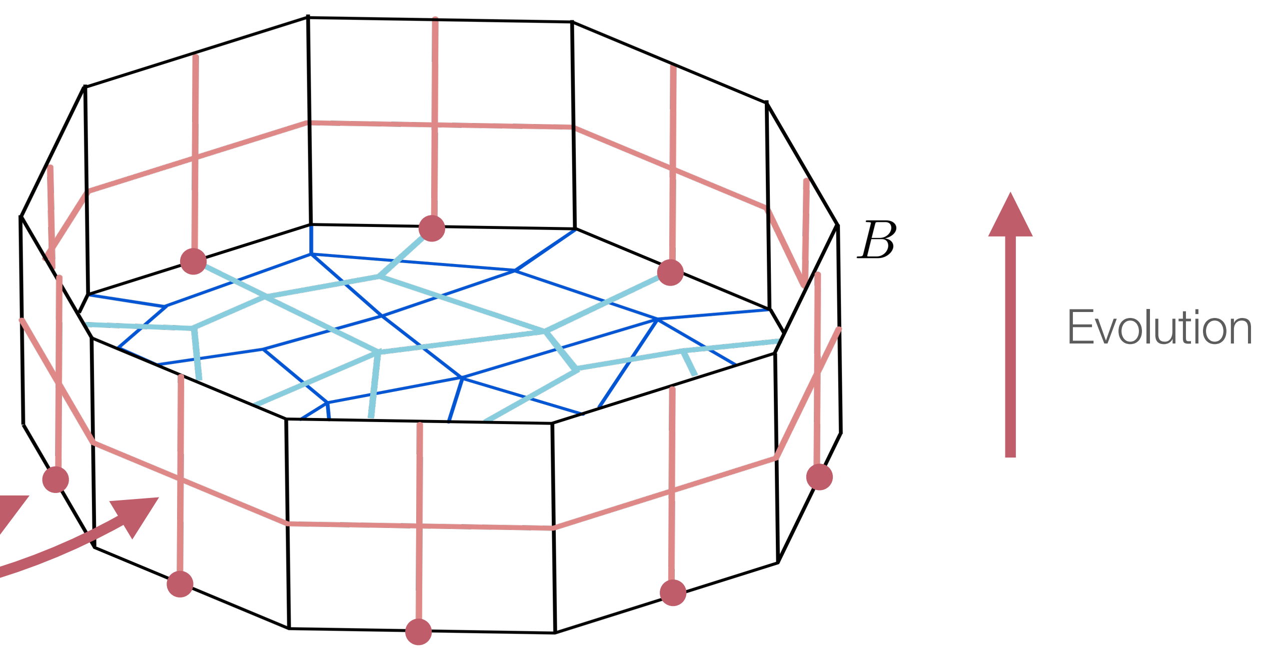






Boundary spin-chain

ψ_B is a spin-network state, i.e. **fixed boundary spins** = fixed intrinsic metric



$$\langle\phi_{\mathrm{f}}|W_{\mathrm{PR}}[\psi_B]|\phi_{\mathrm{i}}\rangle = \sum_{\{m_\partial\}} \phi^*_{\mathrm{f}}(\mathbb{1}, m^{\mathrm{f}}_\partial)\underline{\psi_B(\mathbb{1}, m^{\mathrm{f}}_\partial, m^{\mathrm{i}}_\partial)}\phi_{\mathrm{I}}(\mathbb{1}, m^{\mathrm{I}}_\partial)$$

Recall : initial state is an edge state for the magnetic indices *m*

Thus, edge states are states of a **spin-chain**

One layer of boundary evolution (as prescribed by boundary state) = spin-chain transfer matrix

1/2 Boundary Spins

If all boundary spins are **1/2**, the dual theory is given by the following integrable system:

Heisenberg spin chain [1d QM]

$$H_{XXZ} = -\frac{1}{4}\sum_{n=1}^{N_x}\left(\sigma_n^1\sigma_{n+1}^1 + \sigma_n^2\sigma_{n+1}^2 + \Delta\sigma_n^3\sigma_{n+1}^3\right) \xrightarrow{\Delta \to 1}$$

$\updownarrow$

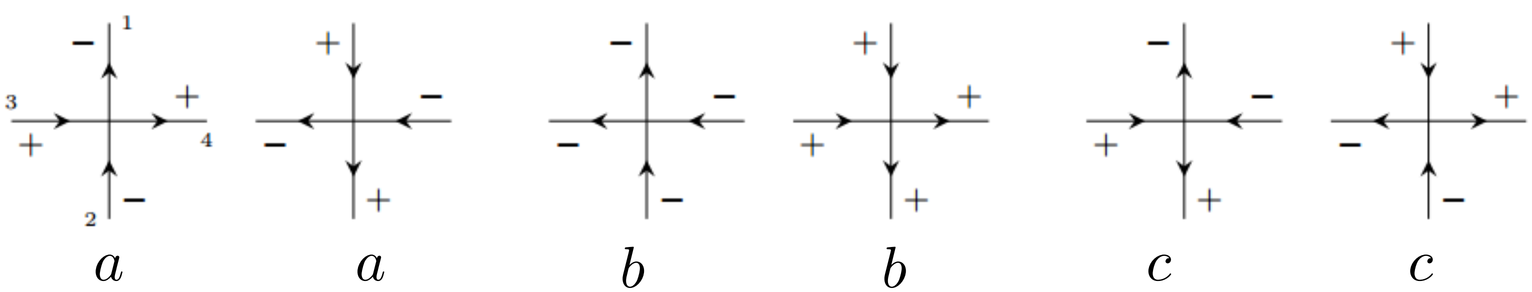
6-vertex model [2d Stat Mech]

isotropic H. spin chain

$$H_{XXX} = -\frac{1}{4}\sum_{n=1}^{N_x}\vec{\sigma}_n^1 \cdot \vec{\sigma}_{n+1}^1$$

$\updownarrow$

stochastic 6-vertex m.



$$\Delta = \frac{a^2 + b^2 - c^2}{2ab}$$

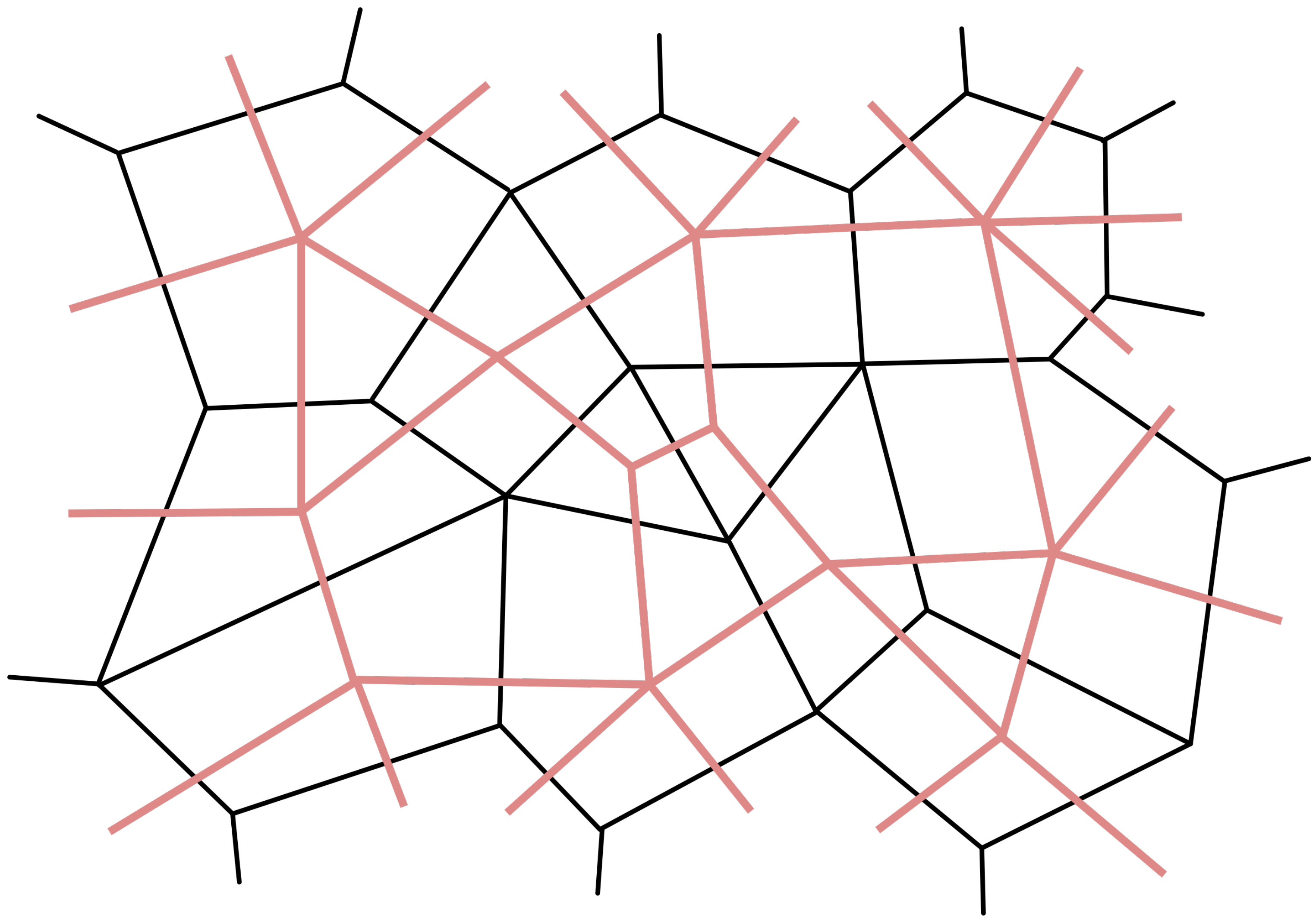
Vertex type # with Boltzmann weight a, b, c ; model is said stochastic if $\boxed{a = b + c}$

intertwiner space $(1/2)^{\otimes 4}$ is 2d

REMARK: there are many techniques to study continuum limits of these models

[see e.g. Reshetikhin & Sridhar 2016] → new tools to study large-spin v. many spins?

Boundary Face-Vertex duality from 3d QG

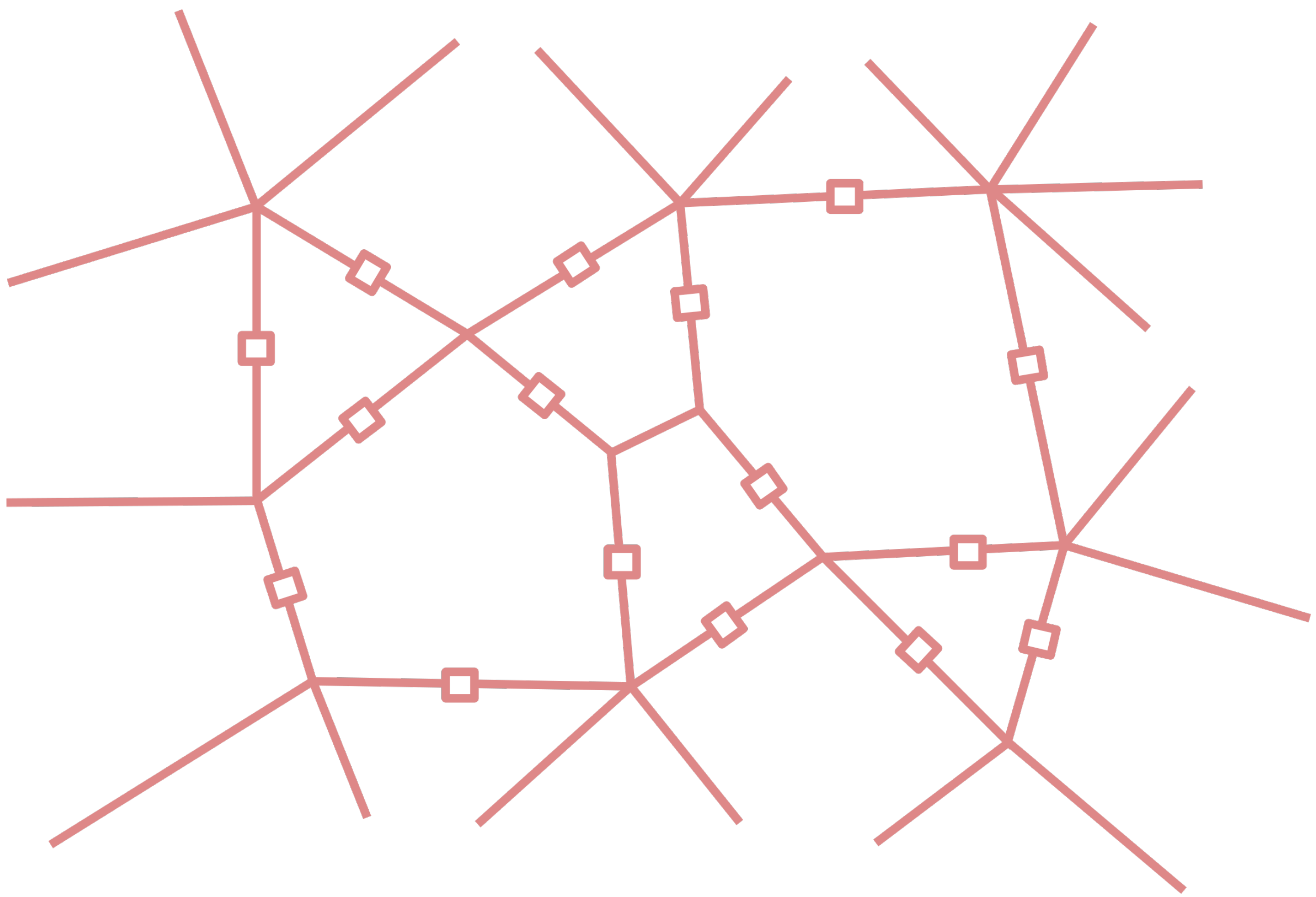


Boundary Face-Vertex duality from 3d QG

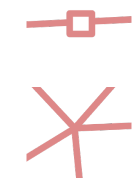
Spin-chain model // Vertex model

[m ~ **Lorentz symmetry** compensator fields]

[boundary spins j = **background**]



1. Boundary dynamics from spin-network evaluation



$$\sum_{m_\partial}$$

$$\iota^{m_\partial \dots}$$

Boundary Face-Vertex duality from 3d QG

Spin-chain model // Vertex model

[*m* ~ **Lorentz symmetry** compensator fields]

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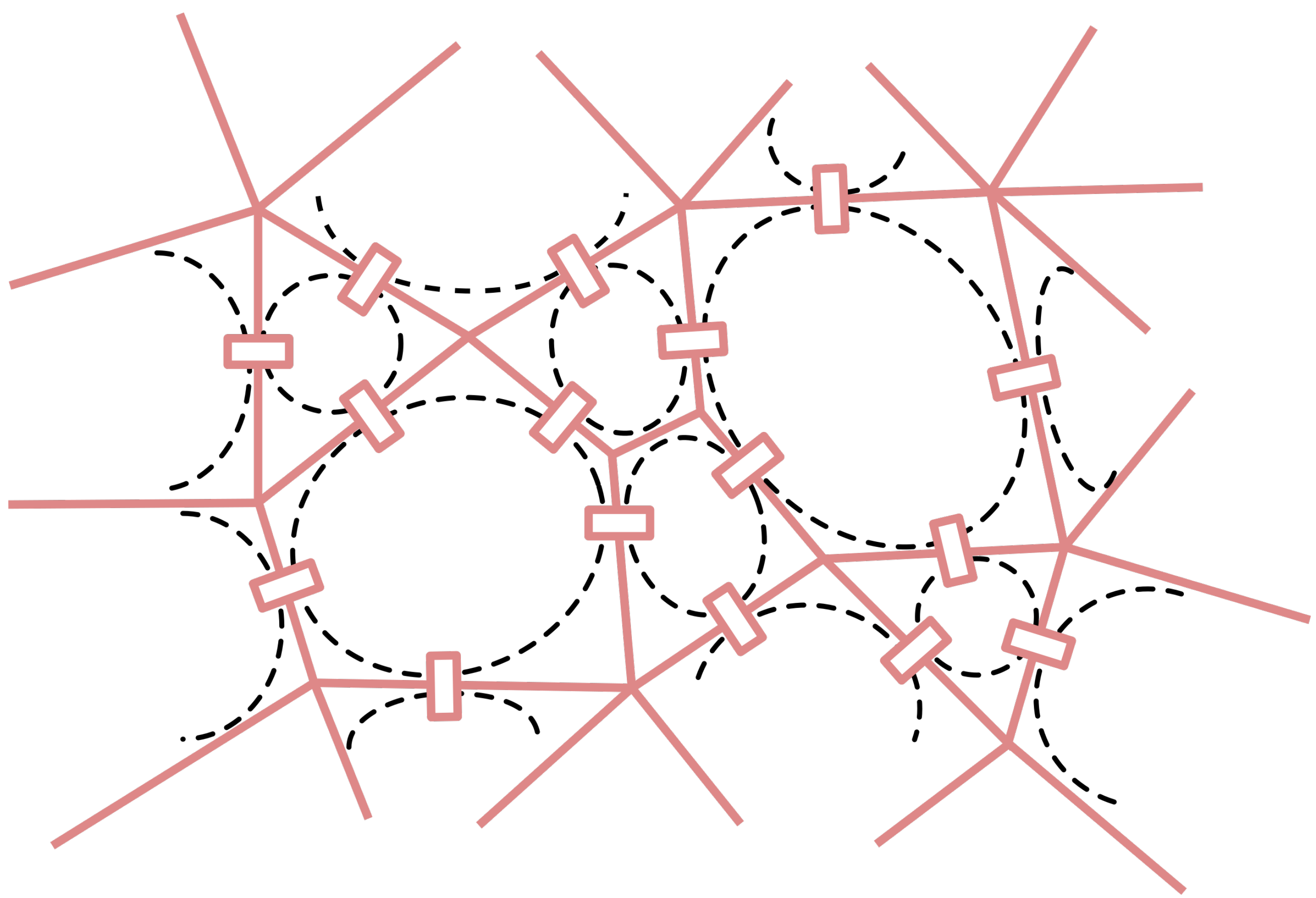

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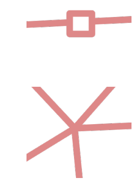

$$\sum_{m_\partial}$$

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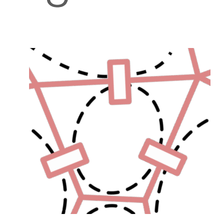
2. Boundary dynamics from boundary delta functions + integration over *h*'s



$$\int \mathrm{d}h_{\ell_\partial}$$

$$\delta\Big(\overleftarrow{\prod} h_{\ell_\partial}\Big)$$

Boundary Face-Vertex duality from 3d QG

Spin-chain model // Vertex model

[m ~ **Lorentz symmetry** compensator fields]

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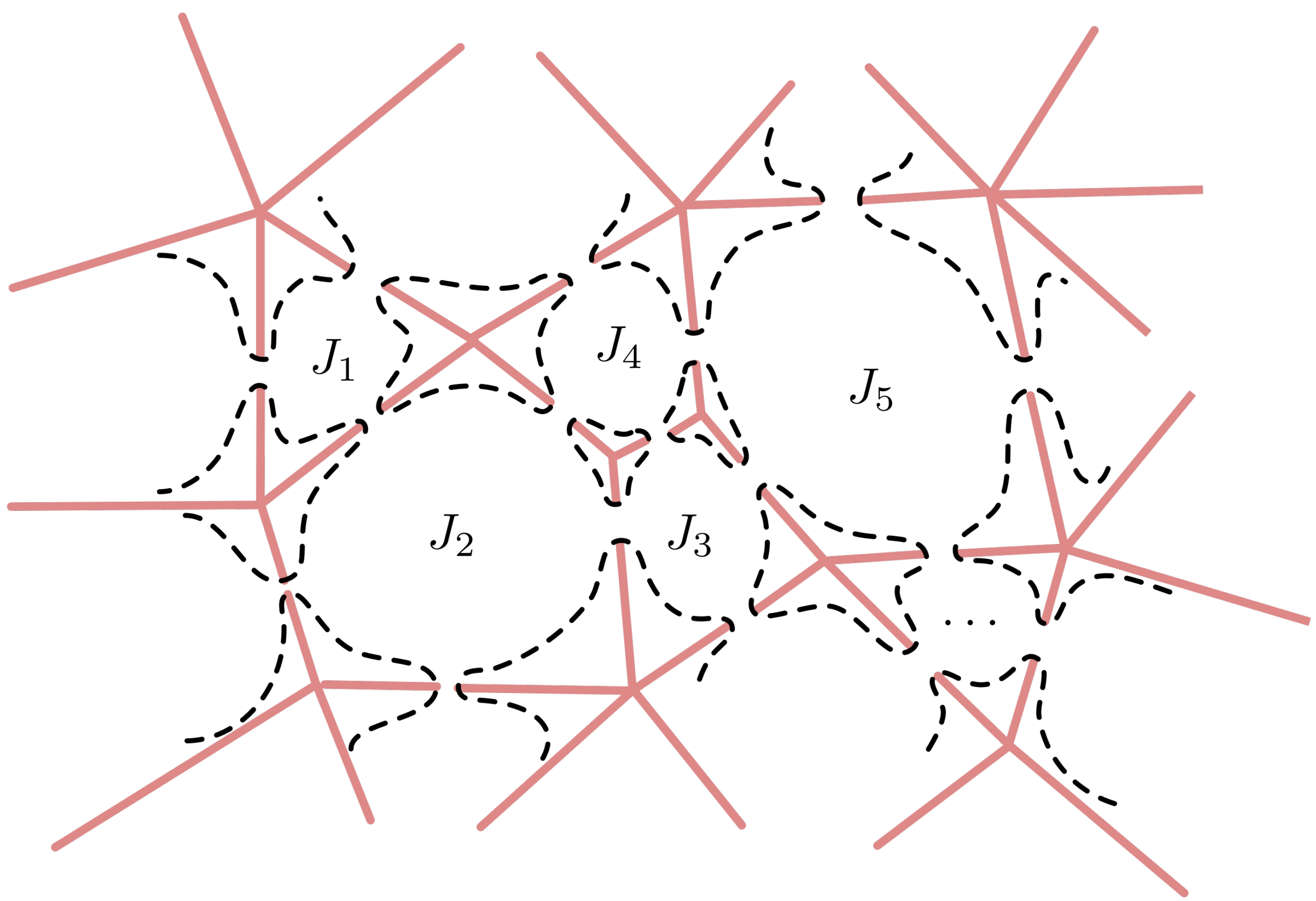

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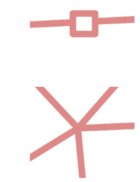

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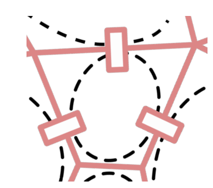
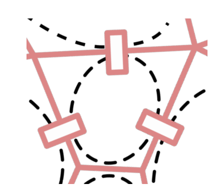


$$\int \mathrm{d}h_{\ell_\partial}$$

$$\delta\Big(\overleftarrow{\prod} h_{\ell_\partial}\Big)$$

3. Peter-Weyl over deltas



$$\textstyle\sum_J d_J \chi^J\Big(\overleftarrow{\prod} h_{\ell_\partial}\Big)$$

Boundary Face-Vertex duality from 3d QG

Spin-chain model // Vertex model

[*m* ~ **Lorentz symmetry** compensator fields]

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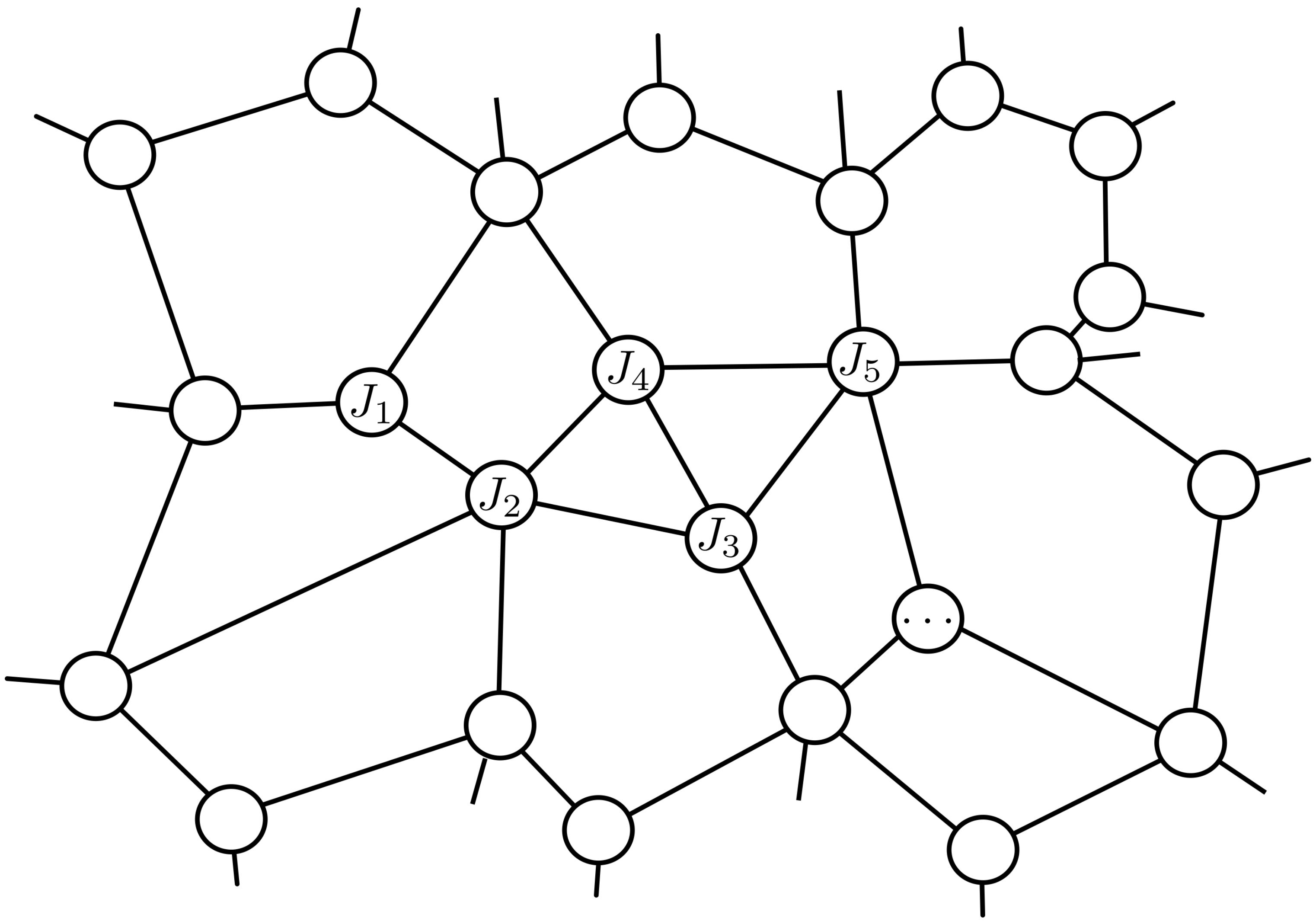

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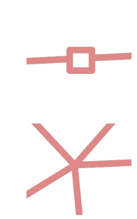

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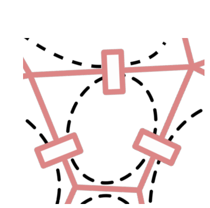

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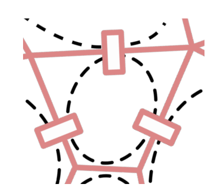

$$\sum_J d_J \chi^J \Big(\overleftarrow{\prod} h_{\ell_\partial}\Big)$$

4. Integrate over *h*’s (recoupling)

$$\begin{Bmatrix} J_1 & J_2 & J_3 \\ j_1 & j_2 & j_3 \end{Bmatrix}$$

Boundary Face-Vertex duality from 3d QG

Spin-chain model // Vertex model

[m ~ **Lorentz symmetry** compensator fields]

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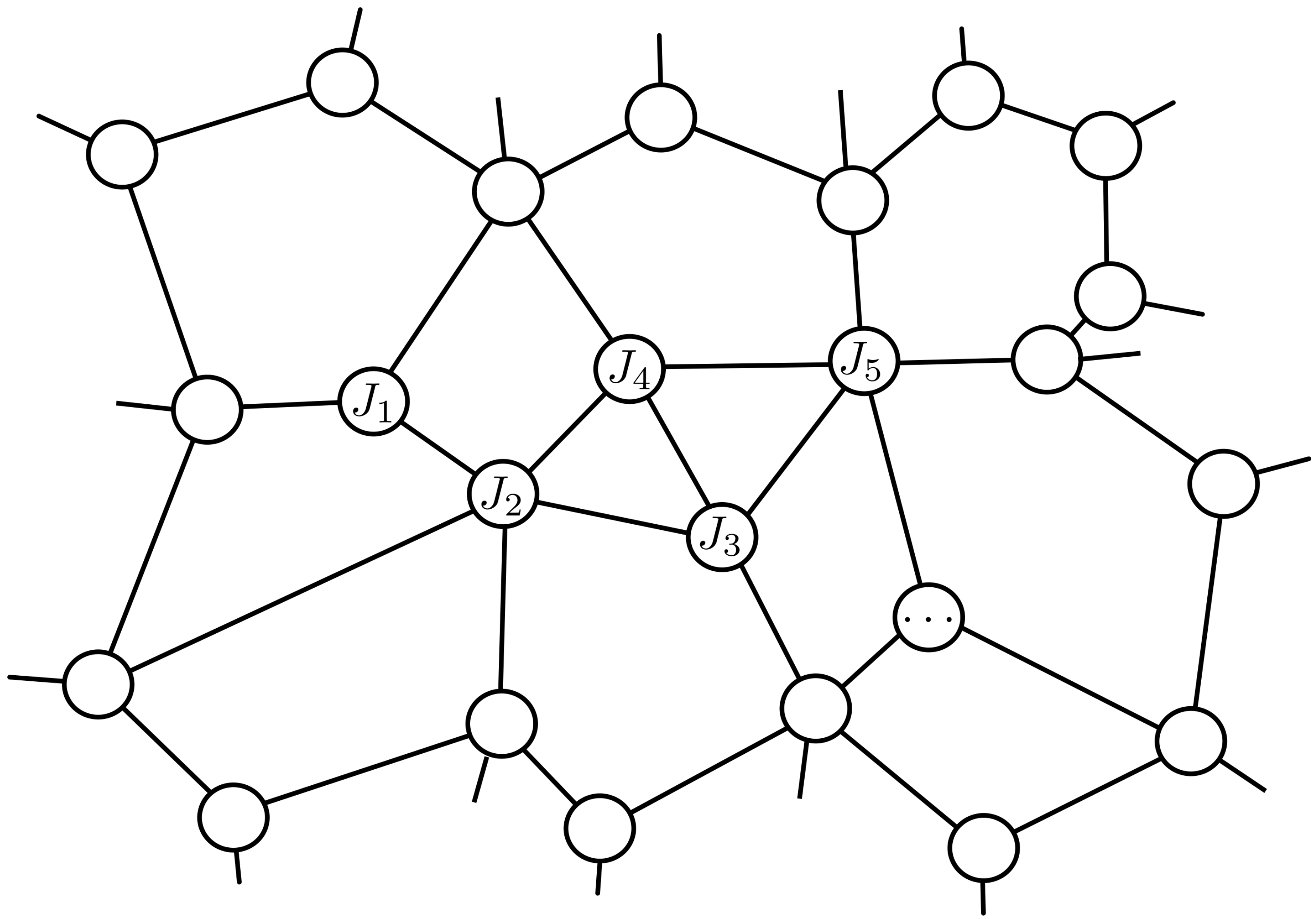

Interaction Round Face model

[J ~ **shift symmetry** compensator fields]

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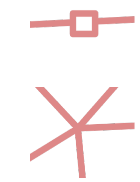

$$\sum_{m_\partial} \qquad \iota^{m_\partial \cdots}$$

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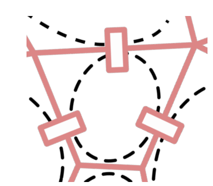

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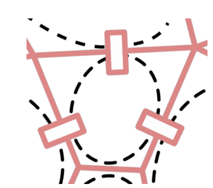

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4. Integrate over h's (recoupling)

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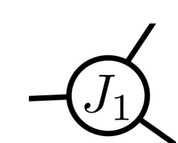
5. Sum over face spins J



$$\sum_{J_\partial}$$

Boundary Face-Vertex duality from 3d QG

[boundary spins j = **background**]

Spin-chain model // Vertex model

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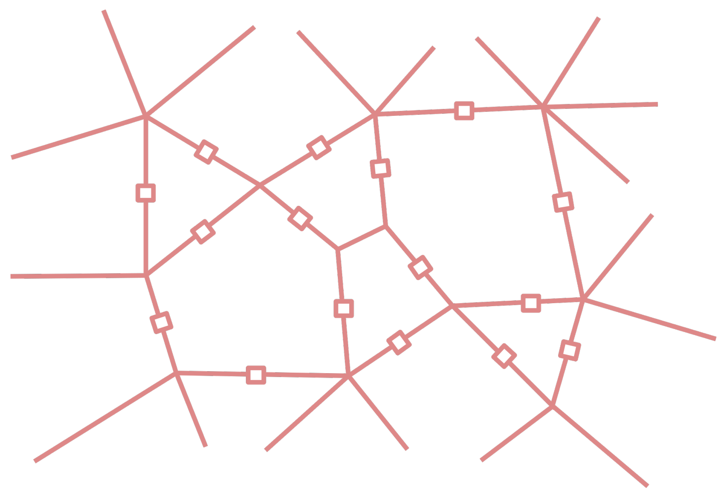

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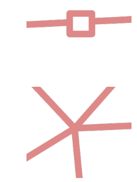
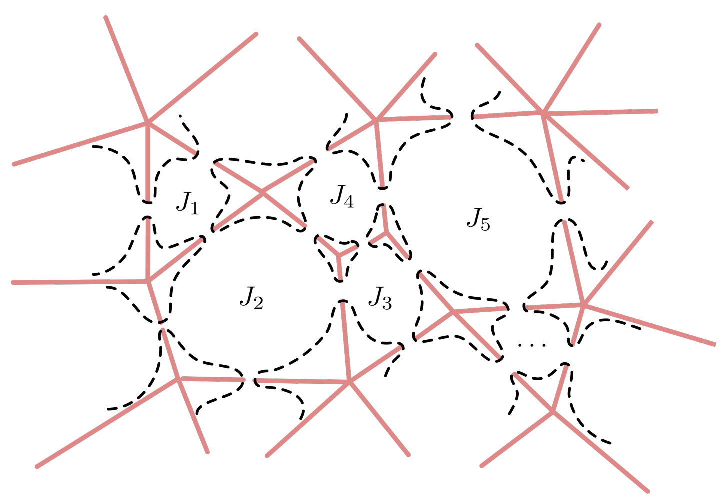


$$\sum_{m_\partial}$$

$$\iota^{m_\partial \cdots}$$

$$W_{\mathrm{PR}}[\psi_B] = \psi_B(\mathbb{1}) = \sum_{m_\partial} \prod_v \iota^{m_\partial \cdots}$$



Interaction Round Face model

[J ~ **shift symmetry** compensator fields]

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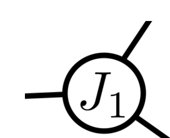

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Boundary Face-Vertex duality from 3d QG

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Spin-chain model // Vertex model

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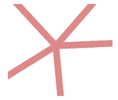
1. Boundary dynamics from spin-network evaluation

$\sum_{m_\partial}$



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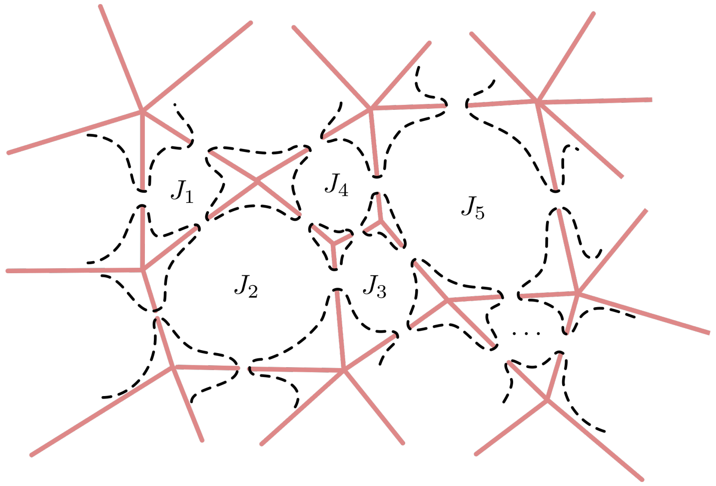

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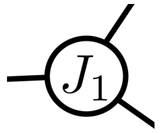

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Interaction Round Face model

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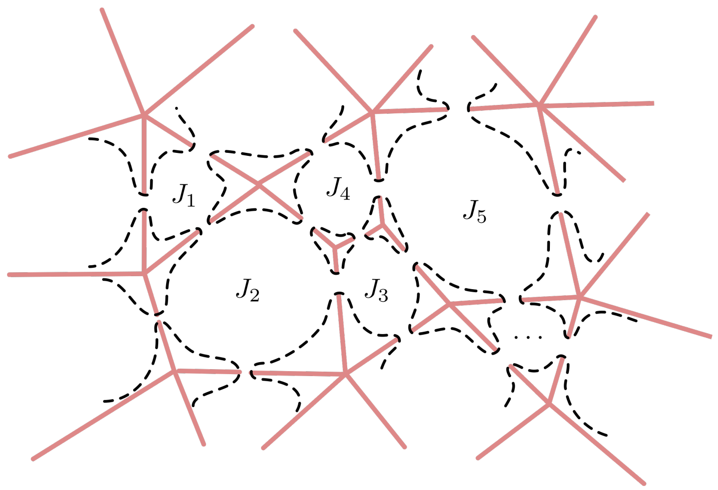
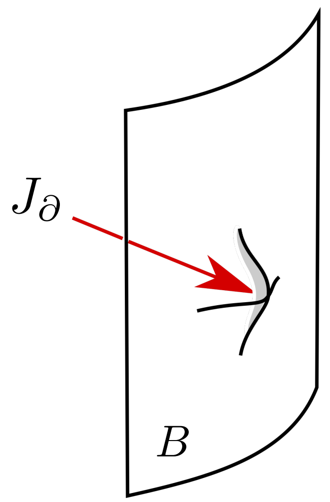
Geometric Interpretation of the Face model

We know that m's = (quantized) Lorentz frames. What is the interpretation of the J's ?





We obtained the J's by "Fourier-transforming" the boundary spin-network evaluation

➞ Dual theory in terms of shift symmetry compensating field

[Lorentz and shift sums are "conjugate ~ Drinfel'd double sym of 3d QG]

"conjugate" symmetries (Poisson-Lie)

$G = \mathrm{T}^*\mathrm{SU}(2)$

Geometrically, J ~ distance of boundary vertex from fiducial "bulk central point"

➞ quantum version of Carlip's would-be-diffeos as boundary dof

Semiclassical analysis in terms of the J's was performed by Dittrich & Bonzom (2016) through q-Regge calculus
Mutatis mutandis, results are compatible with Carlip's analysis (Liouville-like boundary theory)

Also: generalizations to flat sector of 4d gravity (Regge-KBF model) gives similar results [Asante, Dittrich, Haggard 2018]
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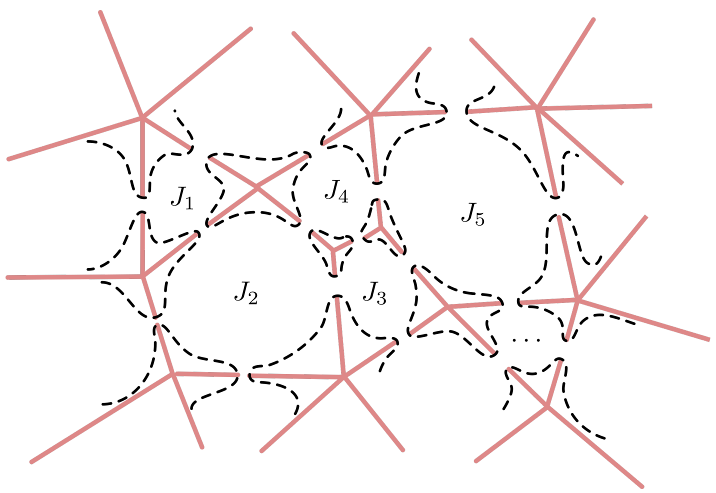

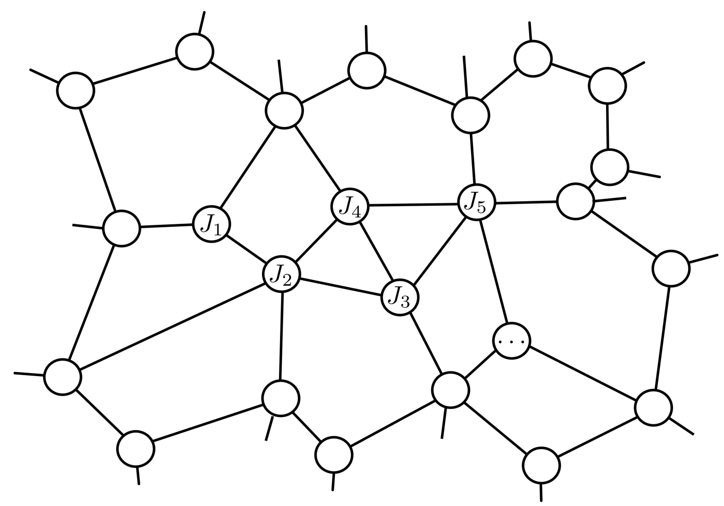

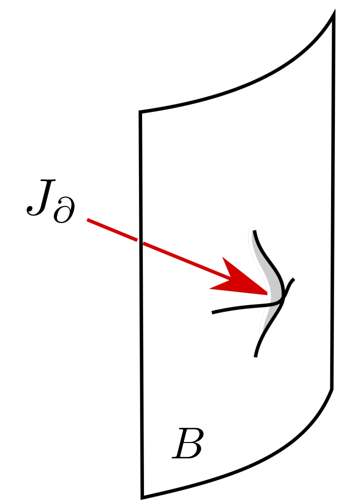

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Other boundary states

Which other boundary states could be interesting to look at?
[For simplicity, we will restrict attention to quadrangulation of boundary]

For fixed spins $j > 1/2$, large choice of boundary intertwiners !

E.g. for semiclassical analysis we can use LS (semi-)coherent states

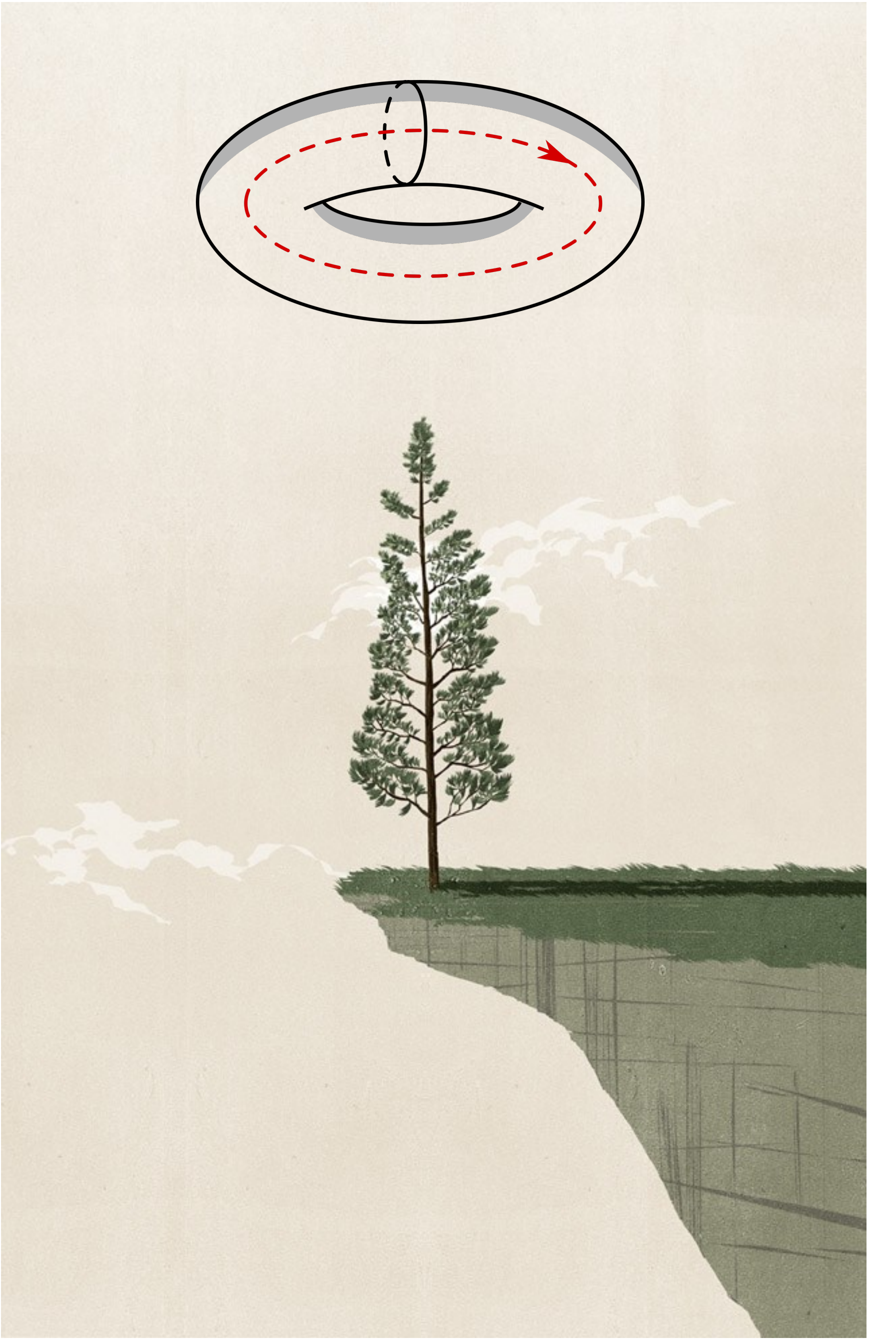
But we can also introduce superpositions of spins :

E.g. nice states closely related to spin-network generating functions (Poisson spin distribution)

[Freidel, Hnybida JMP 13; Bonzom, Livine CQG 13; Bonzom, Costantino, Livine CQG 15; also Dittrich, Hnybida 13]

- Known to be closely related to Ising model on planar trivalent graphs [more generally?]
- Nice geometric interpretation [global scale invariance]
- Exactly computable! [Gaussian when expressed in terms of spinors]

Part IV - Torus' partition function(s)



Guido Scarabottolo

Ponzano-Regge

Thermal partition function

$$Z_{\text{PR}}[\psi_B, \gamma] := \sum_\phi \langle \phi | R[\gamma] W_{\text{PR}}[\psi_B] | \phi \rangle$$

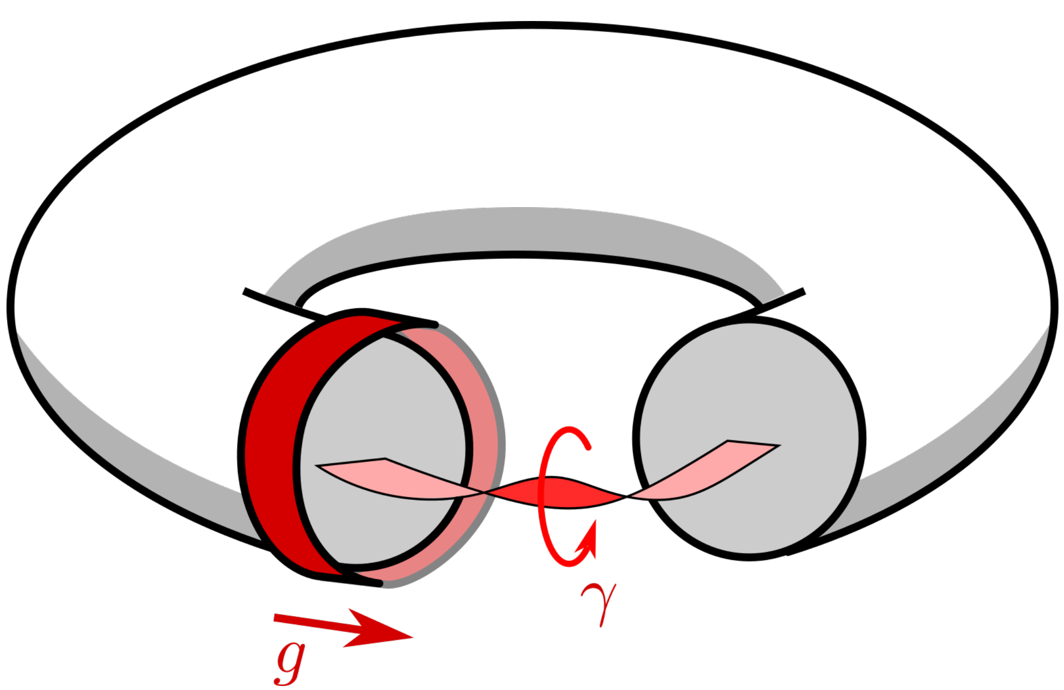
Glue initial and final state

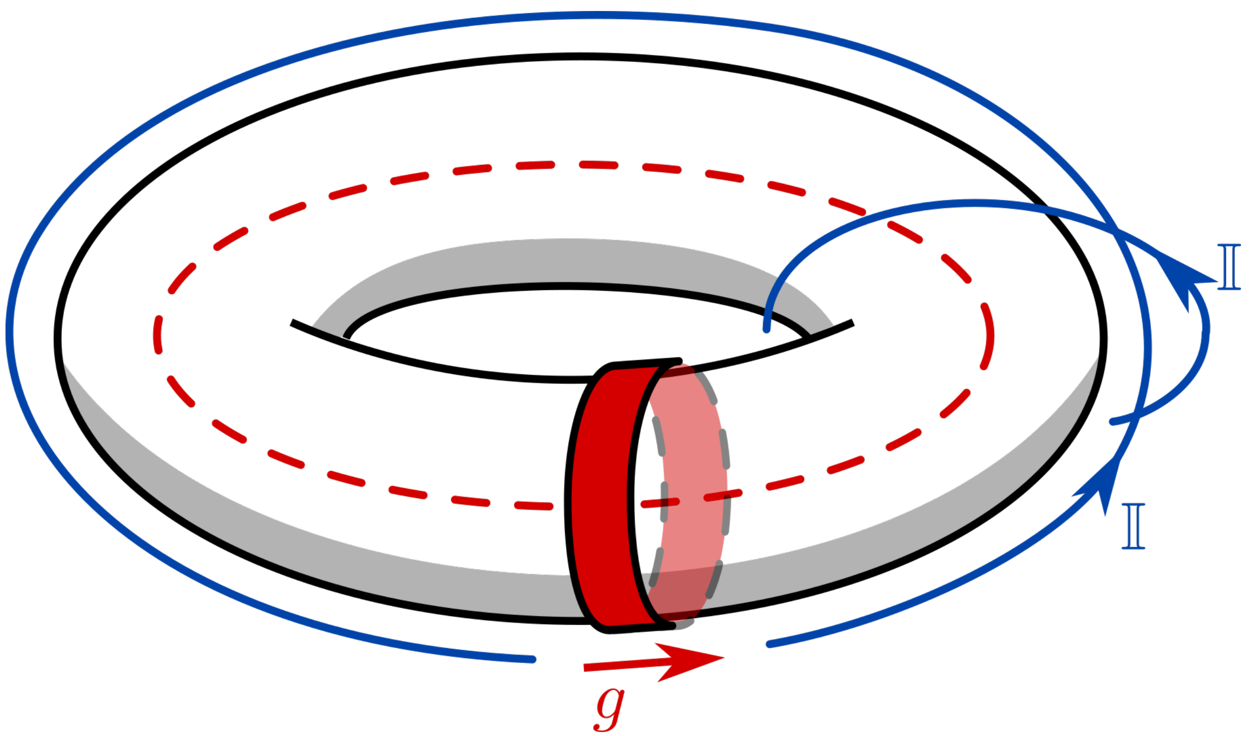
$$= \int \mathrm{d}g \; \psi_B^\gamma(h_{\ell \notin \text{ring}} = \mathbb{1}, h_{\ell \in \text{ring}} = g)$$

Because of gauge invariance, gluing defined up to gauge transf. g

Coupling of boundary theory to bulk dof

New boundary state defined by "closing" ψ_B into a torus with a twist γ





Gluing the cylinder: the spin chain perspective

$$Z_{\mathrm{PR}}[\psi_B,\gamma] = \int \mathrm{d}g\; \psi_B^\gamma(h_{\ell\notin\mathrm{ring}} = \mathbb{1}, h_{\ell\in\mathrm{ring}} = g) = \mathrm{tr}\Big(U_{N_\gamma} T^{N_t} \mathbb{P}_{S=0}\Big)$$

Coupling to bulk dof = Haar projector

Boundary state

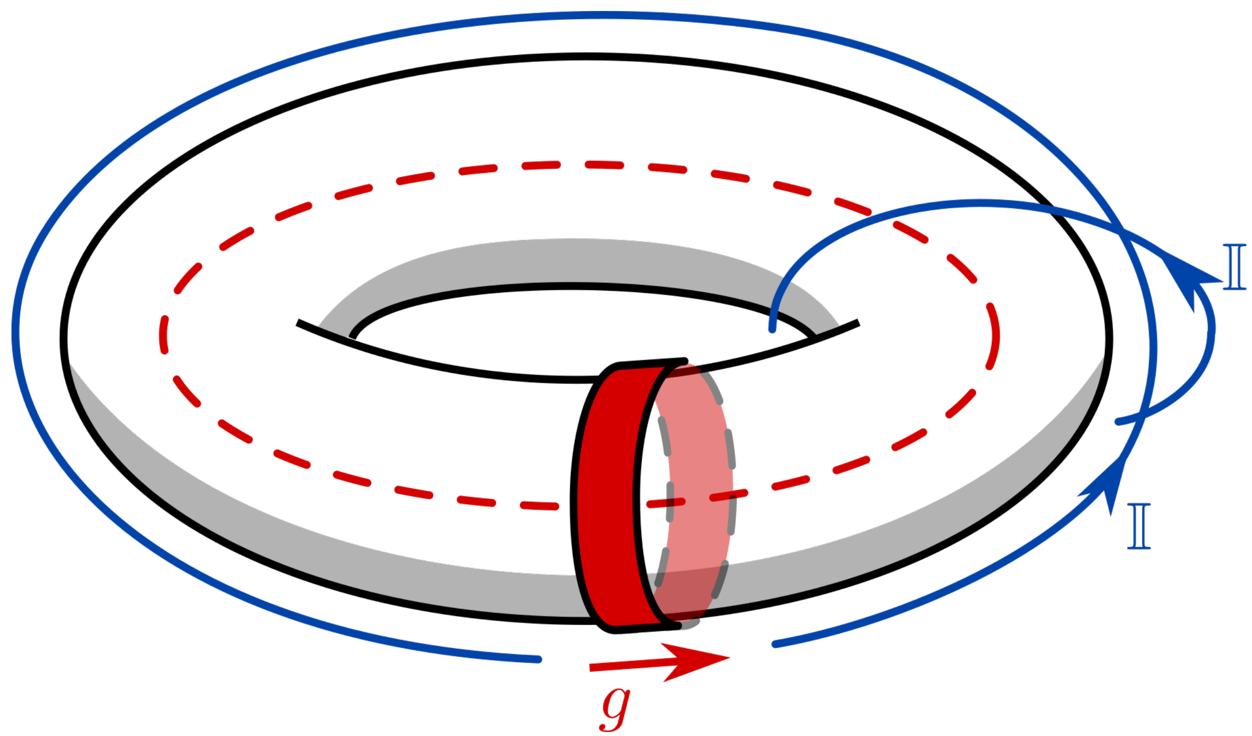
Angle twist = translation by Nγ steps

Transfer-matrix evolution

$$\underbrace{\text{Angle twist = translation by N}_\gamma\text{ steps} \quad \text{Transfer-matrix evolution}}_{\psi_B^\gamma(\mathbb{1}, m_\partial^{\mathrm{f}}, m_\partial^{\mathrm{i}})}$$

Insertion of Haar's intertwiner projects the spin chain's edge state onto its 0-Spin sector

Remark: coupling to g breaks symmetry between two cycles of the torus!
[cf. modular invariance in AdS/CFT
e.g. Maloney & Witten 2007
v. Cotler & Jensen 2019]



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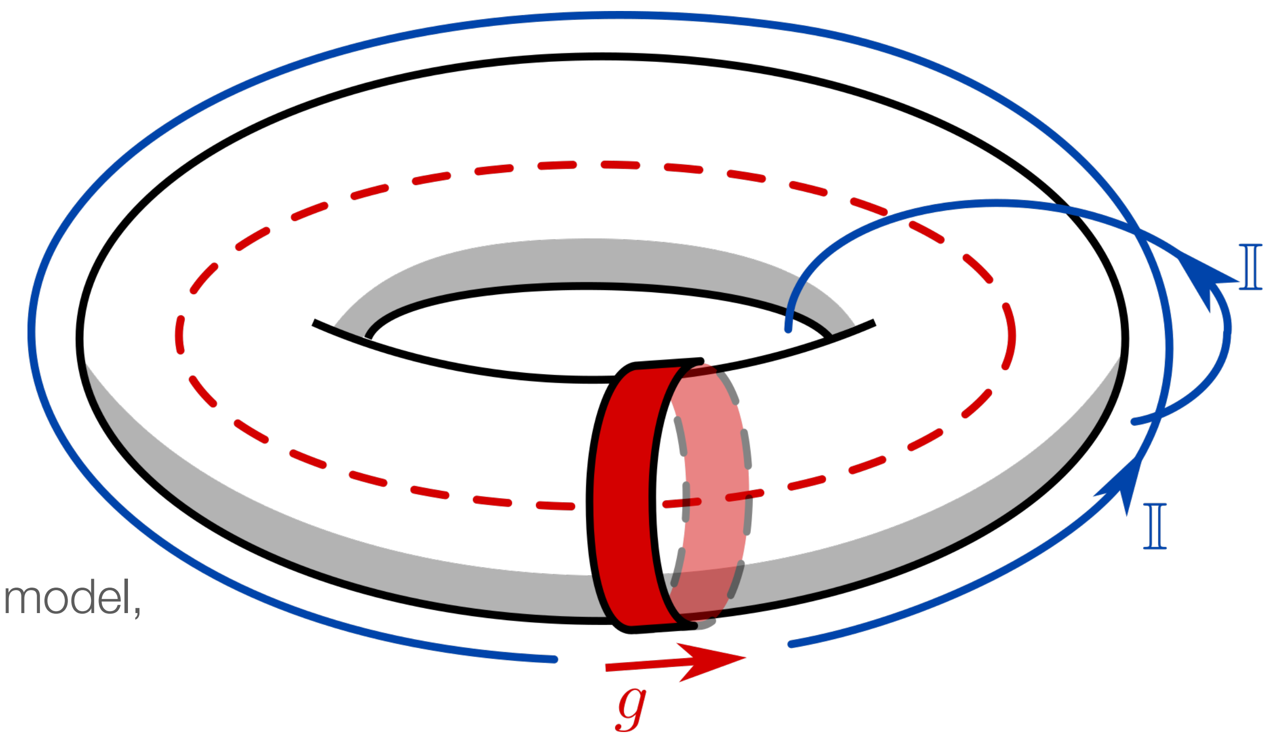

Remark: other ways of coupling boundary to bulk dof is to insert bulk Wilson lines anchored at the boundary
→ insertion of disorder-like operators in the stat model, that create vortex/antivortex pairs
[with Danilo Artigas Guimarey]

Semiclassics: what are we looking for?

Twisted-thermal AdS

$$Z_{\mathrm{TTAdS}}[\beta,\gamma] = \underbrace{e^{-\frac{\pi\beta}{\ell_{\mathrm{Pl}}}}}_{\text{Classical action (with boundary term)}} \underbrace{\prod_{k\geq 2} \frac{1}{\left|1-e^{\mathrm{i}\gamma k - \frac{\beta}{\ell_c}k}\right|^2}}_{\text{1-loop determinant}}$$

Classical action
(with boundary term)

1-loop determinant

[Giombi, Maloney, Yin 08]

OR

Boundary CFT ground state's contribution to

Boundary CFT descendant states' contribution to

$$Z_{\partial\mathrm{CFT}}[\beta,\gamma] = \mathrm{tr}\left(e^{\gamma J}e^{-\beta H}\right)$$

[Maloney, Witten 07]

Virasoro character

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Virasoro character

$\ell_c \to \infty$

Twisted-thermal Minkowski

$$Z_{\text{TTMink}}[\beta,\gamma] = e^{-\frac{(2)\pi\beta}{\ell_{\text{Pl}}}} \prod_{k\geq 2} \frac{1}{\left|1 - e^{i\gamma k}\right|^2}$$

Classical action (with boundary term)

1-loop determinant

[Barnich, Gonzalez, Maloney, Oblak 15]

OR BMS3 character [Oblak 15]

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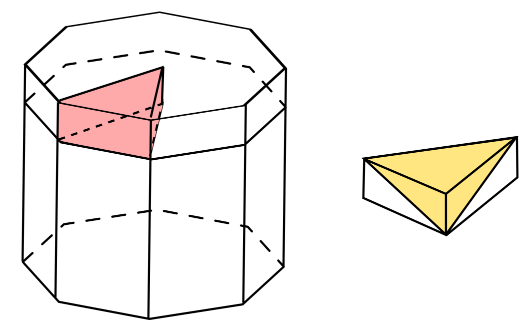
Virasoro character

Quantum Regge calculus on a cylinder

$$Z_{\mathrm{Regge}}[\beta,\gamma] = e^{-\frac{2\pi\beta}{\ell_{\mathrm{Pl}}}} \underbrace{\prod_{k=2}^{\frac{1}{2}(N_x-1)} \frac{1}{|1-e^{\mathrm{i}\gamma k}|^2}}_{\text{1-loop determinant}}$$

Classical action
(with boundary term)

1-loop determinant



[Bonzom, Dittrich 15]

$N_x \to \infty$

$\ell_c \to \infty$

Twisted-thermal Minkowski

$$Z_{\mathrm{TTMink}}[\beta,\gamma] = e^{-\frac{(2)\pi\beta}{\ell_{\mathrm{Pl}}}} \underbrace{\prod_{k\geq 2} \frac{1}{|1-e^{\mathrm{i}\gamma k}|^2}}_{\text{1-loop determinant}}$$

Classical action
(with boundary term)

1-loop determinant

[Barnich, Gonzalez, Maloney, Oblak 15]

OR BMS3 character [Oblak 15]

From Regge calculus to the Ponzano-Regge model

The Ponzano-Regge model is a
full-blown quantum version of Regge calculus

Contrary to perturbative quantum Regge calculus,
it requires no background, only boundary conditions

Question:
Can we recover Bonzom & Dittrich's result in PR?

Answer:
Yes, and one finds more.

What? How?

- We consider a LS (semi-)coherent boundary state to describe a semiclassical cylinder
- We take the boundary large-spin limit (semiclassical *boundary* state: bulk fully resummed)
- We reconstruct the semiclassical geometries and compute the 1-loop determinant (Hessian)
- We find that there are many viable "semiclassical" backgrounds, indexed by a winding number
  coming from the integral over the only bulk dof (holonomy around non contractible cycle)
- The result of Bonzom & Dittrich coincides with the "trivial" background (n=1)
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[Dittrich, Goeller, Livine, AR 17]

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Quantum Regge calculus on a cylinder

$$Z_{\text{Regge}}[\beta,\gamma] = e^{-\frac{2\pi\beta}{\ell_{\text{Pl}}}} \underbrace{\prod_{k=2}^{\frac{1}{2}(N_x-1)} \frac{1}{|1-e^{\mathrm{i}\gamma k}|^2}}_{\text{1-loop determinant}}$$

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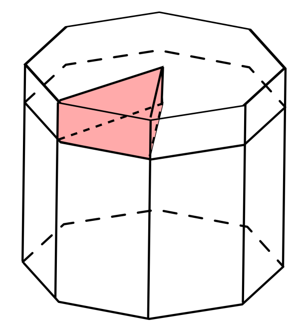

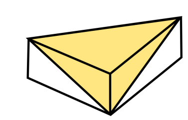

[Bonzom, Dittrich 15]

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$$Z_{\mathrm{PR}}[\beta,\gamma] \approx \sum_n e^{-\frac{2\pi\beta}{\ell_{\mathrm{Pl}}}} \underbrace{\prod_{\substack{k\geq 1\\ k\neq n}}^{\frac{1}{2}(N_x-1)} \frac{1}{|1-e^{\mathrm{i}\gamma k}|^2}}_{\text{1-loop determinant}}$$

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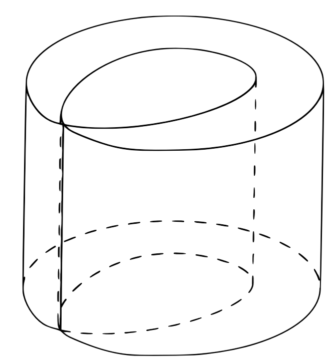

$n=2$

sum over backgrounds

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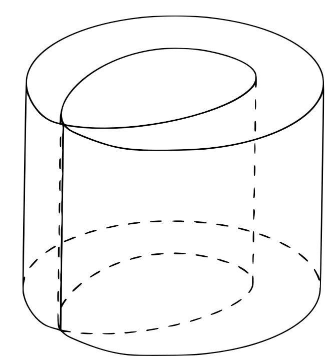

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[Dittrich, Goeller, Livine, AR 17]

Exact computation from PR

Previous result was obtained in the semiclassical limit (for *boundary* spins)

Can we find boundary states that allow for an exact computation?

Motivation:

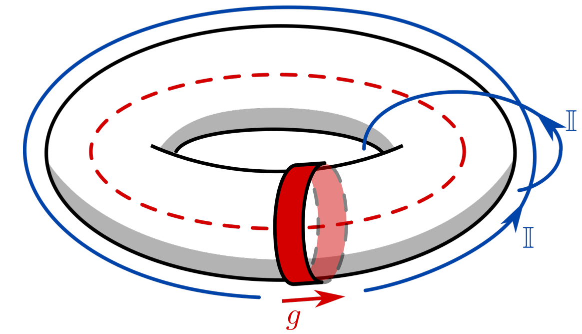
- Asymptotic computations are “exact” and their results reflect the *symmetries of the boundary theory* (Virasoro/BMS3 character): is there a class of states for the *quantum geometry of a finite boundary that encodes a similar correspondence*?
- Can we find a correspondence with a class of discrete *integrable* systems?

These questions are left open for now, but we know of a class of exactly computable states:

$$\psi_{\lambda,\tau}(h^{\text{time}}_{t,x}, h^{\text{space}}_{t,x}) = \underbrace{\sum_{T_{t,x}}}_{\text{“Time”-like spins}} \underbrace{\sum_{L_{t,x}}}_{\text{“Space”-like spins}} \prod_{t,x} \left(\underbrace{\frac{\lambda^{2L_{t,x}}\tau^{2T_{t,x}}}{(2L_{t,x})!(2T_{t,x})!}}_{\text{Poisson(ish) distribution}} (J_{t,x}+1)! \right) \underbrace{\psi^{\text{LS}}_{L_{t,x},T_{t,x}}}_{\text{LS semicoherent states}} (h^{\text{time}}_{t,x}, h^{\text{space}}_{t,x})$$

[Goeller, Livine, AR *to appear*]

Exact computation from PR



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$$Z_{\mathrm{PR}}[\psi^{\gamma}_{\lambda,\tau}] = \frac{1}{\pi}\int_0^{2\pi} \mathrm{d}\varphi\,\sin^2(\varphi)\int_{\mathbb{C}^2}\prod_{t,x=0}^{N_t-1,N_x-1}\frac{\mathrm{d}^4 w_{t,x}}{\pi^2}\,e^{-S_{\lambda,\tau}[\{w_{t,x}\},\varphi]}$$

1 bulk dof
(class angle of *g*)

Boundary dof as spinors

Quadratic action
in *w*

[Goeller, Livine, AR *to appear*]

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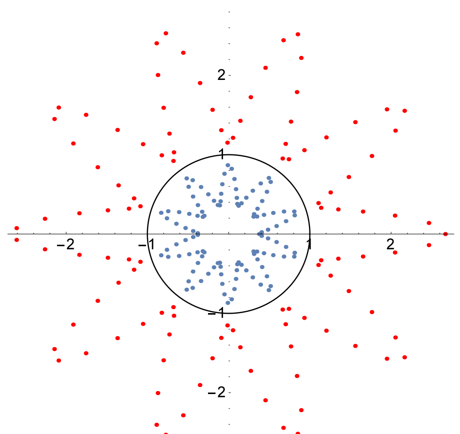


Poles in the plane z = e$^{i\phi}$

$$Z^{\mathrm{PR}}[\psi^{\gamma}_{\lambda,\tau}] = \underbrace{\sum_{n=0}^{N_x-1}}_{\text{sum over backgrounds}} \frac{\sin^2\left(\gamma n + iX_n\right)}{\sinh(X_n)} \underbrace{\prod_{k=1}^{N_x-1} \frac{1}{2(\cosh(X_k) - \cos(k\gamma + iX_k))}}_{\text{Regularized BMS3 character}}$$

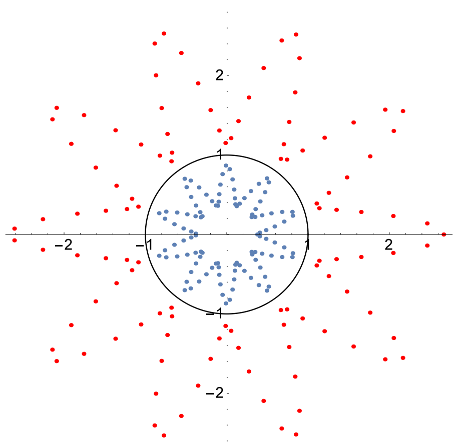
sum over backgrounds

(poles of integrand in ϕ)

Regularized BMS3 character

[Goeller, Livine, AR *to appear*]

Exact computation from PR



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sum over
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(poles of integrand in ϕ)

This should be compared with the AdS regularization of the flat-space result:

$$Z_{\mathrm{TTAdS}}[\beta,\gamma] = \prod_{k=2} \frac{1}{2(1-\cos(k\gamma + i\beta/\ell_c))}$$

In both cases, the theory lives in “box”.
Whereas in AdS the box’s size is controlled by the deformed dynamics ($\Lambda \neq 0$),
in our case the "size of the box” is not related to the dynamics of the theory. Nonetheless around a given background (e.g. n=*1*) regularization takes a *similar* albeit more complicated form.

Questions:

- Can we interpret this result in terms of a dual boundary theory with interesting properties?
- Is the partition function the signature of some special symmetry group?

[Goeller, Livine, AR *to appear*]

Summary

PART I

- Review of 3d gravity and its symmetries

PART II

- We discussed the nature of the edge dof; their relation to the symmetries of the theory; and their different representations as magnetic indices, group elements, spinors

PART III

- We introduced quantum boundary conditions and showed how they induce an edge dynamics (dual spin-chain/statistical model)
- We discussed how the Poisson-Lie symmetry structure of 3d gravity underpins the face-vertex duality of the dual 2d statistical models and how the face models encode a quantum version of Carlip's edge dof as would-be-gauge normal-diffeos.

PART IV

- We briefly presented computations of the twisted torus partition function in the PR model and discussed how they generalize previous QFT/holographic results

Outlook

- Lab for study of renormalization & continuum limit *without* having to solve 4d QG:
  Although 3d QG (PR) is triangulation invariant in the bulk, it isn't at the boundary!
  Dual theory: classical v. asymptotic v. continuum limit? (few large spins v. many small spins?)

- Extension to (A)dS by replacing PR with Tureav-Viro model
  Prima facie difficulty: no group representation

- Clarify status of exactly computable boundary states:
  - do they encode some special/integrable dual theory? (E.g. Ising model?)
  - is the amplitude we computed the character of a symmetry group that deforms BMS3?

- Relations to AdS3/CFT2? And to TTbar holography? [cf Cotler, Jensen 19; Shyam 19; also Freidel 08]
- Relation to AdS/MERA (tensor network Ansatz for AdS/CFT)?
  PR offers a general-covariant, geometric version of this Ansatz [Dittrich, Donnelly, AR wip]

- Study observables and establish nonperturbative holographic dictionary [Artigas Guimarey, AR wip]

- Generalizations to 4d: e.g. for a geometric (topological) theory of flat space
  [Cf. the Korepanov-Baratin-Freidel model — Baratin, Freidel 08; Asante, Dittrich, Haggard 19; Asante, Dittrich, Girelli, AR, Tsimiklis 19]

Thank you

Guido Scarabottolo