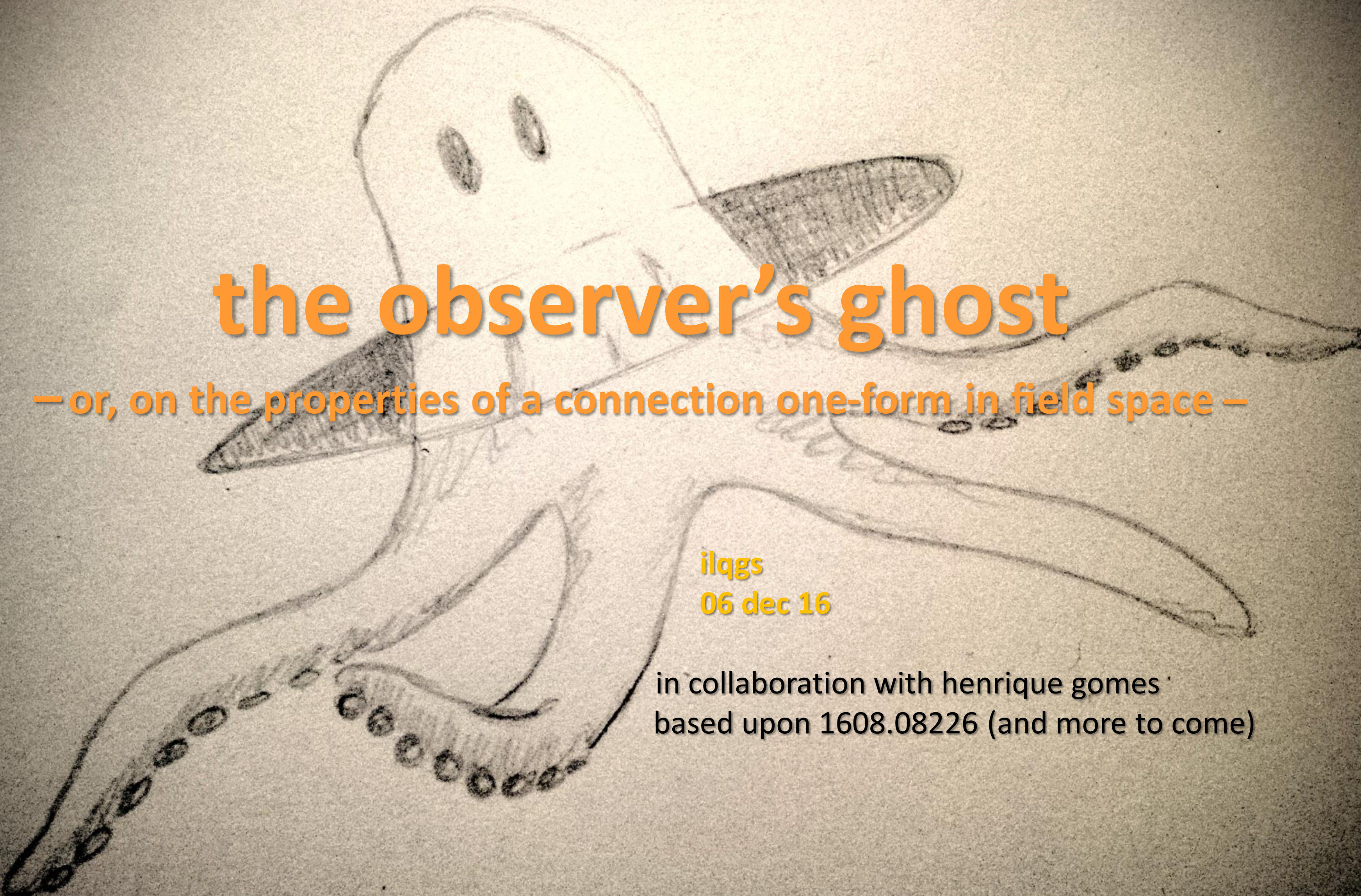


the observer's ghost

– or, on the properties of a connection one-form in field space –

ilqgs
06 dec 16

in collaboration with henrique gomes
based upon 1608.08226 (and more to come)

aldo riello

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

international loop gravity
phone seminar

topic

study theories with **local symmetries** (ym & gr)

from a **field space** perspective

- why field space?

  it is the natural arena for (non-perturbative) qft (covariant path integral picture) [dewitt]

- which tools?

  covariant symplectic framework

  a very flexible and powerful tool to study (pre)symplectic geometry covariantly [see e.g. wald's calculations of noether charges]

- what does *pre*symplectic mean?

  that we work at the **gauge-variant** level

motivations

why gauge-variant? because...

...it is simpler (i.e. it is the only thing we know how to do), and

...because we want to focus on **finite regions** with boundaries and corners

the point is:

gauge theories are **non-local** (gauss constraint $\rightsquigarrow$ systems do not factorize),
and working with gauge variant quantities allows local treatment;
non-local aspects still manifest at boundaries and corners

long term goal: understand **(quantum) gravity and ym in finite regions**

[i also believe this is relevant for ongoing discussions on new (asymptotic) symmetries in gauge theories and gravity, see e.g. strominger et al. 2014-16, but this would be the subject for another talk]

finite regions

why finite regions?

because, they are the **loci of observers** and experimental apparata

[cf. oeckl's «general boundary» program]

furthermore,

their boundaries are where coupling between subsystems takes place

[connections to rovelli's «why gauge?», and of course with the whole extended-tqft framework, e.g. crane 90s]

summary

- study field space geometry of theories with local symmetries (ym & gr)
- using a gauge-variant spacetime-covariant symplectic framework
- in order to emphasize the role of boundaries and corners
- that is where non-locality will manifest itself
- and where the coupling to other subsystems [observers] takes place
- in this talk, for simplicity, we will focus mostly on ym theories

plan of the talk

preliminaries

i. mathematical framework

ii. field-space connection one-forms

iii. a covariant differential in field space

explicit examples of connection one-forms

iv. example one: via new fields

v. example two: no new fields, via fermions

vi. example three: no new fields, via gauge potentials

covariant symplectic geometry

vii. ym

viii. gr

ghosts

ix. relation to geometric brst and the gribov problem

preliminaries

gauge group and field space

ym theory with charge group G with Lie algebra $\mathrm{Lie}(G) = \mathfrak{g}$

gauge group (=group of gauge transformations)

$$\mathcal{G} = \{g(\cdot) : M \to G\}$$ and, infinitesimally

$$\mathfrak{G} = \mathrm{Lie}(\mathcal{G}) = \{Y(\cdot) : M \to \mathfrak{g}\}$$

field space: $\mathcal{F} = \{(A, \psi)\}$

where $\begin{cases} A = A^{\alpha}_{\mu}(x)\tau_{\alpha}dx^{\mu} \in \Lambda^1(M) \otimes \mathfrak{g} \\ \psi = \psi^{Ai}(x)|A\rangle|i\rangle \in \mathcal{C}^{\infty}(M) \otimes \mathbb{C}^2 \otimes V \end{cases}$

elements $g(\cdot) \in \mathcal{G}$ act on $\mathcal{F}$ as

$$\begin{cases} A \mapsto A^g = \mathrm{Ad}_{g^{-1}}A + g^{-1}dg \\ \psi \mapsto \psi^g = g^{-1}\psi \end{cases} \rightsquigarrow \begin{cases} A \mapsto A - \mathrm{ad}_Y A + dY \\ \psi \mapsto \psi - Y\psi \end{cases}$$

mathematical framework

right action and vector fields

more geometrically,

this means there is a right-action of $\mathcal{G}$ on $\mathcal{F}$

$$\begin{array}{rccc} R_{g(\cdot)} : & \mathcal{F} & \to & \mathcal{F} \\ & (A,\psi) & \mapsto & R_{g(\cdot)}(A,\psi) = (A^g, \psi^g) \end{array}$$

the infinitesimal version of this action defines a lifting operator from $\mathfrak{G}$ to $\mathfrak{X}^1(\mathcal{F})$

$$\begin{array}{rccc} {}^\# : & \mathfrak{G} & \to & \mathfrak{X}^1(\mathcal{F}) \\ & Y & \mapsto & Y^\# = \frac{d}{dt}_{|t=0} R^*_{\exp(tY)} \end{array}$$

e.g. $Y^\# f[A] = \frac{d}{dt}_{|t=0} R^*_{\exp(tY)} f[A] = \frac{d}{dt}_{|t=0} f[A + tD_A Y] = \frac{\delta f}{\delta A} D_A Y$

where in the last term we used dewitt's condensed notation

field-dependent gauge transformations

the lift $^\# : \mathfrak{G} \to \mathfrak{X}^1(\mathcal{F})$ generalizes immediately (pointwise)
to transformations which depend on the field configuration $(A, \psi) \in \mathcal{F}$

$$\begin{array}{rccc} \beta : & \mathcal{F} & \to & \mathcal{G} \\ & (A,\psi) & \mapsto & g(\cdot) = \beta[A,\psi](\cdot) \end{array}$$

$$\begin{array}{rccc} \xi : & \mathcal{F} & \to & \mathfrak{G} \\ & (A,\psi) & \mapsto & Y(\cdot) = \xi[A,\psi](\cdot) \end{array}$$

this transformations are useful, e.g. to implement «**gauge fixings**»

from now on, we will always work with this more general gauge transformations

rmk this is a promotion of a global to a local symmetry in field space

differential forms

now, that we have vector fields,

it is only natural to investigate differential forms on $\mathcal{F}$ (cf symplectic geometry!)

the formula for $Y^{\#}f$ suggests the introduction of

a **field-space differential operator**, δ

e.g. on $f : \mathcal{F} \to \mathbb{C}$, it gives $\delta f = \frac{\delta f}{\delta A}\delta A + \frac{\delta f}{\delta \psi}\delta\psi$

it can be extended in the usual way to act on arbitrary forms,

taking care of proper antisymmetrization (wedge products are left understood)

in particular, $\delta^2 \equiv 0$

inclusion (form/vector contraction), $\mathfrak{I}_{\mathfrak{v}}(\cdot) : \Lambda^p(\mathcal{F}) \to \Lambda^{p-1}(\mathcal{F})$, for $\mathfrak{v} \in \mathfrak{X}^1(\mathcal{F})$

e.g.

$$\mathfrak{I}_{\mathfrak{v}}(f_1 \delta f_2) = f_1 \mathfrak{v} f_2$$

lie derivative and convariance

lie derivative on $\Lambda^\bullet(\mathcal{F})$ can be readily defined via cartan's magic formula

$$\mathfrak{L}_{\mathfrak{v}} = \delta \mathfrak{I}_{\mathfrak{v}} + \mathfrak{I}_{\mathfrak{v}} \delta$$

of course, this definition is consistent with the dragging picture, e.g.

$$\mathfrak{L}_{\xi^\#} = \delta \mathfrak{I}_{\xi^\#} + \mathfrak{I}_{\xi^\#} \delta = \frac{d}{dt}_{|t=0} R^*_{\exp(t\xi)}$$

now, notice that even if $\lambda^i \in \Lambda^\bullet(\mathcal{F}) \otimes V_\rho$ transforms «nicely», i.e. **equivariantly**

$$\mathfrak{L}_{\xi^\#} \lambda^i = -\rho(\xi)^i{}_j \lambda^j$$

Its differential does not :
$$\mathfrak{L}_{\xi^\#} \delta\lambda^i = -\rho(\xi)^i{}_j \delta\lambda^j - \rho(\delta\xi)^i{}_j \lambda^j$$

this is the analogue of: $\psi^g = g^{-1}\psi$ while $d\psi^g \neq g^{-1} d\psi$
and to similar problems, there are of course similar solutions...

field-space connection

introduce gauge-algebra valued connection 1-form on field space:

$$\varpi \in \Lambda^1(\mathcal{F}) \otimes \mathfrak{G}$$

such that

$$\begin{cases} \mathfrak{I}_{\xi^\#}\varpi = \xi \\ \mathfrak{L}_{\xi^\#}\varpi = -\mathrm{ad}_\xi \varpi + \delta\xi \end{cases}$$

the first condition is a projector property:
 the kernel of ϖ defines «**horizontality**» in $\mathcal{F}$

the second is an **equivariance** condition:
 intertwines action of $\mathcal{G}$ on $\mathcal{F}$ [lhs] and an «internal» action on $\mathfrak{G}$ [rhs]

remark

my apologizes for this very abstract dewitt-like notation...

anyway, here is the structure of ϖ :

$$\varpi = \varpi^{\alpha}(x)\tau_{\alpha} = \int_M dy \Big(\overset{A}{\varpi}{}^{\alpha\mu}{}_{\beta}(x,y)\delta A(y)^{\beta}_{\mu} + \overset{\psi}{\varpi}{}^{\alpha}{}_{Ai}(x,y)\delta\psi^{Ai}(y)^{\beta}_{\mu} \Big)\tau_{\alpha}$$

Where, $\overset{A}{\varpi}$ and $\overset{\psi}{\varpi}$ can depend on the field content at $y \in M$

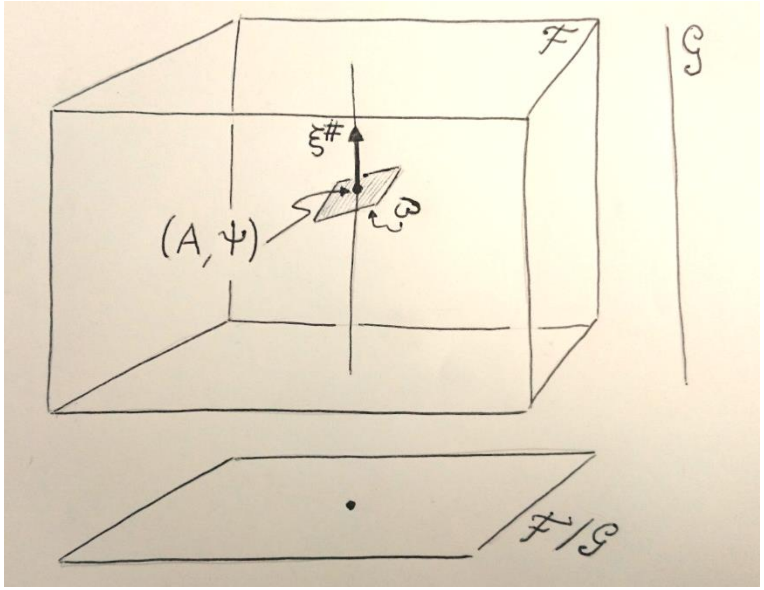
notice that functions on field space are always supposed to be given
by local functionals on spacetime

geometrical picture

the **two fundamental equations**

$$\begin{cases} \mathfrak{I}_{\xi^\#}\varpi = \xi \\ \mathfrak{L}_{\xi^\#}\varpi = -\mathrm{ad}_\xi \varpi + \delta\xi \end{cases}$$

are just (slight generalizations of) the equations for a connection in a PFB



and $\mathfrak{F} = \delta\varpi + \frac{1}{2}[\varpi,\varpi]$ measures the **anholonomicity** of the horizontal planes

covariant differential

with this connection at hand, we can readily define a horizontal differential on $\mathfrak{F}$

for some $\lambda \in \Lambda^\bullet(\mathfrak{F}) \otimes V$

$$\delta_H \lambda = \delta\lambda + \rho(\varpi)\lambda$$
$$\mathfrak{L}_{\xi^\#}\delta_H \lambda = -\rho(\xi)\delta_H\lambda$$

while for A (which transforms inhomogeneously) the appropriate formula is

$$\delta_H A = \delta A + \mathrm{ad}_\varpi A - d\varpi$$
$$\mathfrak{L}_{\xi^\#}\delta_H A = -\mathrm{ad}_\xi \delta_H A$$

by construction: $\mathfrak{I}_{\xi^\#}\delta_H(\cdot) \equiv 0$

meaning of the horizontal differential:

defines «*physical*» (vs pure-gauge) change *with respect to* a choice of ϖ which is non-unique

explicit examples of connection one-forms

explicit examples

via new fields

probably, the simplest way to define a ϖ is to

introduce **new fields** $\mathrm{h} \in \mathcal{C}^{\infty}(M) \otimes G$,
thus $\mathcal{F} \rightsquigarrow \mathcal{F}^{\mathrm{ext}} = \{(A, \psi, \mathrm{h})\}$,
which transform covariantly under the gauge group $\mathcal{G}$

$$\mathrm{h} \mapsto \mathrm{h}^{g} = \mathrm{h}g$$

then, it is immediate to check that

$$\boxed{\varpi_{\mathrm{ext}} = \mathrm{h}^{-1}\delta\mathrm{h}}$$

is such that (i) $\mathfrak{I}_{\xi^{\#}}\varpi_{\mathrm{ext}} = \mathrm{h}^{-1}\xi^{\#}\mathrm{h} = \xi$ ok
and (ii) $\mathfrak{L}_{\xi^{\#}}\varpi_{\mathrm{ext}} = -(\xi\mathrm{h}^{-1})\delta\mathrm{h} + \mathrm{h}^{-1}\delta(\mathrm{h}\xi) = -\mathrm{ad}_{\xi}\varpi_{\mathrm{ext}} + \delta\xi$ ok

[this choice implicitly appears in donnelly & freidel 2016]

explicit examples

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$$h \mapsto h^g = hg$$

this introduces a **new symmetry** $\mathcal{K} \cong \mathcal{G}$

$$h \mapsto h^k = k^{-1}h \quad \text{and} \quad (A, \psi) \mapsto (A, \psi)$$

one can argue that this symmetry manifests itself only at the boundaries
(see later discussion of symplectic geometry)

[this choice implicitly appears in donnelly & freidel 2016]

explicit examples

via matter fields

however, one can also use as a reference frame (aka observer) fields already in $\mathcal{F}$

as a first example, consider

$\mathrm{SU}(2)$ ym theory with matter fields

then, the following defines a viable connection

$$\boxed{\varpi_{\mathrm{mat}} = \varpi^{\alpha}_{\mathrm{mat}}(x)\tau_{\alpha} = \frac{i}{\overline{\psi}\psi}\left(\delta\overline{\psi}^{i}\sigma^{\alpha\, i}{}_{j}\psi^{j} - \overline{\psi}^{i}\sigma^{\alpha\, i}{}_{j}\delta\psi^{j}\right)(x)\tau_{\alpha}}$$

this peculiar example works because

$$[\sigma^{\alpha}, \sigma^{\beta}]_{-} = 2i\epsilon^{\alpha\beta}{}_{\gamma}\sigma^{\gamma} \quad \text{and} \quad [\sigma^{\alpha}, \sigma^{\beta}]_{+} = 2\delta^{\alpha\beta}\mathbb{1}$$

rmk it seems a natural construction in the context of 3d gravity with spinors, where «$\mathrm{SU}(2)$=Lorentz» (and 4d gravity with weyl spinors, or in ashtekar variables)

via the gauge potential

the other field in $\mathfrak{F}$ is the gauge potential A

a connection defined out of A (pure ym) had been proposed first by vilkovisky and then studied in detail by dewitt [his last paper starts by reviewing its construction]

contrary to the previous two examples, such a connection is spacetime **non-local**

the **vilkovisky connection** is defined as the orthogonal projector associated to some «natural» gauge-covariant (needed for equivariance) metric on $\mathfrak{F}$

e.g. in qed the choice $\langle \mathbb{B}, \mathbb{B} \rangle = \int_M \mathrm{B}_\mu \mathrm{B}^\mu$, where $\mathbb{B} = \mathrm{B}_\mu \frac{\delta}{\delta A_\mu} \in \mathrm{T}_{(A,\psi)}\mathfrak{F}$

gives a spacetime non-local, field-space constant connection

$$\boxed{\varpi_{\text{maxwell}} = \Box^{-1} \mathrm{div} \delta A}$$

covariant
symplectic geometry

general review

starting from an action principle $\mathcal{L} : \mathcal{F} \to \mathbb{R}$

one computes the field-space 1-form $\delta\mathcal{L} = \mathrm{EL}(\phi)\delta\phi + \mathrm{d}\vartheta$

this allows to isolate (up to ambiguity in the spacetime cohomology),
the **presymplectic potential density** ϑ

the presymplectic 2-form is then defined by $\Omega = \int_\Sigma \delta\vartheta$

a vector field $\mathfrak{v} \in \mathfrak{X}^1(\mathcal{F})$ is said to be hamiltonian,
if there exists a **charge** functional $\mathrm{Q}[\mathfrak{v}] : \mathcal{F} \to \mathbb{R}$ such that

$$\mathfrak{I}_\mathfrak{v}\Omega = -\delta \mathrm{Q}[\mathfrak{v}]$$

if $\mathfrak{L}_\mathfrak{v}\mathcal{L} = 0$ and $\mathfrak{L}_\mathfrak{v}\vartheta = 0$, then $\mathrm{Q}[\mathfrak{v}] = \int_\Sigma \mathfrak{I}_\mathfrak{v}\vartheta$ is the noether charge

yang-mills

consider ym theory

$$S = \frac{1}{4}\int_M \langle *F[A] \wedge F[A] \rangle$$

the previous procedure gives us the **presymplectic potential density**

$$\vartheta = \frac{1}{2}\langle *F \wedge \delta A \rangle$$

it lives in $\Lambda^{d-1}(M) \otimes \Lambda^1(\mathcal{F})$

it is invariant under the action of $\mathcal{G}$ [since $\delta g(\cdot) \equiv 0$],

but it is *not invariant* under the action of its *field dependent* extension!

$$\begin{aligned} \mathfrak{L}_{\xi^\#}\vartheta &= -\frac{1}{2}\langle (D_A * F)\delta\xi \rangle + \frac{1}{2}d\langle *F\, \delta\xi \rangle \\ &\hat{=} \frac{1}{2}d\langle *F\, \delta\xi \rangle \end{aligned}$$

thus, **non-invariance dictated by (smeared) electric flux across the corner surface**

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gauss : relate to construction of $Q[Y^\#] = \mathfrak{I}_{Y^\#}\theta$

thus, non-invariance dictated by (smeared) electric flux across the corner surface

yang-mills

proposal:

build a gauge-covariant version of symplectic geometry using δ_H:

$$\vartheta_H = \frac{1}{2}\langle *F \wedge \delta_H A \rangle$$

$$\Omega_H = \delta_H \int_\Sigma \vartheta_H$$

is this proposal viable?

(i) naive proposal: use δ_H wherever δ used to appear

problem: $\delta_H^2 = \mathfrak{F}[\varpi] \neq 0$

impose flatness? not viable due to gribov problem

(ii) observe that θ_H is actually **fully invariant**, and thus

$$\Omega_H = \delta_H \int_\Sigma \theta_H = \delta \int_\Sigma \theta_H$$

$$\delta\Omega_H = 0$$

the construction of an invariant potential leads to a closed 2-form covariant symplectic geometry

general relativity

consider general relativity

$$S = \frac{1}{2}\int_M R[g]\epsilon$$

where $\epsilon = *1$ is the volume form. then

$$\vartheta = \frac{1}{2}\nabla_\mu\Big(\delta g^{\mu\nu} - g^{\mu\nu}(g^{\rho\sigma}\delta g_{\rho\sigma})\Big)\epsilon_\nu$$

where $\epsilon_\nu = \iota_{\partial_\nu}\epsilon$

again, it is **covariant** under $\mathcal{G} = \mathrm{Diff}(M)$
but *not under its field dependent* extension! let $\xi \in \mathfrak{G},\ \xi : \mathcal{F} \to \mathfrak{diff}(M)$

$$\mathfrak{L}_{\xi^\#}\vartheta = \pounds_\xi\vartheta + \Big(R^\mu{}_\nu\delta\xi^\mu + \nabla_\nu\nabla^{[\mu}\delta\xi^{\nu]}\Big)\epsilon_\nu$$

$$\hat{=}\ \pounds_\xi\vartheta + \nabla_\mu\Big(\nabla^{[\mu}\delta\xi^{\nu]}\Big)\epsilon_\nu$$

thus, non-invariance dictated by Komar charge associated to $\delta\xi$ at corner surface covariant symplectic geometry

general relativity

introducing δ_H gives

$$\vartheta_H = \vartheta - \tfrac{1}{2}\nabla_\mu\left(\nabla^{(\mu}\varpi^{\nu)} - g^{\mu\nu}\nabla_\rho\varpi^\rho\right)\epsilon_\nu$$

and the action of a diffeomorphism is now

$$\mathfrak{L}_{\xi^\#}\vartheta_H = \pounds_\xi\vartheta_H$$

now we regained **general covariance** of the presymplectic potential density

thus, for the full presymplectic potential

$$\theta_H = \int_\Sigma \vartheta_H$$

to be **invariant** and have a well defined presymplectic framework, one must have

$$\mathfrak{L}_{\xi^\#}\Sigma = \pounds_\xi\Sigma$$

the surface must be defined relationally in terms of physical fields!

general relativity

finally, in defining

$$\Omega_{\rm H} = \delta_{\rm H} \int_\Sigma \vartheta_{\rm H}$$

we notice that the following condition turns out to be *natural*

$$\boxed{\delta_{\rm H}\Sigma = 0}$$

[rmk: this is the best we can require since $\delta\Sigma \neq 0$ because of general covariance]

what does this condition mean?

reacall: $\delta_{\rm H}$ measures «physical» change wrt the functional connection (i.e. reference frame),

thus this condition means that Σ **is fixed wrt the chosen reference frame**!

this reinforces our interpretation of ϖ in terms of an (abstract) observer

ghosts

geometric brst

introduce three more fields: ghost c, antighost $\bar{c}$, Nakanishi–Lautrup field B

brst symmetry mixes the ghost sector with the matter sector

$$
\begin{aligned}
sA &= D_A c \\
s\psi &= -c\psi \\
sc &= -\tfrac{1}{2}[c, c]
\end{aligned}
\qquad\qquad
\begin{aligned}
s\bar{c} &= B \\
sB &= 0
\end{aligned}
$$

where s is the slavnov (or brst) operator

geometric brst

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interpreted geometrically :

$$s \rightsquigarrow \delta_V = \delta - \delta_H$$ vertical differential

$$c \rightsquigarrow \varpi$$ field–space connection

notice that $\delta_V \varpi = -\frac{1}{2}[\varpi, \varpi]$ is equivalent to horizontality of $\mathfrak{F} = \delta_H \varpi$

thus ϖ does not need be flat!

more encompassing than usual treatement,

because now δ defined generally and $\mathfrak{F} \neq 0$

gribov problem

why is it important that $\mathfrak{F} \neq 0$?

because, as shown by gribov and singer, for topological reasons there can be no global horizontal section (i.e. "gauge fixing") in $\mathcal{F}$

<u>reduction ad absurdum:</u>

suppose $\mathfrak{F} = 0$;

this is equivalent to the (frobenius) integrability of the distribution defined by $\mathfrak{v} \in \mathfrak{X}^1(\mathcal{F}) \;:\; \mathfrak{I}_{\mathfrak{v}}\varpi = 0\,;$

this distribution, when integrated, would define a global section $\square$

deformed brst trasformations were proposed in relation with the gribov problem

it is compelling to explain them in terms of ϖ with $\mathfrak{F} \neq 0$

gluing and unitarity (wip)

the most important role of ghosts is to ensure unitariy in feynman diagrams

in particular, for composing transition amplitudes in gauge theories,
the introduction of ghosts is mandatory [this how feynman discovered ghosts]

now, the connection ϖ
(i) not only can be geometrically interpreted as ghosts,
(ii) but also carries crucial information for «gauge invariant» gluings of two field configurations

we are currently working to make this connection mathematically precise in the context of a feynman path integral closing credits

work done in collaboration with

henrique gomes [pi]

references

arXiv:1608.08226
and more to come, stay tuned!

thank you!