On the UV behavior of $\mathcal{N} = 8$ supergravity: Is it a finite theory?

Radu Roiban
Pennsylvania State University

Z. Bern, L. Dixon, R.R. hep-th/0611086

Z. Bern, J.J. Carrasco, L.Dixon
H. Johansson, D. Kosower, R.R., hep-th/0702nnn

Introduction

- $G_N \sim 1/M_{Pl}^2$ is dimensionfull

  $\longrightarrow$ quantization of gravity following usual rules of point-like quantum field theory leads to a nonrenormalizable theory

  - Requires UV completion or Different rules

- String theory:
  - new length scale – extended objects
  - infinitely many fields

- LQG
  - nonperturbative effects

- Is it possible to remain within standard field theory framework?
  - symmetries: supersymmetry can make nongravitational theories perturbatively finite. Can it do the same for gravity?

Most symmetric of supergravities: $\mathcal{N} = 8$ (32 supercharges)

- $\mathcal{N} = 8$ supergravity

deWit, Freedman (1977)

Cremmer, Julia,+Scherk (1978, 1979)

- largest amount of supersymmetry and spins smaller than 2

- classically unique; smallest representation of the symmetry algebra contains the graviton

- spectrum: $2^8 = 256$ massless fields

helicity	-2	$-3/2$	-1	$-1/2$	0	$+1/2$	$+1$	$+3/2$	$+2$
number of fields	1	8	28	56	70	56	58	8	1
names	h^-	ψ_i^-	v_{ij}^-	χ_{ijk}^-	s_{ijkl}	χ_{ijk}^+	v_{ij}^+	ψ_i^+	h^+

- 32 supercharges, $SU(8)$ R-symmetry, E_7 duality symmetry, ⟨add your name here⟩ symmetry

- spectrum = tensor product of 2 $\mathcal{N} = 4$ abelian vector multiplets

helicity	-1	$-1/2$	0	$+1/2$	$+1$
number of fields	1	4	6	4	1
names	v^-	χ_i^-	s_{ij}	χ_i^+	v^+

What do we know about perturbative gravity and supergravity?

- Pure gravity

  - covariance: on-shell counterterms are built out of Riemann tensors together with the condition $R_{\mu\nu} = 0$ and $R = 0$

  - dimensional analysis: $[G_N] = -[R] = -2 \to \#R$–s $= 1 + \#$loops

  - 1-loop: total derivative in 4d: $R_{ijkl}R^{ijkl} - 2R_{ij}R^{ij} + R^2$

    't Hooft, Veltman

  - 2-loops: Feynman diagram calculation⇒nontrivial counterterm

    $S_{\text{ct}}^{2\,\text{loops}} = c_2\, R^{ij}{}_{kl} R^{kl}{}_{pq} R^{pq}{}_{ij}$ with $c_2 \neq 0$ Goroff, Sangotti; van de Ven

- $\mathcal{N} = 8$ supergravity

General reasoning:

- R^3 cannot be supersymmetrized — Grisaru; Tomboulis
- supersymmetric completion of R^4 allowed at 3-loops
  Deser, Kay, Stelle; Kallosh, Howe, Stelle, Townsend

Explicit calculations; improved general arguments

- 1-loop, 4-, 5-, 6-, (7-)points — Bern, Dixon, Dunbar, Perelstein, Rozowski; Bjerrum-Bohr, Dunbar, Ita, etc
- 2-loops 4-point — Bern, Dixon, Dunbar, Perelstein, Rozowski

◇ partial higher loop computations:

first counterterm in $d = 4$ may appear at 5 loops

- confirmed by superspace powercounting — Howe, Stelle

◇ speculation: first counterterm may appear at 6 loops in $d = 4$

String theory; direct and indirect information:

- perturbative
  - 1-loop 4- and 5-points; n-point expression — Green, Schwarz; Montag; 'd Hoker, Phong
  - 2-loops 4-points — 'd Hoker, Phong
  - Vanishing theorems (no $D^{2L}R^4$ above $L \leq 5$) — Berkovits
- non-perturbative: use duality symmetries
  - T-duality ($R \leftrightarrow 1/R$), $SL(2,\mathbb{Z})$, string theory/M-theory duality — Green, Vanhove, Russo, h. Kwon
  - ◇ conjectured exact 4-graviton effective action of string theory from 1- and 2-loop 11d supergravity calculations
- some issues with reliably extracting supergravity/4d information

The question: Is the quantum theory of the low energy limit of string theory the same as the low energy limit of quantum string theory?

Patterns, Field theory calculations and reasons to go to higher loops

Main tools: unitarity method and the KLT relations

- (Generalized) Unitarity method: Bern, Dixon, Dunbar, Kosower

- Feynman diagrammatics:
  - way too general – applies to any field theory
    – allows off-shell external legs
  - (gauge) symmetries restored after summation over diagrams
  - symmetry-based cancellations appear miraculous
  - hides simplicity

e.g. MHV amplitudes in YM theories: $A_n^{\text{tree}}(+\cdots+-_i+\cdots+-_j+\cdots+)$

The sum of arbitrarily-many Feynman diagramms yields

$$A_n^{\text{tree}}(+\cdots+-_i+\cdots+-_j+\cdots+) = \frac{\langle ij\rangle^4}{\prod_{i=1}^{n}\langle i, i+1\rangle}$$

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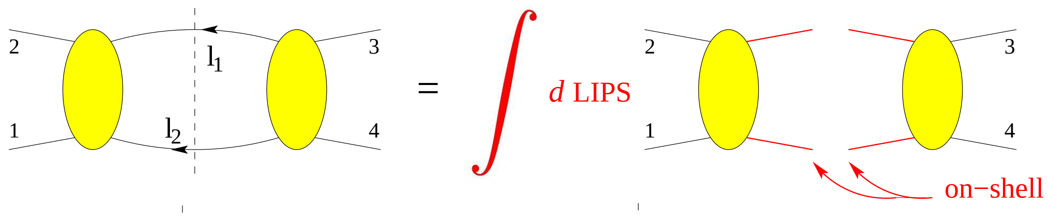
⋆ Solution: stay on-shell

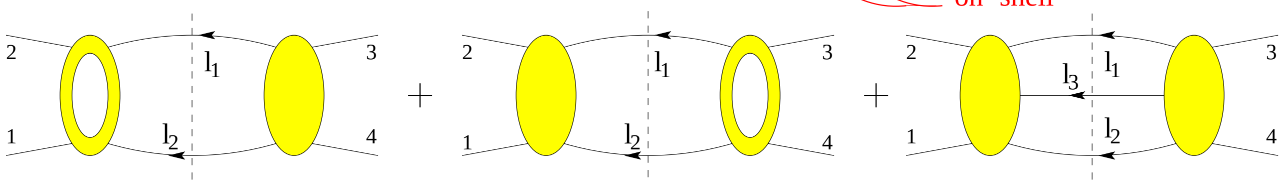
Unitarity: relation between discontinuity of amplitude at some loop order and lower loop amplitudes

$$\mathbb{1} = SS^\dagger \quad \Rightarrow \quad 2\Im T = TT^\dagger$$

1 loop : $2\Im T_4^{1\,\text{loop}} =$



2 loops: $2\Im T_4^{2\,\text{loops}} =$



Field theory calculations, Patterns and reasons to go to higher loops

Main tools: unitarity method and the KLT relations

- (Generalized) Unitarity method: Bern, Dixon, Dunbar, Kosower

◇ Interpret a cut as requiring that the cut propagators are present

→ can "cut" several times the same amplitude

→ on each side of a generalized cut there is an amplitude

◇ reconstruct full amplitude from its generalized cuts

need D-dimensional cuts to ensure no terms are missing

Key technical point: The (generalized) unitarity method reduces the calculation of on-shell loop amplitudes to knowledge of on-shell tree amplitudes and their products

Field theory calculations, Patterns and reasons to go to higher loops

- **KLT relations:** Kawai, Lewellen, Tye

Derived from string theory tree amplitudes and the observation that closed string states are created by bilinears in operators creating open string states; additional factors due to zero modes

⋆ no issues with low energy limit and reduction to 4d

⋆ relate $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ supergravity tree amplitudes

$$
\begin{aligned}
M_4^{\text{tr}}(1,2,3,4) &= -is_{12}A_4^{\text{tr}}(1,2,3,4)A_4^{\text{tr}}(1,2,4,3) \\
M_5^{\text{tr}}(1,2,3,4,5) &= \ is_{12}s_{34}A_5^{\text{tr}}(1,2,3,4,5)A_5^{\text{tr}}(2,1,4,3,5)+(2\leftrightarrow 3) \\
M_6^{\text{tr}} &= \ 12 \text{ terms of the type } s^3A_6A_6
\end{aligned}
$$

⋄ Capture spectrum decomposition $[\mathcal{N}=8]=[\mathcal{N}=4]\otimes[\mathcal{N}=4]$

⋄ benefit from recent advances in tree-level SYM calculation

⋄ Key technical point: in conjunction with the unitarity method, reduce supergravity cuts to SYM cuts!

◇ KLT+unitarity

$$\sum_{\text{gravity sts}} M_{n_1}(...l_1, l_2, l_3...) M_{n_2}(...l_1, l_2, l_3...) = (\text{s}_{\text{ij}}\ \text{factors}) \sum_{\text{KLT terms}}$$
$$\Big[\sum_{\text{SYM sts}} A_{n_1}(...l_1, l_2, l_3...) A_{n_2}(...l_1, l_2, l_3...)\Big]\Big[\sum_{\text{SYM sts}} A'_{n_1}(...l_1, l_2, l_3...) A'_{n_2}(...l_1, l_2, l_3...)\Big]$$

- such a decomposition is not manifest at the level of Lagrangian

- Construction of gravity amplitudes is a 4-step process

0. identify the cuts that uniquely specify the amplitude of interest

1. construct the color-stripped SYM cuts required by the KLT

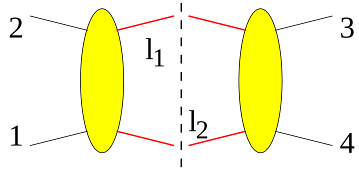
2. construct the gravity cuts

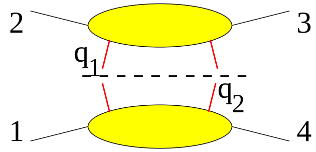
3. identify the functions whose generalized cuts reproduce point 2.

Is there an echo of the finiteness of $\mathcal{N} = 4$ SYM?

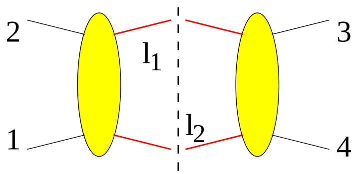
Example: 1-loop SYM vs. 1-loop gravity

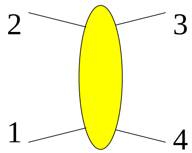
- $\mathcal{N} = 4$ super-Yang-Mills

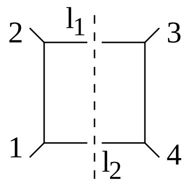
- two 2-particle cuts: 



- at least one cut involves a nontrivial sum over all $\mathcal{N} = 4$ states

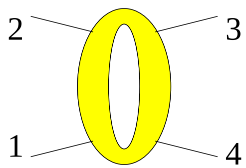
$\sum_{\mathcal{N}=4}$ 

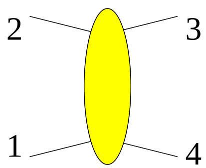
$= -is_{12}s_{23}$ 

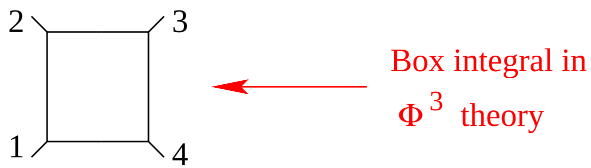


$= -i\, s_{12}s_{23}\, \dfrac{A_4^{\text{tree}}(1,2,3,4)}{(2l_1 \cdot k_2)(2l_2 \cdot k_4)}$

- both cuts contain the same information

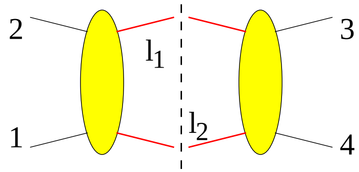
- collect all cuts: 

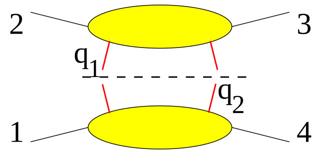
$= i\, s_{12}\, s_{23}$ 



$$A_4^{1\,\text{loop}}(1{,}2{,}3{,}4) = is_{12}s_{23}\, A_4^{\text{tree}}(1{,}2{,}3{,}4) \int \frac{d^dq}{q^2(q-k_1)^2)(q-k_{12})^2(q+k_4)^2}$$

- $\mathcal{N}=8$ supergravity
- two 2-particle cuts:



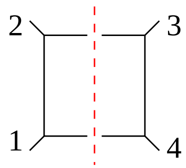
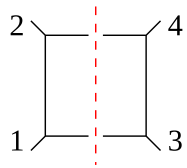


$$\sum_{\mathcal{N}=8} M_4^{\text{tree}}(1,2,l_1,l_2) M_4^{\text{tree}}(l_2,l_1,3,4) =$$

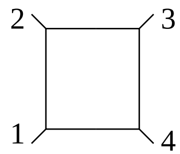
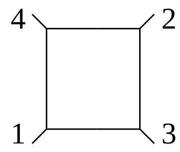
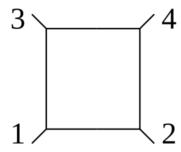
$$= -s_{12}^2 \Big[\sum_{\mathcal{N}=4} A_4^{\text{tree}}(1,2,l_1,l_2) A_4^{\text{tree}}(l_2,l_1,3,4) \Big] \Big[\sum_{\mathcal{N}=4} A_4^{\text{tree}}(2,1,l_1,l_2) A_4^{\text{tree}}(l_2,l_1,4,3) \Big]$$

$$= (s_{12}s_{23})^2 \frac{s_{12} A_4^{\text{tree}}(1,2,3,4)\, s_{12} A_4^{\text{tree}}(2,1,4,3)}{(2l_1\cdot k_2)(2l_2\cdot k_4)(2l_1\cdot k_1)(2l_2\cdot k_3)}$$

$$\xrightarrow{\text{partial fraction}} s_{12}s_{13}s_{23} M_4^{\text{tree}}(1,2,3,4) \Big[\frac{1}{2l_1\cdot k_1} + \frac{1}{2l_1\cdot k_2}\Big] \Big[\frac{1}{2l_2\cdot k_3} + \frac{1}{2l_2\cdot k_4}\Big]$$

$= s_{12}s_{13}s_{23} M_4^{\text{tree}}(1,2,3,4) \Big[$  $+$  $\Big]$

- cuts contain both overlapping and complementary information

$M_4^{1\text{ loop}}(1,2,3,4) = \left[s_{12}s_{23}A_4^{\text{tree}}(1,2,3,4)\right]^2 \Big[$  $+$  $+$  $\Big]$

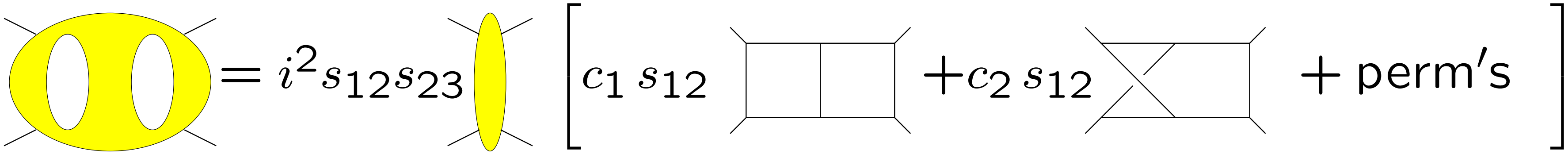
◇ Observation: prefactor is square of SYM one

Bern, Dixon, Dunbar, Perelstein, Rozowski

◇ This pattern persists at 2-loops Bern, Dixon, Dunbar, Perelstein, Rozowski

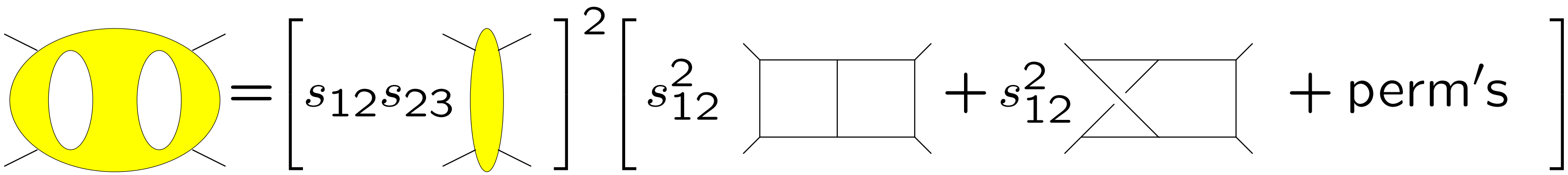
- SYM

Bern, Rozowski, Yan
Bern, Dixon, Dunbar, Perelstein, Rozowski



- supergravity: strip off color and square factors!

Bern, Dixon, Dunbar, Perelstein, Rozowski



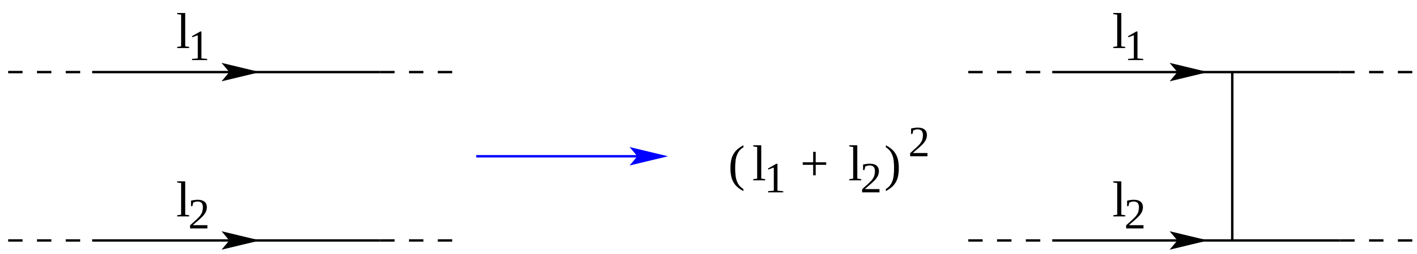
Can this continue? Is it possible to say something about SYM to all loop orders? What are the implications for gravity?

SYM mini-iterative structure: 2-particle cuts can be analyzed to all loops and a function reproducing them may be constructed.

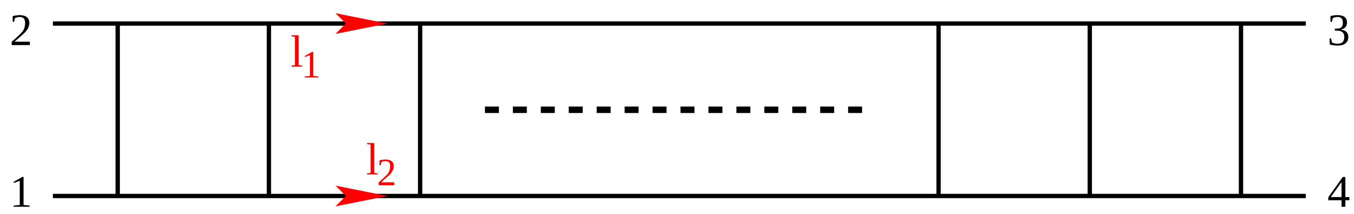
Keys: – two-particle cuts are reproduced by ϕ^3 diagrams

– (special) loop amplitudes proportional to tree amplitudes

• The rung rule: add one rung and put in the numerator the square of the sum of momenta of the lines connected by the rung

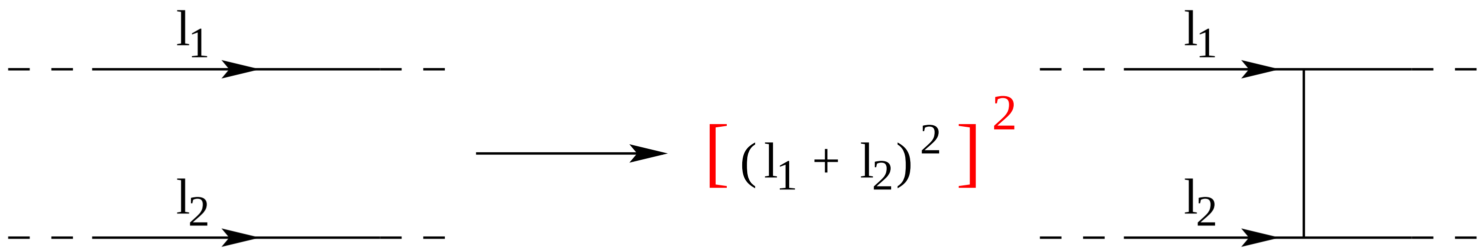


- gives all diagrams at 1-, 2- and 3-loops
- e.g. L-loop ladder diagrams have coefficient $s_{12}s_{23}A_4^{\text{tree}} \times s_{12}^{L-1}$



◇ same can be done for supergravity $\longrightarrow$ gravity rung rule

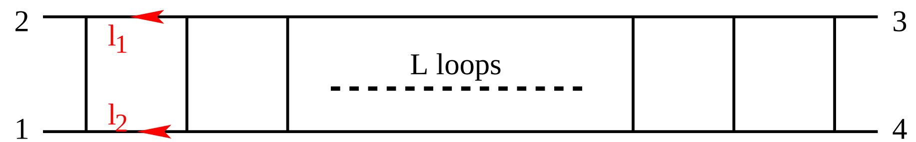
Bern, Dixon, Dunbar, Perelstein, Rozowski



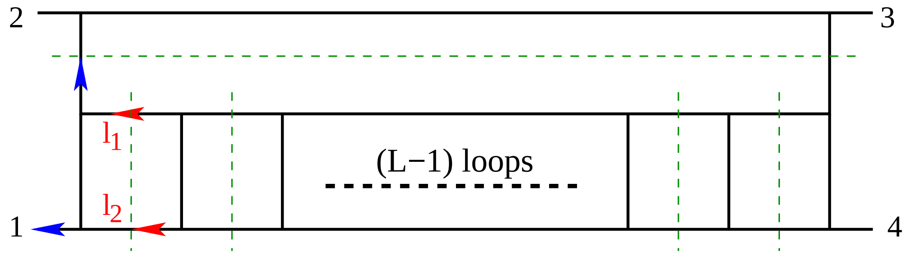
- generates all diagrams which have only two-particle cuts

Examples:

$$\left[s_{12}^{(L-1)}\right]^2$$

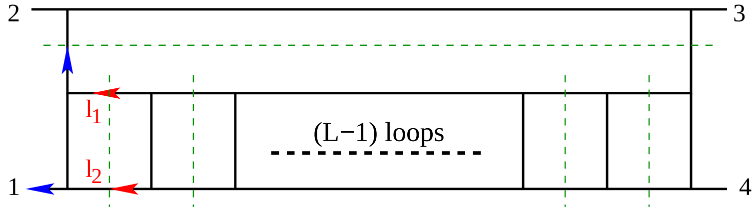


$$\left[s_{23}(l_1 + l_2)^{2(L-2)}\right]^2$$



- Consequences for UV behavior:

$$\left[s_{23}(l_1+l_2)^{2(L-2)}\right]^2$$



$$\propto \quad \underset{D^4}{\underset{\downarrow}{s_{23}^2}} \, (\underset{R^4}{\underset{\downarrow}{s_{12}s_{13}s_{23}M_4^{\text{tree}}}})$$

$$\text{Amplitude scaling} \sim \int d^{DL}l \, \frac{(l^2)^{n(L-2)}}{(l^2)^{3L+1}} \qquad \begin{cases} n=1 & \text{SYM} \\ n=2 & \text{supergravity} \end{cases}$$

$$\text{convergence:} \quad \frac{DL}{2} + n(L-2) < 3L+1 \quad \rightarrow \begin{cases} D_c = 2 + \frac{10}{L} & \mathcal{N}=8 \\ D_c = 4 + \frac{6}{L} & \mathcal{N}=4 \end{cases}$$

⋆ counterterm of the type D^4R^4

⋆ first divergence may appear at $L=5$ in $D=4$

Is this the end?

- Speculations that first divergence at $L=6$ — Howe, Stelle
- String theory hints: higher loop counterterm structure different
  late 2006: Berkovits; Green, Russo, Vanhove
- Field theory: only from 2-particle cuts: further cancellations?

The return of 1-loop gravity: higher point amplitudes

$\mathcal{N}=8$ 1-loop amplitudes analyzed explicitly: 5-, 6- and $\sim$ 7-point:

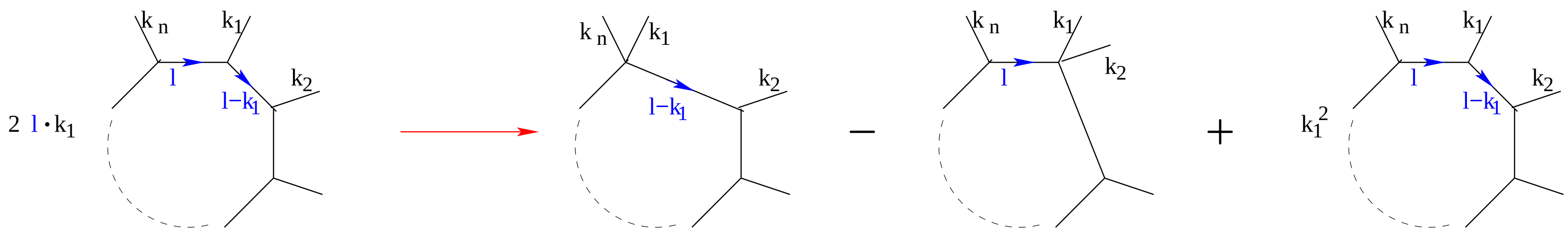
Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager

- all expressed as sum of box integrals
- good arguments that it holds for all 1-loop amplitudes

- formalized in the no-triangle hypothesis — Bern, Bjerrum-Bohr, Dunbar; Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager

- implications for our story follow through integral reduction:

$$\int \frac{d^D l\, 2l\cdot k_1}{l^2(l-k_1)^2\dots} = \int \frac{d^D l\,(l^2-(l-k_1)^2+k_1^2)}{l^2(l-k_1)^2\dots}$$

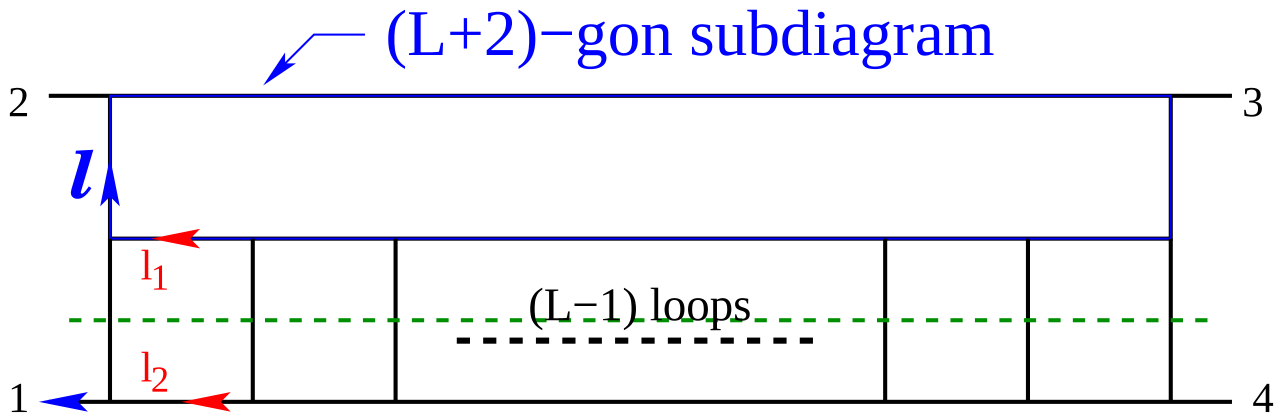


- one factor of l^μ in numerator $\equiv$ one less propagator

N-gon diagram: less that $(N-4)$ loop momentum numerator factors

So what? − analyze different cut of ladder diagram

$$\left[s_{25}(l+k_1)^{2(L-2)}\right]^2$$

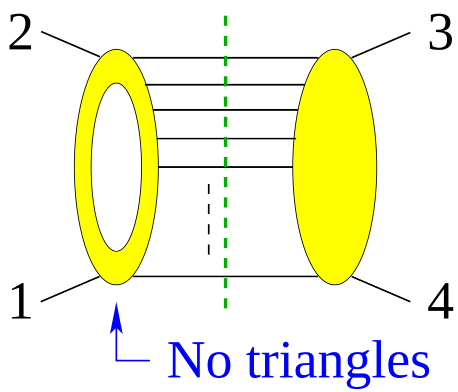


L-particle cut with $(L+2)$-gon diagram on one side and $4(L-2)$ factors of loop momentum!

⋆ contrast with explicit calculation for 5- and 6-point amplitudes

⋆ contrast with no-triangle hypothesis to any loop order

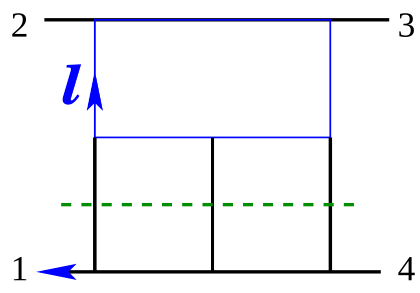
more cancellations ≡ better UV behavior – must be expected too all loop orders!



First place to look: 3-loops – $D_c^{\mathcal{N}=4} \neq D_c^{\mathcal{N}=8}$

3-particle cut of ladder diagram exposes a 1-loop 5-point amplitude with $(l+k_1)^4$ in the numerator

against explicit calculation/no-△ restriction



Slight improvement:

2-particle cut: $l^2 = 0 \to (l+k_1)^4$ might be $(2\, l \cdot k_1)^2$

- still trouble with no-triangle restriction

Possible sources of “major” improvement:

– nonplanar contributions (still 2-particle cut constructible)

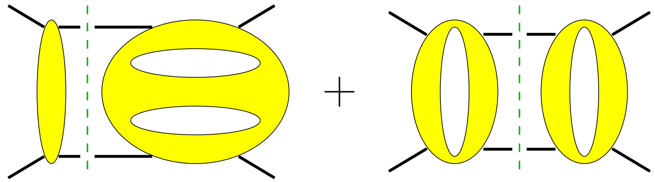
– diagrams not constructable from 2-particle cuts

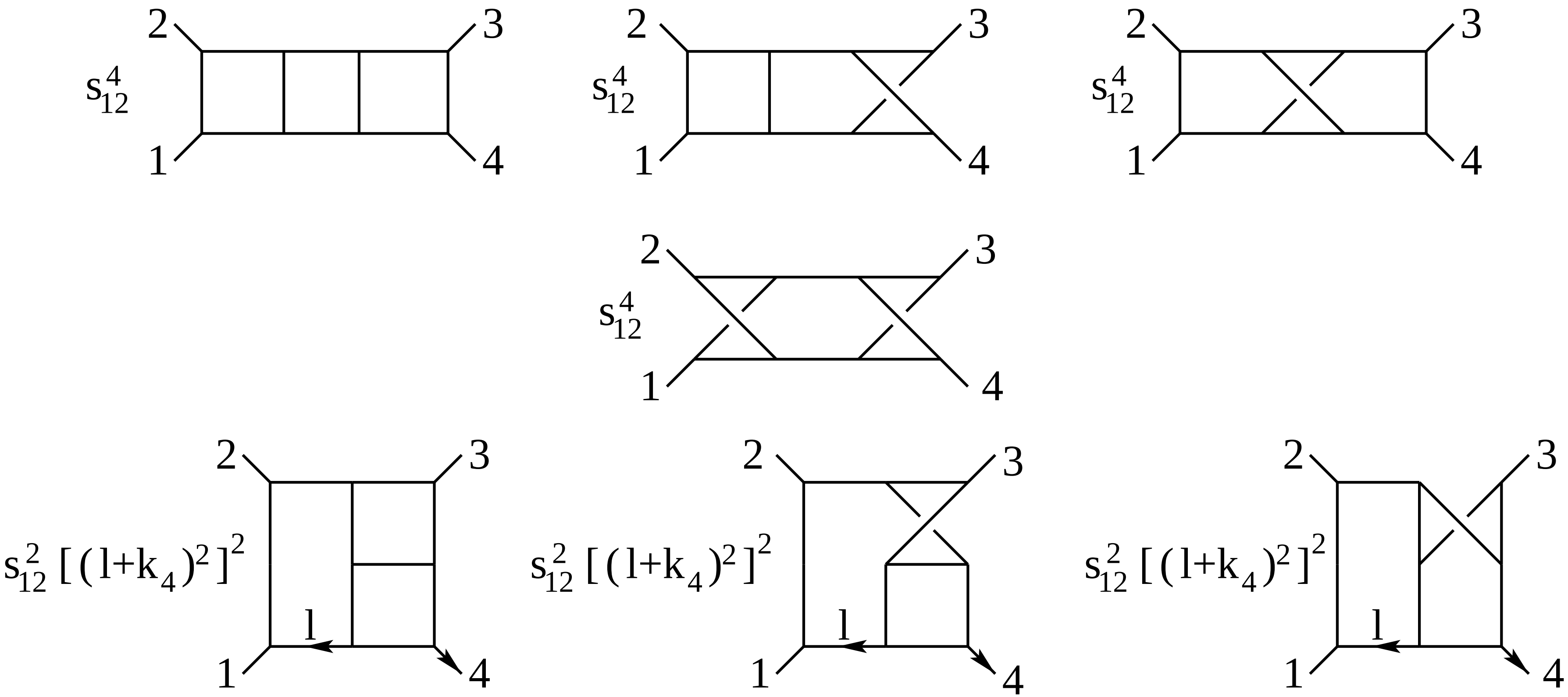
Only one way to be sure: explicitly construct the 3-loop integrand

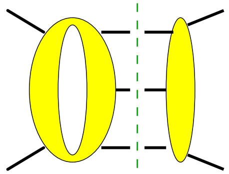
... and so we did

Need to analyze 2-, 3- and 4-particle cuts

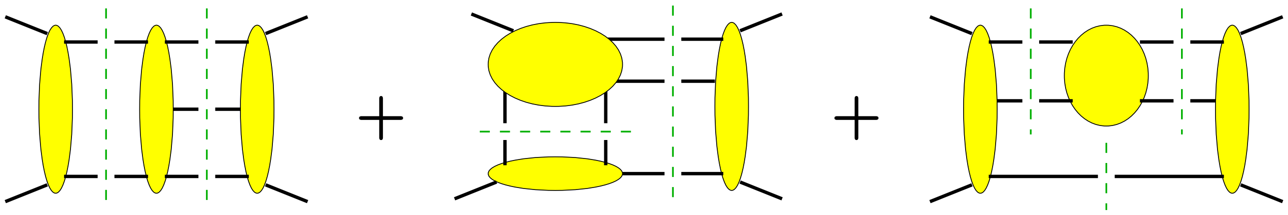
- 2-particle cut constructible diagrams





- 3-particle cuts 

Use generalized cuts: chop down all the way to tree amplitudes

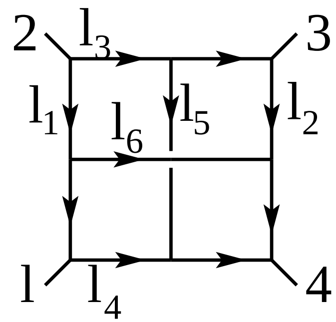


The plan:

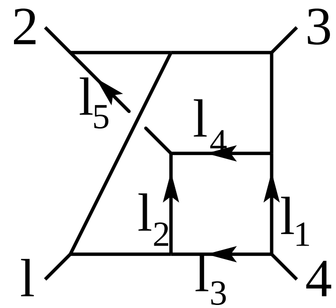
1. use KLT to reduce them to planar and nonplanar $\mathcal{N} = 4$ cuts
2. construct the relevant $\mathcal{N} = 4$ cuts (rung-rule insufficient)
3. reassemble the supergravity cuts
4. identify the integral functions that give these cuts

- additional $\mathcal{N}=4$ diagrams and numerator factors

$$s_{12}\left[(l_1+l_2)^2 - l_5^2\right] \quad + \quad s_{23}\left[(l_3+l_4)^2 - l_6^2\right]$$
$$-s_{12}s_{23}$$



$$s_{12}(l_1+l_2)^2 - s_{23}(l_3+l_4)^2 + \frac{1}{3}\left(s_{12}-s_{23}\right) l_5^2$$



- additional $\mathcal{N} = g$ supergravity diagrams and numerator factors

$$\Big[s_{12}(l_1 + l_2)^2 + s_{23}(l_3 + l_4)^2 - s_{12}s_{23}\Big]^2$$

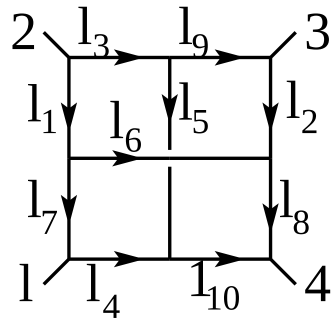
$$-s_{12}^2(2((l_1 + l_2)^2 - s_{23}) + l_5^2)l_5^2$$

$$-s_{23}^2(2((l_3 + l_4)^2 - s_{12}) + l_6^2)l_6^2$$

$$-s_{12}^2(2l_7^2l_2^2 + 2l_1^2l_8^2 + l_2^2l_8^2 + l_1^2l_7^2)$$

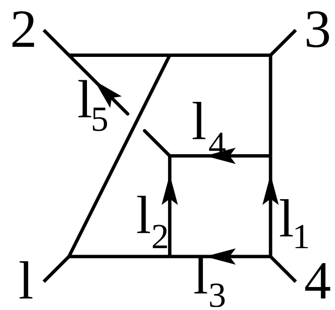
$$-s_{23}^2(2l_3^2l_{10}^2 + 2l_9^2l_4^2 + l_{10}^2l_4^2 + l_3^2l_9^2)$$

$$+2s_{12}s_{23}l_5^2l_6^2$$

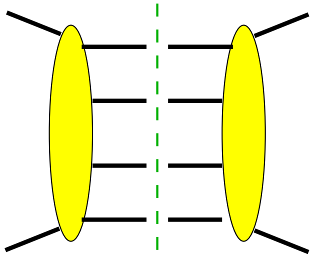


$$\Big(s_{12}(l_1 + l_2)^2 - s_{23}(l_3 + l_4^2)^2\Big)^2$$

$$-\Big(s_{12}^2(l_1 + l_2)^2 + s_{23}^2(l_3 + l_4^2)^2 + \tfrac{1}{3}s_{12}s_{23}s_{13}\Big)\, l_5^2$$



- four-particle cuts:



  - easy to draw and hard to evaluate ($\sim$ 8M)
  - ideally they would be the starting point

  - in present case:
    - powercounting arguments that they bring nothing new
    - higher generalized cuts that support these arguments

◇ calculation: the result from 2- and 3-particle cuts is complete

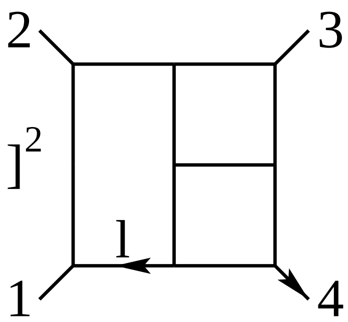
UV behavior

Leading UV divergence $\leftrightarrow$ leading term at small external momenta

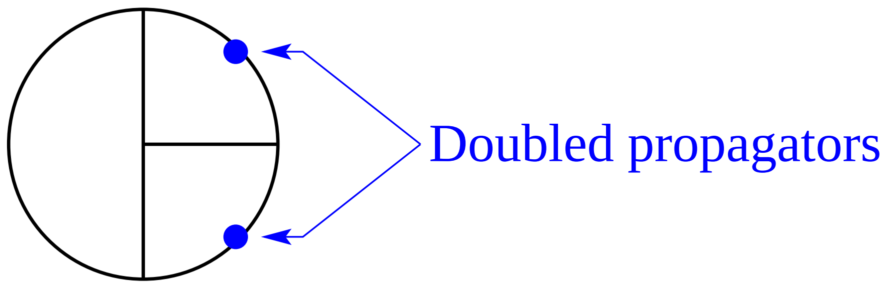
- all diagrams become vacuum diagrams with doubled propagators
- exposes triangle subdiagrams

For example:

$$s_{12}^2\,[(l+k_4)^2]^2$$

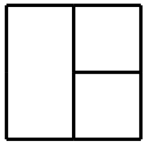
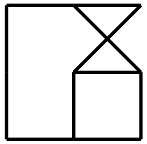
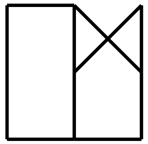
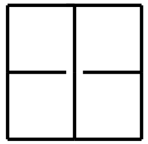
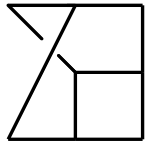
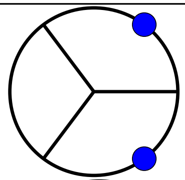
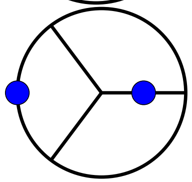
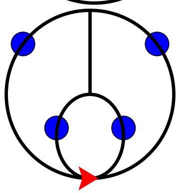
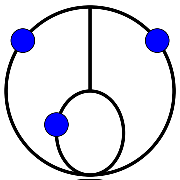
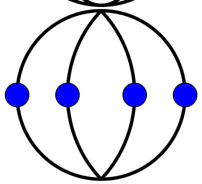


$$\Longrightarrow \quad s_{12}^2$$

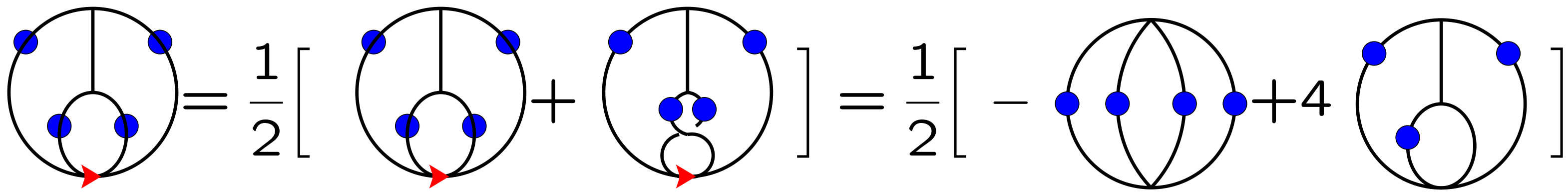


- the diagrams with loop momenta in the numerators determine the leading UV behavior

Vacuum diagrams and their origin

						
	4	0	8	−4	−8	No triangles
	0	4	0	−8	4	unexpected
	0	0	0	−4	0	−4
	0	0	0	0	8	8
	0	0	0	−2	0	−2

Momentum conservation identity:



◇ Leading UV divergence cancels algebraically!

◇ First subleading term also cancels (more complicated)

◇ Cancellations beyond no-triangle behavior

◇ First nonzero contribution has 2 loop momenta in the numerator

– the same as for $\mathcal{N} = 4$ SYM

– D_c consistent with SYM powercounting at 3 loops

◇ extrapolate to all loops w/o further improvement: $D_c = 2 + \frac{12}{L}$

Outlook

- Main message: 2-particle cut power counting

$$D_c = 2 + \frac{10}{L}$$

is improved at 3-loops through rather miraculous cancellations to

$$D_c = 6$$

which to this order is the same as for $\mathcal{N} = 4$ SYM

$$D_c = 4 + \frac{6}{L}$$

- Consistency with no-triangle behavior

- Arguments that improvement persists at higher loops

- Will the $\mathcal{N} = 4$ SYM powercounting hold for $\mathcal{N} = 8$ supergravity to all loop orders? Because of symmetry? dynamics?

- First point-like fine theory of quantum (super)gravity?

"Reports of the death of supergravity are exaggerations. One year everyone believed that supergravity was finite. The next year the fashion changed and everyone said that supergravity was bound to have divergences even though none had actually been found"

S. Hawking