Chiral Super LQG

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$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Marble

Wood

Theorem (Coleman Mandula)

Global symmetries in QFT are of the form

$$\mathfrak{g} = \mathrm{iso}(3,1) \oplus \mathfrak{g}_{\mathrm{int}}$$

(Assumptions: Lie algebra, analyticity properties of the S-matrix etc.)

Ideas for unification

- Kaluza-Klein
- Supersymmetry
- …

Supersymmety

Super Lie algebras

Coleman-Mandula can be avoided if the symmetry algebra is a super Lie algebra:

Theorem (Haag-Luposzanski):
classification of QFT compatible super Lie algebra extensions of iso(3,1).

super Lie algebra:

* $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$
* $[\mathfrak{g}_i, \mathfrak{g}_j] \subseteq \mathfrak{g}_{i+j \mod 2}$ $\qquad [x, y] = -(-1)^{\mathrm{gr}(x)\mathrm{gr}(y)}[y, x]$
* Graded Jacobi-identity

Example from differential geometry

Algebra of operators on $\Omega^*(M) =$ Forms on M

- Lie derivatives $\{\mathcal{L}_v \,|\, v \in \mathrm{T}^*M\} \longmapsto \mathfrak{g}_0$
- Exterior derivative $d \longmapsto \mathfrak{g}_1$
- Interior products $\{\iota_v \,|\, v \in \mathrm{T}^*M\} \longmapsto \mathfrak{g}_1$

form a Lie superalgebra wrt (anti-) commutator. Note for example Cartan identity

$$[\,d, \iota_v\,]_+ \equiv d\iota_v + \iota_v d = \mathcal{L}_v$$

Minimal extension of Poincaré algebra

$$\mathfrak{iso}(\mathbb{R}^{1,3|4}) = \underbrace{\mathbb{R}^{1,3} \ltimes \mathfrak{so}(1,3)}_{\mathfrak{g}_0} \oplus \underbrace{S_{\mathbb{R}}}_{\mathfrak{g}_1}$$

Usual Poincaré generators and relations plus Q_α and

$$[P_I, Q_\alpha]_- = 0$$

$$[M_{IJ}, Q_\alpha]_- = \frac{1}{2} Q_\beta (\gamma_{IJ})^\beta{}_\alpha$$

$$[Q_\alpha, Q_\beta]_+ = \frac{1}{2} (C\gamma_I)_{\alpha\beta} P^I$$

Minimal extension of Anti-deSitter algebra

$$\mathfrak{osp}(4|1) = \mathrm{adS}(3,1) \oplus S_{\mathbb{R}}$$

Usual Poincaré generators and relations plus $Q_\alpha$and

$$[P_I, Q_\alpha]_- = 0 - \frac{1}{2L} Q_\beta (\gamma_I)^\beta{}_\alpha$$

$$[M_{IJ}, Q_\alpha]_- = \frac{1}{2} Q_\beta (\gamma_{IJ})^\beta{}_\alpha$$

$$[Q_\alpha, Q_\beta]_+ = \frac{1}{2} (C\gamma_I)_{\alpha\beta} P^I + \frac{1}{4L} (C\gamma^{IJ})_{\alpha\beta} M_{IJ}$$

Notes on superextensions

Classical:

* Mixes spacetime-trafos with supertranslations:

$\longmapsto$ Symmetries of **supermanifold**

Quantum field theory:

$$Q \mid \text{boson} \rangle = \mid \text{fermion} \rangle$$
$$Q \mid \text{fermion} \rangle = \mid \text{boson} \rangle$$

Extended Superextensions

Extended SUSY:

Not 4 odd generators Q_α but $4 \times \mathcal{N}$: $\quad Q_\alpha \quad \longrightarrow \quad Q^r_\alpha, \qquad r = 1, 2, \ldots, \mathcal{N}$

R-Symmetry:

Additional even generators not commuting with Q. In our case $\mathrm{so}(\mathcal{N})$ generators T^{rs}

$$[Q^r_\alpha, Q^s_\beta]_+ = \frac{1}{2}\,\delta^{rs}\,(C\gamma_I)_{\alpha\beta}P^I + \frac{1}{4L}\,\delta^{rs}\,(C\gamma^{IJ})_{\alpha\beta}M_{IJ} - C_{\alpha\beta}T^{rs}$$

$$\mathrm{osp}(1\,|\,4) \quad \longrightarrow \quad \mathrm{osp}(\mathcal{N}\,|\,4)$$

Chiral decomposition [Eder '20]

(Anti-) selfdual generators

$$T_j^{\pm} = \frac{1}{2}(J_j \pm iK_j) \qquad\qquad \in \mathrm{so}(3,1)_{\mathbb{C}}$$

Decompose Q_α into chiral components $Q_\alpha = (Q_A, Q_{\dot{A}})$ gives super-sub algebra

$$[T_i^+, T_k^+] = \epsilon_{ik}{}^l\, T_l^+$$
$$[T_i^+, Q_A^r] = Q_B^r\, \sigma_i{}^B{}_A$$
$$[T^{pq}, Q_A^r] = \frac{1}{2L}(\delta^{qr} Q_A^p - \delta^{pr} Q_A^q)$$
$$[Q_A^r, Q_B^s]_+ = \frac{1}{L}\, \delta^{rs}\, (\epsilon\sigma^i)_{AB}\, T_i^+ - i\epsilon_{AB}\, T^{rs}$$

Supermanifold

Can be defined via algebra of (anti-)commuting "functions" (—> NC geometry)

Example $\mathbb{R}^{p|q}$

$$f = f_0(x) + f_1(x)^I \,\theta_I + f_2(x)^{I_1 I_2}\,\theta_{I_1}\theta_{I_2} + \ldots + f_q(x)^{I_1\ldots I_q}\,\theta_{I_1}\ldots\theta_{I_q}$$

$$x_\mu x_\nu - x_\nu x_\mu = 0, \qquad \mu,\nu = 1,\ldots,p$$
$$\theta_I\theta_J + \theta_J\theta_I = 0 \qquad I,J = 1,\ldots,q$$

In particular

$$\mathrm{fun}(\mathbb{R}^{0|q}) = \mathrm{fun}(\mathrm{superpoint}) = \mathrm{Gr}(\mathbb{R}^q)$$

Parametrization

Supermanifold map M —> M' := algebra morphism M′ —> M

This imposes severe constraints. In particular

$$\Phi : \mathbb{R}^m \quad \longrightarrow \quad \mathbb{R}^{n|q}, \qquad f = f_0 + f_1^I \theta_I + \dots \quad \Longrightarrow \quad \Phi^* f = \Phi^* f_0$$

Solution: Parametrization by supermanifold Z, for example Z = superpoint

- Supermanifolds $M \quad \longrightarrow \quad M \times Z$
- Maps depend functorially on Z

Supergravity

Global vs. local SUSY

Supersymmetry in (quantum) field theory:

$\mathfrak{iso}(\mathbb{R}^{1,3|4})$, $\mathfrak{osp}(4|1)$ and generalizations appear

* as global symmetries in super-YM theories: YM field + suitable fermions
* as local symmetries in supergravity: Frame field + suitable fermions

N=1 SUGRA

Frame and Rarita-Schwinger fermion:

$$S(e,\psi) = \int \mathrm{d}^4x\; e\left(e^{a\mu}e^{b\nu}R_{\mu\nu ab}(\omega(e)) - \overline{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho - \frac{1}{L}\overline{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \frac{6}{L^2}\right)$$

invariant under

* $\delta e^a_\mu = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu \qquad \delta\psi_\mu = D_\mu\epsilon$ with odd parameter ϵ and
* the usual diffeos.

Can we quantize with LQG methods?
Where does this local symmetry comes from?

some SUSY manifest, somewhat formal

Can one quantize with LQG methods? Yes!

- SUGRA in Ashtekar variables [Jacobson ‘88, Tsuda ‘00]
- enhanced gauge symmetry osp (1|2) [Fülöp ‘94, Gambini + Obregon + Pullin ‘96]
- formal quantization [Gambini + Obregon + Pullin ‘96, Ling + Smolin ‘99]
- Super spin networks, inner boundaries [Ling + Smolin ‘99]
- Spinfoam models in D=3 [Livine + Oeckl ‘03, L+Ryan ’07, Baccetti+L+R ’10]
- Canonical theory for higher D, quantization of RS fields, p-form fields [Bodendorfer+Thiemann+Thurn ‘11]

SUSY not manifest, rigorous formalism

N=1 SUGRA in Ashtekar variables

Ashtekar selfdual connection, frame, fermion and cosmological constant

Canonical theory with constraints (Jacobson '88)

- G, D, H: Gauß, diffeo, Hamilton constraint
- S^L, S^R: SUSY constraints

G, S^L form local $\mathrm{osp}(1\,|\,2)$ algebra! (Fülöp '93, Pullin et al '95)

$$\{G_i, G_k\} = \epsilon_{ik}{}^{l}\, G_l \qquad \{S^L_A, S^L_B\}_+ = \frac{1}{L}\,(\epsilon\sigma^i)_{AB}\, G_i$$

$$\{G_i, S^L_A\} = S^L_B\, \sigma_i{}^{B}{}_{A}$$

Where does this local symmetry comes from? Gauge sym (Fülöp '93, Pullin et al '95)?

MacDowell-Mansouri formulation

AdS-gravity as SO(3,2) gauge theory: $\mathrm{so}(3,2) \simeq \mathbb{R}^{3,1} \oplus \mathrm{so}(3,1)$

$$A_\mu = \mathrm{pr}_{\mathbb{R}^{3,1}} \circ A_\mu + \mathrm{pr}_{so(3,1)} \circ A_\mu =: e_\mu + \omega_\mu$$

Action

$$S[A] = \frac{L^2}{\kappa} \int \mathrm{d}^4 x\, \mathrm{tr}(F(A) \wedge \mathbf{P}\, F(A)) \qquad \mathbf{P} := \mathbf{0} \oplus i\gamma_*/2 : \mathfrak{so}(3,2) \to \mathfrak{so}(3,1)$$

Underlying geometry: Cartan geometry (Wise '06)

Gauge invariance broken down to so(3,1)

MacDowell-Mansouri formulation of N=1 AdS SUGRA

N=1 AdS SUGRA as osp(1|4) gauge theory: $\mathrm{osp}(1,4) \simeq \mathbb{R}^{3,1} \oplus \mathrm{so}(3,1) \oplus S_{\mathbb{R}}$

$$A_\mu = \mathrm{pr}_{\mathbb{R}^{3,1}} \circ A_\mu + \mathrm{pr}_{so(3,1)} \circ A_\mu + \mathrm{pr}_{S_\mathbb{R}} A_\mu =: e_\mu + \omega_\mu + \psi_\mu$$

Action [Castellani-D'Auria-Fré '80, '90]

$$S[A] = \frac{L^2}{\kappa} \int \mathrm{d}^4x\, \mathrm{str}(F(A) \wedge \mathbf{P}\, F(A)) \qquad \mathbf{P} := \mathbf{0} \oplus i\gamma_*/2 \oplus i\gamma_*/2$$

Underlying geometry: Super Cartan geometry (Eder '22)

Gauge invariance broken down to $\mathrm{so}(3,1)$

MacDowell-Mansouri Holst formulation of N=1 AdS SUGRA

N=1 AdS SUGRA as osp(1 | 4) gauge theory: $\mathrm{osp}(1,4) \simeq \mathbb{R}^{3,1} \oplus \mathrm{so}(3,1) \oplus S_{\mathbb{R}}$

$$A_\mu = \mathrm{pr}_{\mathbb{R}^{3,1}} \circ A_\mu + \mathrm{pr}_{so(3,1)} \circ A_\mu + \mathrm{pr}_{S_{\mathbb{R}}} A_\mu =: e_\mu + \omega_\mu + \psi_\mu$$

Action (Eder + HS 22′)

$$S[A] = \frac{L^2}{\kappa} \int \mathrm{d}^4 x \, \mathrm{tr}(F(A) \wedge \mathbf{P}_\beta \, F(A)) \qquad \mathbf{P}_\beta := \mathbf{0} \oplus P_\beta \oplus P_\beta \qquad P_\beta := \frac{\mathbb{1} + i\beta\gamma_*}{2\beta}$$

Underlying geometry: Super Cartan geometry (Eder ′22)

Gauge invariance broken down to $\mathrm{so}(3,1)$

MacDowell-Mansouri Holst formulation of N=2 AdS SUGRA

N=2 AdS SUGRA as osp(2 | 4) gauge theory: $\mathrm{osp}(2,4) \simeq \mathbb{R}^{3,1} \oplus \mathrm{so}(3,1) \oplus S_{\mathbb{R}} \oplus u(1)$

$$A_\mu = \mathrm{pr}_{\mathbb{R}^{3,1}} \circ A_\mu + \mathrm{pr}_{so(3,1)} \circ A_\mu + \mathrm{pr}_{S_{\mathbb{R}}} \circ A_\mu + \mathrm{pr}_{u(1)} \circ A_\mu =: e_\mu + \omega_\mu + \psi_\mu + \widehat{A}_\mu$$

Action (Eder + HS 22′)

$$S[A] = \frac{L^2}{\kappa} \int \mathrm{d}^4 x \, \mathrm{str}(F(A) \wedge \mathbf{P}_\beta \, F(A)) \qquad \mathbf{P}_\beta := \mathbf{0} \oplus P_\beta \oplus P_\beta \oplus (1 + \beta *)/2\beta$$

Underlying geometry: Super Cartan geometry (Eder ’22)

Gauge invariance broken down to $\mathrm{so}(3,1) \oplus \mathrm{u}(1)$

Remarks

- All Super-MM(-H) actions contain boundary terms.
- These terms are the unique ones to guarantee SUSY invariance on the boundary [Andrianopoli+D'Auria '14, KE+HS '21]
- β is literally the parameter of the theta-ambiguity in the U(1) sector
- All super-MM(-H) actions break manifest SUSY down to bosonic subgroup, except…

Chiral supergravity

The chiral case

For $\beta = \pm i \qquad \mathbf{P}_{-i} : \mathfrak{osp}(1|4)_{\mathbb{C}} \to \mathfrak{osp}(1|2)_{\mathbb{C}} \qquad \text{for} \qquad \mathcal{N} = 1$

$$\mathfrak{osp}(2|4)_{\mathbb{C}} \to \mathfrak{osp}(2|2)_{\mathbb{C}} \qquad \text{for} \qquad \mathcal{N} = 2$$

and supersymmetry stays partially manifest. In terms of super-Ashtekar connection

$$\mathcal{A}^{+}_{\mu} = \mathbf{P}_{-i} A^{\mu} = A^{+i}_{\mu} T^{+}_{i} + \psi^{A} Q_{A} + \widehat{A}_{\mu} T$$

action becomes

$$S = \frac{i}{\kappa} \int \mathrm{d}^4 x \left[\mathrm{str}(\mathcal{E} \wedge F(\mathcal{A}^{+})) + \frac{1}{4L^2} \mathrm{str}(\mathcal{E} \wedge \mathcal{E}) \right] + S_{\mathrm{bdy}}$$

with super-electric field $\mathcal{E}$

Thus understand now:

- SUSY can be gauge symmetry 😊
- Ashtekar connection preserves half of SUSY (local osp(1|2) or osp (2|2)) 😊
- Explanation for the observation of Fülöp et al 😊
- Need (negative) cosmological constant 😐
- Need complex variables, reality conditions, for manifest SUSY ☹️

But there is even more

In the presence of boundaries

Chern-Simons theory on the boundary:

$$S_{\mathrm{bdy}}(\mathcal{A}^+) = \frac{k}{4\pi}\int_{\partial M} \langle \mathcal{A}^+ \wedge \mathrm{d}\mathcal{A}^+ + \frac{1}{3}\mathcal{A}^+ \wedge [\mathcal{A}^+ \wedge \mathcal{A}^+]\rangle \qquad k = i\frac{4\pi L^2}{\kappa}$$

and boundary conditions to preserve EOM:

$$\underset{\Leftarrow}{\mathcal{E}} = \frac{i\kappa k}{2\pi} F(\underset{\Leftarrow}{\mathcal{A}^+})$$

See also [Ling + Smolin '99]

Towards chiral super LQG

Needed to extend LQG techniques:

* Holonomies and cylindrical functions ✓
* Invariant derivatives ✓
* Consistent family of measures, Hilbert space ✗
* Super spin networks ✓

Super-holonomies

Parametrized super-connections, super parallel transport on super paths [Eder '20]

For path in body of supermanifold:

$$h_e[\mathcal{A}^+] = h_e[A^+] \cdot \mathcal{P}\mathrm{exp}\left(-\oint_e \mathrm{Ad}_{h_e[A^+]^{-1}}\psi^{(\tilde{s})}\right) : \mathcal{Z} \to \mathcal{G}$$

Holonomies and Fluxes

Graded Poisson relations

$$\left\{\mathcal{E}^a_{\underline{A}}(x), \mathcal{A}^{+\underline{B}}_b(y)\right\} = i\kappa\delta^a_b\delta^{\underline{B}}_{\underline{A}}\delta(x,y)$$

Give graded holonomy-flux algebra generated by

$$\mathcal{E}_n(S) := \int_S \langle n, \mathcal{E}\rangle \qquad h_e[\mathcal{A}^+] = h_e[A^+]\cdot\mathcal{P}\exp\left(-\oint_e \mathrm{Ad}_{h_e[A^+]^{-1}}\psi^{(\tilde{s})}\right)$$

Towards a Hilbert space

Cylindrical functions, generalized connections generalize those of standard LQG.

$$\begin{tikzcd} \mathcal{A}_{\mathcal{S}',\gamma'} \arrow[r] \arrow[d] & \mathcal{A}_{\mathcal{S}',\gamma} \arrow[d] \\ \mathcal{A}_{\mathcal{S},\gamma'} \arrow[r] & \mathcal{A}_{\mathcal{S},\gamma} \end{tikzcd}$$

Super Hilbert space: Ansatz for single edge

$$\mathcal{S}(f|g) := \int_{\mathrm{SL}(2,\mathbb{C})} \mathrm{d}\mu_{\mathrm{SL}(2,\mathbb{C})}(g,\overline{g}) \int_B \mathrm{d}\theta^A \mathrm{d}\overline{\theta}^{A'} \rho(g,\overline{g},\theta,\overline{\theta})\, \overline{f} g$$

But problems with

- Non-compactness
- Positivity (—> consistent Krein structures?)

Super spin networks

[Ling+Smolin ’99, Eder ’22]

Suitable set of irreps:

* Finite or infinite dimensional?
* Tensor category!

In both cases osp(1 | 2), osp(2,2) there seem to exist suitable categories.

Principal series reps of osp(1|2) [Eder+HS '22]

Based on principal series representations of sl(2,R). On super vector space

$$\mathcal{V}_j^{\epsilon} = V_j^{\epsilon} \oplus \Pi V_{j-\frac{1}{2}}^{\epsilon+1} \qquad\qquad j \in \mathbb{C}, \epsilon \in \mathbb{Z}_2$$

Has quadratic casimir

$$\widehat{C}_2^{\mathfrak{osp}} = j\left(j + \frac{1}{2}\right)\mathbb{1}$$

Known finite dimensional irreps recovered for $j \in \mathbb{N}_0/2$

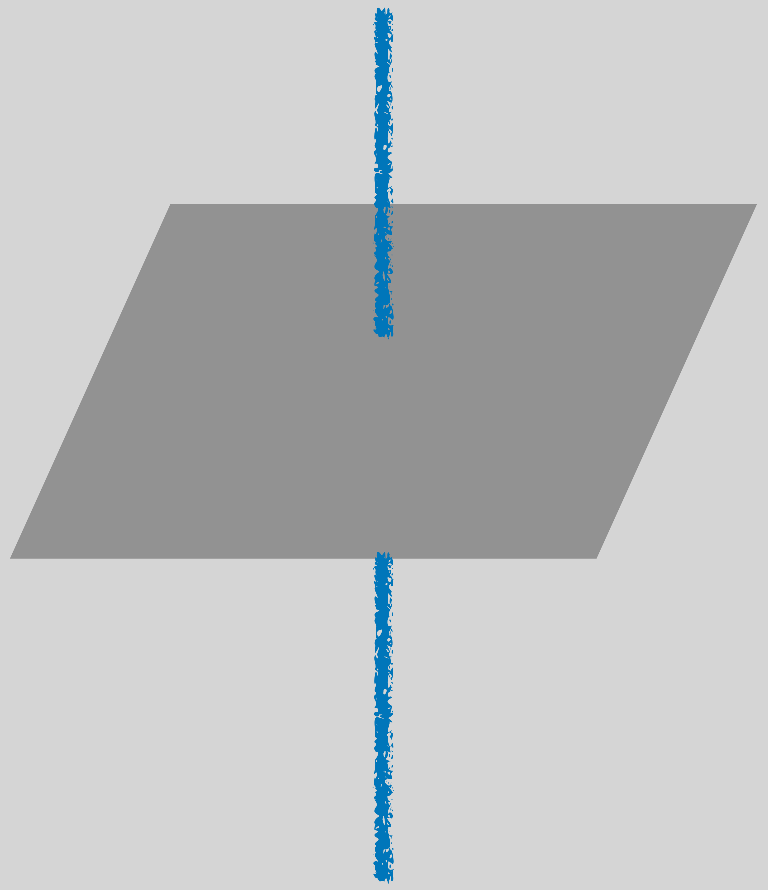
Super area

Regularization following non-supersymmetric theory [Ling+Smolin '99, Eder+HS '22]

Classically:

$$\mathrm{gAr}(S) := \sqrt{2}\int_S \|\mathcal{E}\|$$

In simple situation in quantum theory:



$$\widehat{\mathrm{gAr}}(S) = -8\pi i l_p^2 \sqrt{C_2^{\mathfrak{osp}}}$$

Spectrum:

$$\widehat{\mathrm{gAr}}(S) T_{e,j,m,n} = -8\pi i l_p^2 \sqrt{j\left(j+\frac{1}{2}\right)} T_{e,j,m,n}$$

Representations with real area spectrum

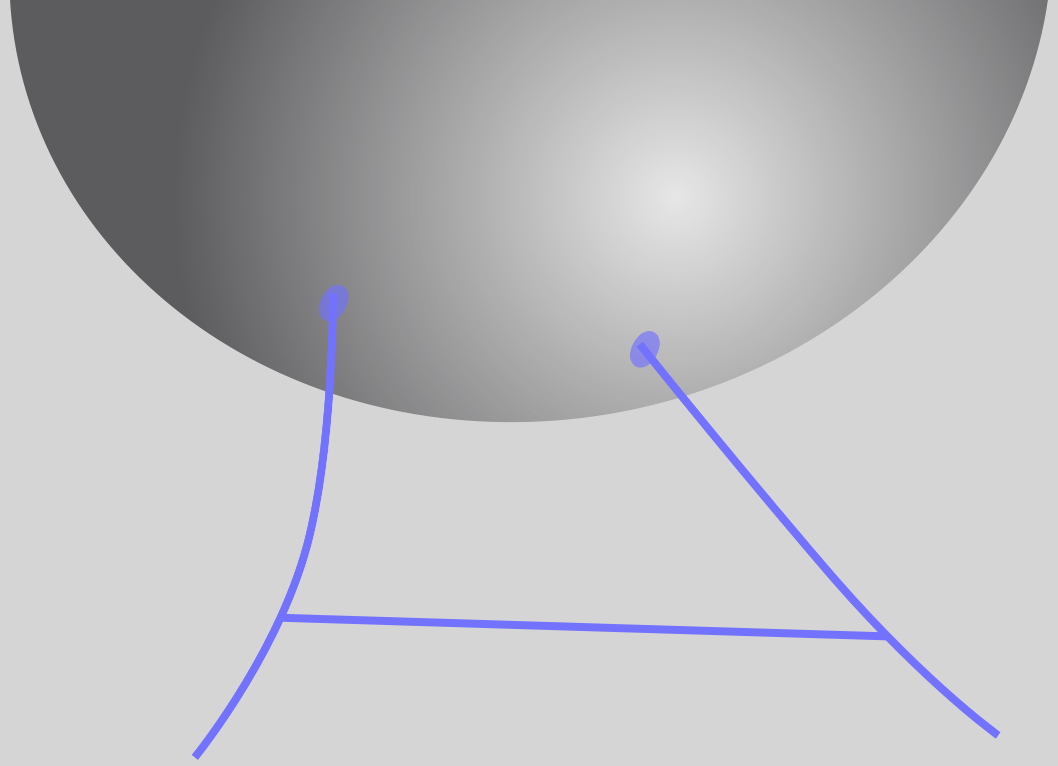
[Eder+HS ′22]

$$j = -\frac{1}{4} + is \text{ with } s \in \mathbb{R}$$

Then

$$\widehat{\mathrm{gAr}}(S) T_{e,j,m,n} = 8\pi l_p^2 \sqrt{s^2 + \frac{1}{16}}\, T_{e,j,m,n}$$

Entropy of spacetime boundaries



Have

- osp(N | 2) Chern-Simons boundary theory
- Boundary condition $$\underset{\Leftarrow}{\mathcal{E}} = \frac{i\kappa k}{2\pi} F(\underset{\Leftarrow}{\mathcal{A}}^{+})$$
- Super Area operator

Can assign entropy to inner boundary and find dependence on gA? Problems:

- CS for non-compact (super) group at level $$k = i\frac{4\pi L^2}{\kappa}$$
- Reality conditions?

State counting at β = i

Ben Achour+Frodden+Geiller+Mouchet+Noui+Perez '14:

- Count states for real Imirzi
- Analytically continue β as well as representation labels…
- … in such a way that area stays real.

What can play the role of SU(2) here?

For OSp(1 | 2): compact real form UOSp(1 | 2) with irreps labeled by $j \in \mathbb{N}_0/2$

Roadmap

* Real level $k = -12\pi/\kappa\Lambda$
* $N(j_1, j_2, \ldots) :=$ dimension of uosp $(1|2)$ superconformal block
* Analytic continuation

$$j \to j = -\frac{1}{4} + is, \qquad s \in \mathbb{R}$$

$$k \to ik$$

UOSp(1|2) characters and Verlinde formula

[Eder, HS ’22]

$$\chi_j = \left(1 - \frac{j}{4}\bar{\eta}\eta\right)\chi_j^{\mathrm{SU}(2)} - \left(1 + \frac{j+\frac{1}{2}}{4}\bar{\eta}\eta\right)\chi_{j-\frac{1}{2}}^{\mathrm{SU}(2)}$$

$$\begin{aligned}\mathcal{N}_\infty(\{n_l, j_l\}_l) &= \mathcal{S}\left(\chi_0 \middle| \prod_{l=1}^{p} (\chi_{j_l})^{n_l}\right) = \int_{\mathrm{UOSp}} \prod_{l=1}^{p} (\chi_{j_l}(\theta))^{n_l} \\ &= \frac{1}{2\pi}\int_0^\pi \mathrm{d}\theta\, \sin^2(2\theta)\left[4 - n + \sum_{i=1}^{p} n_i d_{j_l} \frac{\tan(d_{j_i}\theta)}{\tan\theta}\right] \prod_{l=1}^{p} \left(\frac{\cos(d_{j_l}\theta)}{\cos\theta}\right)^{n_l}\end{aligned}$$

Analytic continuation

$$j \to j = -\frac{1}{4} + is, \qquad s \in \mathbb{R} \qquad\qquad k \to ik$$

finally gives

$$S = \frac{a_H}{4l_p^2} + \dots$$

Under assumption of monochromatic indistinguishable punctures

$$S = \frac{a_H}{4l_p^2} + \nu \ln\left(\frac{2\sigma}{\nu}\right) \frac{\sqrt{a_H}}{l_p} - 2\ln\left(\frac{a_H}{l_p^2}\right) + \mathcal{O}(1)$$

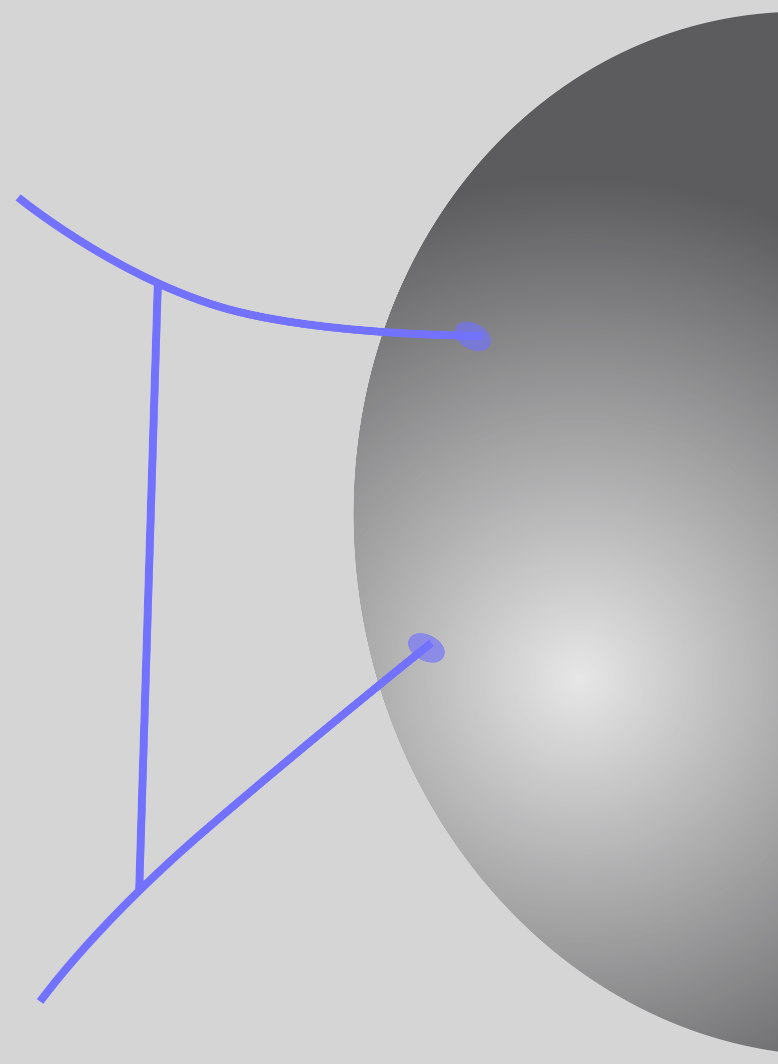
Summary and outlook

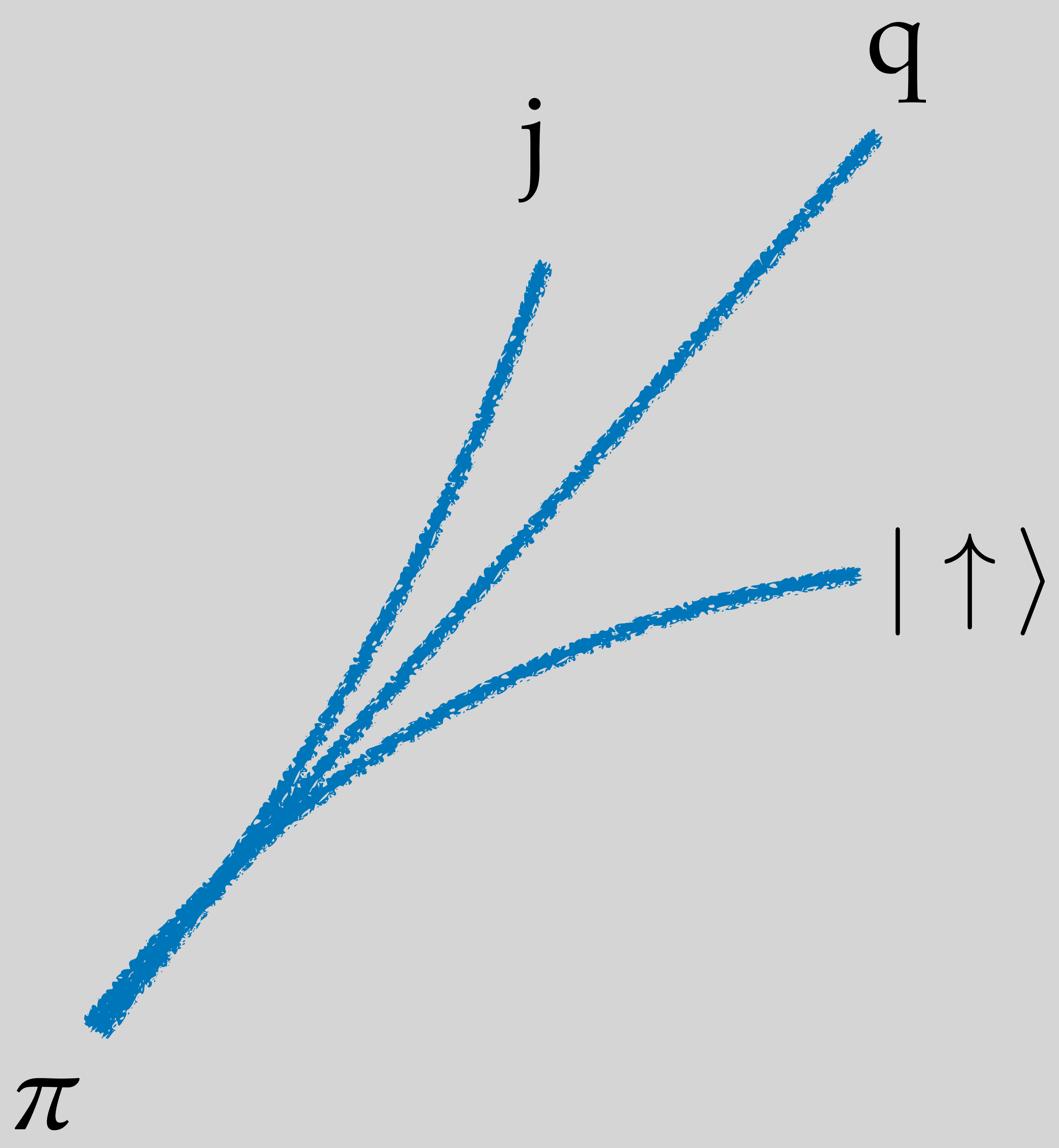
Summary

- Geometric background of super Ashtekar connection
- SUSY partially manifest
- Non-compactness, non-positivity, reality conditions
- Parametrized super connections, superparallel transport
- Steps towards supersymmetric black holes

Outlook

- Supersymmetric (isolated) horizons?
- N=2? Connection to stringy black holes?
- SUSY breaking
- Hilbert space, reality conditions





Based on…

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- Eder, HS: Supersymmetric minisuperspace models in self-dual loop quantum cosmology, JHEP 10.1007/JHEP03(2021)064

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