Renormalization Group Improved Cosmologies

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M. Reuter, F. Saueressig, JCAP **0509** (2005) 012 [hep-th/0507167]

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Big Bang cosmology

Beginning of the universe: $a = 0$

Inflationary phase

Nucleosynthesis and cosmic microwave background

requires classical evolution and radiation domination

today: $\Omega_{\mathrm{Total}} \simeq 1$, $\Omega_M \simeq 0.3$, $\Omega_\Lambda \simeq 0.7$

Crossover to domination of a cosmological constant

asymptotic de Sitter state

$\Omega_M = 0$, $\Omega_\Lambda = 1$

Quantum gravity effects?

seeds for cosmological evolution:

Planck Era $\Longleftrightarrow$ Quantum Gravity effects are important

- address this within “asymptotically safe quantum gravity”

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Outline:

- exact renormalization group (RG) equation for Euclidean Quantum Gravity
  $\Longrightarrow$ non-perturbative RG flow of G, Λ
- RG improvement of cosmological equations
  (= include leading quantum gravity effects)
- analyze flat FRW cosmologies

Exact Renormalization Group approach to Euclidean Quantum Gravity

(M. Reuter, Phys. Rev. D **57** (1998) 971 [arXiv:hep-th/9605030])

exact RG approach in a nut-shell:

- pathintegral approach to quantize theory of Euclidean metrics
  - diffeomorphism invariant
  - background independent
  - non-perturbative

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formulated for: effective average action Γ_k

- Wilsonian effective action
  includes all quantum effects with momenta $p^2 > k^2$
- limits: $k \to \infty$ = bare/classical action S
  $k \to 0$ = ordinary effective action Γ

main ingredient: exact evolution equation for Γ_k

Exact evolution equation for Γ_k

Principle idea: piecewise performance of the path integral

1. Starting point: diffeomorphism invariant gravitational action $S_{\mathrm{grav}}[\gamma_{\mu\nu}]$

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2. Construct: scale dependent gen. fct. for connected Greens functions

$$\exp\{W_k[t^{\mu\nu},\sigma^\mu,\bar{\sigma}_\mu;\bar{g}_{\mu\nu}]\} = \int \mathcal{D}h_{\mu\nu}\mathcal{D}C^\mu\mathcal{D}\bar{C}^\mu \times$$
$$\exp\{-S_{\rm grav}[\bar{g}+h] - S_{\rm gf}[h;\bar{g}] - S_{\rm gh}[h,C,\bar{C};\bar{g}] - S_{\rm source} - \Delta_k S[h,C,\bar{C};\bar{g}]\}$$

- background gauge fixing: $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

  $S_{\rm gf}[h;\bar{g}]$, $S_{\rm gh}[h,C,\bar{C};\bar{g}]$ = standard gauge fixing terms

- $S_{\rm source}$ = Sources for $h_{\mu\nu}, C^\mu, \bar{C}_\mu$

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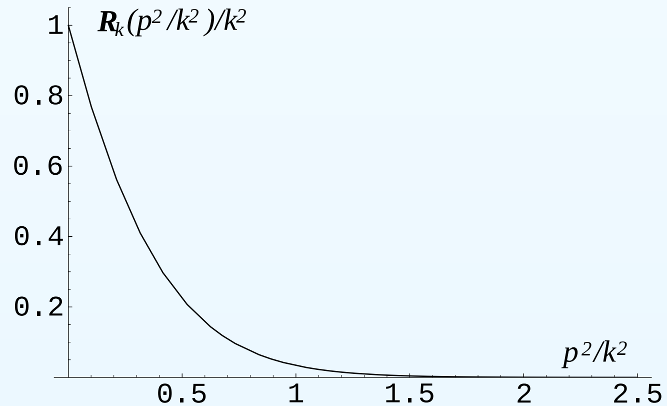
- crucial ingredient: scale dependent mass term

$$\Delta_k S = \int \sqrt{\bar g}\, h_{\mu\nu}\, Z^{\mu\nu\rho\sigma}_{\rm grav}\, \mathcal{R}_k(-\bar D^2)\, h_{\rho\sigma} + \text{ghost term}$$

separates: fast modes $p^2 > k^2$

slow modes $p^2 < k^2$

mode suppression for slow modes



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3. Effective average action Γ_k = (modified) Legendre transform of W_k:
   - Classical fields: $\bar h_{\mu\nu} = \langle h_{\mu\nu}\rangle_k, \quad \xi^\mu = \langle C^\mu\rangle_k, \quad \bar\xi^\mu = \langle \bar C^\mu\rangle_k$

$$\Gamma_k = \int \sqrt{\bar g}\,\left(t^{\mu\nu}\bar h_{\mu\nu} + \bar\sigma_\mu\xi^\mu + \sigma^\mu\bar\xi_\mu\right) - W_k - \Delta S_k\,.$$

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4. exact RG equation for Γ_k:

$$k\partial_k \Gamma_k = \tfrac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k(-\bar{D}^2) \right)^{-1} k\partial_k \mathcal{R}_k(-\bar{D}^2) \right] + \text{ghost contribution}$$

The Einstein-Hilbert truncation

- Catch: exact RG equation cannot be solved exactly
- non-perturbative approximation scheme
  $\Longrightarrow$ truncate the space of all interactions to the “most relevant” ones
- Simplest ansatz: Einstein-Hilbert truncation

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g}\, \{-R + 2\Lambda(k)\} \; + \text{classical gauge fixing}$$

  - running couplings: $G(k), \Lambda(k)$

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  - running couplings: $G(k), \Lambda(k)$
- non-perturbative β-functions $\beta_G(G, \Lambda, k), \beta_\Lambda(G, \Lambda, k)$:

$$k \, \frac{d}{dk} \, G(k) = \eta_N \, G \, ,$$

$$k \, \frac{d}{dk} \, \Lambda(k) = \eta_N \, \Lambda + \frac{1}{8\pi} \, k^4 \, G \left[40\Phi^1_2(-2\,\Lambda/k^2) - 32\Phi^1_2(0) - 20\eta_N \, \tilde{\Phi}^1_2(-2\,\Lambda/k^2) \right]$$

  - η_N: anomalous dimension of Newton's constant
  - $\Phi, \tilde{\Phi}$ encode $\mathcal{R}_k$-dependence

Evidence for asymptotic freedom

Introduce dimensionless coupling constants

$$g(k) \equiv k^2 G(k)\,,\, \lambda(k) \equiv \Lambda(k)/k^2$$

$\Longrightarrow$ autonomous β-functions:

$$k\partial_k g = \beta_g(g,\lambda)\ , \qquad k\partial_k \lambda = \beta_\lambda(g,\lambda).$$

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β-functions have a non-Gaussian fixed point:

- exists $g^* > 0, \lambda^* > 0$ where $\beta_g(g,\lambda) = \beta_\lambda(g,\lambda) = 0$
- IR repulsive in both g, λ
- by now good evidence for existence in full theory

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implications for full theory:

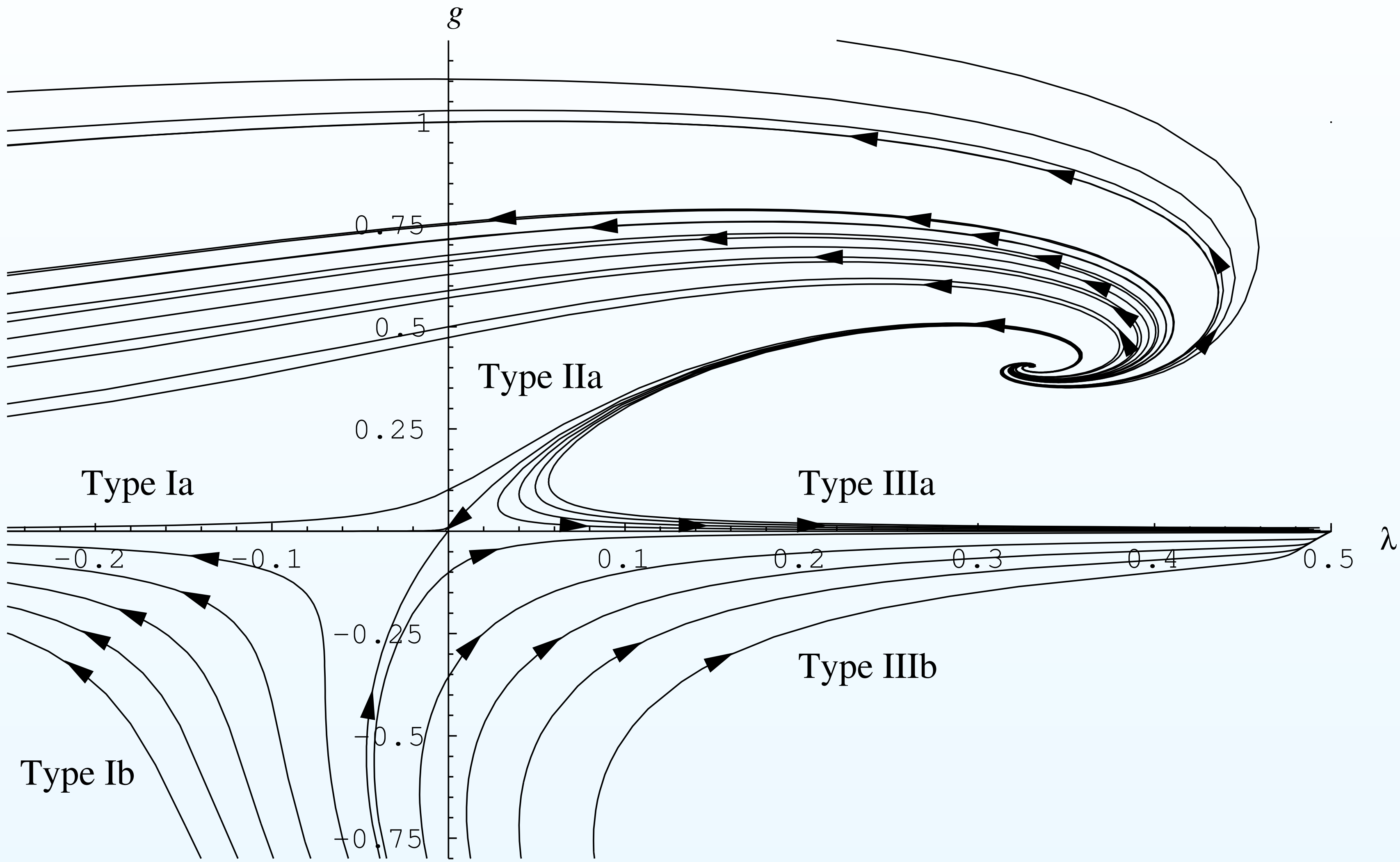
- Euclidean Quantum Gravity is asymptotically free

$$\lim_{k\to\infty} G(k) = \lim_{k\to\infty} \frac{g^*}{k^2} = 0$$

- non-perturbatively renormalizable (Weinberg's asymptotic safety scenario)

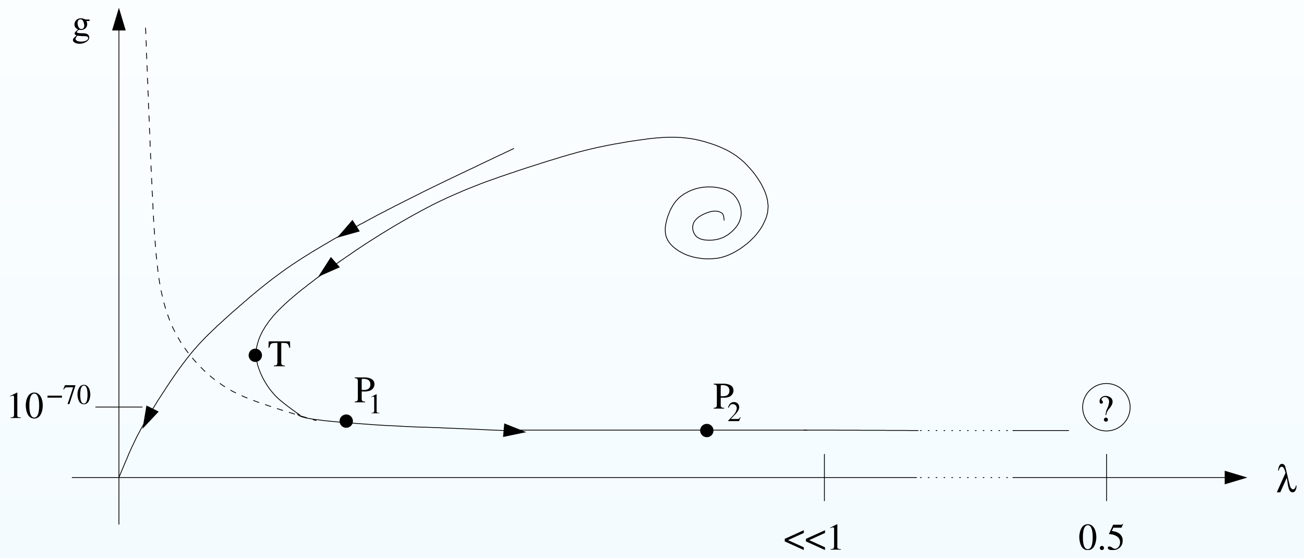
The RG flow of QEG in the Einstein-Hilbert-truncation

(M. Reuter, F. Saueressig, Phys. Rev. D **65** (2002) 065016 [hep-th/0110054])



The RG trajectory realized in Nature

(M. Reuter, H. Weyer, JCAP **0412** (2004) 001 [hep-th/0410119])



- start: NGFP
- linear regime: oscillations around NGFP
- passing *extremely* close to the GFP
- long classical GR regime
- $\lambda \lesssim 1/2$: strong IR renormalization effects?

Flat FRW cosmology

- FRW metric describes spatially flat, homogeneous, isotropic universe

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2\, d\Omega_2^2\right]$$

- dynamics: Einstein's equation

$$G_{\mu\nu} = -\Lambda\, g_{\mu\nu} + 8\pi\, G\, T_{\mu\nu}$$

- conserved energy momentum tensor

$$T_\mu{}^\nu \equiv \mathrm{diag}[-\rho, p, p, p]\,, \quad p(t) = w\rho(t)\,, \quad \nabla^\mu\, T_\mu{}^\nu = 0$$

- Friedmann + matter equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\,\Lambda + \frac{8\pi}{3}\, G\, \rho\,,$$

$$\dot{\rho} + 3\,(1+w)\,\frac{\dot{a}}{a}\,\rho = 0$$

RG improved FRW cosmology (= include quantum gravity effects)

- dynamics: RG scale dependent Einstein's equation:

$$G_{\mu\nu} = -\Lambda(k)\, g_{\mu\nu} + 8\pi\, G(k)\, T_{\mu\nu}$$

- cutoff identification $k \equiv k(t)$

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- Friedmann + matter equation + cutoff identification $k = k(t)$ + RG equations:

$$\begin{aligned}
\left(\frac{\dot a}{a}\right)^2 &= \frac{1}{3}\,\Lambda(k(t)) + \frac{8\pi}{3}\, G(k(t))\, \rho \\
\dot\rho + 3\,(1+w)\,\frac{\dot a}{a}\,\rho &= 0 \\
\dot\Lambda + 8\,\pi\,\rho\,\dot G &= 0 \\
k\partial_k G(k) &= \beta_G(G,\Lambda,k)\,, \qquad k\partial_k \Lambda(k) = \beta_\Lambda(G,\Lambda,k)
\end{aligned}$$

Solving the RG improved FRW equations

$$
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solve equations for given $k = k(t)$:

$$
\rho(t) = -\,\frac{1}{8\pi}\,\frac{\dot{\Lambda}(t)}{\dot{G}(t)}\,, \qquad a(t) = \left[-\,\mathcal{M}\,\frac{\dot{G}(t)}{\dot{\Lambda}(t)}\right]^{1/(3+3w)}
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Friedmann equation provides constraint on $k(t)$

$$
\frac{d\,k}{d\,t} = \frac{a}{a'}\,\left[\frac{\Lambda}{3} + \frac{8\pi}{3}\,G\,\rho\right]^{1/2}
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$$\frac{d\,k}{d\,t} = \frac{a}{a'}\,\left[\frac{\Lambda}{3} + \frac{8\pi}{3}\,G\,\rho\right]^{1/2}$$

- $t = k(t)^{-1}$ increases monotonically with decreasing k
- NGFP ($k = \infty$) = origin of the universe ($a = 0$)

RG improved cosmologies on λ-g space

$$
\begin{aligned}
\Omega_M &\equiv \frac{\rho}{\rho_{\mathrm{crit}}} = -\frac{1}{3H^2}\,\frac{\Lambda'}{G'/G} = \Omega_M(g,\lambda) \\
q &\equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(3w+1)\Omega_M - \Omega_\Lambda
\end{aligned}
$$

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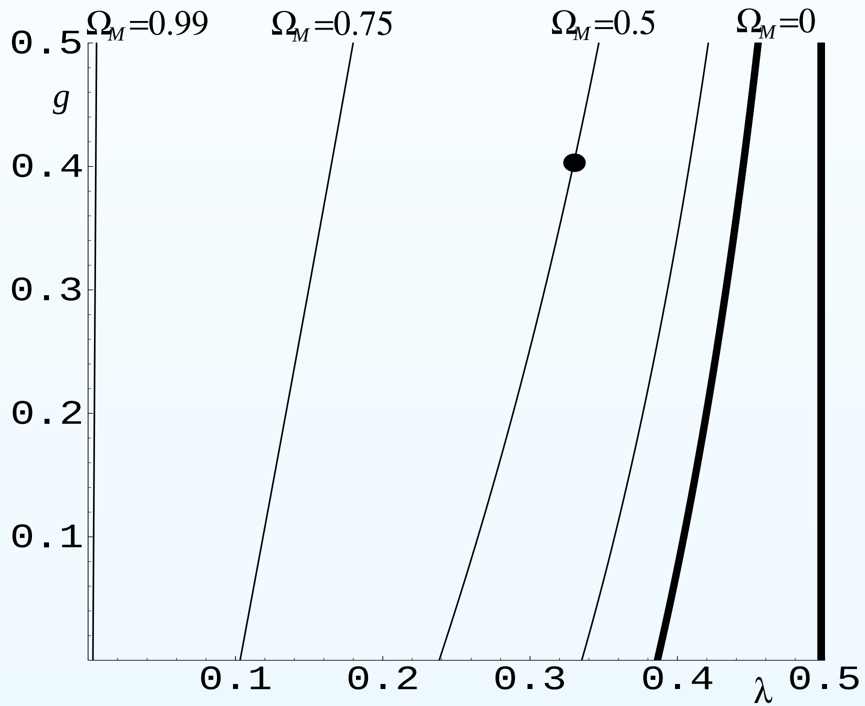
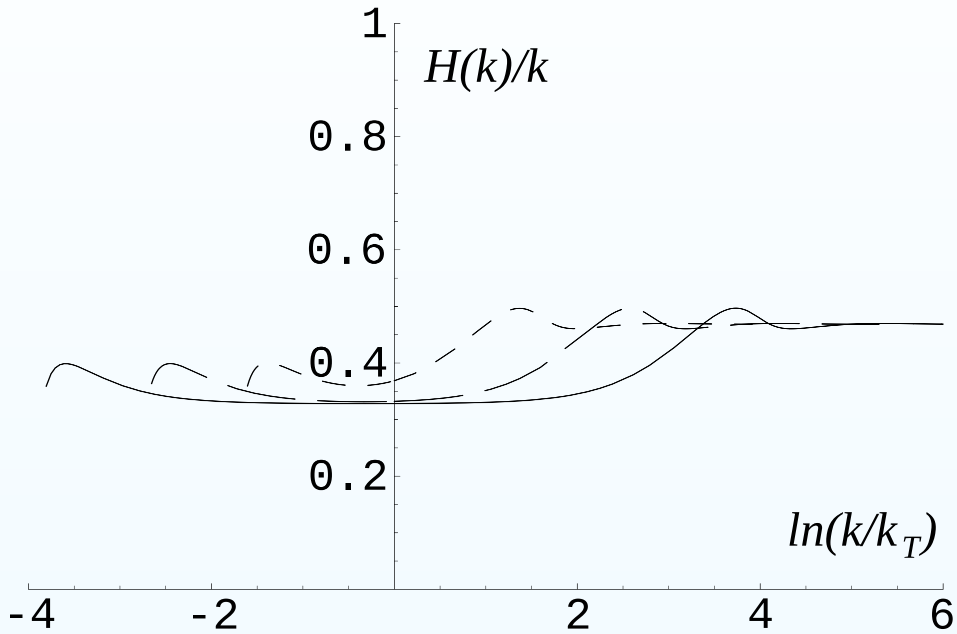
Matter density on λ-g space:



Physical interpretation of the dynamical cutoff identification

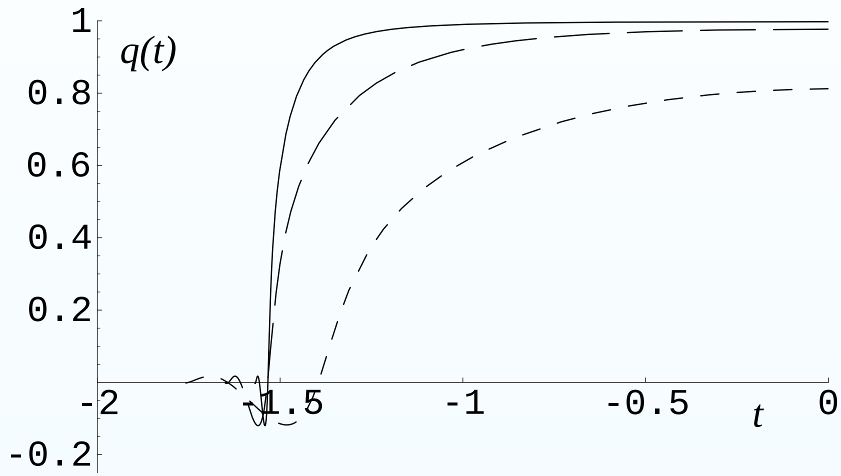
natural candidate: $k \propto H(k)$

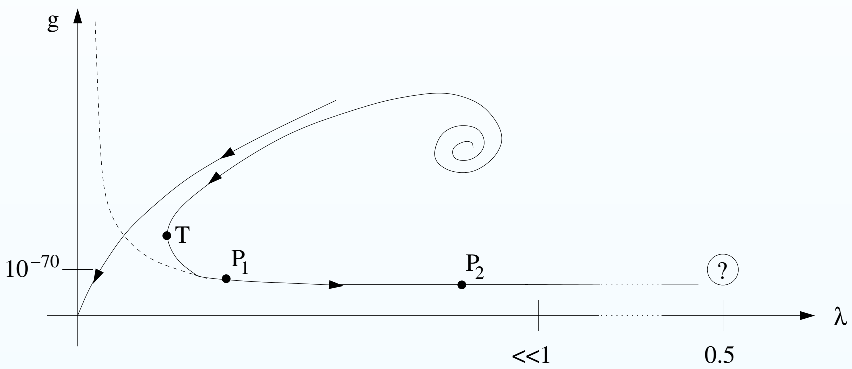


- almost constant in both UV, IR
  $\Longrightarrow$ dynamically generated cutoff is $\propto$ Hubble radius

Cosmological evolution in the quantum regime (pre-nucleosynthesis)

assume equation of state: $w = 1/3$

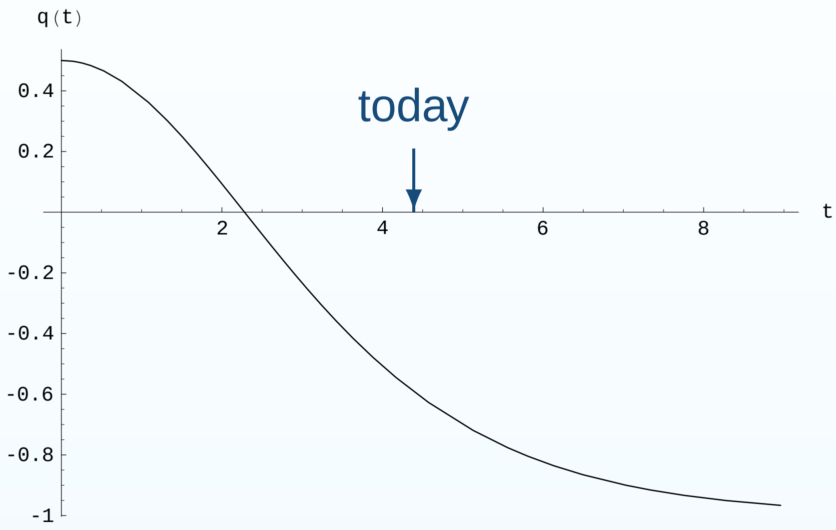


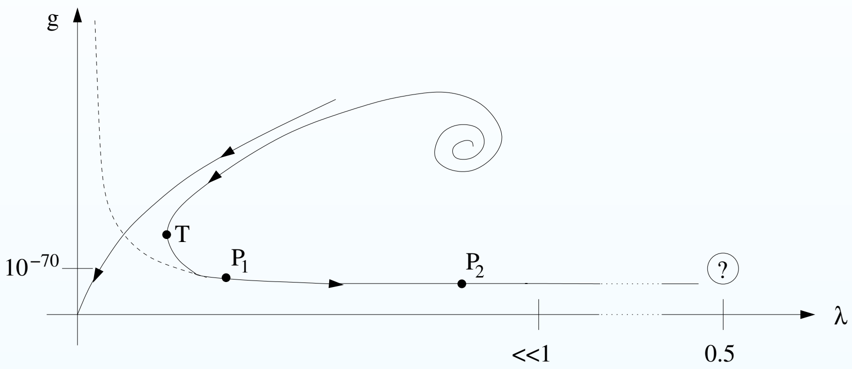


- evolution is (almost) universal
- Non-Gaussian fixed point: origin of the universe
- linear regime: oscillatory inflation
  one e-fold expansion during last acceleration cycle
- turning point (GFP): radiation domination $\Omega_M \simeq 1$

Cosmological evolution in the classical regime (nucleosynthesis onwards)

assume equation of state: $w = 0$





- $\Omega_M = 1$ at beginning of classical era
- crossover at cosmological time scales
- evolution stops before entering in regime where RG effects become strong

Summary and Outlook

dynamical cutoff identification:

- generates physical identification $k(t) \propto H(t)$

characteristics of RG improved cosmologies:

- universal initial conditions at the Big Bang $\Omega_M = \Omega_\Lambda = 1/2$
- period of oscillatory inflation
- matter domination at the beginning of the classical regime $\Omega_M \simeq 1, \Omega_\Lambda \simeq 0$
- cosmological evolution ends before strong RG effects set in

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Outlook:

- Continuous inflationary phase? (Work in progress)