

Renormalization Group Improved Cosmologies

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M. Reuter, F. Saueressig, JCAP 0509 (2005) 012 [hep-th/0507167]

ILQGS and Stückelberg Workshop

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Big Bang cosmology

Beginning of the universe: $a = 0$

Inflationary phase

Nucleosynthesis and cosmic microwave background
requires classical evolution and radiation domination

today: $\Omega_{\text{Total}} \simeq 1$, $\Omega_M \simeq 0.3$, $\Omega_\Lambda \simeq 0.7$

Crossover to domination of a cosmological constant

asymptotic de Sitter state

$$\Omega_M = 0, \Omega_\Lambda = 1$$

Quantum gravity effects?

seeds for cosmological evolution:

Planck Era \iff Quantum Gravity effects are important

- address this within “asymptotically safe quantum gravity”

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Outline:

- exact renormalization group (RG) equation for Euclidean Quantum Gravity
 \implies non-perturbative RG flow of G, Λ
- RG improvement of cosmological equations
(= include leading quantum gravity effects)
- analyze flat FRW cosmologies

Exact Renormalization Group approach to Euclidean Quantum Gravity

(M. Reuter, Phys. Rev. D 57 (1998) 971 [arXiv:hep-th/9605030])

exact RG approach in a nut-shell:

- pathintegral approach to quantize theory of Euclidean metrics
 - diffeomorphism invariant
 - background independent
 - non-perturbative

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formulated for: effective average action Γ_k

- Wilsonian effective action
includes all quantum effects with momenta $p^2 > k^2$
- limits: $k \rightarrow \infty =$ bare/classical action S
 $k \rightarrow 0 =$ ordinary effective action Γ

main ingredient: exact evolution equation for Γ_k

Exact evolution equation for Γ_k

Principle idea: piecewise performance of the path integral

1. Starting point: diffeomorphism invariant gravitational action $S_{\text{grav}}[\gamma_{\mu\nu}]$

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2. Construct: scale dependent gen. fct. for connected Greens functions

$$\exp\{W_k[t^{\mu\nu}, \sigma^\mu, \bar{\sigma}_\mu; \bar{g}_{\mu\nu}]\} = \int \mathcal{D}h_{\mu\nu} \mathcal{D}C^\mu \mathcal{D}\bar{C}^\mu \times \\ \exp\{-S_{\text{grav}}[\bar{g} + h] - S_{\text{gf}}[h; \bar{g}] - S_{\text{gh}}[h, C, \bar{C}; \bar{g}] - S_{\text{source}} - \Delta_k S[h, C, \bar{C}; \bar{g}]\}$$

- background gauge fixing: $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$S_{\text{gf}}[h; \bar{g}], S_{\text{gh}}[h, C, \bar{C}; \bar{g}] = \text{standard gauge fixing terms}$

- $S_{\text{source}} = \text{Sources for } h_{\mu\nu}, C^\mu, \bar{C}_\mu$

Exact evolution equation for Γ_k

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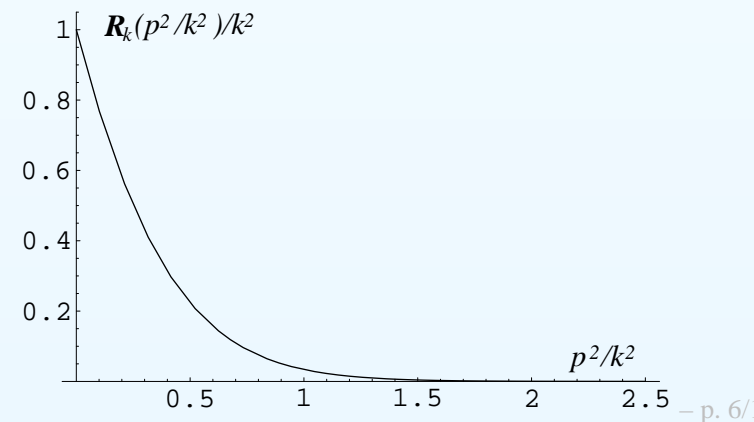
- crucial ingredient: scale dependent mass term

$$\Delta_k S = \int \sqrt{\bar{g}} h_{\mu\nu} Z_{\text{grav}}^{\mu\nu\rho\sigma} \mathcal{R}_k(-\bar{D}^2) h_{\rho\sigma} + \text{ghost term}$$

separates: fast modes $p^2 > k^2$

slow modes $p^2 < k^2$

mode suppression for slow modes



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3. Effective average action $\Gamma_k =$ (modified) Legendre transform of W_k :

- Classical fields: $\bar{h}_{\mu\nu} = \langle h_{\mu\nu} \rangle_k$, $\xi^\mu = \langle C^\mu \rangle_k$, $\bar{\xi}^\mu = \langle \bar{C}^\mu \rangle_k$

$$\Gamma_k = \int \sqrt{\bar{g}} (t^{\mu\nu} \bar{h}_{\mu\nu} + \bar{\sigma}_\mu \xi^\mu + \sigma^\mu \bar{\xi}_\mu) - W_k - \Delta S_k .$$

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4. exact RG equation for Γ_k :

$$k\partial_k \Gamma_k = \frac{1}{2} \text{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k(-\bar{D}^2) \right)^{-1} k\partial_k \mathcal{R}_k(-\bar{D}^2) \right] + \text{ghost contribution}$$

The Einstein-Hilbert truncation

- Catch: exact RG equation cannot be solved exactly
- non-perturbative approximation scheme
⇒ truncate the space of all interactions to the “most relevant” ones
- Simplest ansatz: Einstein-Hilbert truncation

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g} \{-R + 2\Lambda(k)\} + \text{classical gauge fixing}$$

- running couplings: $G(k), \Lambda(k)$

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- running couplings: $G(k), \Lambda(k)$
- non-perturbative β -functions $\beta_G(G, \Lambda, k), \beta_\Lambda(G, \Lambda, k)$:

$$k \frac{d}{dk} G(k) = \eta_N G,$$

$$k \frac{d}{dk} \Lambda(k) = \eta_N \Lambda + \frac{1}{8\pi} k^4 G \left[40\Phi_2^1(-2\Lambda/k^2) - 32\Phi_2^1(0) - 20\eta_N \tilde{\Phi}_2^1(-2\Lambda/k^2) \right]$$

- η_N : anomalous dimension of Newton's constant
- $\Phi, \tilde{\Phi}$ encode \mathcal{R}_k -dependence

Evidence for asymptotic freedom

Introduce dimensionless coupling constants

$$g(k) \equiv k^2 G(k), \lambda(k) \equiv \Lambda(k)/k^2$$

\implies autonomous β -functions:

$$k\partial_k g = \beta_g(g, \lambda), \quad k\partial_k \lambda = \beta_\lambda(g, \lambda).$$

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β -functions have a non-Gaussian fixed point:

- exists $g^* > 0, \lambda^* > 0$ where $\beta_g(g, \lambda) = \beta_\lambda(g, \lambda) = 0$
- IR repulsive in both g, λ
- by now good evidence for existence in full theory

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implications for full theory:

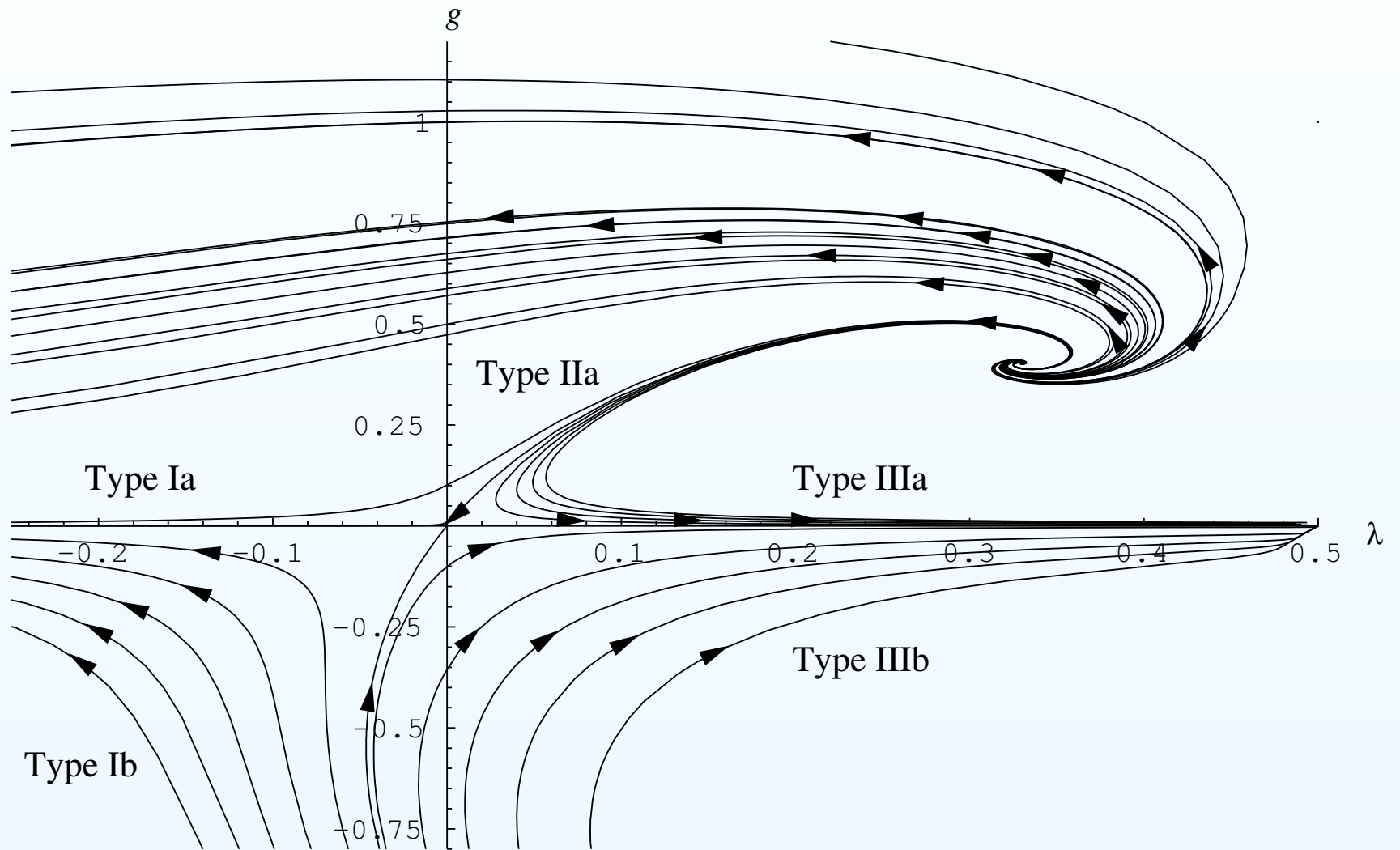
- Euclidean Quantum Gravity is asymptotically free

$$\lim_{k \rightarrow \infty} G(k) = \lim_{k \rightarrow \infty} \frac{g^*}{k^2} = 0$$

- non-perturbatively renormalizable (Weinberg's asymptotic safety scenario)

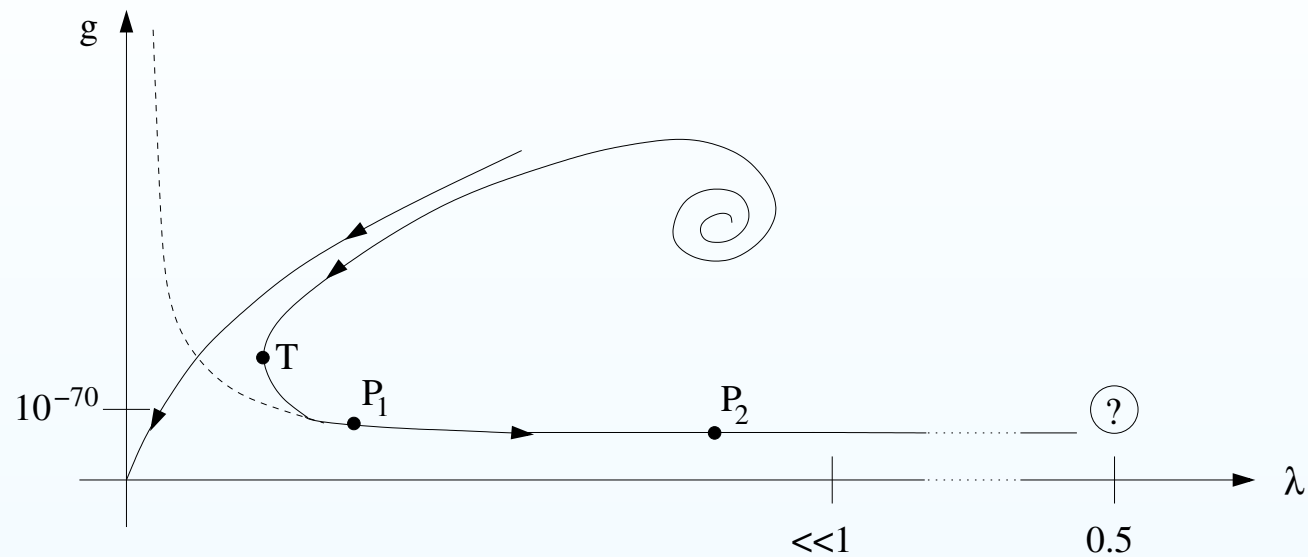
The RG flow of QEG in the Einstein-Hilbert-truncation

(M. Reuter, F. Saueressig, Phys. Rev. D 65 (2002) 065016 [hep-th/0110054])



The RG trajectory realized in Nature

(M. Reuter, H. Weyer, JCAP 0412 (2004) 001 [hep-th/0410119])



- start: NGFP
- linear regime: oscillations around NGFP
- passing *extremely* close to the GFP
- long classical GR regime
- $\lambda \lesssim 1/2$: strong IR renormalization effects?

Flat FRW cosmology

- FRW metric describes spatially flat, homogeneous, isotropic universe

$$ds^2 = -dt^2 + a(t)^2 [dr^2 + r^2 d\Omega_2^2]$$

- dynamics: Einstein's equation

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}$$

- conserved energy momentum tensor

$$T_{\mu}^{\nu} \equiv \text{diag}[-\rho, p, p, p], \quad p(t) = w\rho(t), \quad \nabla^{\mu} T_{\mu}^{\nu} = 0$$

- Friedmann + matter equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\Lambda + \frac{8\pi}{3}G\rho,$$
$$\dot{\rho} + 3(1+w)\frac{\dot{a}}{a}\rho = 0$$

RG improved FRW cosmology (= include quantum gravity effects)

- dynamics: RG scale dependent Einstein's equation:

$$G_{\mu\nu} = -\Lambda(k) g_{\mu\nu} + 8\pi G(k) T_{\mu\nu}$$

- cutoff identification $k \equiv k(t)$

RG improved FRW cosmology (= include quantum gravity effects)

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- **cutoff identification** $k \equiv k(t)$
- Friedmann + matter equation + **cutoff identification** $k = k(t)$ + **RG equations**:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3} \Lambda(k(t)) + \frac{8\pi}{3} G(k(t)) \rho$$
$$\dot{\rho} + 3(1+w) \frac{\dot{a}}{a} \rho = 0$$
$$\dot{\Lambda} + 8\pi \rho \dot{G} = 0$$
$$k \partial_k G(k) = \beta_G(G, \Lambda, k), \quad k \partial_k \Lambda(k) = \beta_\Lambda(G, \Lambda, k)$$

Solving the RG improved FRW equations

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= \frac{1}{3} \Lambda(k(t)) + \frac{8\pi}{3} G(k(t)) \rho \\ \dot{\rho} + 3(1+w) \frac{\dot{a}}{a} \rho &= 0 \\ \dot{\Lambda} + 8\pi \rho \dot{G} &= 0 \\ k\partial_k G(k) &= \beta_G(G, \Lambda, k), \quad k\partial_k \Lambda(k) = \beta_\Lambda(G, \Lambda, k)\end{aligned}$$

solve equations for given $k = k(t)$:

$$\rho(t) = -\frac{1}{8\pi} \frac{\dot{\Lambda}(t)}{\dot{G}(t)}, \quad a(t) = \left[-\mathcal{M} \frac{\dot{G}(t)}{\dot{\Lambda}(t)} \right]^{1/(3+3w)}$$

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Friedmann equation provides constraint on $k(t)$

$$\frac{dk}{dt} = \frac{a}{a'} \left[\frac{\Lambda}{3} + \frac{8\pi}{3} G \rho \right]^{1/2}$$

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- $t = k(t)^{-1}$ increases monotonically with decreasing k
- NGFP ($k = \infty$) = origin of the universe ($a = 0$)

RG improved cosmologies on λ - g space

$$\Omega_M \equiv \frac{\rho}{\rho_{\text{crit}}} = -\frac{1}{3H^2} \frac{\Lambda'}{G'/G} = \Omega_M(g, \lambda)$$

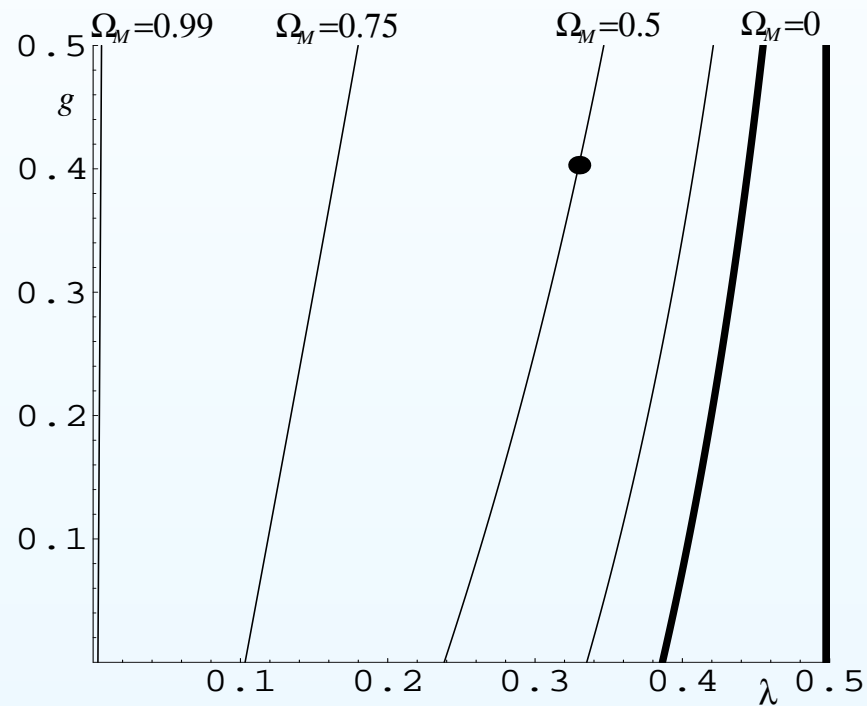
$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(3w + 1)\Omega_M - \Omega_\Lambda$$

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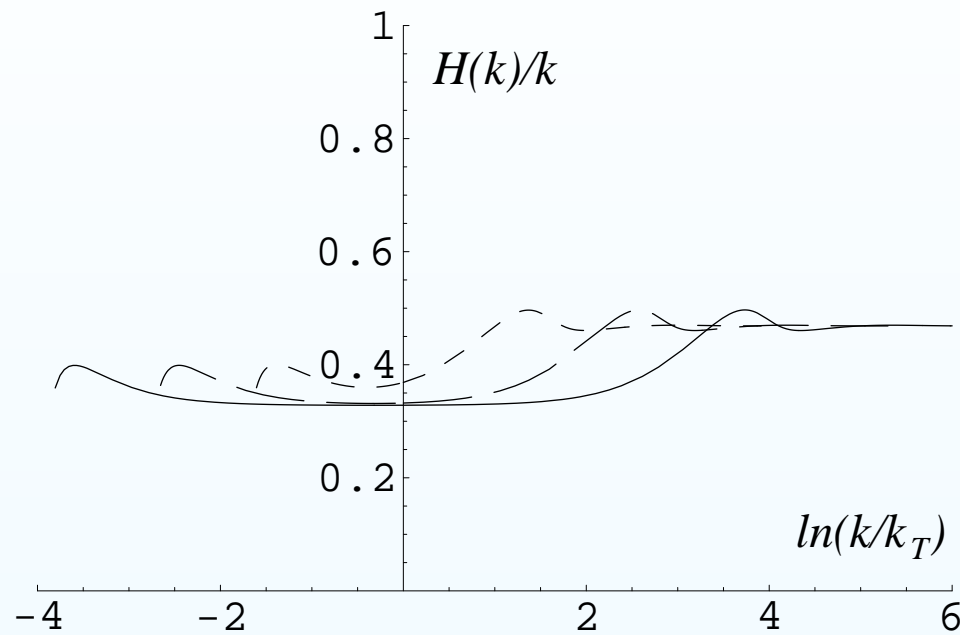
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Matter density on λ - g space:



Physical interpretation of the dynamical cutoff identification

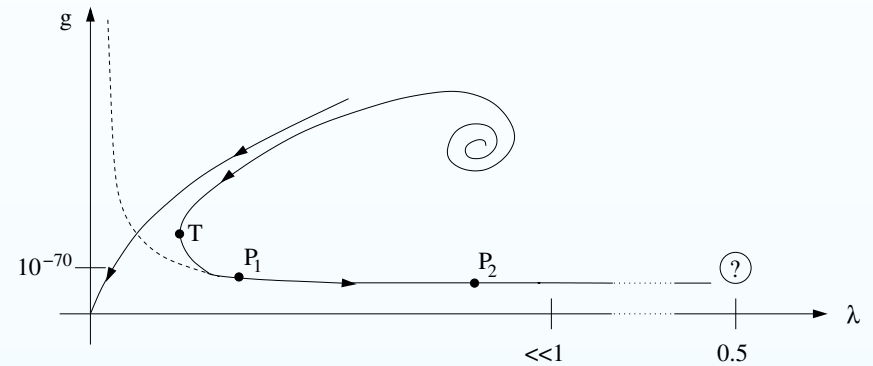
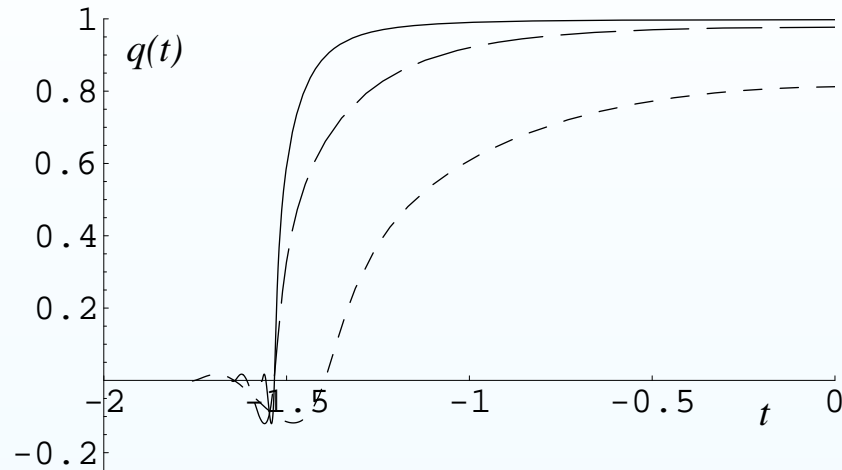
natural candidate: $k \propto H(k)$



- almost constant in both UV, IR
⇒ dynamically generated cutoff is \propto Hubble radius

Cosmological evolution in the quantum regime (pre-nucleosynthesis)

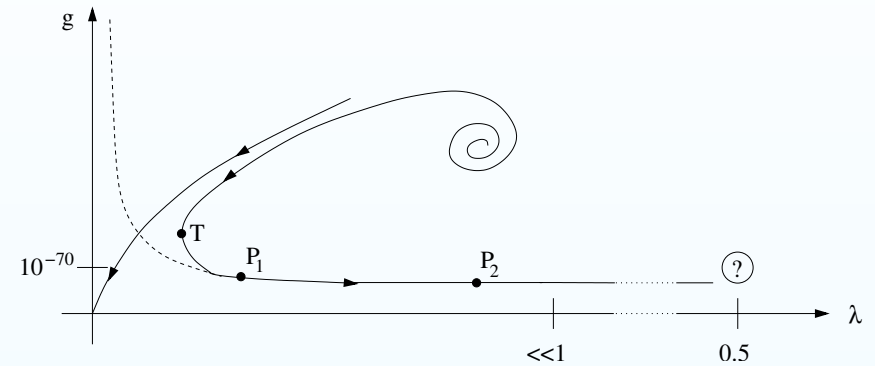
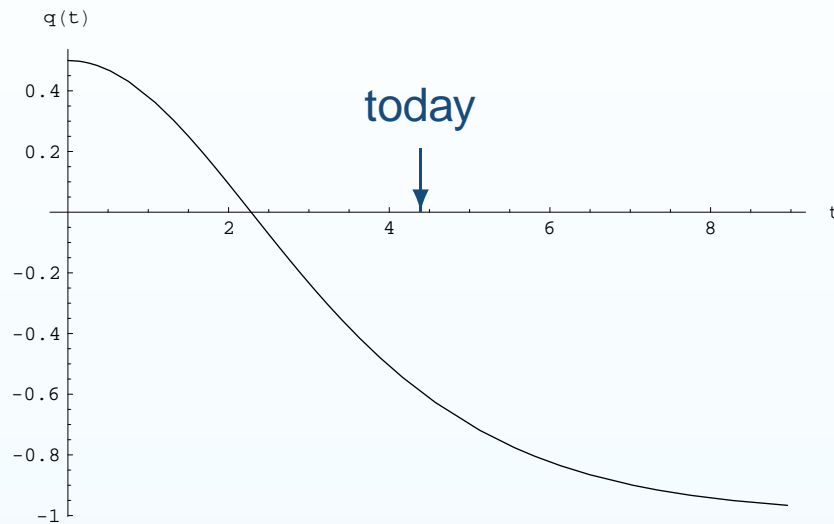
assume equation of state: $w = 1/3$



- evolution is (almost) universal
- Non-Gaussian fixed point: origin of the universe
- linear regime: oscillatory inflation
one e-fold expansion during last acceleration cycle
- turning point (GFP): radiation domination $\Omega_M \simeq 1$

Cosmological evolution in the classical regime (nucleosynthesis onwards)

assume equation of state: $w = 0$



- $\Omega_M = 1$ at beginning of classical era
- crossover at cosmological time scales
- evolution stops before entering in regime where RG effects become strong

Summary and Outlook

dynamical cutoff identification:

- generates physical identification $k(t) \propto H(t)$

characteristics of RG improved cosmologies:

- universal initial conditions at the Big Bang $\Omega_M = \Omega_\Lambda = 1/2$
- period of oscillatory inflation
- matter domination at the beginning of the classical regime $\Omega_M \simeq 1, \Omega_\Lambda \simeq 0$
- cosmological evolution ends before strong RG effects set in

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Outlook:

- Continuous inflationary phase? (Work in progress)