2+1D Loop Quantum Gravity on the Edge (arXiv:1811.04360)

Barak Shoshany
Perimeter Institute
Waterloo, Ontario, Canada

In collaboration with
Laurent Freidel (Perimeter Institute)
Florian Girelli (University of Waterloo)

International Loop Quantum Gravity Seminar
January 22, 2019

Motivation and background

- We are interested in the discretization of gauge theories, and gravity in particular, such that gauge invariance is preserved. Doing this for gravity is a major unsolved problem.
- Our work is based on the procedure used in [Freidel, Geiller, Ziprick 1110.4833] and later developed in [Dupuis, Freidel, Girelli 1701.02439], which separates discretization into two steps:
  - *Subdivision*, or decomposition into subsystems.
  - *Truncation*, or coarse graining of the subsystems.
- We combine this with the formalism described in [Donnelly, Freidel 1601.04744] and [Geiller 1703.04748] (see also [Rovelli 1308.5599]), where the presence of boundaries in gauge theories introduces:
  - New degrees of freedom (*edge modes*), which may be used to *dress* observables and render them gauge invariant.
  - New boundary symmetries, which transform the edge modes and control the gluing map between subsystems.

Summary of results

- In this work [Freidel, Girelli, Shoshany 1811.04360] we rigorously discretize 2+1 gravity (with zero cosmological constant), while keeping track of the boundaries of the subsystems and the corresponding edge modes, symmetries and charges.
- As will be shown in detail below, we find relations between:
  - The continuous phase space of piecewise flat and torsionless geometries,
  - The discrete spin network phase space of loop gravity,
  - The phase space of a collection of particle-like curvature and torsion defects.

Flat 2+1D gravity ($\Lambda = 0$)

- Let G be a Lie group and $\mathfrak{g}$ its Lie algebra. A possible choice is $G = \mathrm{SU}(2)$ and $\mathfrak{g} = \mathfrak{su}(2)$. **Bold font** denotes algebra elements.
- Let M be a 2+1-dimensional manifold and let Σ be a 2-dimensional spatial manifold such that $M = \Sigma \times \mathbb{R}$.
- We define the following geometric variables:
  - $\mathbf{A}$: $\mathfrak{g}$-valued connection 1-form,
  - $\mathbf{E}$: $\mathfrak{g}^*$-valued frame field 1-form,
  - $\mathbf{F} \equiv \mathrm{d}\mathbf{A} + \frac{1}{2}[\mathbf{A}, \mathbf{A}]$: $\mathfrak{g}$-valued curvature 2-form,
  - $\mathbf{T} \equiv \mathrm{d}_{\mathbf{A}}\mathbf{E} \equiv \mathrm{d}\mathbf{E} + [\mathbf{A}, \mathbf{E}]$: $\mathfrak{g}^*$-valued torsion 2-form.
- Let the dot product denote the Killing form: $\mathbf{E} \cdot \mathbf{F} \equiv E^i \wedge F_i$. The action and equations of motion are:

$$S = \int_M \mathbf{E} \cdot \mathbf{F}, \qquad \mathbf{F} = 0, \qquad \mathbf{T} = 0.$$

- The symplectic potential is:

$$\Theta = -\int_\Sigma \mathbf{E} \cdot \delta\mathbf{A}.$$

Edge modes

- Consider a generalized Euclidean gauge transformation

$$\mathbf{A} \mapsto g^{-1}\mathbf{A}g + g^{-1}\mathrm{d}g, \qquad \mathbf{E} \mapsto g^{-1}\left(\mathbf{E} + \mathrm{d}_{\mathbf{A}}\mathbf{z}\right)g,$$

  where the *rotation* g is a G-valued 0-form, and the *translation* $\mathbf{z}$ is a $\mathfrak{g}^*$-valued 0-form.
- On-shell, S is invariant, but Θ transforms with a boundary term:

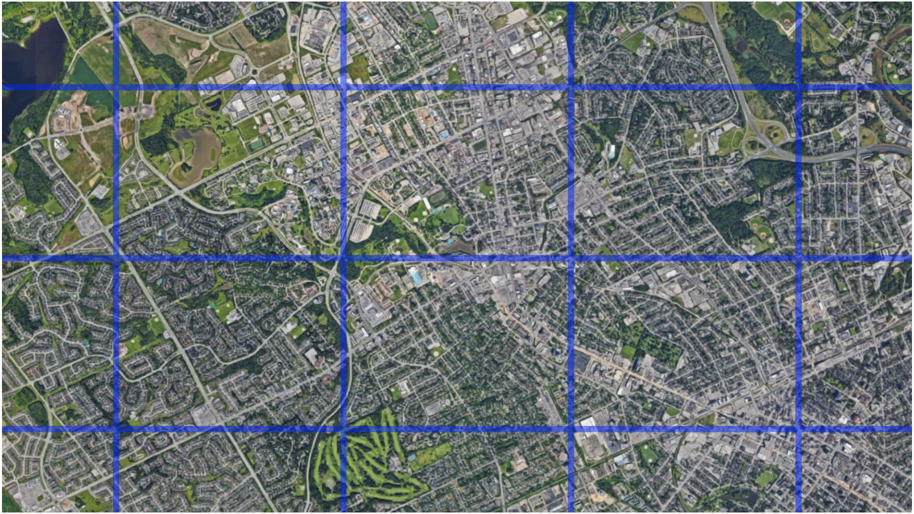
$$\Theta \mapsto \Theta - \int_{\partial\Sigma} \left(\mathbf{z} \cdot \delta\mathbf{A} - \left(\mathbf{E} + \mathrm{d}_{\mathbf{A}}\mathbf{z}\right) \cdot \mathbf{\Delta}g\right),$$

  where $\mathbf{\Delta}g \equiv \delta g g^{-1}$ is a shorthand for the Maurer-Cartan form on field space.
- $\mathbf{A}$ and $\mathbf{E}$ can be *dressed* by adding new boundary degrees of freedom (*edge modes*) in order to make them (and thus Θ) invariant. A new boundary symmetry algebra is introduced, which transforms the edge modes.

A map of Waterloo

Represents the spatial manifold Σ.

Decomposition into subsystems



The blue graph represents the spin network Γ.

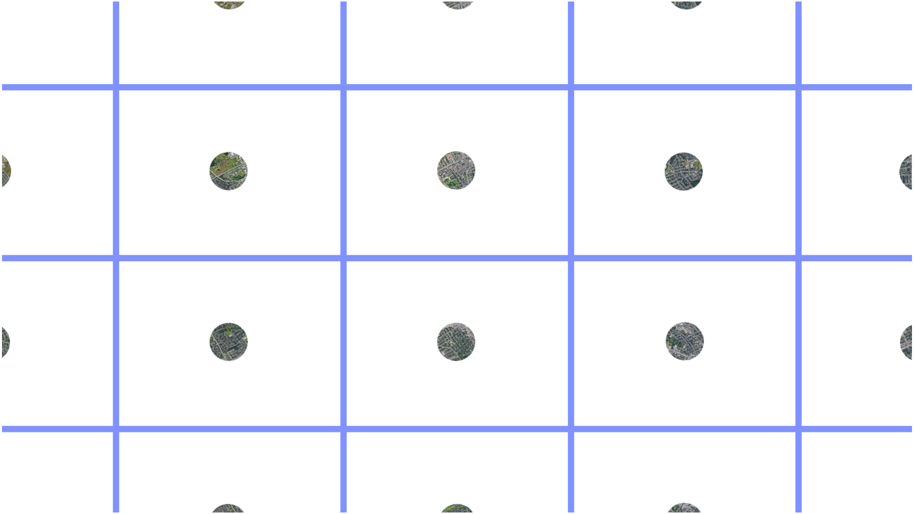
Truncating / coarse graining

- We would like to keep only the relevant gauge-invariant observables from each subsystem.
- Assume that the only way to probe the geometry (curvature and torsion) is using holonomies along loops of the spin network. Then, as far as observables are concerned, the following are equivalent:

Curvature and torsion defects

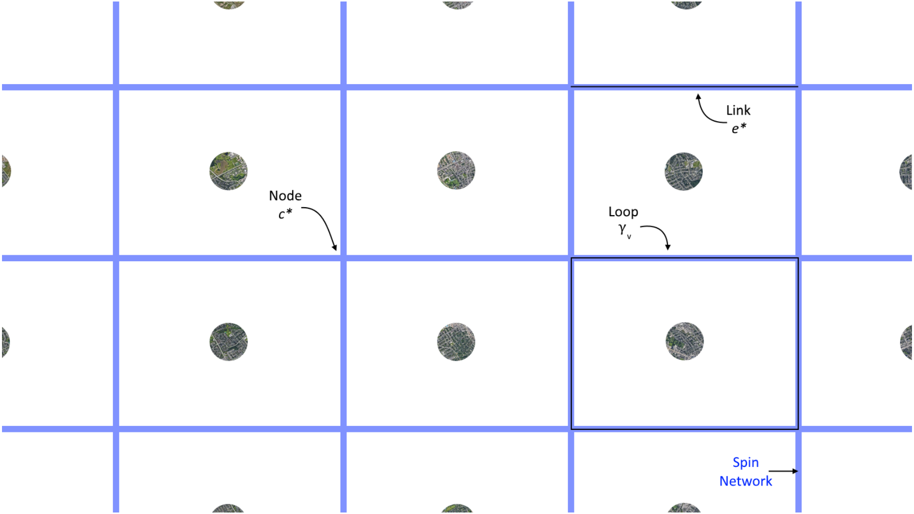
- More precisely:
  - Let γ_v be a loop in the spin network.
  - Let $\mathfrak{h}, \mathfrak{h}^*$ be the Cartan subalgebras of $\mathfrak{g}, \mathfrak{g}^*$.
  - Define Cartan elements $\mathbf{M}_v \in \mathfrak{h}$ and $\mathbf{S}_v \in \mathfrak{h}^*$. They encode all the gauge-invariant information we can obtain about the geometry inside the loop.
  - We will see below that $\mathbf{M}_v$ and $\mathbf{S}_v$ are the charges of the edge mode symmetries.
  - $\mathbf{M}_v$ and $\mathbf{S}_v$ have the same value whether the geometry inside the loop is continuous, concentrated only at one point, or anything in between.

The truncated geometry



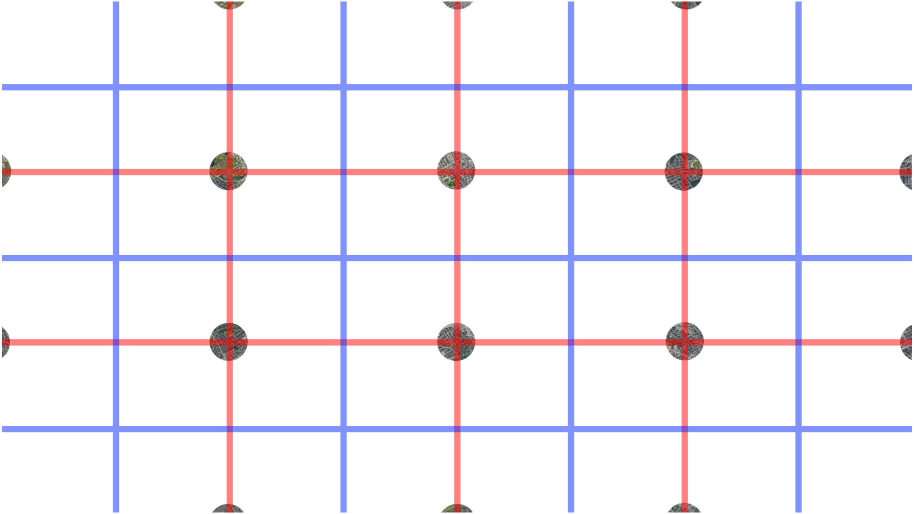
The disks represent $\mathbf{M}_v, \mathbf{S}_v$ for each loop.

Map legend



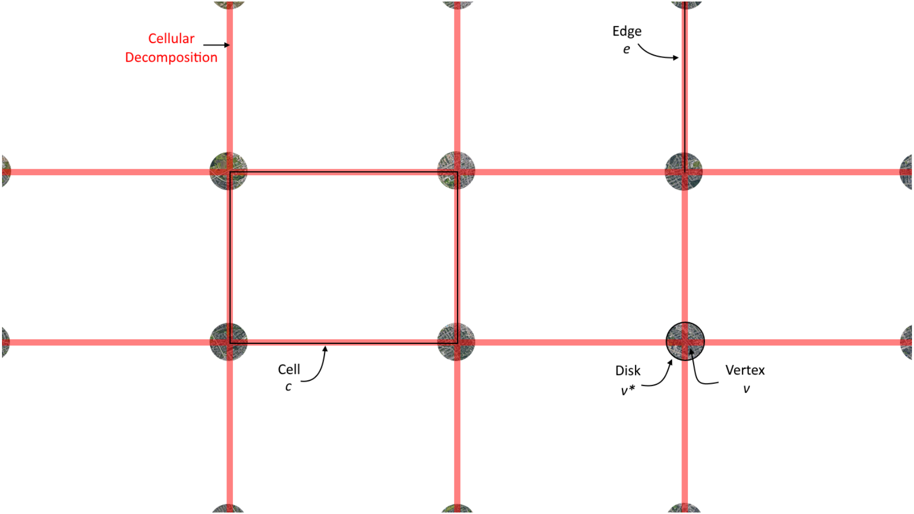
The spin network Γ is composed of loops γ_v, links e^* and nodes c^*.

The dual cellular decomposition Δ



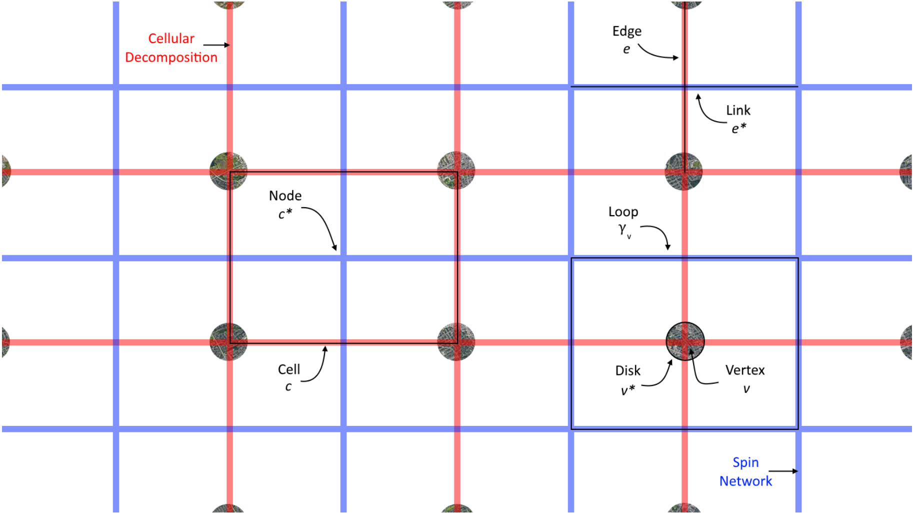
To each blue link e^* we assign a red *dual edge* e.

Dual legend



Δ is composed of cells c, edges e and vertices v.

Everything together



The spin network Γ and the cellular decomposition Δ.

Summary

- We have a spin network graph Γ dual to a cellular decomposition Δ such that:
  - Each link e^* of Γ is dual to an edge e of Δ,
  - Each node c^* of Γ is dual to a cell c of Δ,
  - Each vertex v of Δ is regularized by an infinitesimal disk v^* and dual to a loop γ_v.
- The cells describe a *piecewise flat and torsionless geometry*.
  - The geometry is completely flat and torsionless everywhere on the interior of each cell.
  - The curvature and torsion are concentrated at the vertices v, are distributional:

$$\mathbf{F} \propto \delta\left(v\right), \qquad \mathbf{T} \propto \delta\left(v\right),$$

    and depend on the Cartan elements $\mathbf{M}_v$, $\mathbf{S}_v$ as will be shown below.

The holonomy variables inside the cells

- Inside each cell c, we define:
  - $h_c(x)$: a dressed rotational holonomy from the node c^* (at the center of c) to a point $x \in c$.
  - $\mathbf{y}_c(x)$: a dressed translational holonomy from the node c^* (at the center of c) to a point $x \in c$.
- Then $\mathbf{A}$ and $\mathbf{E}$ are given inside c by:

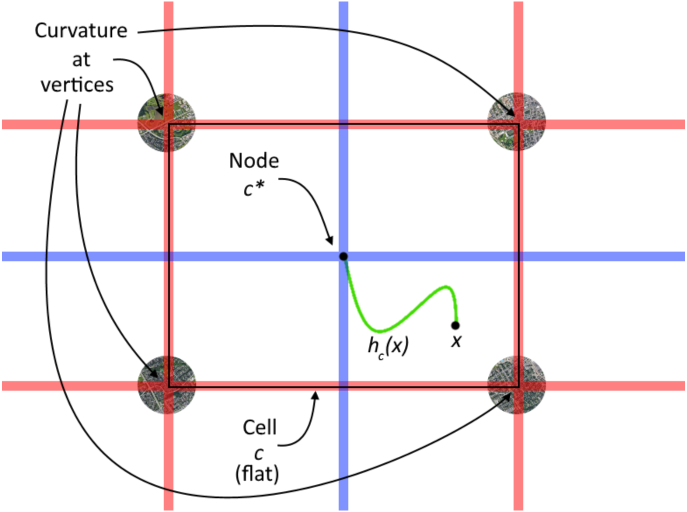
$$\mathbf{A}\Big|_c = h_c^{-1}\mathrm{d}h_c, \qquad \mathbf{E}\Big|_c = h_c^{-1}\mathrm{d}\mathbf{y}_c h_c.$$

  - By construction, $\mathbf{F} = \mathbf{T} = 0$ inside c.
  - The relation for $\mathbf{A}$ in terms of h_c may be inverted to find:

$$h_c(x) = h_c(c^*)\,\overrightarrow{\exp}\int_{c^*}^{x}\mathbf{A}.$$

    $h_c(c^*)$, the value of the dressed holonomy h_c at the node c^*, conveys *extra information* that cannot be obtained from the connection alone. Note that in general $h_c(c^*) \neq 1$.
  - Similarly, in general $\mathbf{y}_c(c^*) \neq 0$.

The holonomy variables inside the cells



The holonomy variables inside the disks

- Inside each disk v^*, we analogously define:
  - $h_v(x)$: a dressed rotational holonomy from the vertex v (at the center of v^*) to a point $x \in v^*$.
  - $\mathbf{y}_v(x)$: a dressed translational holonomy from the vertex v (at the center of v^*) to a point $x \in v^*$.
  - In general, $h_v(v) \neq 1$ and $\mathbf{y}_v(v) \neq 0$.
- Then $\mathbf{A}$ and $\mathbf{E}$ are given inside v^* by:

$$\mathbf{A}\Big|_{v^*} = h_v^{-1}\mathrm{d}h_v + h_v^{-1}\mathbf{M}_v h_v \,\mathrm{d}\phi_v,$$

$$\mathbf{E}\Big|_{v^*} = h_v^{-1}\mathrm{d}\mathbf{y}_v h_v + h_v^{-1}\left(\mathbf{S}_v + [\mathbf{M}_v, \mathbf{y}_v]\right) h_v \,\mathrm{d}\phi_v,$$

  where ϕ_v is the angular coordinate on the disk.
  - By construction, $\mathbf{F} = \mathbf{p}_v \,\delta(v)$ and $\mathbf{T} = \mathbf{j}_v \,\delta(v)$ inside v^*, where

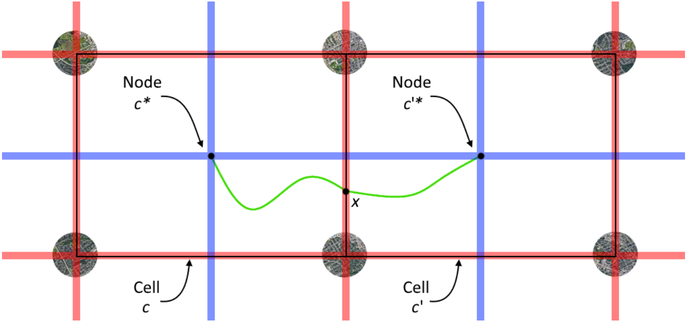
$$\mathbf{p}_v \equiv h_v^{-1}\mathbf{M}_v h_v, \qquad \mathbf{j}_v \equiv h_v^{-1}\left(\mathbf{S}_v + [\mathbf{M}_v, \mathbf{y}_v]\right) h_v.$$

  - This means that $\mathbf{F} = \mathbf{T} = 0$ everywhere inside v^*. Note that v^* is a *punctured* disk, so v itself is not inside v^*, it is on its boundary!

The continuity conditions on the cells

- **A** and **E** must be continuous when moving from a cell c to an adjacent cell c'. This is obtained by introducing
  - $h_{cc'} = h_{c'c}^{-1}$: a constant G element,
  - $\mathbf{y}_c^{c'} = -h_{cc'}\mathbf{y}_{c'}^{c}h_{c'c}$: a constant $\mathfrak{g}^*$ element.
- Then we have, for x on the boundary between the cells:

$$h_{c'}\left(x\right) = h_{c'c}h_c\left(x\right), \qquad \mathbf{y}_{c'}\left(x\right) = h_{c'c}\left(\mathbf{y}_c\left(x\right) - \mathbf{y}_c^{c'}\right)h_{cc'}.$$

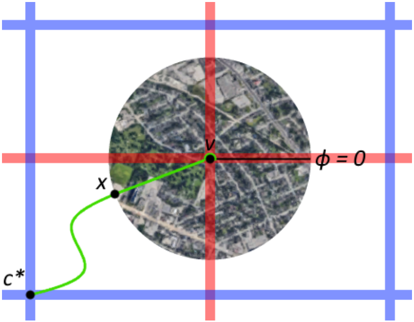


The continuity conditions on the disks

- Similarly, for a cell c and a disk v^* we introduce:
  - $h_{vc} = h_{cv}^{-1}$: a constant G element,
  - $\mathbf{y}_v^c = -h_{vc}\mathbf{y}_c^v h_{cv}$: a constant $\mathfrak{g}^*$ element.
- Then for x on the boundary between the cell and the disk:

$$h_c(x) = h_{cv}\,\mathrm{e}^{\mathbf{M}_v \phi_v(x)}\,h_v(x)\,,$$

$$\mathbf{y}_c(x) = h_{cv}\left(\mathrm{e}^{\mathbf{M}_v \phi_v(x)}\left(\mathbf{y}_v(x) + \mathbf{S}_v \phi_v(x)\right)\mathrm{e}^{-\mathbf{M}_v \phi_v(x)} - \mathbf{y}_v^c\right)h_{vc}.$$



The right action

- The dressed holonomies $h_c, \mathbf{y}_c, h_v, \mathbf{y}_v$ may be acted upon by two commuting actions.
- The right action, with 0-form parameters $g(x)$ and $\mathbf{z}(x)$, is:

$$h_c \mapsto h_c g, \qquad \mathbf{y}_c \mapsto \mathbf{y}_c + h_c \mathbf{z} h_c^{-1},$$

$$h_v \mapsto h_v g, \qquad \mathbf{y}_v \mapsto \mathbf{y}_v + h_v \mathbf{z} h_v^{-1}.$$

It imposes the gauge transformation we have seen before:

$$\mathbf{A} \mapsto g^{-1} \mathbf{A} g + g^{-1} \mathrm{d} g, \qquad \mathbf{E} \mapsto g^{-1} \left(\mathbf{E} + \mathrm{d}_{\mathbf{A}} \mathbf{z} \right) g.$$

The left action

- Let H be the Cartan subgroup of G and $\mathfrak{h}$ the Cartan subalgebra. Define constant rotation parameters $g_c \in G$, $g_v \in H$ and constant translation parameters $\mathbf{z}_c \in \mathfrak{g}^*$, $\mathbf{z}_v \in \mathfrak{h}^*$ for each cell c and disk v^*.
- The left action is:

$$h_c \mapsto g_c h_c, \qquad \mathbf{y}_c \mapsto \mathbf{z}_c + g_c \mathbf{y}_c g_c^{-1},$$

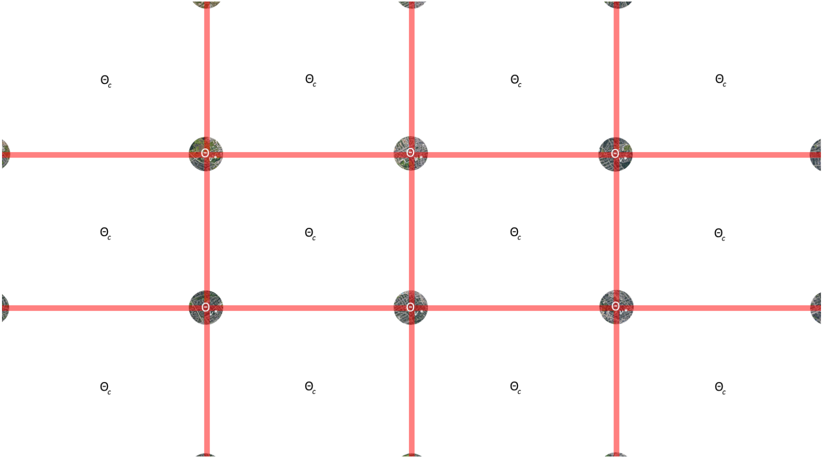
$$h_v \mapsto g_v h_v, \qquad \mathbf{y}_v \mapsto \mathbf{z}_v + g_v \mathbf{y}_v g_v^{-1}.$$

  It leaves $\mathbf{A}$ and $\mathbf{E}$ invariant. It represents a new boundary symmetry for the edge mode degrees of freedom. On the vertices, it is generated by the charges $\mathbf{M}_v$ and $\mathbf{S}_v$. (Note that $g_c, \mathbf{z}_c$ commute with $\mathbf{M}_v, \mathbf{S}_v$.)
- Since the left and right action commute, the charges $\mathbf{M}_v$ and $\mathbf{S}_v$ are gauge-invariant.

The discretized symplectic potential

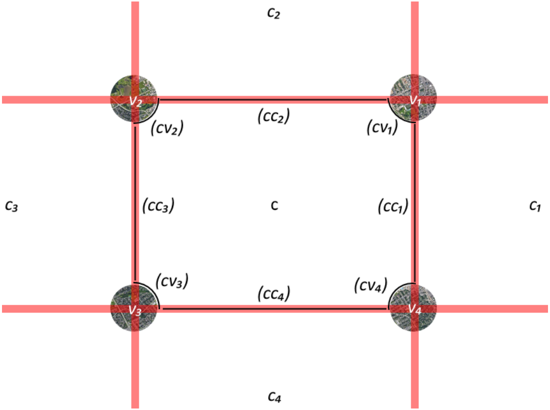
- We decompose Θ into a sum over cells and disks:

$$\Theta = \sum_c \Theta_c + \sum_v \Theta_{v^*}, \qquad \Theta_c \equiv -\int_c \mathbf{E} \cdot \delta \mathbf{A}, \qquad \Theta_{v^*} \equiv -\int_{v^*} \mathbf{E} \cdot \delta \mathbf{A}.$$



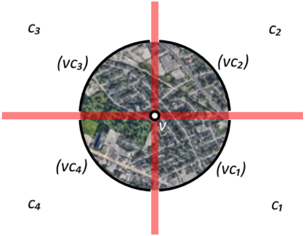
The boundary of the cells

- The cell c does not contain the disks surrounding it. Thus the boundary ∂c of each cell is composed of:
  - Edges, labeled (cc') where c' is an adjacent cell. Note that $(cc') = (c'c)^{-1}$.
  - Arcs, labeled (cv) where v is an adjacent disk.



The boundary of the disks

- Each disk v^* is punctured and so does not contain the vertex v. Thus the boundary ∂v^* is composed of:
  - The vertex v,
  - Arcs, labeled (vc) where c is an adjacent cell. Note that $(cv) = (vc)^{-1}$.



- In conclusion, we may rearrange the sums as:

$$\Theta = \sum_c \Theta_c + \sum_v \Theta_{v^*} \underset{\text{Stokes'}}{=} \sum_{(cc')} \Theta_{cc'} + \sum_{(vc)} \Theta_{vc} + \sum_v \Theta_v.$$

The edge term

- On each cell, we plug in $\mathbf{A} = h_c^{-1}\mathrm{d}h_c$ and $\mathbf{E} = h_c^{-1}\mathrm{d}\mathbf{y}_c h_c$ and get:

$$\Theta_c = \int_{\partial c} \mathrm{d}\mathbf{y}_c \cdot \mathbf{\Delta} h_c,$$

  where $\mathbf{\Delta} h_c \equiv \delta h_c h_c^{-1}$ is a shorthand for the Maurer-Cartan form on field space.
- After rearranging the sum, each edge (cc') has two contributions, one from each of the two cells c, c' sharing the edge, with opposite orientation since $(cc') = (c'c)^{-1}$:

$$\Theta_{cc'} = \int_{(cc')} \left(\mathrm{d}\mathbf{y}_c \cdot \mathbf{\Delta} h_c - \mathrm{d}\mathbf{y}_{c'} \cdot \mathbf{\Delta} h_{c'}\right).$$

- Using the continuity conditions, this simplifies to

$$\Theta_{cc'} = \mathbf{\Delta} h_c^{c'} \cdot \int_{(cc')} \mathrm{d}\mathbf{y}_c,$$

  where $\mathbf{\Delta} h_c^{c'} \equiv \delta h_{cc'} h_{c'c}$ is a constant.

The holonomy-flux algebra

- Defining the flux along the edge,

$$\mathbf{X}_c^{c'} \equiv \int_{(cc')} \mathrm{d}\mathbf{y}_c,$$

we get

$$\Theta_{cc'} = \mathbf{\Delta} h_c^{c'} \cdot \mathbf{X}_c^{c'}.$$

- We see that we have obtained the familiar spin network phase space T^*G,

$$\Theta_{cc'} = \mathbf{\Delta} h_c^{c'} \cdot \mathbf{X}_c^{c'} \equiv \mathrm{Tr}\left(\delta h_{e^*} h_{e^*}^{-1} \mathbf{X}_e\right),$$

where:

  - $h_{e^*} \equiv h_{cc'}$ is the holonomy along the link e^* connecting the node c^* to the node c'^*,
  - $\mathbf{X}_e \equiv \mathbf{X}_c^{c'}$ is the flux along the edge e (dual to the link e^*) which is the boundary between the cells c and c'.

The arc term

- Similarly, on the arcs we obtain

$$\Theta_{vc} = \mathbf{\Delta} h_v^c \cdot \mathbf{X}_v^c,$$

where

$$\mathbf{\Delta} h_v^c \equiv \delta h_{vc} h_{cv}, \qquad \mathbf{X}_v^c \equiv \int_{(vc)} \mathrm{d} \left(\mathrm{e}^{\mathbf{M}_v \phi_v} \, \mathbf{y}_v \, \mathrm{e}^{-\mathbf{M}_v \phi_v} + \mathbf{S}_v \phi_v \right).$$

- This is again the spin network phase space T^*G, where now
  - h_{vc} is the holonomy from the vertex v to the node c^*,
  - $\mathbf{X}_v^c$ is the flux along the arc (vc).

The vertex term

- On the vertices, we obtain after some calculations
$$\Theta_v = \mathbf{X}_v \cdot \delta \mathbf{M}_v - \left(\mathbf{S}_v + [\mathbf{M}_v, \mathbf{X}_v]\right) \cdot \mathbf{\Delta} h_v(v),$$
where the "vertex flux" $\mathbf{X}_v$ depends on $\mathbf{y}_v(v)$.
- If we define the "position" $\mathbf{q}_v$ and "momentum" $\mathbf{p}_v$:
$$\mathbf{q}_v \equiv h_v^{-1} \mathbf{X}_v h_v \in \mathfrak{g}, \qquad \mathbf{p}_v \equiv h_v^{-1} \mathbf{M}_v h_v \in \mathfrak{g}^*,$$
we can write
$$\Theta_v = \mathbf{q}_v \cdot \delta \mathbf{p}_v - \mathbf{S}_v \cdot \mathbf{\Delta} h_v(v).$$
This may be interpreted as the phase space of a relativistic particle with mass $\mathbf{M}_v$ and spin $\mathbf{S}_v$. The phase space variables contain the edge modes $h_v(v)$ and $\mathbf{y}_v(v)$.
- Since each edge is shared by two cells, and each arc is shared by a cell and a disk, the edge mode contributions from each side of the edge or arc cancel. However, the edge modes on the vertices (the corners of the cells) have nothing to cancel with.

The full discretized potential

- In conclusion, the full discretized symplectic potential is:

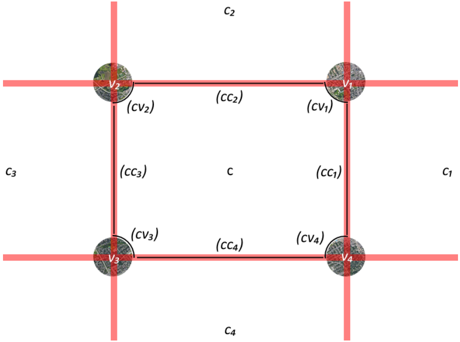
$$\Theta = \sum_{(cc')} \mathbf{\Delta} h_c^{c'} \cdot \mathbf{X}_c^{c'} - \sum_{(vc)} \mathbf{\Delta} h_c^{v} \cdot \mathbf{X}_c^{v} + \\ + \sum_{v} \left(\mathbf{X}_v \cdot \delta \mathbf{M}_v - \left(\mathbf{S}_v + \left[\mathbf{M}_v, \mathbf{X}_v \right] \right) \cdot \mathbf{\Delta} h_v \left(v \right) \right).$$

  - First term: Spin network phase space T^*G per link (cc').
  - Second term: Spin network phase space T^*G per arc (vc).
  - Third term: Particle-like defects. These degrees of freedom are the edge modes.

The Gauss constraint

- In the continuum, the Gauss constraint imposes $\mathbf{T} = 0$ everywhere except the vertices.
- In the discrete theory, we have one constraint on each cell and disk. On each cell c:

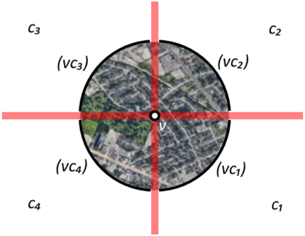
$$\sum_{c' \ni c} \mathbf{X}_c^{c'} = \sum_{v \ni c} \mathbf{X}_c^{v}.$$



The Gauss constraint

- On each disk v^*:

$$\sum_{c\in v}\mathbf{X}_v^c=\mathbf{S}_v+\mathrm{e}^{\mathbf{M}_v}\,\mathbf{y}_v\,(v)\,\mathrm{e}^{-\mathbf{M}_v}\,-\mathbf{y}_v\,(v)\,.$$



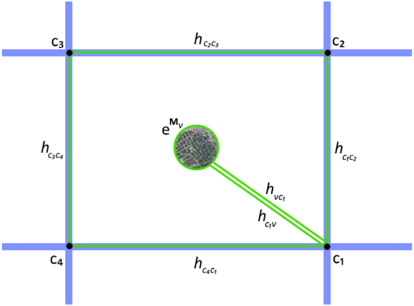
- The Gauss constraint generates the rotational part of the symmetry transformations corresponding to the left action. Recall that this is the edge mode symmetry which leaves **A** and **E** invariant.
- The constraint is satisfied identically in our construction.

The curvature constraint

- In the continuum, the curvature constraint imposes $\mathbf{F} = 0$ everywhere except the vertices.
- In the discrete theory, we have for each vertex v:

$$h_{c_1c_2} \cdots h_{c_Nc_1} = h_{c_1v}\, \mathrm{e}^{\mathbf{M}_v}\, h_{vc_1}.$$

- It generates the translational part of the symmetry corresponding to the left action, and is also satisfied identically.



Shrinking the disks

- Using the curvature constraint, one can choose a particular transformation parameter which considerably simplifies things:
  - $\mathbf{X}_v$ becomes simply the edge mode $\mathbf{y}_v\,(v)$,
  - The vertex Gauss constraint becomes simply $\sum_{c\ni v}\mathbf{X}_v^c = \mathbf{S}_v$.
- If we now assume that $\mathbf{S}_v = 0$ for all v, that is, there are no torsion defects, then we may shrink the disks to points.
  - The length of all arcs becomes zero, and we can take $\mathbf{X}_v^c = 0$ for all v and c. Then the cell Gauss constraint simplifies to $\sum_{c'\ni c}\mathbf{X}_c^{c'} = 0$, which is the correct constraint for loop gravity. The vertex potential becomes that of a spinless particle: $\Theta_v = \mathbf{q}_v \cdot \delta\mathbf{p}_v$.
  - Therefore, the torsionless case corresponds to the usual loop gravity phase space, along with a collection of curvature defects:

$$\Theta = \sum_{(cc')} \mathbf{\Delta} h_c^{c'} \cdot \mathbf{X}_c^{c'} + \sum_v \mathbf{q}_v \cdot \delta\mathbf{p}_v.$$

Freezing the edge modes

- We can now also "freeze" the edge modes.
  - Recall that the "position" $\mathbf{q}_v$ and "momentum" $\mathbf{p}_v$ are:

$$\mathbf{q}_v = h_v^{-1}(v)\,\mathbf{y}_v(v)\,h_v(v)\,, \qquad \mathbf{p}_v = h_v^{-1}(v)\,\mathbf{M}_v h_v(v)\,.$$

  - We interpret $\mathbf{y}_v(v)$ as the position in the rest frame and $\mathbf{M}_v$ as the rest mass, while $h_v(v)$ acts like a Lorentz transformation.
  - By setting $\mathbf{y}_v(v) = 0$ and $h_v(v) = 1$, we are "undressing" the holonomies. Then $\mathbf{q}_v = 0$ and $\mathbf{\Delta} h_v(v) = 0$, so

$$\Theta_v = \mathbf{q}_v \cdot \delta \mathbf{p}_v - \mathbf{S}_v \cdot \mathbf{\Delta} h_v(v) = 0.$$

    The vertex (particle-like) potential has vanished.
  - In conclusion, the loop gravity phase space is a special case of the full phase space, where the geometry is torsionless and the edge modes are frozen:

$$\Theta = \sum_{(cc')} \mathbf{\Delta} h_c^{c'} \cdot \mathbf{X}_c^{c'}, \qquad \sum_{c' \ni c} \mathbf{X}_c^{c'} = 0, \qquad \prod_i h_{c_i c_{i+1}} = \mathrm{e}^{\mathbf{M}_v}\,.$$

Shrinking the cells

- Instead of shrinking the disks, we may shrink the cells (or equivalently, expand the disks).
  - The boundary of each disk is identified with the spin network loop that contained it.
  - The edges (cc') now have zero length, and thus the fluxes $\mathbf{X}_c^{c'} \equiv \int_{(cc')} \mathrm{d}\mathbf{y}_c$ vanish.
  - Therefore, the spin network symplectic potential $\Theta_{cc'} = \mathbf{\Delta} h_c^{c'} \cdot \mathbf{X}_c^{c'}$ vanishes, and we are left with

$$\Theta = -\sum_{(vc)} \mathbf{\Delta} h_c^v \cdot \mathbf{X}_c^v + \sum_v \left(\mathbf{q}_v \cdot \delta \mathbf{p}_v - \mathbf{S}_v \cdot \mathbf{\Delta} h_v \right).$$

  - If we furthermore assume $\mathbf{S}_v = 0$, then we can again take $\mathbf{X}_c^v = 0$ for all v and c, and get

$$\Theta = \sum_v \mathbf{q}_v \cdot \delta \mathbf{p}_v.$$

  - In conclusion, in the spinless case, the phase space may be reduced to a collection of particle-like curvature defects.

Summary

- We have found a relation between:
  - The continuous phase space of piecewise flat and torsionless geometries,
  - The discrete spin network phase space of loop gravity coupled to a collection of particle-like curvature and torsion defects.
- In the torsionless case, we further found a relation between:
  - The loop gravity phase space alone,
  - The phase space of particle-like curvature defects.

Future Plans

- The defect picture in our formalism suggests a new perspective for addressing the problem of the continuum limit, possibly related to that of [Delcamp, Dittrich, Riello 1607.08881].
- In 2+1D, our formalism can be extended to include a “dual polarization”, in which the holonomies and fluxes switch places. (Paper to appear soon.)
- The most challenging task is to generalize our formalism to the physically relevant case of 3+1D.
  - The cells will now be 3-dimensional, and the spin network loops will encircle the edges of the cells.
  - This is work in progress, but we have already been able to show that piecewise flat and torsionless geometries in 3+1D satisfy the Gauss, vector and scalar constraints of loop quantum gravity.
- Thank you for listening!