Physics of Bianchi models in Loop Quantum Cosmology

Based on arXiv:1109.6636 (with Brajesh Gupt);

arXiv:1112.6391, and work in progress

Parampreet Singh

Louisiana State University

ILQGS

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Introduction

- In recent years, quantization of isotropic models in LQC, has revealed very rich physics at the Planck scale. Rigorous understanding reached using extensive analytical and numerical methods and phenomenological models. (Important contributions by many colleagues and groups around the world.)
- The classical big bang singularity is replaced by a big bounce in the Planck regime. Underlying quantum geometry, via non-local nature of field strength of the connection, removes the classical boundaries of spacetime. Examples of isotropic models where singularity resolution has been explicitely shown at the quantum level: massless scalar in spatially flat and curved spacetime, in presence of positive and negative cosmological constants, massive scalar etc.
- Effective spacetime description captures details of the underlying quantum theory to an excellent precision. In particular, for universes which bounce at scales greater than Planck volume. Many physical implications studied. Also used to study cosmological perturbations and inhomogeneous models.
- Using effective dynamics, it was recently proved that there exist no strong singularities in the spatially flat isotropic model (PS (09)). These include exotic singularities other than the big bang. However, there can exist events where curvature invariants diverge, as first shown by Cailleteau, Cardoso, Vandersloot, Wands (08). These were proved to be weak singularities, which can occur in certain dark energy scenarios. These results extend to spatially curved isotropic models (PS, Vidotto (10)).

Going beyond isotropy

Due to non-zero Weyl curvature, Bianchi models provide the simplest yet a very non-trivial test of the physics as understood in isotropic LQC. Very rich physics. Many important lessons can be potentially drawn, including for qualitative dynamics, BKL behavior, cosmological perturbations and generic resolution of singularities.

Quantization of Bianchi-I, Bianchi-II and Bianchi-IX models performed (Ashtekar, Wilson-Ewing (08-10)). Various subtleties in quantization addressed. Limitations of previous works overcome. What is the resulting physics of these models? (For Bianchi-I model, some implications studied by Chiou, Vandersloot (07); Maartens, Vandersloot (08); Calleteau, PS, Vandersloot (09); Artymowski, Lalak (11))

Some of the questions we will address in this talk:

- In isotropic models, bounces occur when energy density reaches a maximum value. What is the nature of bounces in Bianchi models? Note that in the isotropic models, due to zero Weyl curvature, for matter with a fixed equation of state, all curvature invariants can be written in terms of energy density. This is no longer true in Bianchi models.
- What are the properties of energy density, expansion and shear scalars? Are they bounded? Recall that these quantities determine the fate of geodesic extendability via Raychaudhuri equation in GR, and also of curvature components.
- What is the fate of strong singularities in Bianchi-I model?

Outline

- Nature of singularities – a very brief overview
- A brief re-cap of isotropic model (of needed results)
- Towards the resolution of strong singularities in Bianchi-I model. For matter with a vanishing anisotropic stress, i.e. $\rho = \rho(p_1 p_2 p_3)$, (no magnetic fields)
  - All known strong singularities are resolved
  - Weak singularities can occur
  - Examples: perfect fluid with equation of state $w > -1$.
- Physics of Bianchi-II and Bianchi-IX models
  - Bounds on energy density, expansion and shear scalars
  - Role of energy conditions in Bianchi-II model
  - Non-trivial role of inverse triad corrections for spatially compact topology
- Applications: massive scalar field in Bianchi models (and how to make pancakes from cigars).

Nature of singularities

Different types of Singularities

Classification of events where curvature invariants diverge in isotropic models. Can also occur in Bianchi models.

- Big Bang/Big Crunch: The scale factor vanishes in finite time causing the energy density blows up. Curvature invariants diverge. Occurs for all matter satisfying null energy condition (except Λ): $\rho + P \geq 0$.
- Type I singularities (Big Rip): The scale factor, energy density and pressure diverge at a finite time in future. The universe rips apart. Dominant energy condition $(\rho > |P|)$ is violated.
- Type II singularities (Sudden): Discovered by Barrow and co-workers (01-04). Occur at finite value of scale factor and a finite time. As the singularity is approached the energy density vanishes but pressure diverges, causing divergence in spacetime curvature. Generalization to anisotropic models by Barrow, Tsagas (04).
- Type III singularities (Big freeze): Singularity occurs at a finite value of scale factor and in finite time. Energy density, pressure and curvature invariants diverge.

Strength of Singularities

Not all singularities where curvature invariants diverge, necessarily signal the breakdown of laws of physics. It is important to understand whether these are strong or weak. Strong singularities: Big bang/big crunch, Big rip; Weak singularities: Sudden singularities, shell-crossing singularities.

To determine the stength of singularities, one uses necessary and sufficient conditions by Clarke and Królak (1985) (based on the seminal work of Tipler (77) and Królak (80's)). These involve integrals of Ricci and Weyl curvature tensors over geodesics.
For a spacetime, admitting an incomplete, inextendible null geodesic parameterized by affine parameter τ, a singularity at $\tau = \tau_o$ is Królak strong, if and only if

$$\int_0^{\tau} d\tau' R_{\alpha\beta} u^{\alpha} u^{\beta}$$

or

$$\int_0^{\tau} d\tau' (\int_o^{\tau''} d\tau'' |C_{\alpha\beta\mu\nu} u^{\beta} u^{\nu}|)^2$$

diverges as $\tau \to \tau_o$.

A singularity can be strong by Królak's criteria, yet it can be weak according to Tipler's criteria. However, all singularities which are strong by Tipler's criteria are also strong by Królak's criteria.

Geometry of Singularities

Structure of singularities is much richer in the anisotropic models. Classified for various matter models in Bianchi spacetimes in 60's (Doroshkevich, Thorne, Ellis, Jacobs, McCallum, ...) using behavior of three scale factors.

- Point-like or isotropic singularities: All scale factors vanish as the singularity is reached. In general, requires matter with a stiff equation of state (or a massless scalar). $a_1, a_2, a_3 \to 0$.
- Barrel singularities: One of the scale factors approaches a finite value. Other two scale factors vanish. Example: $a_1 \to \text{finite}$, $a_2, a_3 \to 0$.
- Cigar singularities: One of the scale factors diverges. Other two approach zero. Example: $a_1 \to \infty$, $a_2, a_3 \to 0$
- Pancake singularities: One of the scale factors vanishes. Other two approach a finite value. Example: $a_1 \to 0$, $a_2, a_3 \to \text{finite}$

All of these singularities occur with a divergence in ρ, and expansion (θ) and shear (σ^2) scalars. At these singularities curvature invariants diverge and geodesic equations break down. These are strong singularities a la Tipler and Królak's conditions.

A brief re-cap of isotropic models

Hamiltonian Constraint (flat model)

$$C_{\rm grav} = - \int_{\mathcal{V}} d^3x \, N \, \varepsilon_{ijk} \, F^i_{ab} \, (E^{aj} E^{bk} / \sqrt{|\det E|})$$

Procedure: Express $C_{\rm grav}$ in terms of elementary variables and their Poisson brackets

– Classical identity of the phase space: (Thiemann (98))

$$\varepsilon_{ijk} (E^{aj} E^{bk} / \sqrt{|\det E|}) \longrightarrow {\rm Tr}(h_k^{(\mu)} \{h_k^{(\mu)-1}, V\} \tau_i)$$

(Peak tied to the fiducial volume of the cell introduced to define symplectic structure)

– Express field strength in terms of holonomies and quantize. Leads to quantum difference equation.

Two types of quantum modifications:

(i) Curvature modifications from field strength. Solely responsible for bounce at $\rho = \rho_{\rm crit} \sim 0.41 \rho_{\rm Pl}$.

(ii) Inverse triad corrections (also for the matter part). Not tied to any curvature scale in the flat model. Only physically meaningful for spatially compact models (eg. a closed universe).

Effective Dynamics

Based on gemoetrical formulation of quantum mechanics. Effective dynamics turns out to be an excellent approximation to quantum dynamics for various models. Extensive numerical simulations performed over last 6 years.
Examples: massless scalar, inflationary potential, closed universe, old quantization of Bianchi-I model.

Can be obtained using coherent state techniques under controlled approximations.

Derived for different matter sources: Massless scalar (Taveras (08)), Dust (Willis (04)) (Another approach developed by Bojowald et al)

Caveat for this talk: We assume the validity of effective dynamics in Bianchi models.

Effective Hamiltonian: In terms of symmetry reduced connection c and triad p, for lapse $N = |p|^{3/2} = V$

$$\mathcal{H}_{\rm eff} = -\frac{3V}{8\pi G\gamma^2}\,\frac{\sin^2(\bar\mu c)}{\bar\mu^2}|p|^{1/2} + \mathcal{H}_{\rm matt} V$$

where $\bar\mu = \lambda/\sqrt{|p|}$, with $\lambda^2 = 4\sqrt{3}\pi\gamma l_{\rm Pl}^2$.
Modified Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm crit}}\right) + O(\epsilon^2), \quad \rho_{\rm crit} = \frac{3}{8\pi G\gamma^2\lambda^2} = 0.41\rho_{\rm Pl}$$

- Phase of super-inflation when $\rho_{\rm max} < \rho < \rho_{\rm max}/2$. Many interesting implications, including for the probability of inflation (Ashtekar, Sloan (10); Corichi, Karami (10))
- Allowed range of energy density $0 \leq \rho \leq \rho_{\rm crit}$ implies that the expansion scalar of geodesics, $\theta = \dot V/V$ has an upper bound: $\theta_{\rm max} = 3/(2\gamma\lambda)$. This bound is responsible for completeness of geodesics in the effective spacetime PS(09).
- Resulting physics leads to uniqueness of underlying quantization. Only one loop or loop inspired quantization of isotropic cosmology has well defined UV and IR limits and a bounded θ (Corichi, PS (09)). This is the improved dynamics (Ashtekar, Pawlowski, PS (06))
- Spacetime curvature:

$$R = 6\left(H^2 + \frac{\ddot a}{a}\right) = 8\pi G\rho\left(1 - 3w + 2\frac{\rho}{\rho_{\rm crit}}(1+3w)\right)$$

It is bounded above in effective spacetime unless $w = P/\rho \to \pm\infty$. There can be situations where the curvature invariants diverge. (Sudden singularities).

However, these are weak singularities and geodesics can be extended beyond them. These are in general not resolved by quantum geometry. (Some exceptions found for spatially curved models (PS, Vidotto (10))
All strong singularities are generically resolved in isotropic flat LQC.

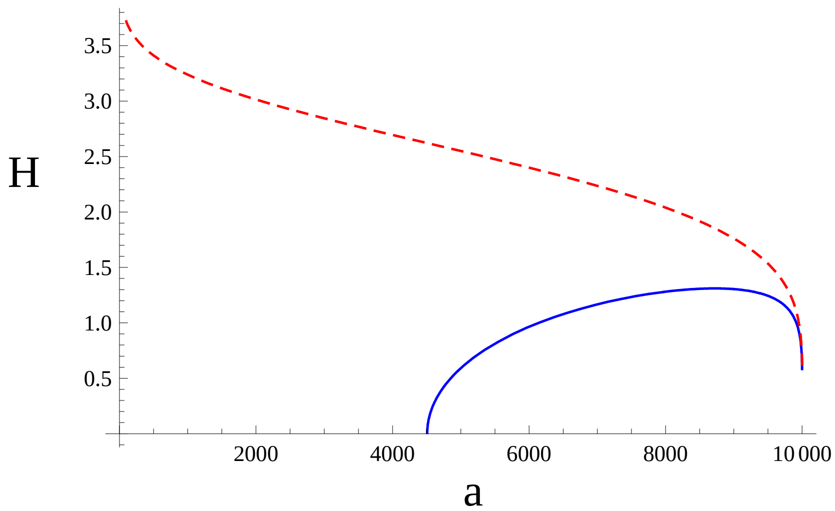
Example of a weak singularity in isotropic LQC

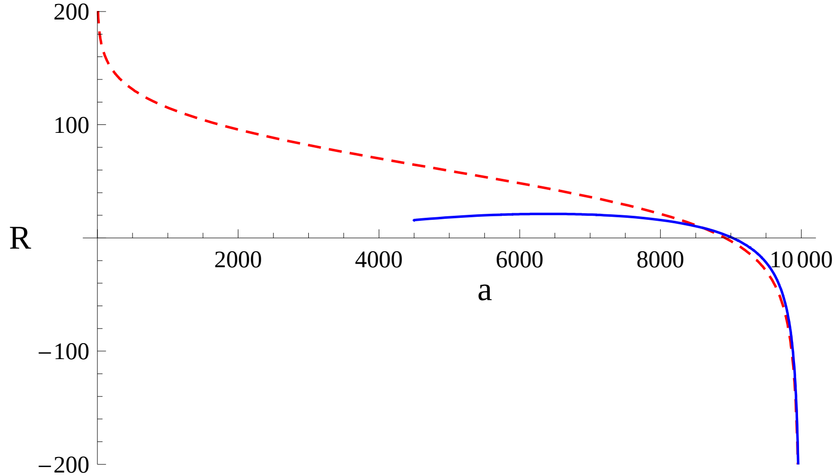
With an appropriate choice of equation of state, evolution can lead to a singularity where pressure diverges at a finite energy density.

$$P = -\rho - f(\rho), \quad f(\rho) = \frac{AB\rho^{2\alpha-1}}{A\rho^{\alpha-1} + B}$$

For $A/B < 0$, we obtain in classical theory: $(a \to \alpha_s, \rho \to \rho_s, |P| \to \infty)$.

The divergence in pressure causes curvature invariant to blow up at a finite scale factor. This is an example of a sudden singularity.





Strong singularities and Bianchi-I model

Bianchi-I model

Spacetime metric: $ds^2 = -dt^2 + a_1^2\, dx^2 + a_2^2 dy^2 + a_3^2 dz^2$

The scale factors are related to triads as:

$$p_1 \;=\; \varepsilon_1\, l_2\, l_3\, |a_2\, a_3|,\;\; p_2 \;=\; \varepsilon_2\, l_1\, l_3\, |a_1\, a_3|,\;\; p_3 \;=\; \varepsilon_3\, l_2 l_1 |a_1 a_2|$$

Directional Hubble rates:

$$H_i = \frac{\dot{a}_i}{a_i} = \frac{1}{2}\left(\frac{\dot{p}_j}{p_j} + \frac{\dot{p}_k}{p_k} - \frac{\dot{p}_i}{p_i}\right)$$

Expansion scalar: $\theta \;=\; (H_1 + H_2 + H_3)$

Shear scalar $\sigma^2 = \sigma^{\alpha\beta}\sigma_{\alpha\beta}$

$$\sigma^2 \;=\; \sum_{i=1}^{3} (H_i - \theta)^2 \;=\; \frac{1}{3}\left((H_1 - H_2)^2 + (H_2 - H_3)^2 + (H_3 - H_1)^2\right)$$

These scalars determine the fate of geodesic extendibility in spacetime (Raychaudhuri equation). $\theta' = -\theta^2/3 - \sigma^{\alpha\beta}\sigma_{\alpha\beta} - R_{\alpha\beta}u^\alpha u^\beta$

A divergence of these quantities, also implies associated divergence of spacetime curvature components.

Bianchi-I model in LQC

Quantization: Early works (Chiou, Szulc, Martin-Benito, Mena-Marugan, Pawlowski). Improved quantization: (Ashtekar, Wilson-Ewing (09)). Leads to a non-singular quantum difference equation. Isotropic LQC obtained by integrating out anisotropic degrees of freedom.

Effective Hamiltonian: Similar treatment as in the isotropic models. Captures underlying quantum geometric effects of the quantum constraint in a continuum effective spacetime description.

$$\mathcal{H}_{\rm eff} = -\frac{1}{8\pi G\gamma^2 V}\left(\frac{\sin(\bar\mu_1 c_1)}{\bar\mu_1}\frac{\sin(\bar\mu_2 c_2)}{\bar\mu_2} p_1 p_2 + \text{cyclic terms}\right) + \mathcal{H}_{\rm matt}$$

where

$$\bar\mu_1 = \lambda\sqrt{\frac{p_1}{p_2 p_3}}, \quad \bar\mu_2 = \lambda\sqrt{\frac{p_2}{p_1 p_3}}, \quad \text{and } \bar\mu_3 = \lambda\sqrt{\frac{p_3}{p_1 p_2}} .$$

Matter Hamitlonian treated as in Fock quantization.
Vanishing of the Hamiltonian constraint $\Rightarrow$

$$\rho = \frac{1}{8\pi G\gamma^2\lambda^2}\left(\sin(\bar\mu_1 c_1)\,\sin(\bar\mu_2 c_2) + \text{cyclic terms}\right) \le 0.41\rho_{\rm Planck} .$$

The maximum of energy density coincides with that of the isotropic model.

Expansion and Shear scalars

Using Hamilton's equations,

$$\dot{p}_i = \frac{p_i}{\gamma\lambda}\left(\sin(\bar{\mu}_j c_j) + \sin(\bar{\mu}_k c_k)\right)\cos(\bar{\mu}_i c_i)$$

It is bounded above.

Immediately leads to universal bounds on directional Hubble rate, θ and shear scalar.

Maximum of Hubble rates and expansion scalar: $H_i^{\max} = 3/(2\gamma\lambda) = \theta_{\max}$

Maximum of shear scalar:

$$\sigma^2_{I \ \ \max} = \frac{10.125}{3\gamma^2\lambda^2}$$

As for the energy density, the upper bound on expansion and shear scalars is independent of the matter content.

Upper bounds on ρ, H_i, θ and σ^2 are direct consequences of the underlying quantum geometry. In the limit $\lambda \to 0$, (ρ, H_i, θ,σ^2) $\to \infty$.
Bounds are not saturated in evolution. Bounces occur before their maximum values are reached.

Does the boundedness of above physical quantities, guarantee no singularities?

Curvature invariants in Bianchi-I model

Ricci scalar

$$R = 2\left(H_1H_2 + H_2H_3 + H_3H_1 + \sum_{i=1}^{3}\frac{\ddot{a}_i}{a_i}\right)$$

Kretschmann scalar

$$K = 4\left(H_1^2H_2^2 + H_1^2H_3^2 + H_2^2H_3^2 + \sum_{i=1}^{3}\frac{\ddot{a}_i^2}{a_i}\right)$$

Square of the Weyl curvature

$$\begin{aligned} C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu} \quad &= \quad \frac{4}{3}\Bigg[H_1^2H_2^2 + H_2^2H_3^2 + H_3^2H_1^2 - H_1H_2H_3(H_1 + H_2 + H_3) \\ &\quad + \left\{\frac{\ddot{a}_1}{a_1}\left(\frac{\ddot{a}_1}{a_1} - H_1H_2 - H_3H_1 + 2H_2H_3 - 1\right) + \text{cyclic terms}\right\}\Bigg] \end{aligned}$$

In the classical theory, all curvature invariants diverge as the point, barrel, pancake or cigar singularities are approached.

In effective dynamics, H_i are universally bounded. Fate of divergence controlled by $\ddot{a}_i/a_i$ terms.

Ricci scalar in LQC

$$\begin{aligned} R \;=\;& -24\pi G(\rho + P) + \frac{1}{V}\left(\frac{\dot{p}_1}{p_1}(\kappa_{12}+\kappa_{13}) + \frac{\dot{p}_2}{p_2}(\kappa_{23}+\kappa_{21}) + \frac{\dot{p}_3}{p_3}(\kappa_{31}+\kappa_{32})\right) \\ & + \frac{1}{2\gamma^2\lambda^2}\Bigg[3 + \cos^2(\bar{\mu}_1 c_1)\left(\sin^2(\bar{\mu}_3 c_3) + 4\sin(\bar{\mu}_2 c_2)\sin(\bar{\mu}_3 c_3) - \cos(2\bar{\mu}_2 c_2)\right) \\ & \quad + \cos^2(\bar{\mu}_2 c_2)\left(\sin^2(\bar{\mu}_1 c_1) + 4\sin(\bar{\mu}_1 c_1)\sin(\bar{\mu}_3 c_3) - \cos(2\bar{\mu}_3 c_3)\right) \\ & \quad + \cos^2(\bar{\mu}_3 c_3)\left(\sin^2(\bar{\mu}_2 c_2) + 4\sin(\bar{\mu}_1 c_1)\sin(\bar{\mu}_2 c_2) - \cos(2\bar{\mu}_1 c_1)\right) \\ & - \left(\sin^2(\bar{\mu}_1 c_1)\sin^2(\bar{\mu}_2 c_2) + \sin^2(\bar{\mu}_1 c_1)\sin^2(\bar{\mu}_3 c_3) + \sin^2(\bar{\mu}_2 c_2)\sin^2(\bar{\mu}_3 c_3)\right)\Bigg]. \end{aligned}$$

Energy density and $\dot{p}_i/p_i$ are univerally bounded in LQC. $\kappa_{ij} = (c_i p_i - c_j p_j)$ are constants of motion.

Ricci scalar can only diverge if in evolution, volume becomes zero and/or pressure becomes infinite, at a finite value of ρ, θ and σ^2

Analysis of expressions for Kretschmann scalar and square of the Weyl curvature also lead to above conclusion.

For all matter, considered so far in Bianchi models in GR, curvature invariants turn out to be bounded in effective dynamics.

Geodesics and Strength of singularities

What is the nature of singularities where curvature invariants diverge in Bianchi-I LQC? Anaylsis of null geodesics and Królak's conditions for strong singularities leads to following conclusions:

- In GR, point-like, barrel, cigar and pancake singularities involve one of the scale factors to vanish and an associated divergence of directional Hubble rates. In LQC, due to boundedness of H_i, all these singularities are avoided. The values of the scale factors at which geodesic evolution breaks down in GR are excluded from the effective spacetime of LQC.
- All known types of strong singularities of the classical theory in Bianchi-I model are generically resolved.
- Curvature invariant diverging events due to infinite pressure at finite values of scale factors: Geodesic equations are well behaved. Divergence occurs only in $\ddot{a}_i/a_i$ terms. Integrals in Królak's conditions are finite. These are generalizations of (sudden) weak singularities in isotropic LQC.
- Curvature invariant diverging events due to vanishing volume: If in an evolution, curvature invariants diverge at a finite value of ρ, θ and σ^2 at a finite value of the affine parameter and a vanishing volume, then geodesic equations in effective spacetime break down. Strength of singularity depends on details of such a 'solution.' There is no known solution satisfying these properties in Bianchi models.

Fate of singularities: physical examples

For a perfect fluid with a constant equation of state $w > -1$ $(w = P/\rho)$, boundedness of energy density implies that physical volume never becomes smaller than $V_{\rm min}$:

$$V_{\rm min} = \left(\frac{8\pi G\gamma^2\lambda^2 C}{3}\right)^{\frac{1}{1+w}}$$

where C is a constant determined by intial conditions.
For such a matter,

- Geodesic evolution never breaks down. In contrast, in GR, for perfect fluid with $w > -1$, geodesics are incomplete.
- Tipler and Królak's conditions are never satisfied. Recall that in GR, such a perfect fluid always leads to a strong singularity.

Exotic matter: If we assume that as in the isotropic models, for an exotic choice of equation of state, big rip, sudden and big freeze singularities exist in classical Bianchi-I model. Then, for such a matter model,

- As in isotropic LQC, no generalized big rip or big freeze singularities occur in Bianchi-I LQC.
- Generalized sudden singularities can occur in Bianchi-I LQC, as in isotropic models. However, these are harmless. Geodesics can be extended beyond them and their strength is weak.

Bounds on ρ, θ and σ^2 in Bianchi-II and Bianchi-IX models

Bianchi-II model

Important subtleties in quantization in comparison to the Bianchi-I model (Ashtekar, Wilson-Ewing (10)). Physical implications: general features with arbitrary matter (Gupt, PS (11)); massless scalar (Corichi, Montoya (12))
Effective Hamiltonian:

$$\mathcal{H}_{\rm eff} = -\frac{p_1 p_2 p_3}{8\pi G\gamma^2\lambda^2}\left[\sin(\bar\mu_1 c_1)\sin(\bar\mu_2 c_2) + \text{cyclic terms}\right] - \frac{1}{8\pi G\gamma^2\lambda^2}\left[\frac{\alpha}{\lambda}\frac{(p_2p_3)^{3/2}}{\sqrt{p_1}}\sin(\bar\mu_1 c_1) - \frac{(1+\gamma^2)}{4}\left(\frac{p_2p_3}{p_1}\right)^2\right] + \mathcal{H}_{\rm matt}$$

Properties of energy density:

$$\rho = \frac{1}{8\pi G\gamma^2\lambda^2}\left[\sin(\bar\mu_1 c_1)\sin(\bar\mu_2 c_2) + \text{cyclic terms}\right] + \frac{1}{8\pi G\gamma^2}\left[\frac{x}{\lambda}\sin(\bar\mu_1 c_1) - \frac{(1+\gamma^2)x^2}{4}\right]$$

where $x = \alpha\sqrt{\frac{p_2p_3}{p_1^3}}$.
Energy density has a global maximum: $\rho \le \rho_{\rm max} \approx 0.54\rho_{\rm Pl}$.

If one assumes the validity of effective dynamics till arbitrary small values of triads, then energy density can take arbitrarily large negative values. This is also under the assumption that solutions probing arbitrarily small triads exist. In practice such solutions have not been found.

Behavior of energy density in Bianchi-II model provides insights for the first time on the role of energy conditions in LQC. Recall, that in GR, energy conditions are used to eliminate potentially unphysical solutions. So far, we did not put any restriction on the matter to be physical in effective theory. As in GR, one can assume the validity of weak energy condition (WEC), which restricts the choice of matter.

Expansion scalar:

$$\theta = \frac{1}{2\gamma\lambda}\left(\sin(\bar{\mu}_1 c_1 + \bar{\mu}_2 c_2) + \sin(\bar{\mu}_2 c_2 + \bar{\mu}_3 c_3) + \sin(\bar{\mu}_3 c_3 + \bar{\mu}_1 c_1) + \lambda x \cos\left(\bar{\mu}_1 c_1\right)\right)$$

If solutions exist such that $x \to \infty$ in effective dynamics, then expansion scalar can grow without bound. However, for all matter satisfying WEC, the maximum allowed value of θ is:

$$\theta_{\max} \approx \frac{6.05}{2\gamma\lambda} \approx \frac{5.60}{l_{\rm Pl}}$$

Shear scalar:

Has similar dependence on x as the expansion scalar. For matter satisfying WEC,

$$\sigma^2_{\rm II\ max} \approx \frac{57.58}{3\gamma^2\lambda^2} \approx \frac{65.82}{l^2_{\rm Pl}}$$

In Bianchi-II model, energy conditions play an important role on the values of upper bounds of expansion and shear scalars. Only when the energy density is unbounded below, the expansion and shear scalars are unbounded. For all other matter, θ and σ^2 are bounded.

Bianchi-IX model

Quantization: Wilson-Ewing (10)
Some details of physics: PS, Gupt (11)

The topology of the manifold is spatially compact S^3. Fiducial volume $V_o =: \ell_o^3 = 16\pi^2$. Inverse scale factor effects, a la Thiemann (98) play an important role.

Effective Hamiltonian:

$$\begin{aligned}\mathcal{H}_{\rm eff} \quad = \quad & -\frac{p_1 p_2 p_3}{8\pi G \gamma^2 \lambda^2}\left[\sin(\bar\mu_1 c_1)\sin(\bar\mu_2 c_2) + \text{cyclic terms}\right] \\ & -\frac{\varepsilon \ell_o}{8\pi G\gamma^2 \lambda}\Big[(p_1 p_2)^{3/2} f(p_3)\sin\bar\mu_3 c_3 + (p_2 p_3)^{3/2} f(p_1)\sin(\bar\mu_1 c_1) + (p_3 p_1)^{3/2} f(p_2)\sin(\bar\mu_2 \\ & -\frac{\ell_o^2(1+\gamma^2)}{32\pi G\gamma^2}\left[2(p_1^2+p_2^2+p_3^2) - (p_1 p_2)^2 f(p_3)^4 - (p_2 p_3)^2 f(p_1)^4 - (p_3 p_1)^2 f(p_2)^4\right] + \mathcal{H}_{\rm m}\end{aligned}$$

Analysis of energy density reveals that if the approach to the singularity is isotropic, then there exists a universal maximum: $\rho_{\rm max} \approx 11.74\rho_{\rm Pl}$.
For anisotropic approach to singularity, ρ is finite, except for an isolated case where two of the triads vanish and the third diverges. If effective dynamics is true even when triads vanish and such an isolated solution exists, then energy density in principle can be unbounded. In practice, ρ does not saturate the bound.

Expansion scalar: For the isotropic approach to singularity, expansion scalar has a global maxima

$$\theta_{\rm max} \approx \frac{47.72}{2\gamma\lambda} \approx \frac{44.18}{l_{\rm Pl}}$$

For anisotropic approach to singularity, expansion scalar is finite except for an isolated case (same as in the case of the energy density).

Shear scalar: Global maximum occurs for isotropic approach to the singularity.

$$\sigma^2_{\rm IX\,max} \approx \frac{2165.91}{3\gamma^2\lambda^2} \approx \frac{2476.04}{l^2_{\rm Pl}}$$

If the approach to singularity is anisotropic, then above maximum can be violated. If an isolated case of one triad vanishing and two diverging in a physical evolution exists in the effective dynamics, then the shear scalar is unbounded.

Role of the choice of lapse: (Gupt, PS (12)) Unlike any other quantization in LQC, the choice of lapse affects the bounds on ρ, θ and σ^2.
For $N = 1$,

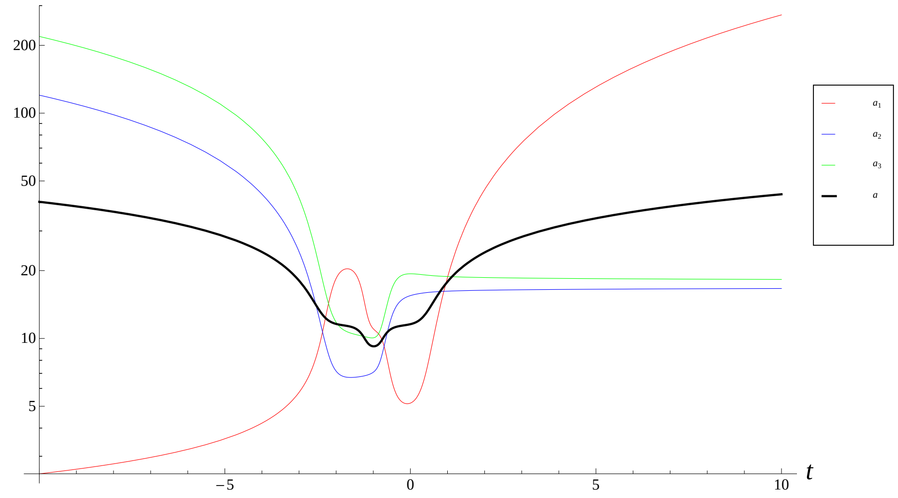
$$\rho_{\rm IX\,max} \approx 11.68\rho_{\rm Pl}, \quad \theta_{\rm max} \approx \frac{46.14}{2\gamma\lambda} \approx \frac{42.73}{l_{\rm Pl}}, \quad \sigma^2_{\rm max} \approx \frac{1723.65}{3\gamma^2\lambda^2} \approx \frac{1969.63}{l^2_{\rm Pl}}$$

This dependence is tied to the important role played by inverse triad corrections in the Bianchi-IX model.

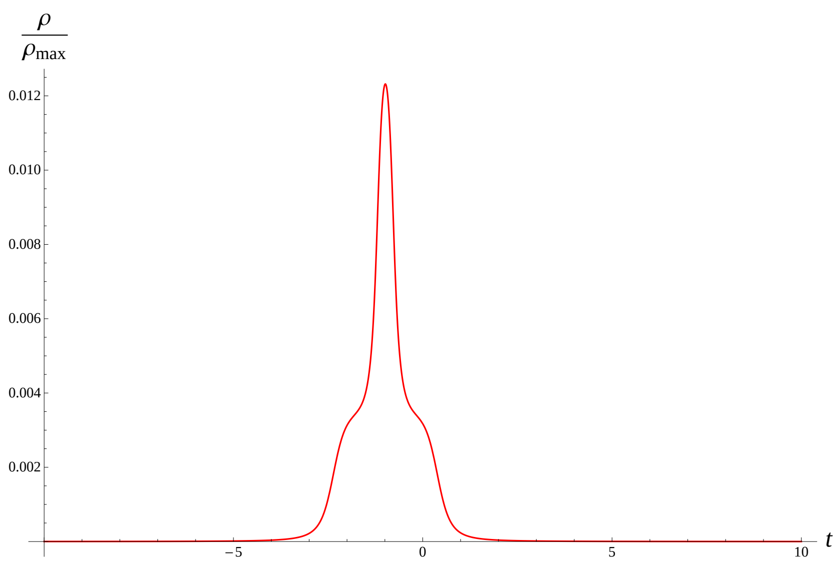
Massive scalar field in Bianchi models
(Upcoming work with B. Gupt)

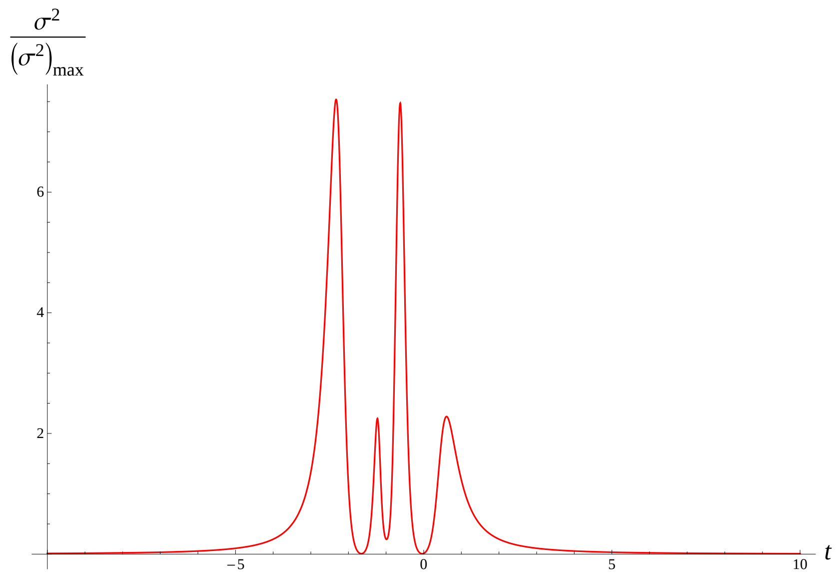
Massive scalar field in Bianchi-I model

Examples of (non-inflationary) solutions for ϕ^2 potential. (Gupt, PS (12))



'Cigar' to 'pancake' transition in Planck regime

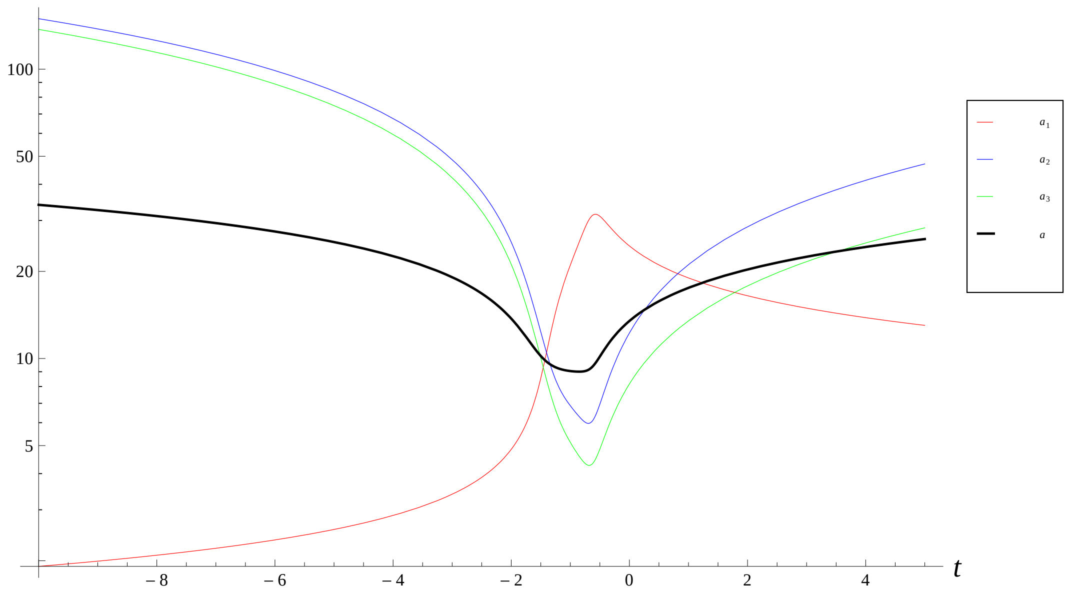


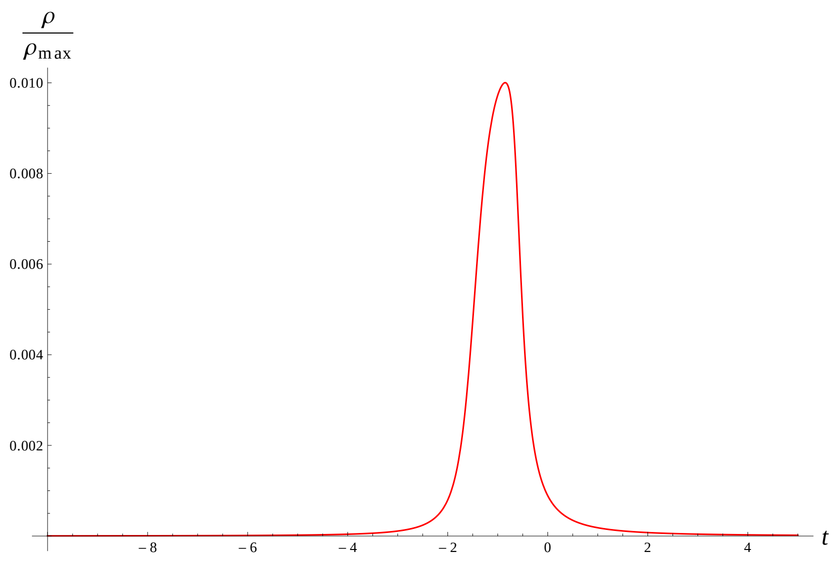


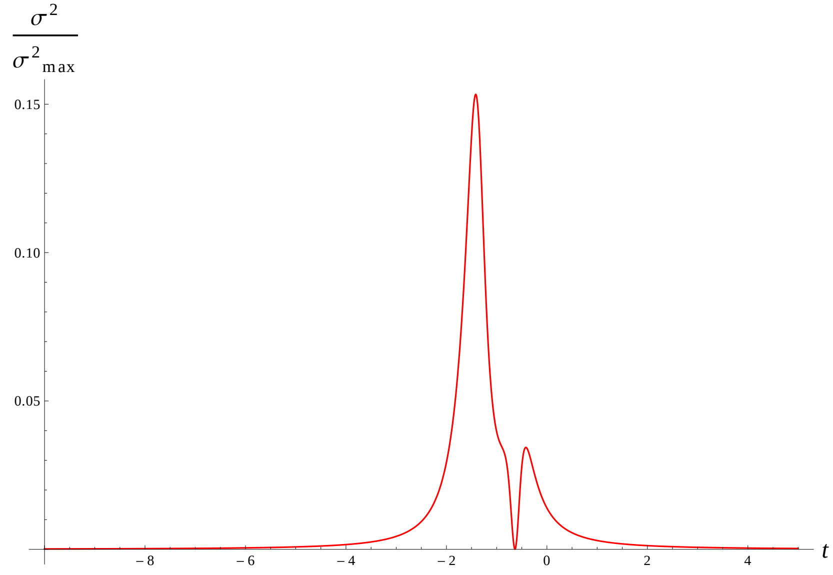
Energy density and shear scalar are not saturated by their maximum values.

Massive scalar field in Bianchi-II model

(Gupt, PS (12))

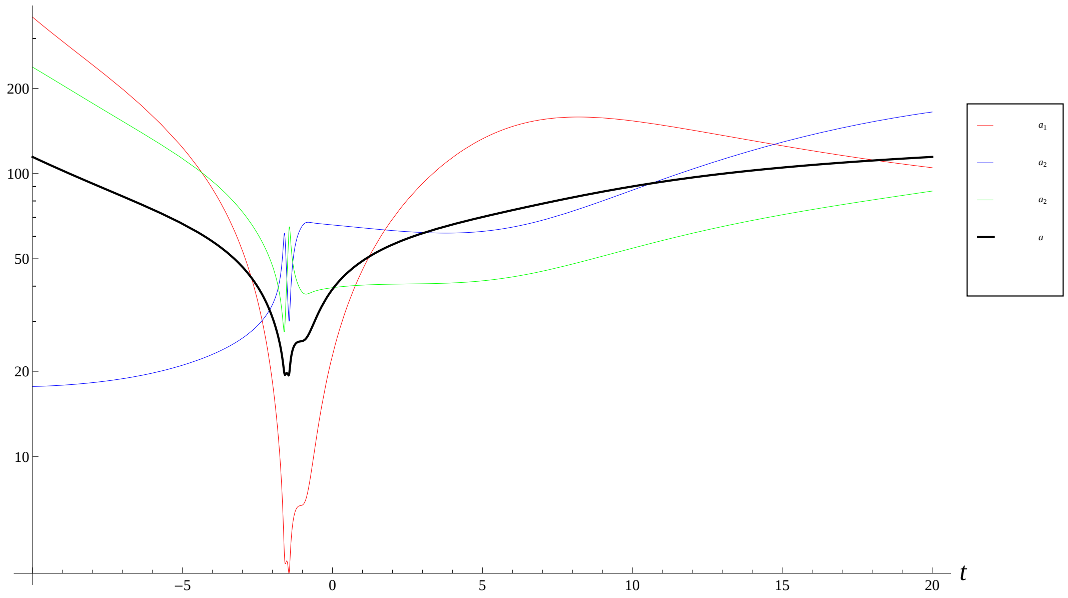


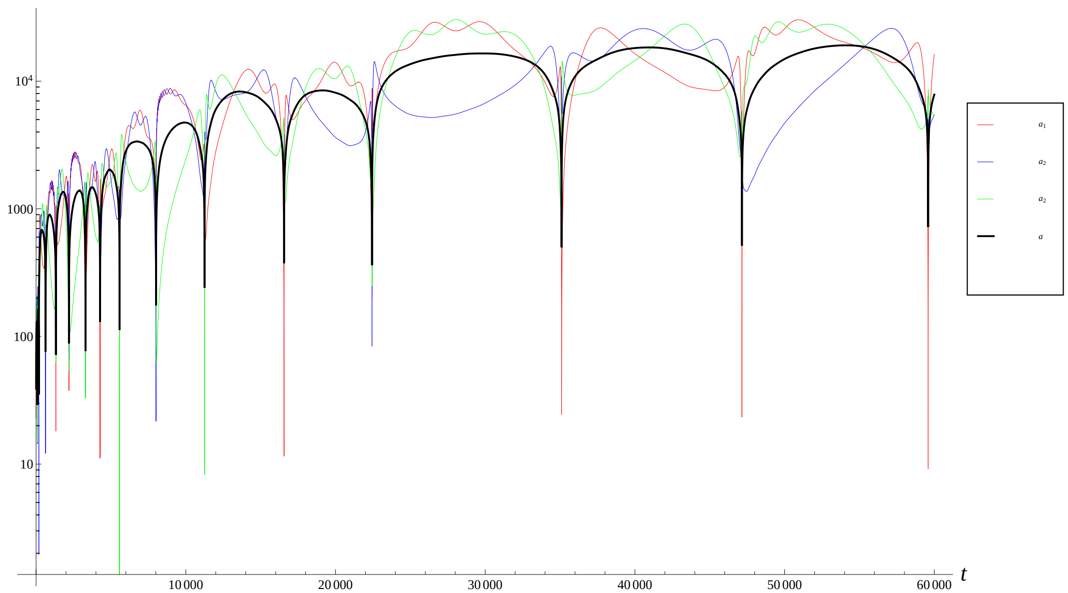




Massive scalar field in Bianchi-IX model

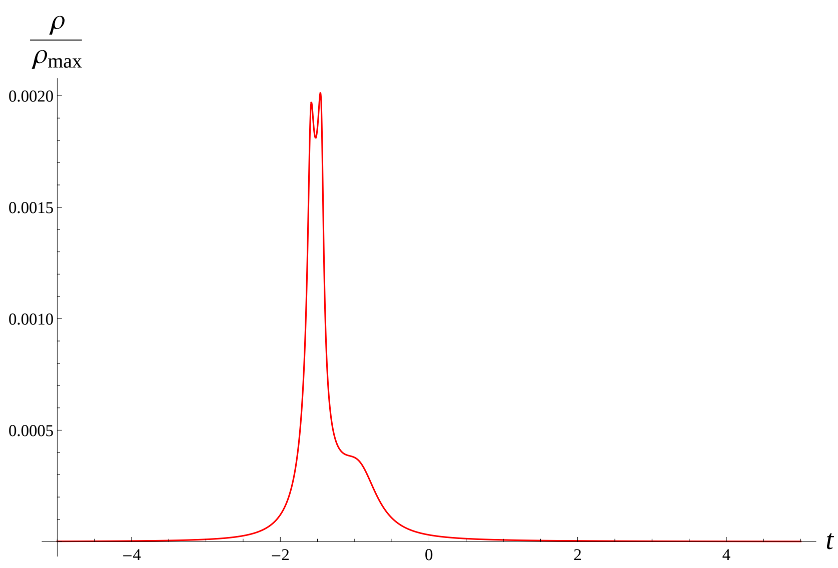
Behavior of scale factors: a cyclic universe (Gupt, PS (12))

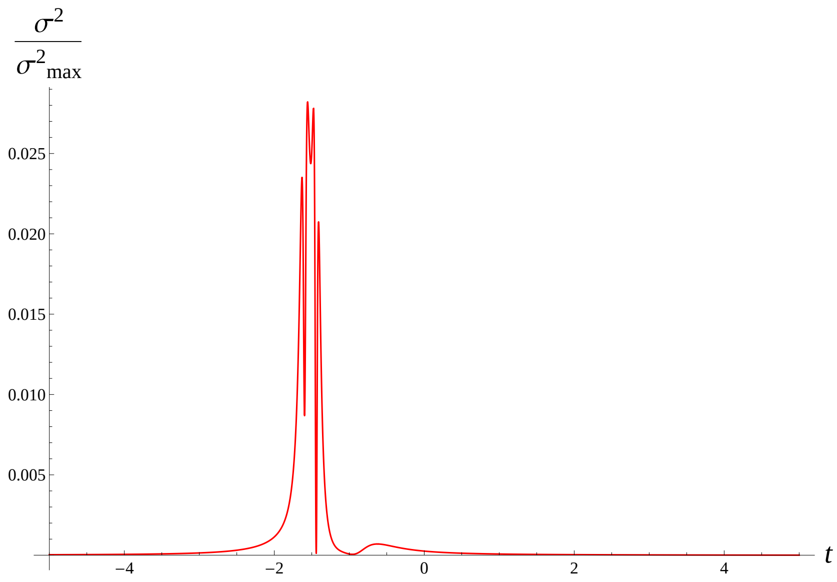




Massive scalar field in Bianchi-IX model

Behavior of energy density and shear scalar





Note that in this case the approach to singularity is highly anisotropic. Even then energy density and shear are bounded below the values corresponding to those for the isotropic approach to singularities.

Summary

- Physics of Bianchi models is very rich compared to the isotropic models in LQC. We analyzed various physical implications assuming the validity of effective dynamics at all scales. Results obtained here, put an emphasis on future tests on the validity of effective dynamics in extreme situations.
- Non-local effects from the field strength again play a crucial role in bounce. For models with spatial compactness, inverse scale factor effects are also important.
- Results on the resolution of strong singularities in Bianchi-I model indicate that generic resolution of singularities may extend to anisotropic spacetimes. One needs to understand details of singularities in vacuum spacetime and matter with anisotropic stress.
- Weak singularities can arise in Bianchi-I model, as in isotropic LQC.
- Energy conditions play an important role on the way singularities are resolved in Bianchi-II model. Does this signal that a "non-singularity theorem", if it exists, will require certain energy conditions (as singularity theorems in GR do)? Does quantum geometry ignore 'unstable' matter configurations (which violate WEC)? Is it an artifact of breakdown of effective theory as scale factors approach zero? Or does it point to some yet to be understood subtle features of the models?
- Upper bounds on ρ, θ and σ^2 are different for different Bianchi models. These bounds signal that geodesic equations are well behaved in effective spacetime. Extensive simulations with various potentials show that even in highly anisotropic situations, bounds are never saturated.